## SMITH CHART – STUB MATCHING

### Single Stub Matching

**Determine the stub length and the distance of the stub from the load. Given that a complex load $Z_L = 50 - j100$ is to be matched to a 75Ω transmission line using a short–circuited stub.**

***Given :***

Characteristic impedance of the transmission line $Z_0 = 75\Omega$

Load impedance to be matched to the transmission line $Z_L = 50 - j100$

***Steps :***

1) The normalized impedance is determined by dividing the load impedance by the characteristic impedance of the transmission Line.

$$z_L = \frac{Z_L}{Z_0} = \frac{50 - j100}{75} = 0.667 - j1.33$$

2) The normalized impedance, $z_L$ is plotted on the Smith chart by determining the point of intersection between the constant R circle with R = 0.667 and constant X circle with X = 1.33.

   The impedance circle is drawn.

   Because stubs are connected in parallel with the load, admittances can be much easily used rather than impedances to simplify the calculations.

3) The normalized admittance y is determined from the Smith chart by simply rotating the impedance plot, by 180°. This is simply done by drawing a line from point A through the center of the chart to the opposite side of the circle, point B.

4) The admittance point is rotated clockwise to a point on the impedance circle where it intersects the R = 1 circle, at point C. The real component of the input impedance at this point is equal to the characteristic impedance $Z_0$. At the point C, the admittance is $y = 1 + j1.7$.

5) The distance from point B to point C, in terms of the wavelength is how far from load the stub must be placed, i.e, *d*.

   The stub must have a zero resistive component impedance and a susceptance that has the opposite polarity.

6) To determine the length of the shorted stub that has an opposite reactive component to the input admittance, the outside of the Smith Chart (R=0) is moved around with the starting point at D {since at point D t = 0 and hence $\gamma = \infty$}, until an admittance y = 1.7 is found.

7) The distance between point D and E is the length of the stub. For this quantity the notation is, *l*.

From the Smith Chart ,

***length of the stub*** $l = 0.086\ \lambda$ ***; position of the stub from the load*** $d = 0.088\ \lambda$.

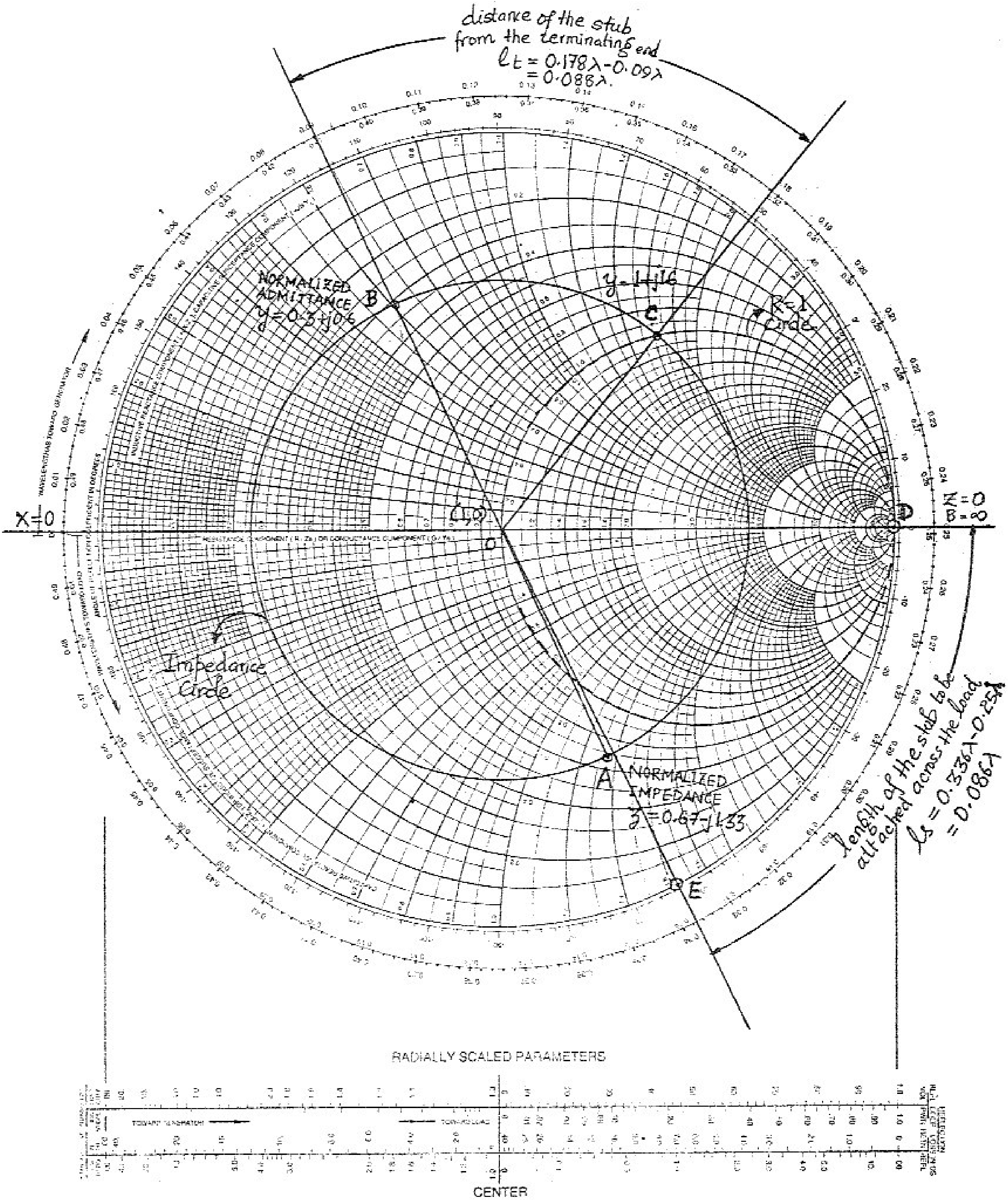



*Fig. 2.14 Single Stub Matching.*

## Double Stub Matching

**Using Double stub matching, match a complex load of $Z_L = 18.75 + j56.25$ to a line with characteristic impedance $Z_0 = 75\Omega$.**

**Determine the stub lengths, assuming a quarter wavelength spacing is maintained between the two short circuited stubs.**

A spacing of $\lambda/4$ is maintained between the stubs, stub 2 and stub 1. For smooth line operation of the transmission line the input admittance looking into the terminals 2, 2 of the line should be,

$$Y_{2,2} = {}^{1}/_{Z_0}$$

i.e. the line beyond the point 2, 2, should appear to be a pure resistance of value '$Z_0$'. {considering $Z_0$ is purely resistive}. Similar, to the single stub matching, the admittance (normalized) at the point 2,2 must be,

$$\frac{Y_S}{G_0} = 1 \pm jba$$

The stub at 1, 1 must be capable to transform the admittance at the terminating impedance end to the circle B which is displaced from the circle A ; R = 1 by '$\lambda/4$'.

The quarter wavelength line will further transform the admittance into a value at 2,2 which will plot on the circle A. Thus the line to load distance between position 2, 2 is not required to be determined.

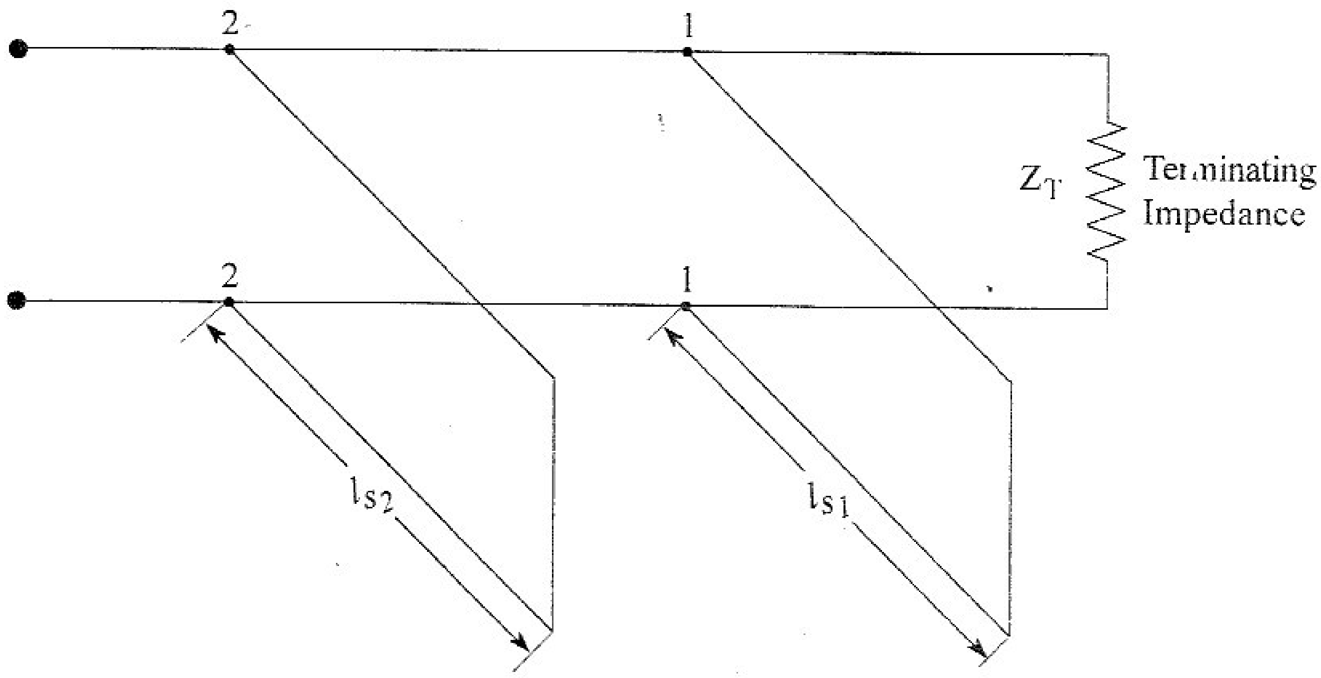



*Fig. 2.15 Double Stub Matching*

1. The normalized load impedance $z_L = \frac{Z_i}{Z_0} = \frac{18.75 + j\,56.25}{75}$

$$z_L = 0.25 + j0.75$$

plotting the normalized impedance on the Smith Chart, the impedance circle is drawn with distance between the point (1, 0) and the point of the normalized impedance as the radius {distance, OA}

2. Moving by 180° (0.25λ) on the impedance circle, i.e., at a diametrically opposite point to the point A, i.e., point B will give the normalized admittance.

   From the smith chart $y_L = 0.4 - j1.2$

3. Circle A is the constant R circle for R = 1. Circle B is the locus of all the points on the circle A displaced by λ/4, quarter wavelength. The stub 1 adds a susceptance (1/reactance) in parallel, this is done to change the value of $y$ to such a value that it plots on the circle B.

   Since stub 1 cannot alter the conductance (1/resistance), to a point on the circle B, point C,

   $y$(at point C) = 0.4 − j0.5

   It is observed that the susceptance is changed by + j0.7.

4. Transforming the point C to the point D on the circle A, since the line between 1, 1 and 2. 2 is a quarter wave line that transforms the admittance at 1, 1 to 2, 2 such that the conductance equals the characteristic conductance, $1/Z_0$.

   $y$(at point D) = 1.0 + j1.2

5. The stub length at 2, 2 should cancel the imaginary part of the above admittance.

   The susceptance of the stub at 2, 2 must be −j1.2.

6. To find the length of the stub with an admittance,

   (a) + j 0.7 and (b) −j 1.2

   The outside circle of the Smith Chart (the circle, R = 0), is moved around having a reference at point P, until

   an admittance $y$ = −1.2 is found at point E and

   an admittance $y$ = +0.7 is found at point F.

7. From the Smith Chart,

   Length of the stub 1 = distance between P and F $l_{S1}$ = **0.348λ**.

   Length of the stub 2 = distance between P and E $l_{S2}$ = **0.11λ**.

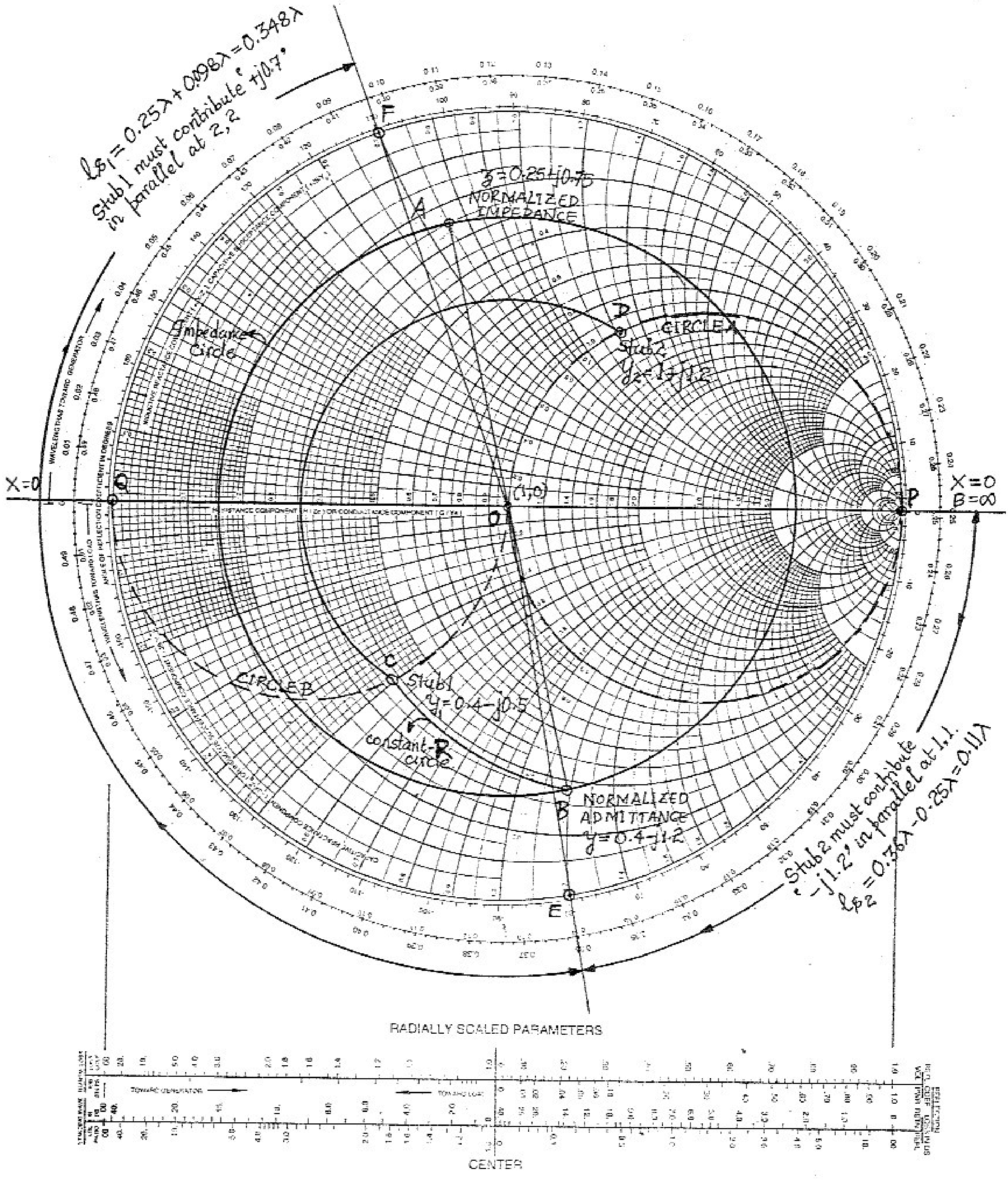



*Fig. 2.16 Double Stub Matching.*

**Note** Point P is chosen as a reference because a short circuited stub is used, which has zero resistance (R=0) and consequently a susceptance B = ∞. Point P is such a point. If an open stub were used, the rotation would begin at the opposite of the X = 0 line, point Q.

## SOLVED EXAMPLES

**Example:1 Determine the following :**

**(a) Standing wave ratio (SWR)**

**(b) Load admittance**

**(c) Distance between load and the first voltage minimum along the transmission line for a line with a characteristic impedance of 300Ω and terminated in a load of 175 + j207Ω. An electrical signal of 200MHz is transmitted along the line in free space.**

*Given :*

Characteristics impedance, $Z_0 = 300\Omega$

Load impedance, $Z_R = Z_L = 175 + j207\Omega$

Frequency of transmission, $f = 200\text{MHz}$

Wavelength of transmission, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ m}$

(a) Normalizing the load impedance with respect to the characteristic impedance hence, the normalized load impedance

$$\frac{Z_L}{Z_0} = z_L = \frac{175 + j\,207}{300} = 0.5833 + j0.69.$$

Plotting the point P on the Smith Chart (Smith Chart – I) at the intersection of the circles R = 0.5833 and X = 0.69.

Taking the centre point O (1 + j0) and the radius as (OP) the distance between the centre and the normalized load impedance point, the Impedance Circle is drawn.

The distance between the centre and the point S where the impedance circle crosses the horizontal axis on the right side of the chart, i.e., OS is the 'Standing Wave Ratio'

SWR = 2.8

(b) The point Q diametrically opposite to the normalized impedance point on the impedance circle is the admittance of the load, normalized w.r.t the characteristic admittance.

$$\text{Normalized admittance } \frac{Y}{G_0} = YZ_0 = 0.72 - j0.86$$

$$\text{hence, } Y = \frac{1}{300}(0.72 - j0.86) \text{ mho} = 0.0024 - j\,0.00286 \text{ mho}$$

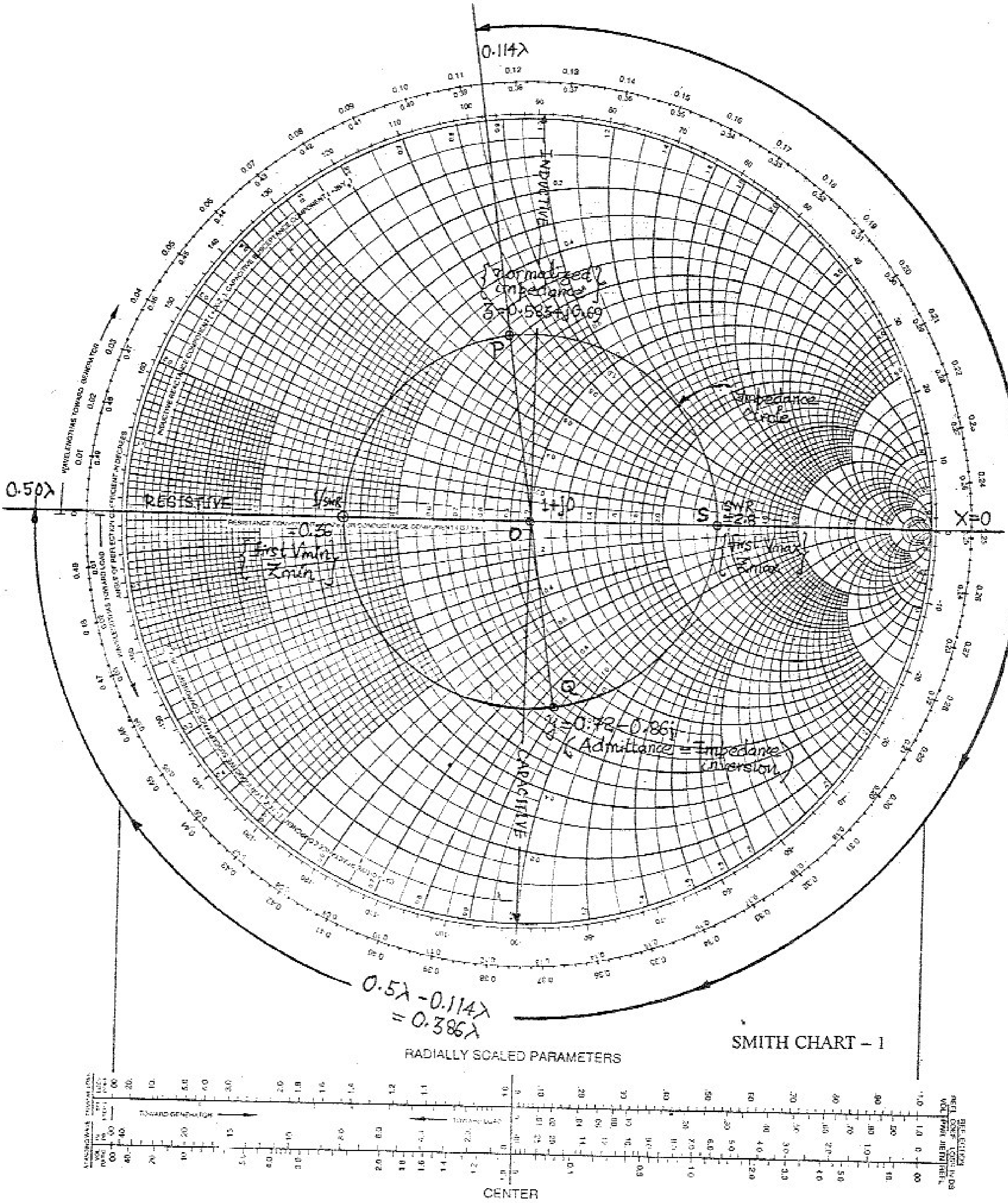
(c) The first voltage minimum from the load lies along the horizontal axis to the left side of the chart.

From the chart the distance between load and the first voltage maximum can be found interms of wavelength $= 0.386\lambda$.

Hence, the distance between the load and the first load impedance $= 0.386 \times 1.5\text{m}$

$= 0.579\text{m}$

$= 0.58\text{m}.$




SMITH CHART – 1

**Example:2 Determine the following :**

**(a) Standing Wave Ratio (VSWR)**

**(b) Load Admittance**

**(c) Impedance of the transmission line at the maximum and minimum of the stationary waves along the line**

**(d) Distance between load and first voltage maximum**

**for a transmission line with characteristic impedance of 50Ω with a receiving end impedance of 100 + 121j. The wavelength of the electrical signal along the line is 2.5m.**

***Given :***

Characteristic impedance $Z_0 = 50\Omega$

Load impedance $Z_L = 100 + j121\Omega$

Wavelength of the electrical signal $\lambda = 2.5\text{m}$

(a) Normalised load impedance $= \dfrac{100 + j121}{50} = 2 + j\,2.42$

Plotting the point P on the Smith Chart (Smith Chart – 2). The impedance circle is drawn with O (1+j0) centre and radius as (OP), the distance between centre and the normalised impedance point. From this SWR is found out.i.e., OS = 5.

Voltage Standing Wave Ratio = 5

(b) The point Q diametrically opposite to the normalized impedance point on the impedance circle is the normalized admittance of the load.

$$\text{Load admittance } \frac{Y}{G_0} = 0.22 - j0.25$$

$$YZ_0 = 0.22 - j0.25$$

$$\therefore \quad Y = \frac{1}{50}(0.22 - j0.25)$$

$$= 0.0044 - j\,0.005 \text{ mho}$$

(c) Impedance at the first voltage maximum from load $= 5 \times Z_0$

$= 250\Omega$

Impedance at the first voltage minimum $= 0.2 \times Z_0$

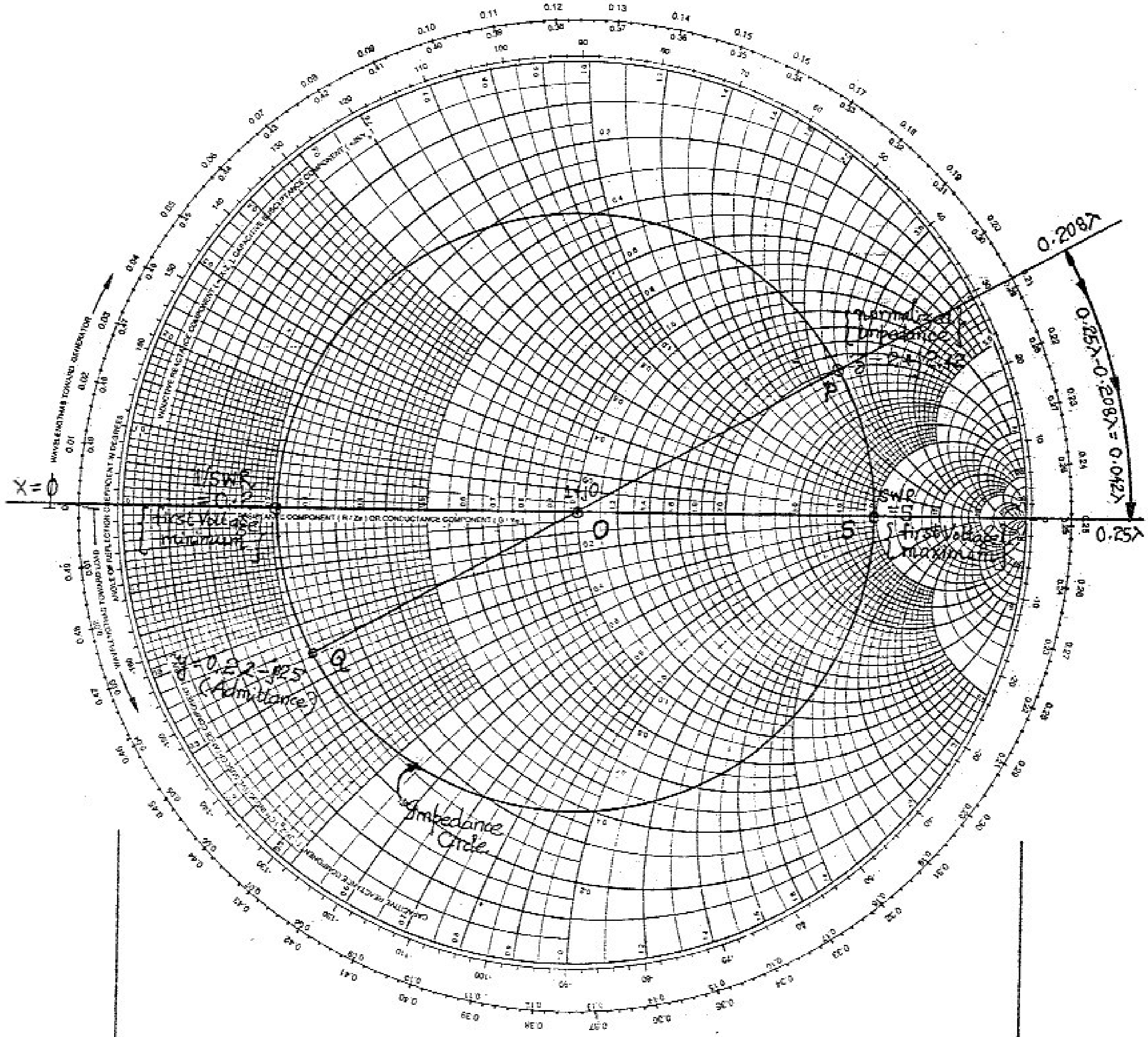
$= 10\Omega$

Distance between load and first voltage maximum $= 0.042\lambda$

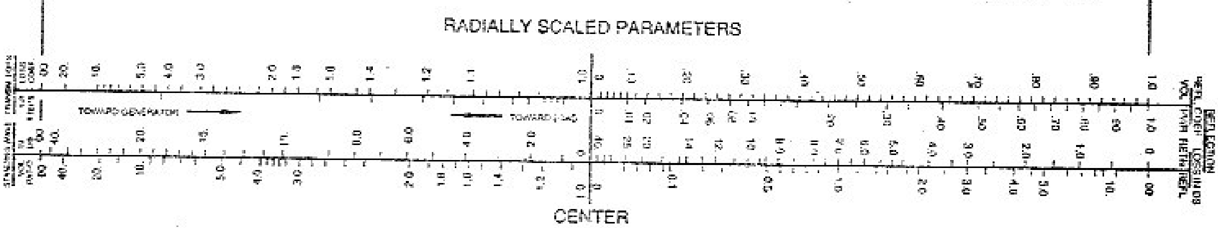
$= 0.042 \times 2.5\text{m}$

$= 0.105\text{m}$

$= 105\text{cm}.$




SMITH CHART – 2



**Example:3 Determine the impedance along the line for the open circuited and short circuited line at a distance from the load :**

**(a) d = 0**

**(b) d = 0.125, . . . . .(n +1) λ/8 n = 0, 1, . . . .**

**(c) d = (n +1) λ/4 n = 0, 1, . . . .**

**(d) d = (n +1) 3λ/8 n = 0, 1, . . . .**

**(e) d = (n +1) λ/2 n = 0, 1, . . . .**

As in the Smith Chart (Smith Chart – 3)

(I) For an open circuited load, R = ∞, X = ∞

(a) For d = 0, Z = ∞Ω

(b) For d = (n +1) λ/8, Z = –jΩ

(c) For d = (n +1) λ/4, Z = 0Ω

(d) For d = (n +1) 3λ/8, Z = jΩ

(e) For d = (n +1) λ/2, Z = ∝Ω

(II) For a short circuited load, Z = 0, R = X = 0.

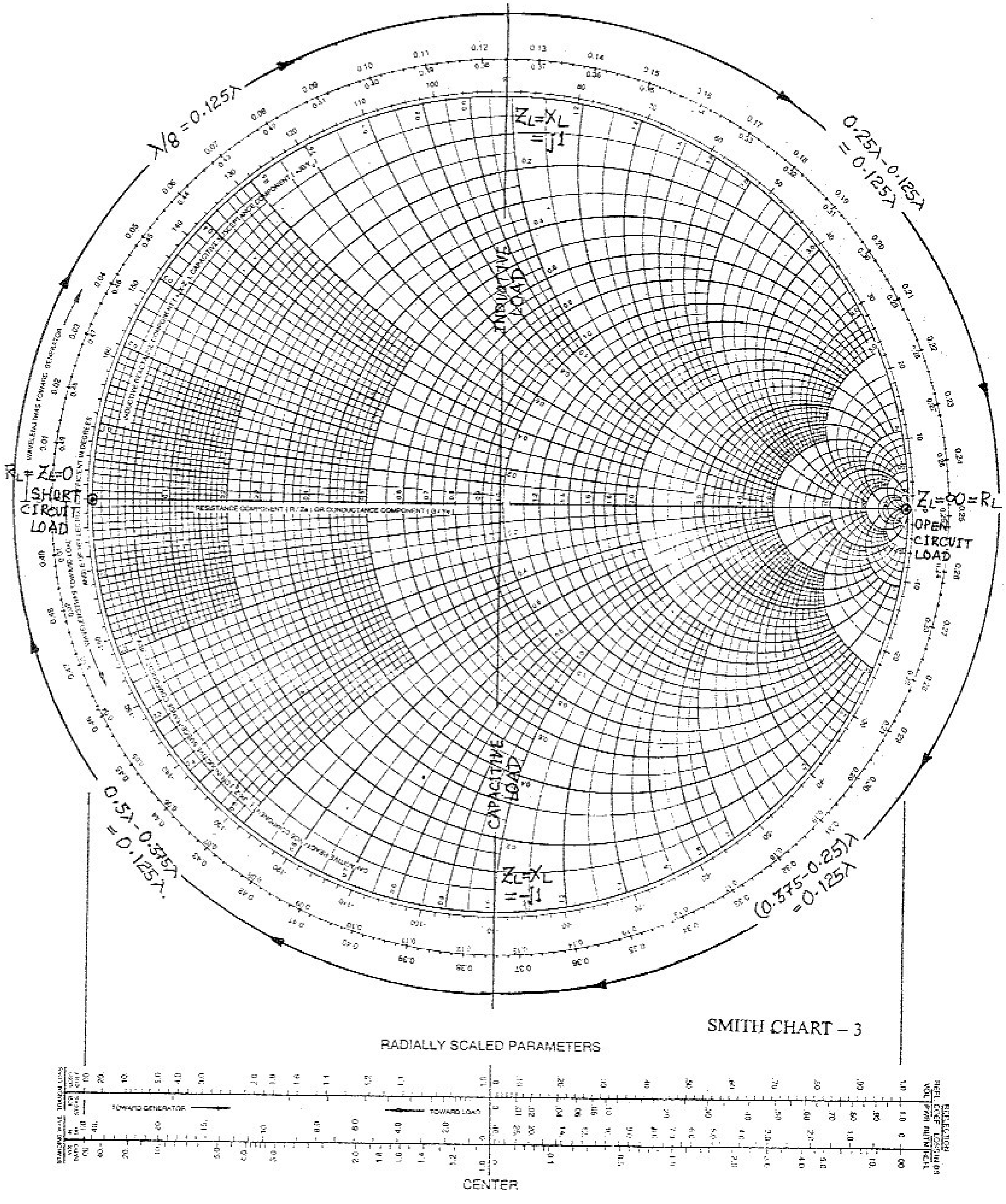
(a) For d = 0, Z = 0Ω

(b) For d = (n +1) λ/8, Z = j Ω

(c) For d = (n +1) λ/4, Z = ∞Ω

(d) For d = (n +1) 3λ/8, Z = –jΩ

(e) For d = (n +1) λ/2, Z = 0Ω




SMITH CHART – 3

**Example:4 Determine the input impedance and SWR for distance 0.21λ from the load towards generator with a characteristic impedance $Z_0$ = 50Ω and a load impedance, $Z_L$ = 80 – j30Ω.**

*Given :*

Characteristic impedance $Z_0 = 50\Omega$

Load impedance $Z_L = 80 - j30\Omega$

Distance from load at which impedance is to be found = 0.21 λ

Normalised load impedance $\frac{Z_L}{Z_0} = \frac{80 - j30}{50} = 1.6 - j\,0.6.$

The normalized impedance is plotted at P on the Smith Chart (Smith Chart – 4) and the Impedance circle is drawn.

SWR is directly read from the intersection of the impedance circle and the X = 0 line on the right side, i.e., OS = 1.9

$\therefore$ SWR = 1.9

The straight line OP is drawn from centre 0 and is extended to $P^1$ on the outer circle at 0.295λ. It is rotated in clockwise direction by 0.21λ to the point $Q^1$ at 0.505λ. The straight line $OQ^1$ is drawn and it cuts the impedance circle at $Q^1$.

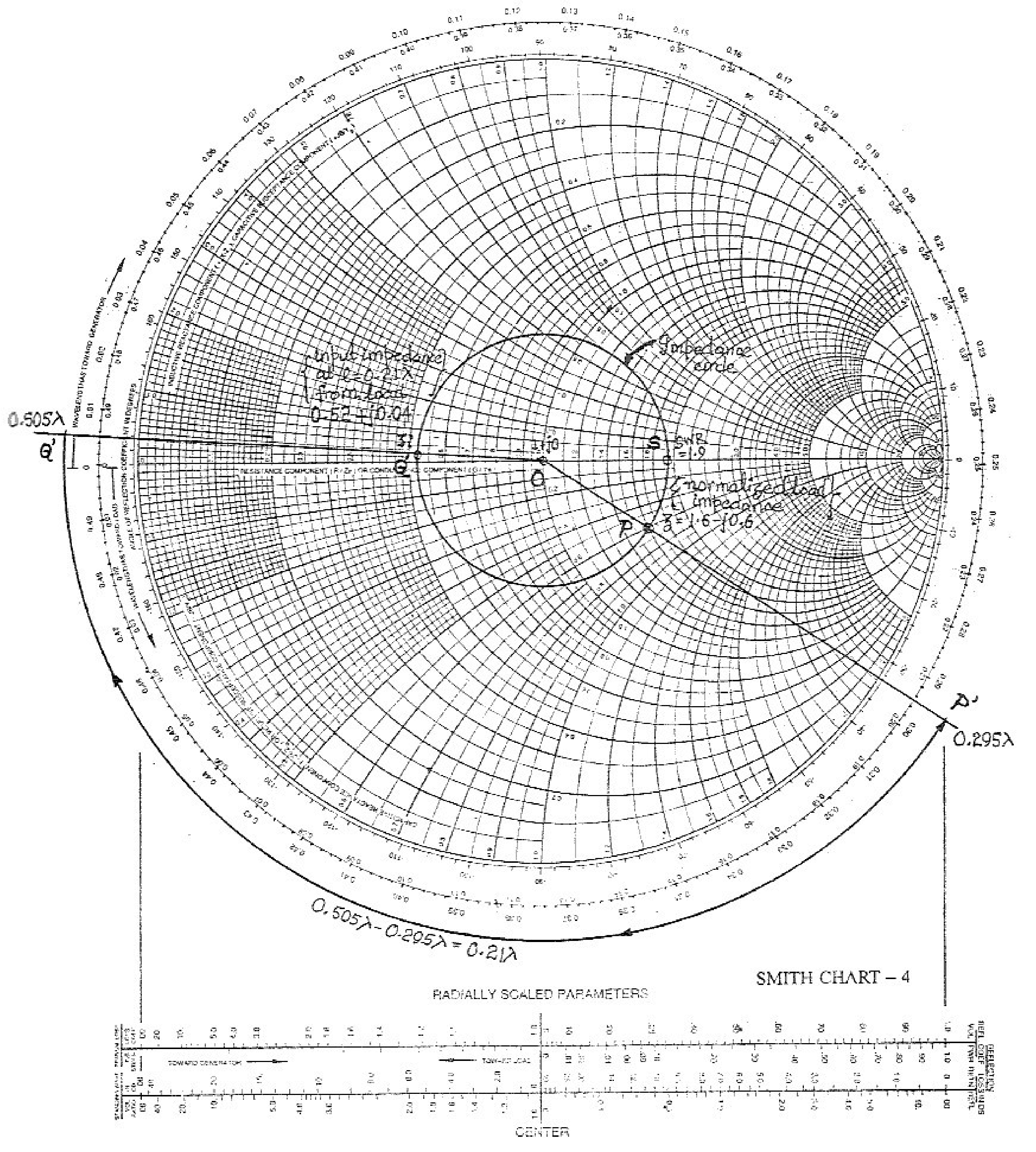
The point $Q_1$ where the distance on outside circle in a clockwise direction is 0.2[illegible], 0.505λ is joined to the centre.

The point where this line cuts the impedance circle is the normalized impedance at that point,

$$\frac{Z_{in}}{Z_0} = 0.52 + j0.04$$

$$Z_{in} = (0.52 + j\,0.04) \times 50\Omega$$

$$Z_{in} = 26 + j2\ \Omega.$$




SMITH CHART – 4

**Example:5 Determine the input impedance and SWR for a 1.25λ long transmission line at the sending end with a characteristic impedance, $Z_0$ = 50Ω and a load impedance $Z_L$ = 30 + j40Ω.**

*Given :*

Characteristic impedance $Z_0 = 50\Omega$

Load impedance $Z_L = 30 + j\,40\Omega$

Length of the transmission line = 1.25 λ

Normalised load impedance $\dfrac{Z_L}{Z_0} = \dfrac{30 + j40}{50} = 0.6 - j\,0.8$

The normalised impedance is plotted at P on the Smith Chart (Smith Chart–5) and impedance circle is with Q as centre is drawn. SWR is directly read from intersection of the impedance circle and the X = 0 line on the right side.i.e., OS = 3

$\therefore$ SWR = 3

A straight line OP is drawn from O and is extended to $P^1$ on the outer circle at 0.127λ and rotated in a clockwise direction by 0.25λ to the point $Q^1$ at 0.377.

[$\because$ 0.25λ is equivalent to 1.25λ as it takes two revolutions in excess]

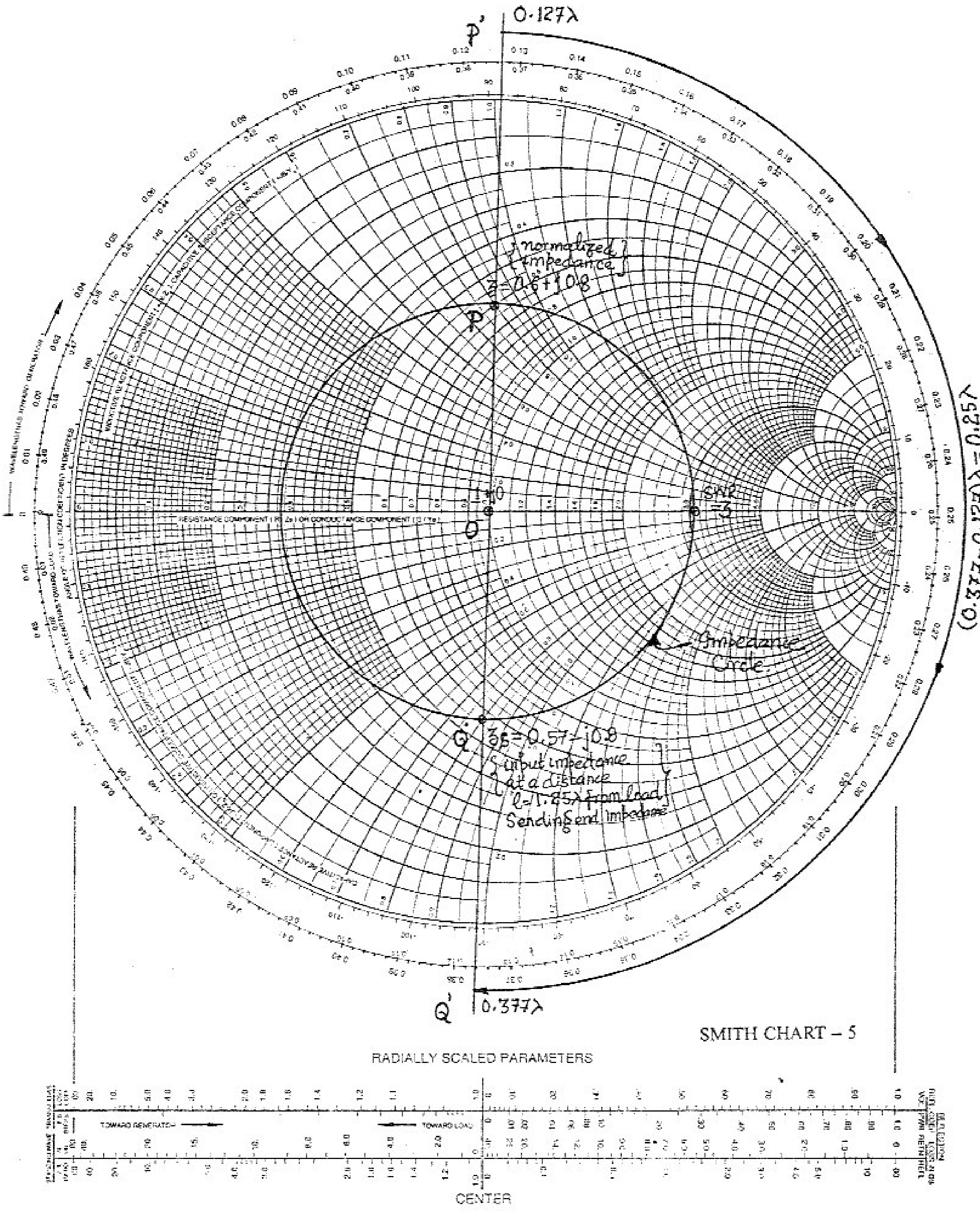
The straight line $OQ^1$ is drawn and it cuts the impedance circle at Q.

The point where this line cuts the impedance circle is the normalised input impedance at that point. i.e., 0.57 – j0.8.

Normalised input impedance $\dfrac{Z_S}{Z_0} = 0.57 - j0.8$

$$Z_S = (0.57 - j0.8) \times 50$$

$$= 28.5 - j\,40\Omega.$$




SMITH CHART – 5

**Example:6 Consider the line with $Z_0$ = 100 Ω terminated by an unknown impedance. The SWR = 2.5 and first voltage minimum at 16cm from termination, when the frequency is 100MHz. Determine the terminating impedance by use of Smith Chart assuming the line is placed in free space.**

*Given :*

Standing Wave Ratio = 2.5

Characteristic impedance $Z_0 = 100\Omega$

Frequency of the electrical signal = 100MHz

Wavelength of the electrical signal $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3\text{m}$

The impedance circle is drawn with (1+j0) centre and 2.5 as radius on Smith Chart (Smith Chart – 6).

The first voltage minimum lies on the left of the Smith Chart along the X = 0 axis.

The distance between the load and first voltage minimum = 16cm

Equivalently in wavelengths $= \frac{16}{300}\lambda$

$= 0.053\lambda.$

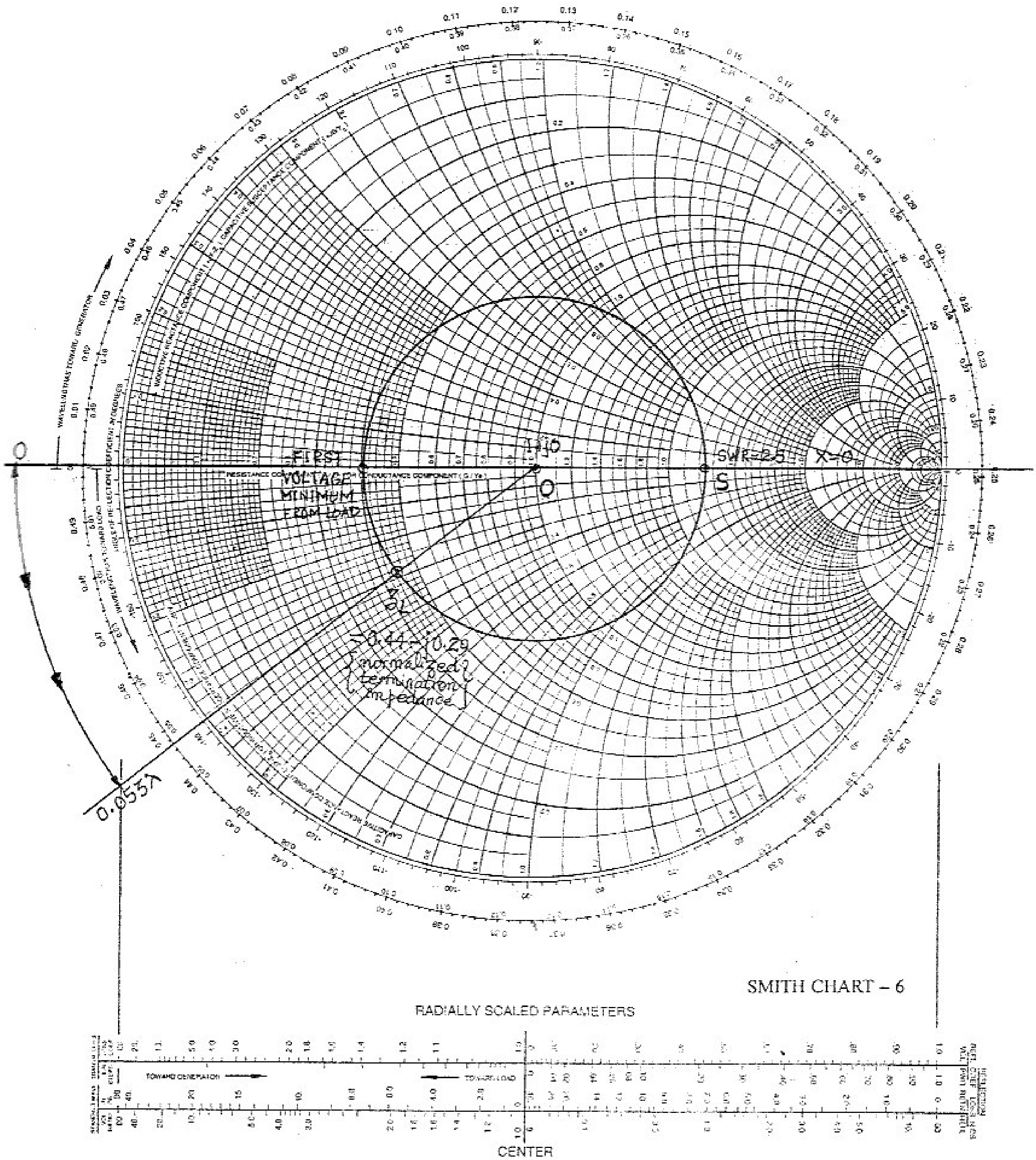
Moving in counter clockwise direction, i.e., from the first voltage minimum towards theload,

Normalised load impedance $z_L = \frac{Z_L}{Z_0}$

$= 0.44 - j\,0.29.$

Hence, the

terminating end impedance $Z_L = 44 - j29\Omega.$




SMITH CHART – 6

**Example :7 On a transmission line a standing wave pattern is observed and the voltage standing wave ratio is found to be 3.0 and it is also noted that two successive voltage minima are 30cm apart. The first voltage minimum is found to be 12 cm from the load. The length of the line is 85.2 cm and the characteristic impedance is 200Ω.**

**Attenuation lossless can be neglected at such high frequencies. Find**

**(i) Load impedance**

**(ii) sending end impedance**

*Given :*

Characteristic impedance $Z_0 = 200\Omega$

Standing Wave Ratio (SWR) = 3

Distance between two successive voltage minima $= \frac{\lambda}{2} = 30$ cm.

hence, $\lambda = 60$cm

$l_{min}$ = distance between first voltage minimum and load = 12 cm

$$l_{min} = \frac{12}{60}\lambda = 0.2\lambda.$$

$l$ = length of the given line = 85.2 cm

$$l = \frac{85.2\lambda}{60} = 1.42\lambda.$$

The impedance circle is drawn with (1+j0) as centre and 3 as radius on the Smith Chart (Smith Chart – 7).

(i) Moving from the first voltage minimum, (towards the left of the X = 0 axis of the Smith Chart) towards the load in the counter clockwise direction, by 0.2λ to $P^1$. The line $OP^1$ is drawn. It cuts the impedance circle at P. This point shows the normalised load impedance.

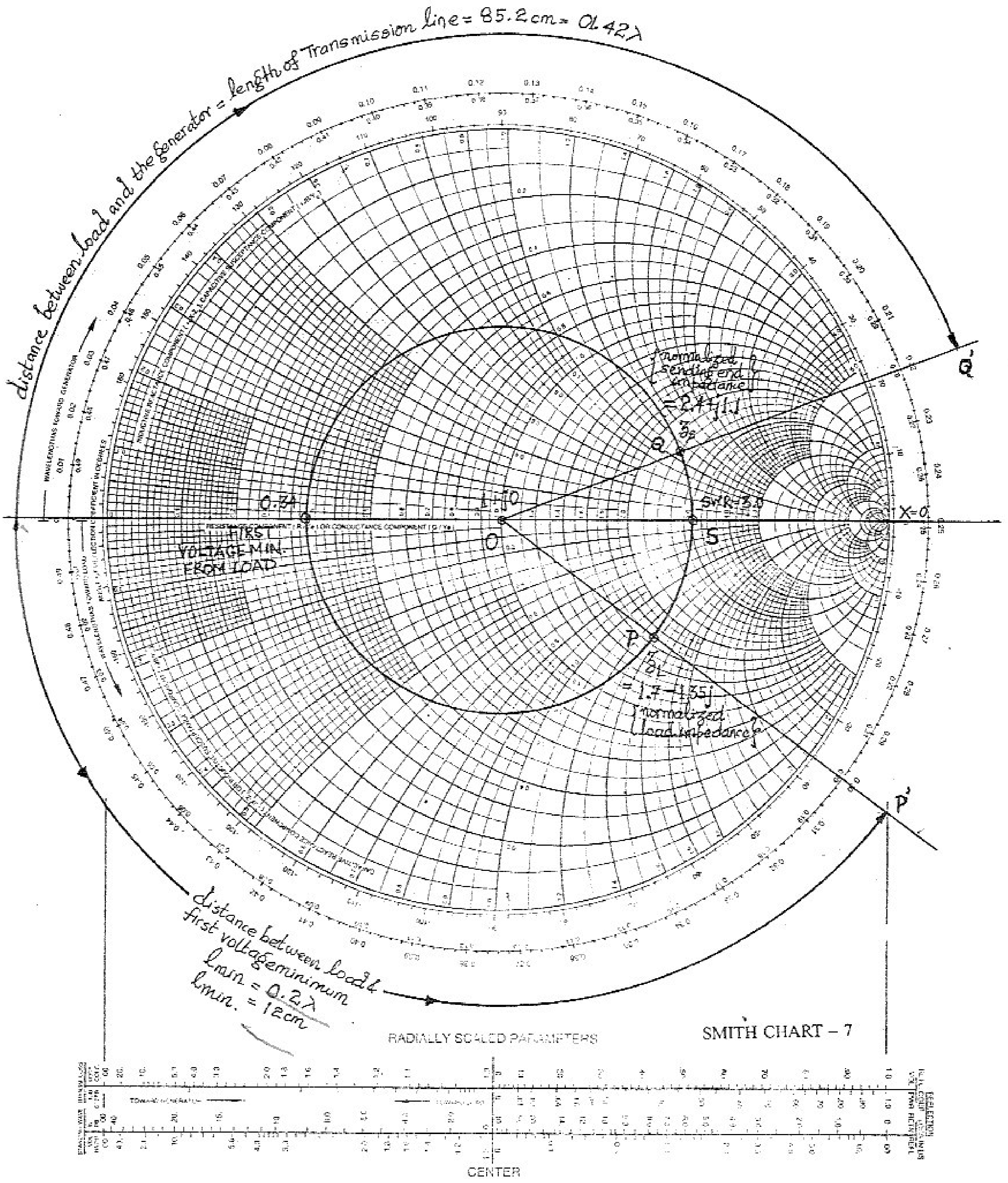
$$\text{Normalised load } z_L = \frac{Z_L}{Z_0} = 1.7 - j1.35$$

Load impedance $Z_L = 340 - j270\Omega$

(ii) Moving towards the generator from the load in a clockwise direction by 0.42 {as '1λ' is equivalent to 2 complete revolution} to $Q^1$. The line $OQ^1$ is drawn. It cuts the impedance circle at Q. This point shows the normalised sending end impedance.

$$\text{Normalised sending end impedance } z_S = \frac{Z_S}{Z_0} = 2.4 - j\,1.1$$

Sending end impedance $Z_S = 480 + 220j\ \Omega$


distance between load and the generator = length of Transmission line = 85.2cm = 0.42λ
Q'
normalized sending end impedance = 2.4+j1.1
Q
0.34
1+j0
SWR=3.0
X=0
FIRST VOLTAGE MIN. FROM LOAD
O
S
P
= 1.7 − 1.35j
normalized load impedance
P'
distance between load & first voltage minimum
lmin = 0.2λ
lmin. = 12cm
SMITH CHART – 7
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
CENTER