Example :8 A Transmission line is terminated in Z_L. Measurements indicate that the standing wave voltage maxima are 102 cm apart and that the last maxima is 35cm from the load end of the line.

The value of the voltage standing wave ratio is 2.4 and the value of the input impedance at sending end is 250Ω when the receiving end is terminated in same impedance.

Find

(a) frequency being transmitted along the line, if they are placed in the free space.

(b) the real and reactive components of the terminating impedance.

Given :

Characteristic impedance $Z_0 = 250\Omega$

SWR = 2.4

Distance between two successive voltage maxima $\frac{\lambda}{2} = 102\text{cm}$

(a) Wavelength $\lambda = 204\text{cm}$

$$\text{frequency } f = \frac{C}{\lambda} = \frac{3 \times 10^8}{2.04} = 147.1\text{MHz}$$

(b) l_{max} = distance between last voltage maxima and load = 35cm

$$l_{max} = \frac{35\text{cm}}{204\text{cm}} \lambda$$

$$l_{max} = 0.172\lambda.$$

The impedance circle is drawn with (1 + j0) as radius and 2.4 as radius on Smith Chart (Smith Chart – 8)

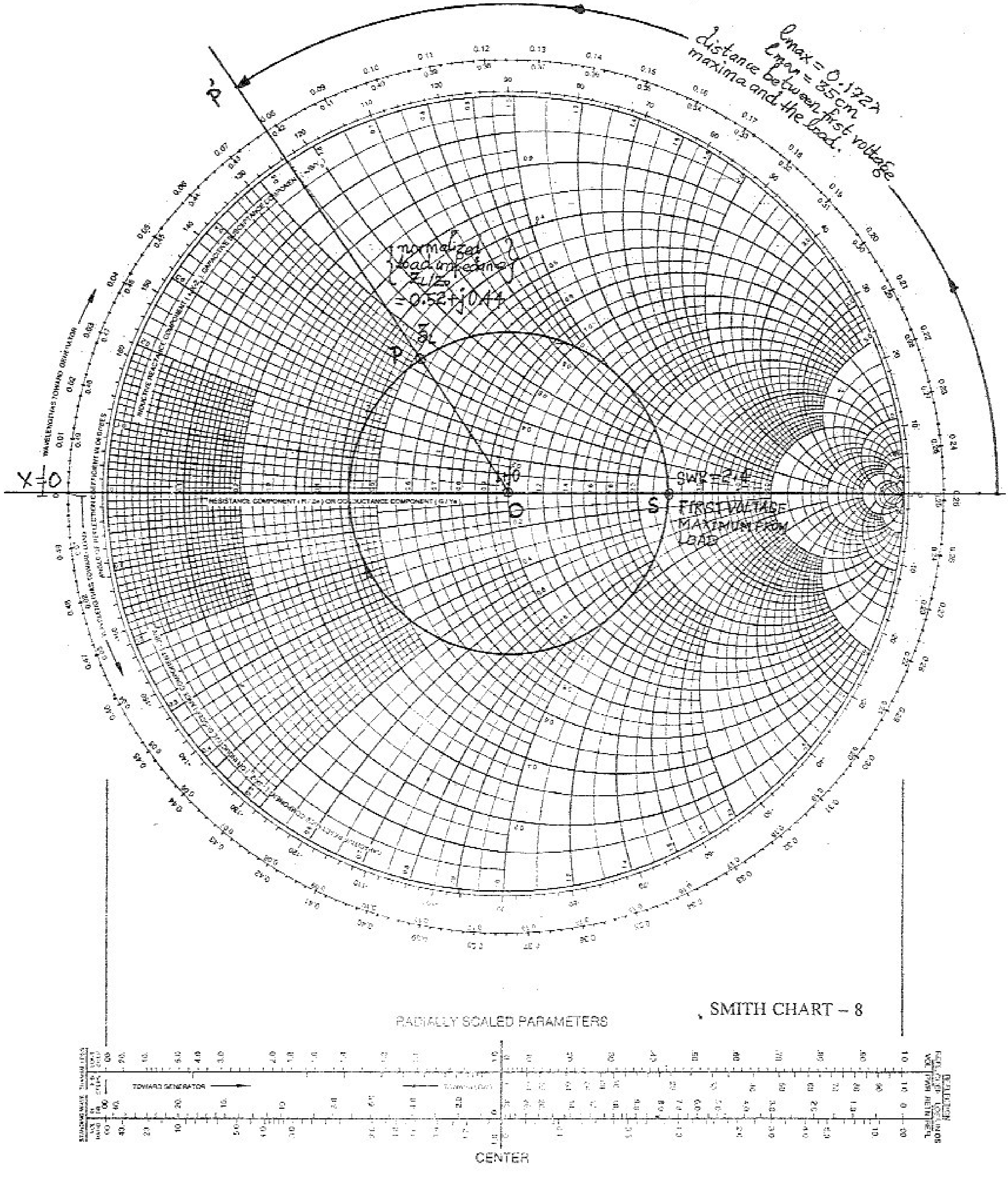
The first voltage maxima from load or the last from the generator lies along X = 0 on the right of the Smith Chart.

Moving towards the load from the first voltage maxima by 0.172λ in a counter clockwise direction, to point P^1. The straight line OP^1 is drawn. It cuts the impedance circle at P. This point indicates the normalised terminating impedance.

$$\text{Normalised terminating impedance } z_L = \frac{Z_L}{Z_0} = 0.52 + j0.44$$

$$Z_L = (0.52 + j0.44) \times 250\Omega$$

$$\text{Terminating impedance } Z_L = 130 + 110\,j\Omega.$$



SMITH CHART – 8

Example:9 Determine the SWR, characteristic impedance of the quarter wave transformer, and the distance the transformer must be placed from the load to match a 75Ω transmission line to a load $Z_L = 25 - j50\Omega$.

Given :

Characteristic impedance $Z_0 = 75\Omega$

Load impedance $Z_L = 25 - j50\Omega$

Normalized load impedance $\dfrac{Z_L}{Z_0} = 0.33 - j\,0.66$.

The normalised load impedance is plotted at P on Smith Chart (Smith Chart – 9) and the normalized impedance circle is drawn. It cuts the X = 0 line on the right side at S

SWR = 4.6 ($\because$OS = 4.6)

The characteristic impedance of an ideal transmission line is purely resistive. Therefore, if the QWT is located at a distance from the load where the input impedance is purely resistive, the transformer can match the transmission line to the load.

There are two points S^1 and S on the impedance circle where the input impedance can be resistive; where the circle intersects X = 0 line.i.e., (0.22, 0) and (4.6, 0).

Therefore, the distance from the load to a point where the input impedance is purely resistive, is calculated in wavelength to the shortest of the two resistive input impedance point.

$$d = 0.098\,\lambda.$$

$$z_i = 0.22 = \frac{Z_i}{Z_0}$$

$$Z_i = 0.22 \times 75$$

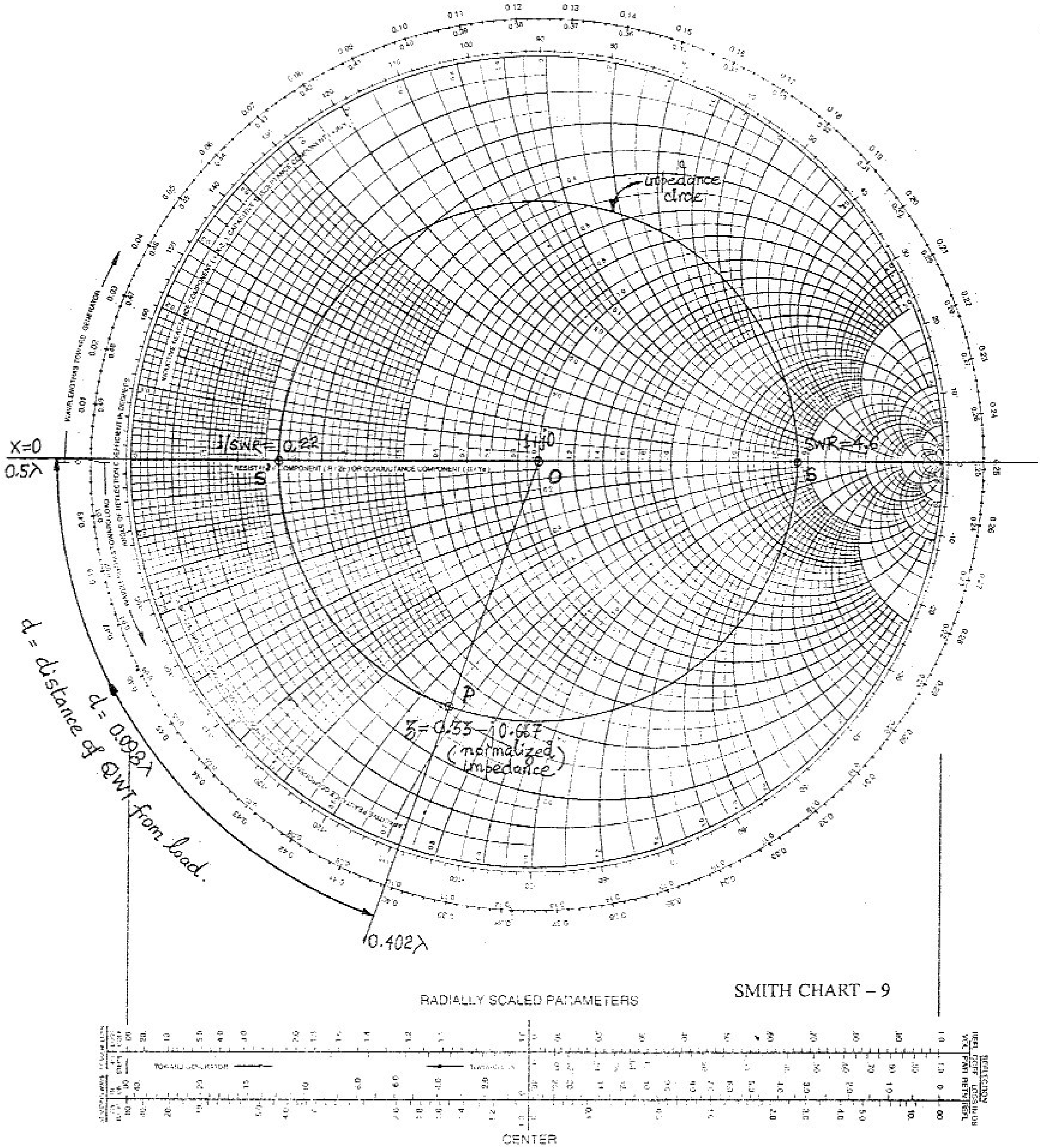
$$= 16.5\ \Omega.$$

The characteristic impedance of quarter wave transformer is found from the formula,

$$Z_0' = \sqrt{Z_0 . Z_i}$$

$$= \sqrt{75 \times 16.5}$$

$$= 35.2\Omega.$$



SMITH CHART – 9

Example:10 Determine the SWR, characteristic impedance of a QWT, and the distance the transformer must be placed to achieve a smooth line with characteristic impedance $Z_0 = 50\Omega$, with a load $Z_L = 75 + j60\Omega$.

Given :

Characteristic impedance $Z_0 = 50\Omega$

Load impedance $Z_L = 75 + j\,60\Omega$

Normalized load impedance $\dfrac{Z_L}{Z_0} = \dfrac{75 + j60}{50} = 1.5 + j\,1.2$

The normalised load impedance is plotted at P on the Smith Chart (Smith Chart–10) and normalized impedance circle is drawn. It cuts X = 0 line at S

From the Smith Chart,

$$SWR = 2.9$$

There are two points S^1 and S on the impedance circle where the input impedance can be resistive, where the impedance circle intersects X = 0 line i.e., 0.36 and 2.9.

The distance from the load to a point where the input impedance is purely resistive is calculated in wavelength to the shortest of the two resistive input impedance point (i.e., S = 2.9)

Distance at which the QWT must be placed from the load = 0.06λ.

$$z_i = \frac{Z_i}{Z_0} = 2.9$$

$$Z_i = 2.9\,Z_0$$

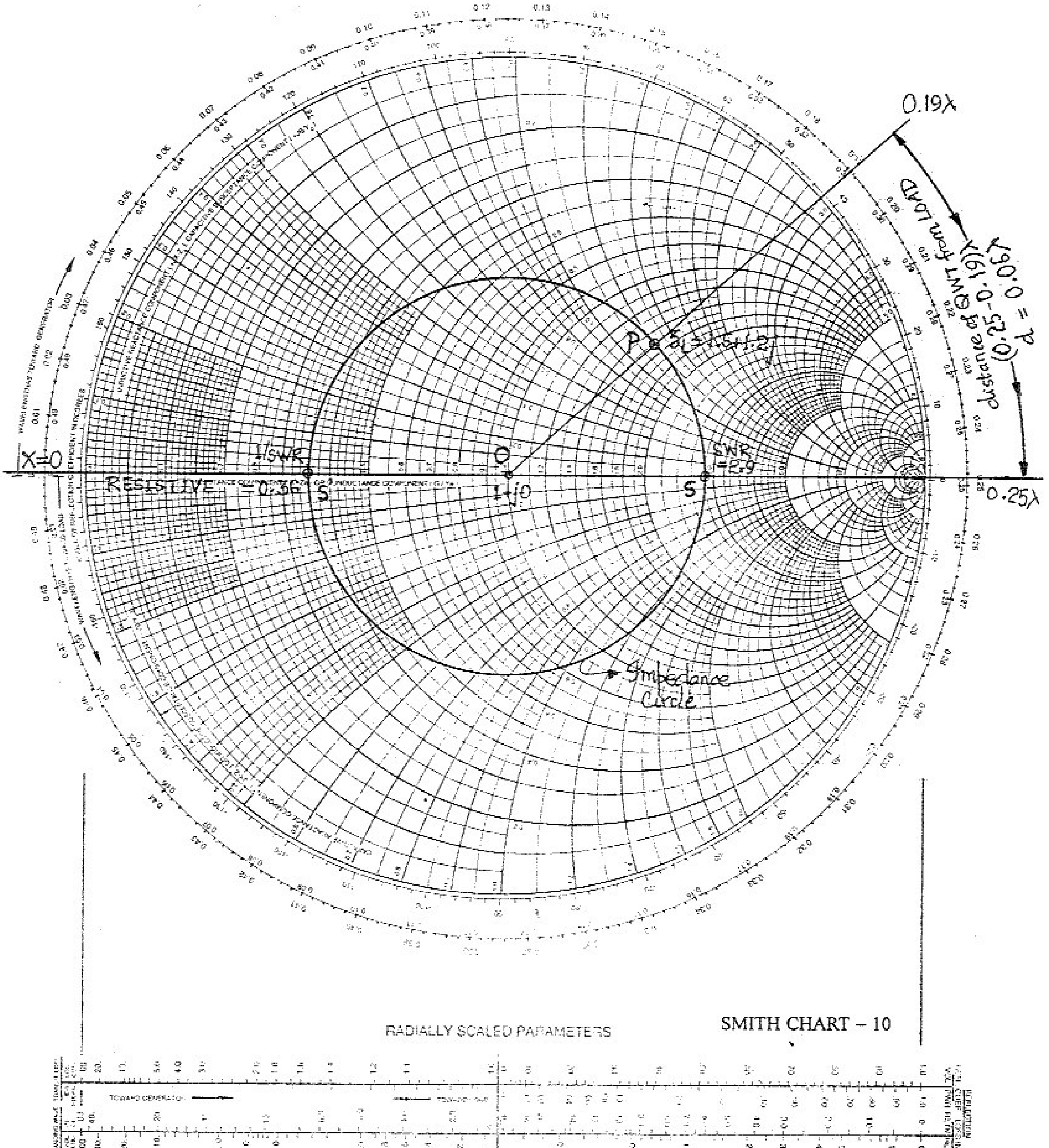
Impedance of the QWT position $Z_i = 2.9 \times 50\ \Omega$

$$Z_i = 145\Omega$$

Characteristic impedance of QWT,

$$Z_0' = \sqrt{Z_i \times Z_0}$$

$$= 85.15\Omega.$$



SMITH CHART – 10

Example :11 For a transmission line with a characteristic Z_0 = 300Ω, and a load with a complex impedance Z_L = 390 + j600Ω. Determine SWR, the distance a shorted stub must be placed from the load to smoothen the line, and the length of the stub.

Given :

Characteristic impedance $Z_0 = 300\Omega$

load impedance $Z_L = 390 + j\,600\Omega$

normalized load impedance $z_L = \dfrac{390 + j600}{300} = 1.3 + j2.$

The normalized load impedance is plotted on Smith Chart (Smith Chart – 11) at A and the impedance circle is also drawn. Because stubs are shunted across the load, admittances are used rather than impedances to simplify the calculations.

From the Chart SWR = 4.8

The normalized load admittance is diametrically opposite to the normalized load impedance at B. i.e., $Y_L = 0.22 - j0.35$

The admittance point is rotated clockwise to a point C on the impedance circle where it intersects the R = 1 circle. At this point admittance $y = 1 + j2.2$

The distance between this point where $\dfrac{\text{Re}(Z_L)}{Z_0} = 1$ and the load admittance is the distance from the load, at which the shorted stub must be placed $d = (0.682 - 0.442)\lambda = 0.24\lambda$.

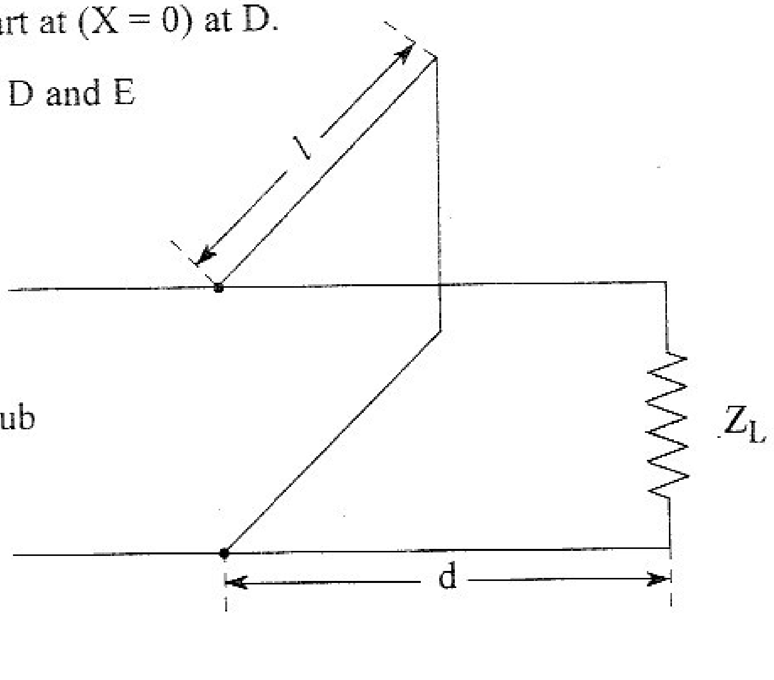
The stub must have zero resistance and an susceptance that has an exactly opposite value at E. i.e., $y_{stub} = 0 - j2.2$

The stub is short circuited to avoid radiation losses hence the distance/length of stub is calculated from the right side of chart at (X = 0) at D.

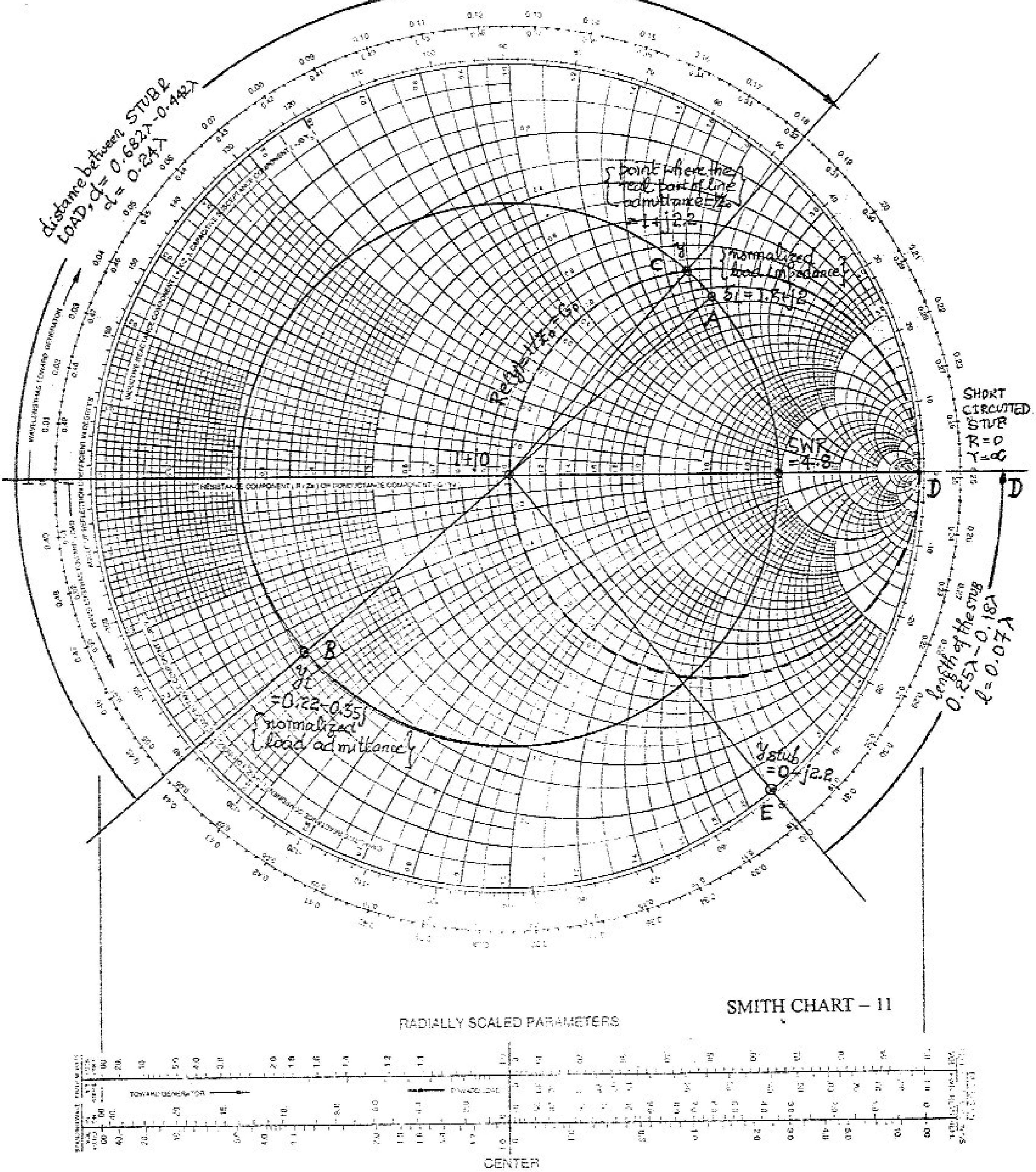
Length of the stub distance between D and E

i.e., $l = 0.25\lambda - 0.18\lambda$

$= 0.07\lambda$



Single Stub



SMITH CHART – 11

Example:12 For a transmission line with a characteristic Z_0 = 75Ω, and a load with a capacitive impedance Z_L = 45 – j100Ω. Determine SWR, the distance a shorted stub must be placed from the load to match the load to the line, and the length of the stub.

Given :

Characteristic impedance $Z_0 = 75\Omega$

load impedance $Z_L = 45 - j100\Omega$

Normalized load impedance $z_L = \frac{Z_L}{Z_0} = \frac{45 - j100}{50}$

The normalized load impedance is plotted on the Smith Chart (Smith Chart–12) at A and the impedance circle is drawn.

From the Smith Chart,

$$SWR = 4.8$$

The normalised load admittance is diametrically opposite to the normalised load impedance at B.i.e., $Y_L = 0.3 + j0.6$.

The admittance point is rotated in a clockwise direction to a point C on the impedance circle. Where it intersects the R = 1 circle. At this point C admittance. $y = 1 + j1.7$

The distance between these two points B and C is the distance from the load to the location of the stub.

$$\text{i.e., } d = (0.18 - 0.094)\lambda$$

$$= 0.086\lambda$$

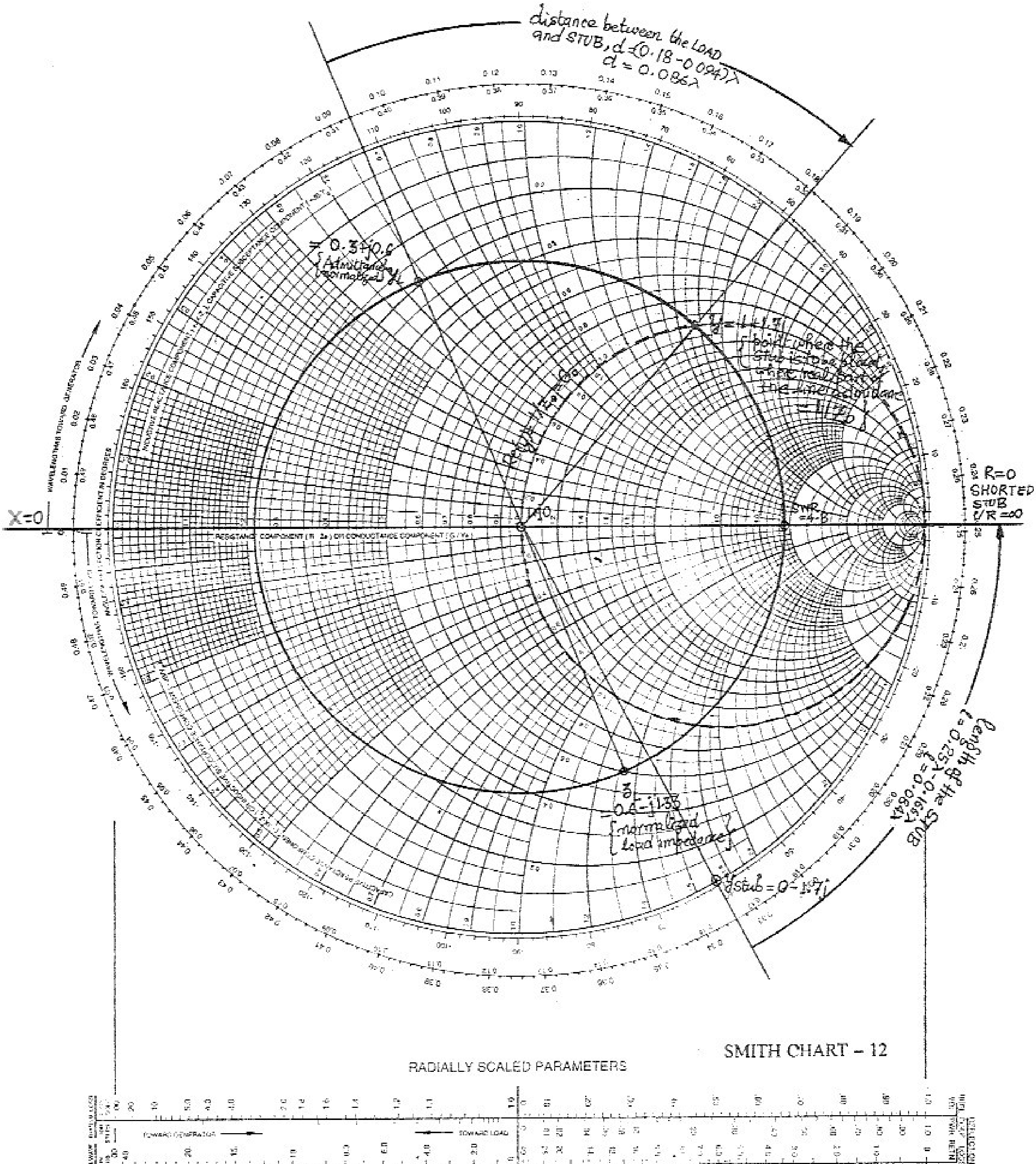
The stub must have zero resistance and an susceptance that has an exactly opposite value at E

$$\text{i.e., } y_{stub} = 0 - j1.7$$

The length of the stub is measured from the right side of the chart (X=0) at D to the point E.

$$l = 0.25\lambda - 0.166\lambda$$

$$= 0.084\ \lambda$$



SMITH CHART – 12

Example:13 What are the short–comings of the single stub matching? What lengths of shorted stub must be attached to the line with a characteristic impedance of 300Ω to achieve a smooth line and maximize the received power, if load is 75 + 225jΩ.

Single–stub impedance matching requires that the stub be located at a definite point on the line.

This is undesirable since this point may be physically inaccessible.

For a coaxial line, it is not possible to determine the location of the voltage minimum without a slotted section, so the location of the stub be an exact required point that is virtually impossible.

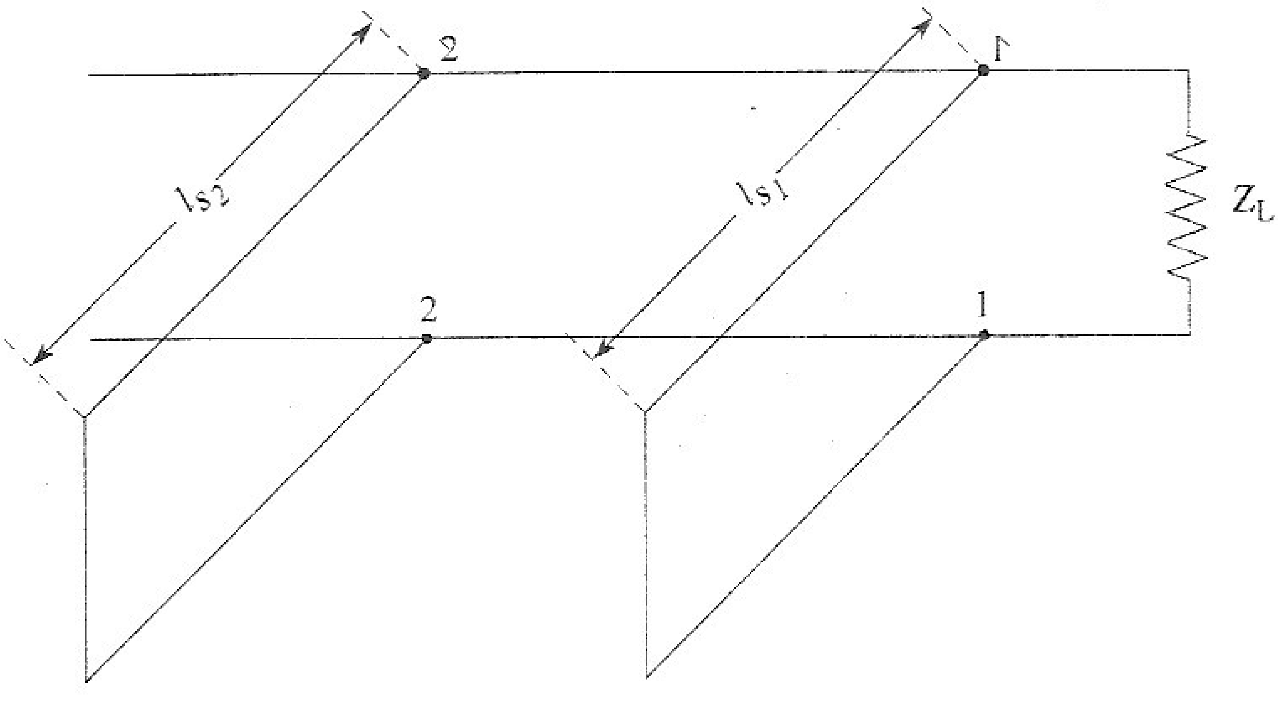
In case of the single stub matching there are two requirements :

(i) distance of the stub from the load.

(ii) length of the stub.

Double stub matching requires the stub lengths but their exact position can be arbitrary.

The spacing is frequently kept as λ/4.



For Smooth line operation,

the input impedance at 2 – 2 must be,

$$Z_{in} = Z_0$$

There is no possibility of reflection

Hence, the point 2 – 2 must have an admittance

(since, stub is shunted hence it is easy to deal in terms of admittance)

$$\frac{Y_{in}}{Y_0} = 1 \pm jb_a$$

Since, if a stub with an admittance $\pm\, b_a$ is shunted at 2 – 2 then the input impedance at 2 – 2 is the characteristic impedance.

A circle A for Y/G = 1 which is the locus of all points for which the real part of the line conductance is unity, is drawn in dotted lines.

$$\text{i.e.,} \quad \frac{Y_{in}}{Y_0} = 1 \pm jb_a$$

Anywhere on this circle the impedance can be equalled to characteristic impedance by cancelling out jba.

If the distance between the two stub is 'λ/4' then shifting all the points by 0.25λ = 180° to the LHS of the Smith Chart the locus of second circle B as shown in dotted lines is got.

Hence, stub 1 must transform the impedance of the remaining line and load to its right to an input impedance at 1 – 1 such that it lies on the circle on the RH of the Smith Chart.

The line transforms this on to the circle on LH, length of the line being 'λ/4' there by following the impedance circle.

Stub 2 can now cancel the reactive part of the admittance, allowing the smooth line operation.

Characteristic impedance	Z_0	$= 300\Omega$
load impedance	Z_L	$= 75 + j225\Omega$
Normalized load impedance		$= 0.25 + j0.75$
Normalized load admittance	y_L	$= \frac{Y_L}{Y_0} = \frac{Z_0}{Y_L}$

$$= \frac{300}{75+j225} = 0.4 - j1.2 \text{ mho}$$

This normalised load admittance is entered in the Smith Chart (Smith Chart–13).

Stub 1 adds a susceptance in parallel and this must change $\frac{Y_L}{Y_0}$ to a value $\frac{Y_1}{Y_0}$ which lies on locus circle B at P. Stub 1 cannot alter the conductance, so following a constant conductance circle from load admittance y_L locates point P on the circle B.

$$\text{i.e.,} = y_{s_1} = 0.4 - 0.5\lambda$$

A circle with 'O' as center and 'OP' as radius is drawn.

It cuts circle A at Q (diametrically opposite to P)

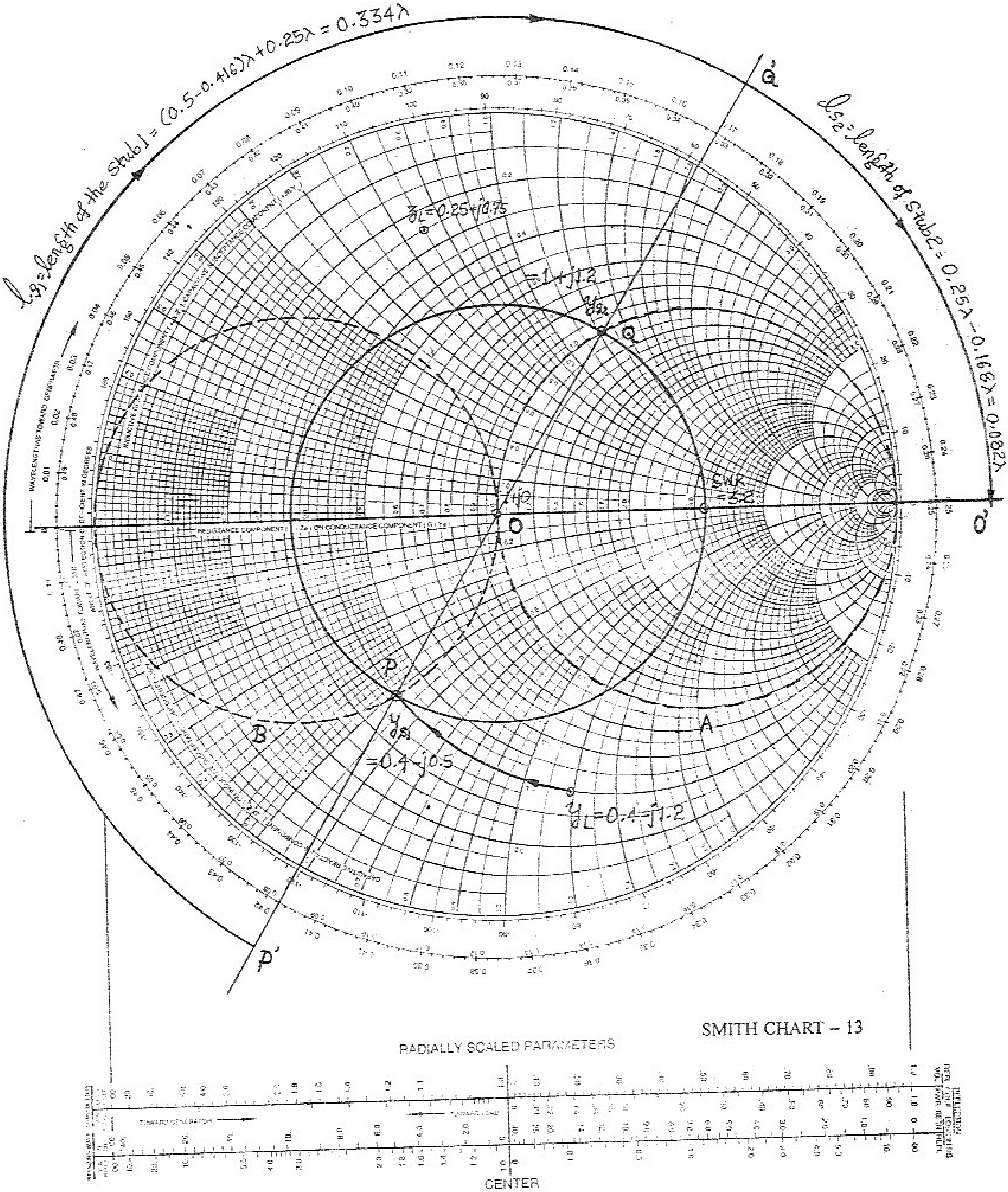
$$\text{i.e.,}\ y_{s_2} = 1 + j\,1.2$$

To provide proper termination at 2.2, $\frac{Y_2}{Y_0}$ should be 1 + j0.

The lengths of stub can be found from the Chart.

length of the stub 1 = P' Q' = l_{s_1} = 0.334 λ

length of the stub 2 = Q' O' = l_{s_2} = 0.082 λ



SMITH CHART – 13

Example:14 What lengths of short circuited stubs must be attached to the line with a characteristic impedance of 75Ω to achieve the maximum power transfer to a load of 37.5 + j97.5 Ω.

Given :

Characteristic impedance $Z_0 = 75\Omega$

Load impedance $Z_L = 37.5 + j\,97.5\Omega$

Normalized load impedance $z_L = \dfrac{Z_L}{Z_0}$

$$= \frac{37.5 + j97.5}{75} = 0.5 + j1.3$$

Normalised load admittance $y_L = \dfrac{Y_L}{Y_0} = \dfrac{Z_0}{Z_L}$

$$= \frac{75}{37.5 + j97.5} = 0.25 - j0.68$$

The dotted circle A for Y/G = 1 is drawn.

Since the distance between the two stub is $\lambda/4$, this circle is rotated by $\lambda/4 = 180°$. The new circle B is drawn in dotted line.

Normalised load admittance $y_L = 0.25 - j0.68$ is entered in the Smith Chart (Chart-14).

Stub 1 adds a susceptance in parallel and this must change $\dfrac{Y_L}{Y_0}$ to a value $\dfrac{Y_L}{Y_0}$ which lies on circle B. Stub 1 cannot alter the conductance, so following a constant–conductance circle from load admittance y_L locates point ys1 on circle B.

$$\frac{Y_1}{Y_0} = 0.25 - j0.44$$

∴ The normalised input admittance at 1–1 = 0.25 – j0.44.

A circle with 'O' as center and Oy_{s_1} as radius is drawn. It cuts circle A at y_{s_2} (diametrically opposite to y_{s_1}).

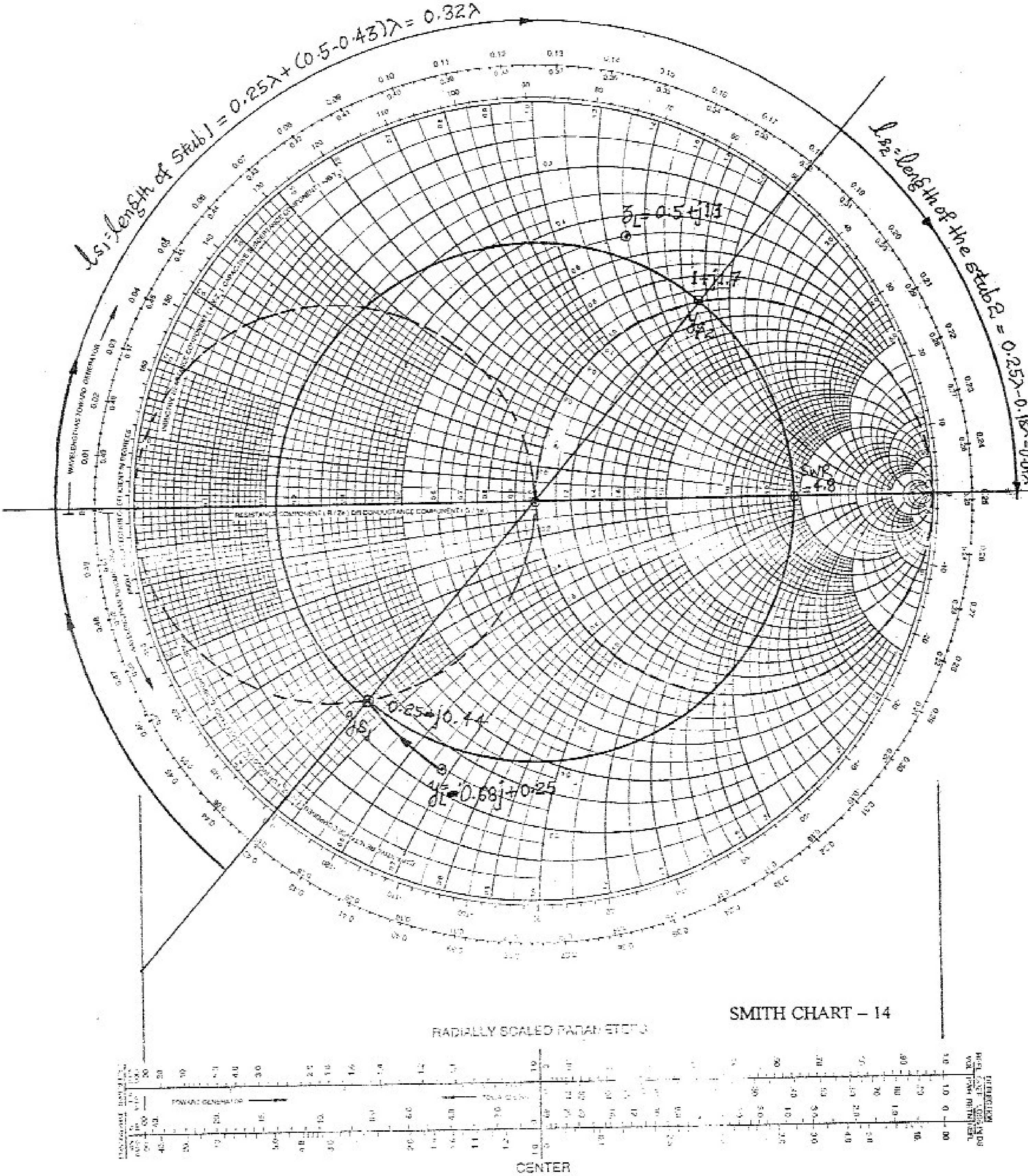
$$\text{i.e., } y_{s_2} = 1 + j1.7$$

To provide proper termination at 2-2, $\dfrac{Y_2}{Y_0}$ should be $1 + j0$.

The lengths of the stub can be found from the chart

length of the stub 1 = $P'O' = l_{s_1} = 0.32\,\lambda$

length of the stub 2 = $Q'O' = l_{s_2} = 0.07\,\lambda$



SMITH CHART – 14

Example:15 A lossless transmission line of length 0.434λ and characteristic 100Ω is terminated in an impedance 260 + j 180Ω. Find the standing wave ratio reflection, coefficient the input impedance and the location of a voltage maximum on the line.

Given :

Length of the transmission line = 0.434λ

Characteristic impedance $Z_0 = 100\ \Omega$

Load impedance $Z_L = 260 + j180\ \Omega$

Normalised load impedance $z_L = \frac{Z_L}{Z_0}$

$$= \frac{260 + j\,180}{100} = 2.6 + j1.8$$

This normalised load impedance is entered in the Smith Chart (Chart-15) at P.

An impedance circle with O as center and OP as radius is drawn. SWR (Standing Wave Ratio) is directly read from the intersection of the impedance circle and the X = 0 line on the right side i.e., OS = SWR = 4.2.

Reflection coefficient $|\Gamma| = \frac{S-1}{S+2}$

$$= \frac{4.2-1}{4.2+1}$$

$$= 0.615$$

A straight line OP is drawn from O and is extended to P' on the outer circle at 0.218λ and rotated in a clockwise direction by 0.434λ to the point Q' at 0.152λ

$[(0.5-0.218)\lambda + 0.152\lambda = 0.434\lambda] = (0.25-0.218)\,4\pi$

The phase angle of reflection coefficient = $0.128\pi = 23°$

Reflection coefficient = $0.615\ \angle 23°$

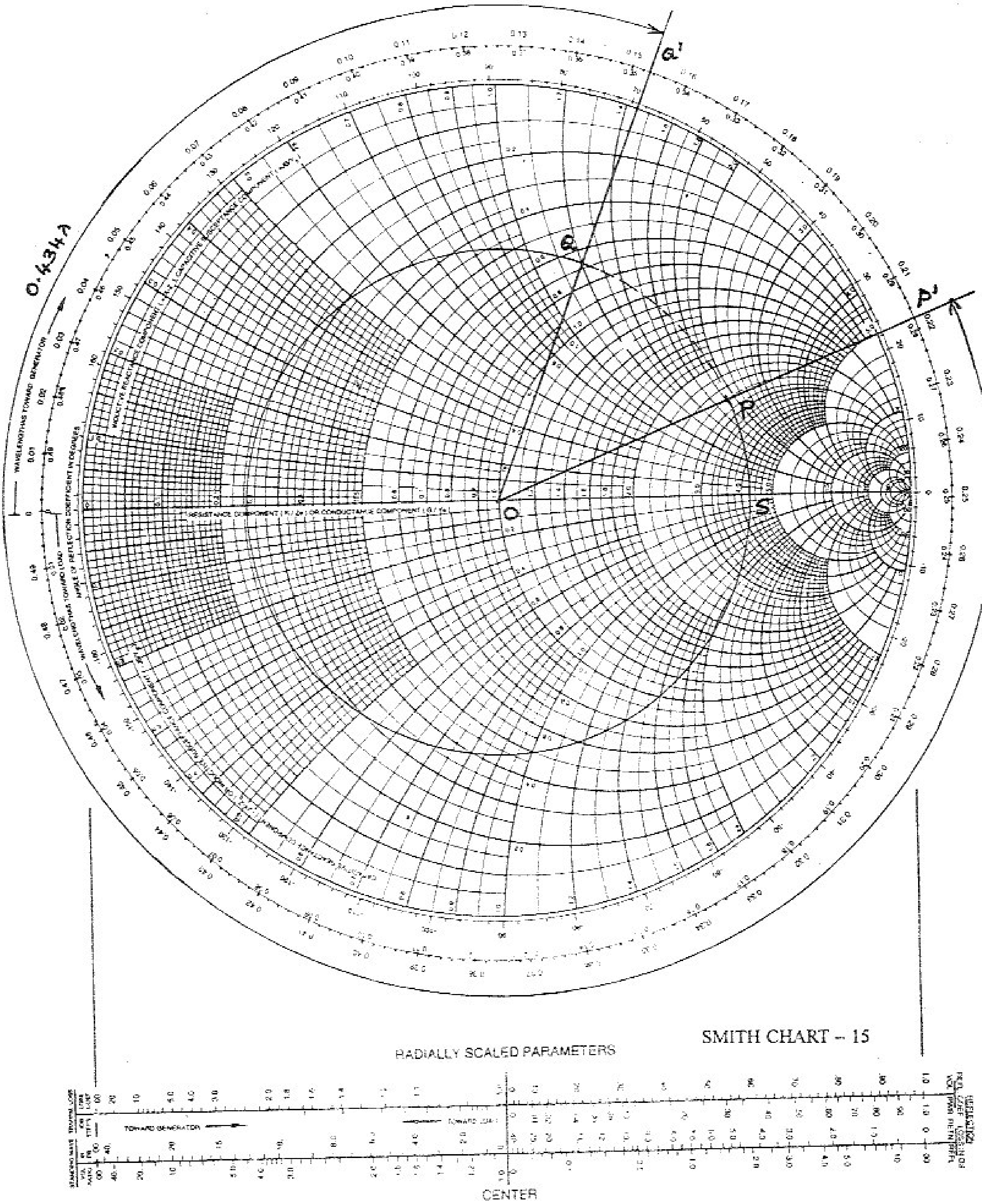
The straight line OQ' is joined and it cuts the impedance circle at Q. The point where this line cuts the impedance circle is the normalised input impedance at that point i.e., 0.66 + j1.2

The input impedance $Z_i = 100\,(0.66 + j1.2)$

$$= 66 + j120\Omega.$$

The voltage maximum occurs at the distance of voltage maximum from the load and can be found out interms of wavelength from this chart = $(0.25 - 0.218)\lambda$

$$= 0.032\ \lambda.$$



SMITH CHART – 15

Example:16 The input impedance of a short circuited lossy transmission line of length 2m and the characteristic impedance 75Ω is 45 + j225Ω

(a) Find α and β of a line.

(b) Determine the input impedance if the short circuit is replaced by a load impedance of 67.5 – j45Ω.

Given :

$$Z_L = 67.5 - j\,45\ \Omega$$

$$Z_0 = 75\ \Omega$$

$$Z_i = 45 + j225\ \Omega$$

$$l = 2\text{m}$$

(a) The short circuit load is entered as point P_{SC} on the extreme left of the Smith Chart (Chart-16).

Normalised input impedance $z_i = \dfrac{Z_i}{Z_0}$

$$= \frac{45 + j\,225}{75} = 0.6 + j3\ \Omega$$

Normalised impedance is entered as point P_1 in the Smith Chart

A straight line OP_1 is drawn from the origin O and extended to P_1' on the outer circle at 0.197λ.

The ratio OP_1 and OP_1' i.e., $\dfrac{OP_1}{OP_1'} = \dfrac{7.5}{8.5} = 0.88$

$$e^{-2\alpha l} = 0.88$$

$$\alpha = \frac{1}{2l}\, ln\left[\frac{1}{(0.88)}\right]$$

$$= \frac{1}{2}\, ln\left[\frac{1}{(0.88)}\right]$$

$$= 0.032\ \text{Np/m}.$$

$$l = 0.197$$

$$2\beta l = 2 \cdot \frac{2\pi}{\lambda}\, l$$

$$= \frac{4\pi}{\lambda}\, l$$

$$= 4\pi\,(0.197) = 0.788\,\pi$$

$$\beta = \frac{0.788\,\pi}{2l}$$

$$= \frac{0.788\,\pi}{4}$$

$$= 0.197\,\pi \text{ rad/m.}$$

(b) Normalised load impedance $z_i = \frac{Z_L}{Z_0}$

$$= \frac{67.5 + j\,45}{75}$$

$$\doteq 0.9 + j0.6\ \Omega$$

The normalised load impedance is entered in the Smith Chart (Chart-16) at point P_2

A straight line OP_2 is drawn from the origin and extended to P_2' on the outer circle at 0.364λ.

An impedance circle with 'O' as center and OP_2 as radius is drawn.

The point P_2 is shifted to P_3 along the outer circle by 0.197λ (wavelength to ward generator) at 0.061λ .

Point P_3' and origin O are joined by a straight line.

It cuts impedance circle at P_3

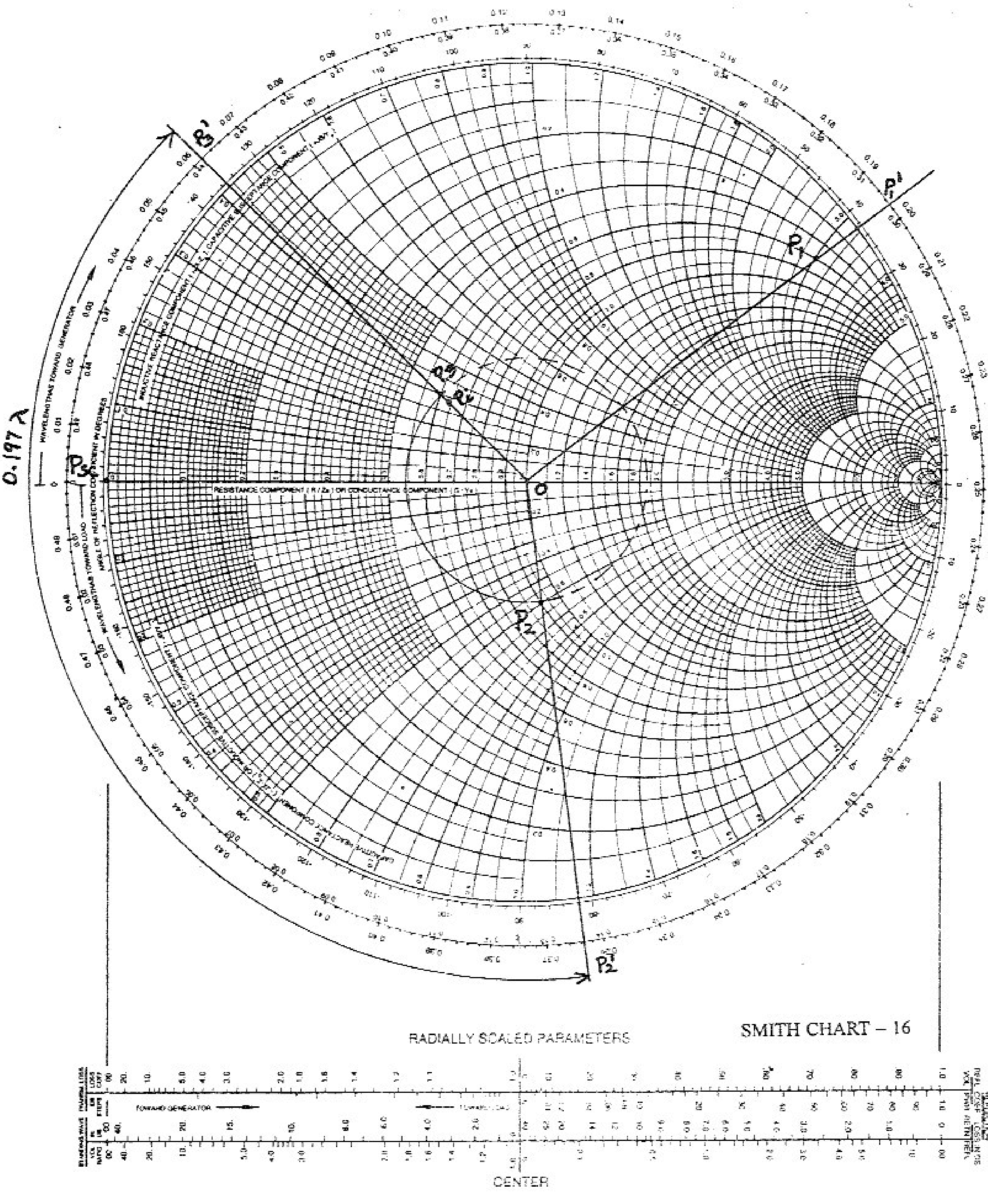
A point P_i is marked on OP_3 such that $\frac{OP_i}{OP_3} = 0.88$

Point P_i shows $z_i = 0.65 + j\,0.28$

Input impedance $Z_i = z_i Z_0$

$$= (0.65 + j0.28)75$$

$$Z_i = 48.75 + j21\ \Omega.$$



SMITH CHART – 16

mpedance is 100Ω is to be matched at a
ıaving characteristic impedance of 600 be
ation and length of the stub.

$\beta = \frac{0.78}{}$

(b) Nor

2.71

$$= \frac{3}{2\pi}\tan^{-1}\sqrt{\frac{100}{600}}$$

$$= \frac{3}{2\pi} \times \frac{22.2^\circ}{180} \times \pi$$

($\because$ To convert 22.2° in to radians, multiply $\frac{\pi}{180}$)

$$l_s = 0.185\text{m}.$$

Length of the stub is

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}\right)$$

$$= \frac{3}{2\pi}\tan^{-1}\left(\frac{\sqrt{100 \times 600}}{100 - 600}\right)$$

$$= \frac{3}{2\pi}\tan^{-1}(-04899)$$

$$= \frac{3}{2\pi} \times (-26.1^\circ)$$

$$= \frac{3}{2\pi}(180^\circ - 26.1^\circ)$$

(Because of negative, angle will be subtracted from 180°.)

$$l_t = \frac{3}{2\pi} \times \frac{153.9^\circ}{180} \times \pi$$

$$l_t = 1.28\text{m}.$$