Example:17 A dipole antenna whose input impedance is 100Ω is to be matched at a frequency of 100MHz to a transmission lines having characteristic impedance of 600 be means of short circuit stub. Determine the location and length of the stub.

Given :

$$Z_R = 100\ \Omega$$

$$Z_0 = 600\Omega$$

$$f = 100\ \text{MHz}$$

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{100 \times 10^6}$$

$$= 3\text{m}$$

Location of the stub is

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

$$= \frac{3}{2\pi} \tan^{-1} \sqrt{\frac{100}{600}}$$

$$= \frac{3}{2\pi} \times \frac{22.2^\circ}{180} \times \pi$$

($\because$ To convert 22.2° in to radians, multiply $\frac{\pi}{180}$)

$$l_s = 0.185\text{m}.$$

Length of the stub is

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0} \right)$$

$$= \frac{3}{2\pi} \tan^{-1} \left(\frac{\sqrt{100 \times 600}}{100 - 600} \right)$$

$$= \frac{3}{2\pi} \tan^{-1} (-04899)$$

$$= \frac{3}{2\pi} \times (-26.1^\circ)$$

$$= \frac{3}{2\pi} (180^\circ - 26.1^\circ)$$

(Because of negative, angle will be subtracted from 180°.)

$$l_t = \frac{3}{2\pi} \times \frac{153.9^\circ}{180} \times \pi$$

$$l_t = 1.28\text{m}.$$

Example:18 A UHF lossless transmission line working at 1GHz is connected to an unmatched line producing a reflection coefficient of 0.5 ∠30°. Determine the location and length of the stub to match the line.

Given :

$$f = 1 \times 10^9 Hz$$

$$\lambda = \frac{C}{f}$$

$$= \frac{3 \times 10^8}{1 \times 10^9} = 0.3m$$

$$K = 0.5 \angle 30°$$

$$|K| = 0.5$$

$$\phi = \frac{\pi}{6} = 30°.$$

Location of the stub is

$$l_s = \frac{\lambda}{4\pi} [\phi + \pi - \cos^{-1}|K|]$$

$$= \frac{0.3}{4\pi}\left[\frac{\pi}{6} + \pi - \cos^{-1} 0.5\right]$$

$$= \frac{0.3}{4\pi}\left[\frac{7\pi}{6} - \frac{\pi}{3}\right]$$

$$= \frac{1}{16} = 0.0625m$$

Length of the stub is

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1+|K|^2}}{2|K|}$$

$$= \frac{0.3}{2\pi} \tan^{-1} \frac{\sqrt{1-(0.5)^2}}{2 \times 0.5}$$

$$= \frac{0.3}{2\pi} \tan^{-1} 0.866$$

$$= \frac{0.3}{2\pi} \times 40.9° \times \frac{\pi}{180}$$

$$= 3.41 \text{ cm}.$$

Example:19 A single stub is to match a 400 ohm line to a load of 200 – j 100Ω. The wavelength is 3m. Determine the position and length of the short circuited stub.

Given :

$$Z = 400\Omega$$
$$Z_R = 200 - j100\Omega$$
$$\lambda = 3m.$$

$$K = \frac{Z_R - Z_0}{Z_R - Z_0}$$
$$= \frac{200 - j100 - 400}{200 - j100 + 400}$$
$$= \frac{-200 - j100}{600 - j100}$$
$$= \frac{-2 - j}{6 - j} = -0.3 - j0.2$$
$$= -0.36 \angle 213.7°$$
$$|K| = 0.36$$
$$\phi = 213.7° = 1.17\pi.$$

Location of the stub is

$$l_s = \frac{3}{4\pi}[\phi + \pi - \cos^{-1} 0.36]$$
$$= \frac{3}{4\pi}\left[1.17\pi + \pi - 63.9 \times \frac{\pi}{180}\right]$$
$$= \frac{3}{4\pi}(2.17\pi - 0.38\pi)$$
$$= 1.34m$$

Length of the stub is

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1 - |K|^2}}{2|K|}$$
$$= \frac{3}{2\pi}\tan^{-1}\frac{\sqrt{1 - (0.36)^2}}{2 \times 0.36}$$
$$= \frac{3}{2\pi}\tan^{-1} 1.11$$
$$= \frac{3}{2\pi} \times \frac{48}{180} \times \pi$$
$$= 0.4m.$$

Example:20 An aerial of (300–j300)Ω is to be matched with 600Ω line. The matching is to be done by means of low loss 600 stub line. Find the position and length of the stub line if the operating wavelength is 20m.

Given :

$$Z_R = 300 - j300\Omega$$

$$Z_0 = 600\Omega$$

$$\lambda = 20\text{m}.$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{300 - j300 - 600}{300 - j300 + 600}$$

$$= \frac{-300 - j300}{900 - j300}$$

$$= \frac{-1 - j}{3 - j}$$

$$= -0.2 - j\,0.4$$

$$K = 0.4472 \angle 63.435°$$

$$|K| = 0.4472$$

$$\phi = 63.435°$$

The position of the stub line is

$$l_s = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}$$

$$= \frac{\lambda}{4\pi}\,[\phi + \pi - \cos^{-1}(0.4472)\,]$$

$$= \frac{\lambda}{4\pi}\,[0.352\,\pi + \pi - \frac{\pi}{180} \times 63.44]$$

$$= \frac{\lambda}{4\pi}\,(0.352\,\pi + \pi - 0.352\pi)$$

$$= \frac{\lambda}{4}$$

$$= \frac{20}{4}$$

$$l_s = 5\text{ m}$$

Length of the stub is

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$= \frac{20}{2\pi} \tan^{-1} \frac{\sqrt{1-(0.4472)^2}}{2(0.4472)}$$

$$= \frac{20}{2\pi} \times 45^\circ \times \frac{\pi}{180}$$

$$= \frac{20}{2\pi} \cdot \frac{\pi}{4}$$

$$l_t = 2.5 \text{ m.}$$

Example :21 A lossless transmission line with $Z_0 = 75\Omega$ and electric length $l = 0.3\lambda$ is terminated with a complex load impedance of $Z_R = 40 + j\,20\Omega$. Determine reflection coefficient and VSWR of line.

Given :

$$Z_0 = 75\Omega$$
$$Z_R = 40 + j20\Omega$$
$$l = 0.3\,\lambda$$

Reflection coefficient is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{40 + j20 - 75}{40 + j20 + 75}$$

$$= \frac{-35 + j20}{115 + j20}$$

$$= \frac{40.3113 \angle -29.74^\circ}{116.726 \angle 9.866^\circ}$$

$$K = 0.3453 \angle -39.606^\circ$$

$$\text{VSWR} = \frac{1+|K|}{1-|K|}$$

$$= \frac{1+0.3453}{1-0.3453}$$

$$\text{VSWR} = 2.0548$$

Example:22 The terminating load of UHF transmission line working at 300MHz is 50 + j50Ω. Calculate VSWR and the position of the voltage minimum nearest to the load if the characteristic impedance of the line is 50Ω.

Given :

$$Z_R = 50 + j50\ \Omega$$

$$Z_0 = 50\Omega$$

$$f = 300\text{ MHz}$$

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{300 \times 10^6}$$

$$= 1\text{ m.}$$

Reflection coefficient is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{50 + j50 - 50}{50 + j50 + 50}$$

$$= \frac{j50}{100 + j50}$$

$$= \frac{j}{2 + j}$$

$$K = 0.4472 \angle 63.5°$$

$$|K| = 0.4472$$

$$\phi = 63.5°$$

$$\text{VSWR} \quad S = \frac{1 + |K|}{1 - |K|}$$

$$= \frac{1 + 0.4472}{1 - 0.4472}$$

$$S = 2.62$$

Position of the voltage minima nearest to the load is

$$2\beta x_{min} - \phi = (2n + 1)\pi$$

$$= 0 \qquad [\because n = 0]$$

$$2 \times \frac{2\pi}{\lambda} x_{min} - 63.5° = \pi$$

$$\frac{4\pi}{\lambda} x_{min} - \frac{63.5°}{180°} \times \pi = \pi$$

$$\frac{4\pi}{1}\, x_{min} = \frac{63.5^\circ}{180^\circ}\,\pi + \pi = \pi\left[1 + \frac{63.5}{180}\right]$$

$$x_{min} = 0.3382\text{m}.$$

Example:23. The VSWR measured on UHF transmission line at a frequency of 300MHz found to be 2. If the distance between load and voltage minimum is 0.8m, determine the normalized load impedance.

Given :

$$\text{VSWR} \quad S = 2$$

$$x_{min} = 0.8\text{m}$$

$$f = 300 \text{ Hz}$$

$$\lambda = \frac{3 \times 10^8}{300 \times 10^6}$$

$$= 1\text{m}$$

$$|K| = \frac{S-1}{S+1}$$

$$= \frac{2-1}{2+1}$$

$$= \frac{1}{3}$$

For voltage minima

$$2\beta x_{min} - \phi = (2n+1)\pi$$

$$= \pi \qquad [\because n = 0]$$

$$\phi = 2\beta x_{min} - \pi$$

$$= 2\left(\frac{2\pi}{\lambda}\right) 0.8 - \pi$$

$$= 3.2\,\pi - \pi = 2.2\,\pi$$

$$= 396^\circ \text{ or } 36^\circ$$

$$|K|\, e^{j\phi} = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\frac{1}{3} e^{j36°} = \frac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1}$$

$$= \frac{z_r - 1}{z_r + 1} \qquad \left[\because z_r = \frac{Z_R}{Z_0}\right]$$

$$\frac{1}{3}(\cos 36 + j \sin 36) = \frac{z_r - 1}{z_r + 1}$$

$$0.2677 + j\,0.1959 = \frac{z_r - 1}{z_r + 1}$$

$$(z_r + 1)(0.2677 + j0.1959) = z_r - 1$$

$$z_r [0.2677 + j0.1959 - 1] = -1 - 0.2677 - j0.1959$$

$$z_r = \frac{1.2677 + j0.1959}{0.7323 - j\,0.1959}$$

$$= \frac{1.29 \angle 8.9°}{0.76 \angle 18.3°}$$

$$\frac{Z_R}{Z_0} = 1.7 \angle{-6.4°}$$

TWO MARK QUESTIONS AND ANSWERS

1. What are constant S circles ?

The input impedance equation for a dissipationless line if expressed in terms of standing wave ratio S, results in the form of a circle. These circles are called as constant S circle.

Since, the minimum value of S is unity, S circles surround the 1, 0 point.

2. Explain how Smith Chart can be used as an admittance chart.

The Smith Chart may be used as an admittance chart, the R and X axes become g and b axes, with the usual implication that capacitive susceptance is positive above and inductive susceptance below the V or real axis. The point at the left of the conductance of g axis then represents zero conductance or an open circuit, while the point at extreme right represent infinite conductance or a short circuit.

3. What are the advantages of double stub matching over single stub matching?

(i) Double stub matching does not require that the stub should be placed at definite point on the line like single stub matching.

(ii) Double stub matching requires only the length of the stubs being changed while the position (or location) of the stubs over the transmission line can be arbitrary. This is a definite advantage over single stub matching.

4. What is stub matching?

For the maximum transfer of power, the sending end impedance and the receiving end impedance of a transmission line should be matched perfect. In practical cases, this impedance matching is not perfect. So, a stub is placed on the transmission line and its length position is adjusted for maximum power transfer. This is called as stub matching.

5. Give the names of circles on Smith Chart.

The names of circles on the Smith Chart are

(i) Constant – R circles

(ii) Constant – X circles.

6. Mention two applications of Smith Chart.

(i) Determination of SWR, sending end impedance and load admittance.

(ii) The solution of the stub matching problem may be easily carried out using a Smith Chart.

7. List the advantages of Smith Chart.

(i) It can be used to find the impedance and admittance of a transmission line.

(ii) The position of voltage minima and maxima can be easily found out.

(iii) The position and location of stub in stub matching problem can be solved using smith chart.

8. What are the limitations of single stub matching?

Limitations of single stub matching.

(i) It requires the stub should be placed at a definite point on the line.

(ii) It requires that two adjustment should be made, these being location and length of the stub.

9. What is the transformation utilized for formulating the Smith Chart?

The transformation utilized for formulating the Smith chart is called as Bilinear or Molius transformation.

10. Why short circuited stub is preferred to an open circuited stub?

The short circuited stub is preferred over an open circuited stub for the reason that the length of stub is easily alterable by moving the short at any desired length of stub.

Single Stub Matching	Double Stub Matching
(i) It has one stub to match the transmission line impedance.	It requires two stub for impedance matching.
(ii) It necessities both length and location of stub to be altered for matching.	It requires only to alter the length of stubs for matching.
(iii) It requires stub should be placed at a definite place on a line.	The location of stub is arbitrary.

11. Distinguish between single and double stub matching.

12. On what mathematical formulation are the curves, circles, etc : of a Smith Chart obtained?

The curves, circles of the Smith Chart are obtained by the bilinear transformation.

13. Why is single stub matching inaccurate on coaxial line?

For a coaxial line, it is not possible to determine the location of a voltage minimum without a slotted line section, so that the placement of a stub is extremely difficult at the required point. Hence the single stub matching is inaccurate on a coaxial line.

14. What is the importance of a quarter wave line? or Mention the uses of quarter line?

The impedance of a quarter wave line is that it matches the load with the source and ensures that maximum power is being transferred to load. The quarter wave line may be used as

(i) an impedance inverter

(ii) couple a transmission line to a resistive load such as antenna

(iii) serve as an insulator to support an open wire line.

15. What is the functional operation of quarter wave transformer?

A quarter – wave section of line may be thought as a transformer to match a load of Z_R ohms to a source of Z_S, ohms. Such a match can be obtained if the characteristic impedance R_0 of matching line is given as $R_0' = |\sqrt{Z_S Z_R}|$

16. How tapped λ/4 line can be used as an impedance transformer?

A quarter wave line may be considered as an impedance inverter in that it can transform a low impedance into a high impedance and vice versa. This effect is illustrated by λ/4 short circuited line in transforming the zero impedance short–circuited termination to an apparent open circuit and of the open circuited λ/4 line in transferring the open circuited termination to a low value of an apparent short circuit.

17. What is bilinear transformation? Mention its utility with respect to the Smith Chart.

A linear variable if related to another linear variable linearly, then the relation between those to variable transforms one set of variable in one domain to another set of variables. This is called as bilinear transformation.

The idea of bilinear transformation is useful in deriving the constant R and constant X circles. These circles constitute the Smith Chart.

18. Write down the expression to determine the length of stub.

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{1 - |K|^2}{2|K|}$$

or

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_R Z_0}}{(Z_R - Z_0)}$$

where

K is the reflection coefficient, λ is the wavelength

Z_R is the load impedance Z_0 is the characteristic impedance.

19. Write down the expression to determine the position of the stub.

$$l_s = \frac{\lambda}{2\pi} [\phi + \pi - \cos^{-1} |K|]$$

or

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

where

φ is the angle of reflection co–efficient. K is the reflection coefficient

λ is the wavelength Z_R is the load impedance

Z_0 is the characteristic impedance.

EXERCISES

1. Describe an experimental set up for the determination of VSWR of an RF transmission.
2. Starting with the expression for the input impedance of a dissipationless transmission line, derive the basic equation of a Smith circle.
3. Discuss how a Smith Chart is constructed and explain its applications.
4. Originating from the bilinear transformation derive the analytical formulation of the Smith Chart for a lossless transmission line.
5. Derive from first principles how the Smith Chart analytical equations can be obtained from a bi–linear transformation. Is the Smith chart an approximation?
6. Enumerate the advantages of Smith chart, single stub and double stub matching on a line.
7. Explain, with diagrams, the method of deriving constant S circles and constant β_S circles used for impedance determination. Illustrate one application.
8. Derive from first principles, the resistance and reactance circles converging through the centre of the Smith Chart.
9. Obtain from first principles, the Smith Chart formulation using the bilinear transformation. Explain how double stub matching is undertaken.
10. Explain the following :

    (a) Single stub matching (b) Double stub matching.
11. What are the features of a quarter wave transformer? Discuss its properties.
12. A single stub tuner is to match a lossless line of 400Ω to a load of $800 - j300\Omega$. The frequency is 3GHz. Find the distance from the load and determine the length of the stub. (1.42m, 0.0135m).
13. A line of Z0 = 300Ω is connected to a load of 73Ω, for a frequency of 40MHz. Find the length and location of the nearest load of a single stub to produce an impedance match. (69cm, 106.9m)
14. A 300Ω line feeding an antenna has a standing wave ratio of 4 and the distance from the load of the first voltage minima is 6cm. If the frequency is 150MHz, design a single stub matching the system to eliminate the standing wave from the maximum possible length of the line. (81cm, 51cm).
15. A lossless line $^3/_8\lambda$ long has a normalised input impedance of $1.2 + j\,0.95$. Find the normalised load impedance and standing wave ratio.

16. An antenna, as load on a transmission line, produces a standing wave ratio of 3 with a voltage minimum 0.12λ from the antenna terminals. Find the antenna impedance and the reflection coefficient at the antenna, if characteristic impedance is 300Ω for the line.

17. A lossless line terminated in a resistance is found to have a standing wave ratio of 4. The characteristic impedance is 100Ω. A short circuited stub that matches the line to the load is placed less than λ/8 from the load.

    (a) What is the value of the load resistance?

    (b) What is the stub length in wavelengths?

18. Determine the sending end impedance of a lossless line whose data is given below

    $Z_0 = 70\Omega,\ Z_R = 20 + j100\Omega,$ length of the line $= 4.2\lambda$.

19. A load impedance of 40–j80Ω is connected to a 100Ω line. Calculate the reflection coefficient at the load, input impedance of the line of length 0.7λ connected to the above load impedance. Determine the length of the stub required to connect the susceptance utilising the Smith Chart.

20. A SWR on a lossless line is found to be 5 and the successive voltage minimum are 40cm apart. The first voltage minimum is observed to be 15 cm from the load. The length of the line is 160cm and the characteristic impedance is 300Ω. Using Smith Chart determine

    (a) The load impedance (b) The Sending end impedance.

21. Using Smith Chart, find the length, termination and location nearest to the load of a single stub to produce impedance matching for a line of characteristic impedance of 500 Ω connected to a load of 63Ω in the frequency is 50 MHz.