2

THE LINE AT RADIO FREQUENCIES

2.1 INTRODUCTION

When a line, either open wire or of coaxial type is used at radio frequencies the following assumptions are made.

- The current is considered as flowing on the surface of the conductor in a skin of very small depth.
- Lω >> R because of skin effect.
- G may be considered as zero.

Radiation losses in an open wire become appreciable for frequencies above 300 MHz. However coaxial cables can handle frequencies upto 3GHz. For open wire at high frequencies, the skin effect is considerably high i.e. current is flowing essentially on the surface of the conductor in a skin of very small depth. For coaxial cable at high frequencies, because of skin effect, the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor. The internal flux and internal inductance are reduced nearly to zero. The capacitance of the coaxial line is not affected by skin effect.

2.2 STANDING WAVES AND STANDING WAVE RATIO

When the transmission line is not matched to its load i.e., load impedance is not equal to the characteristic impedance ($Z_R \neq Z_0$), the energy delivered to the load is reflected back to the source. The combination of incident and reflected waves give rise to the standing waves. Fig.2.1(a) shows the standing waves along the length of transmission line termined in a load other than Z_0 (not properly matched) Fig.2.1.(b) shows the standing waves along the line for either open or short circuit.

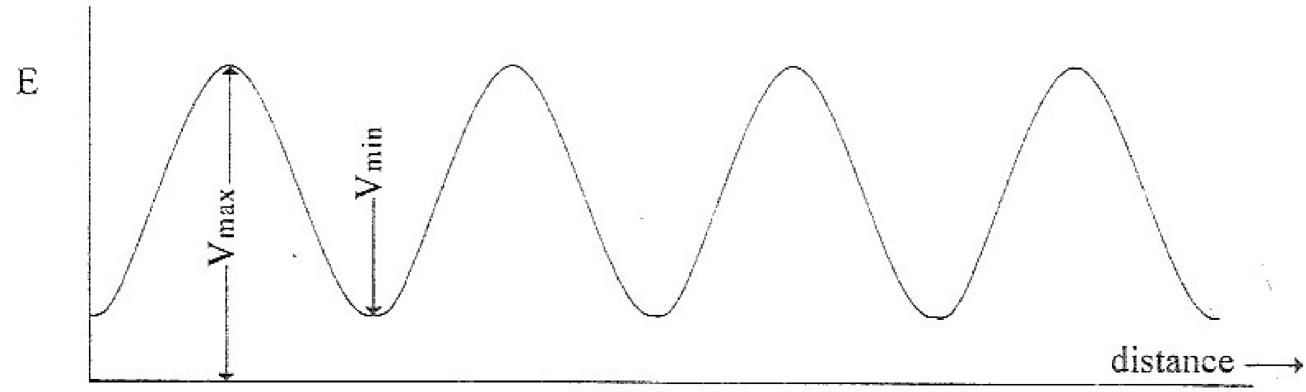

Fig. 2.1 (a) Standing waves on a dissipationless line terminated in a load not equal to R_0

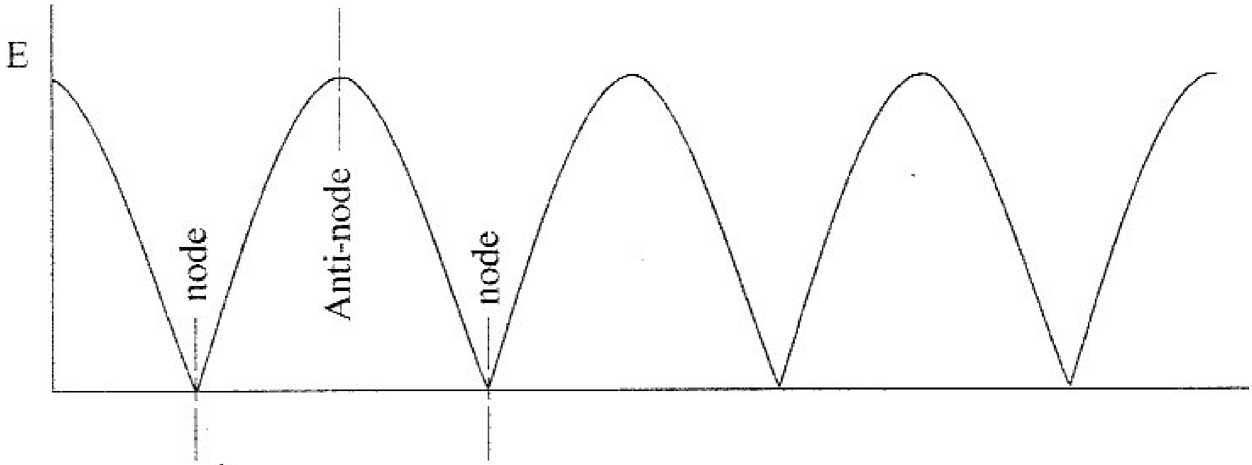

Fig. 2.1 (b) Standing waves on a line having open–or short–circuit termination.

The plot of current variation along the line is the same as that of voltage varriation except for a $\lambda/4$ shift in the position of maximum and minimum. The maximum and minimum values on a line are labeled as in Fig. 2.1(a), where as the nodes and antinodes are labeled in Fig.2.1(b). Nodes are points of zero voltage or current in the standing wave systems, antinodes or loops are points of maximum voltage or current. A line terminated in its characteristic impedance has no standing waves and thus no nodes. It is called a smooth line. For open circuit voltage nodes occur at distances of $\lambda/4$, $3\lambda/4$, $5\lambda/4$ and so on, from the open end. For short circuit voltage nodes occur at $0, \lambda/2$, λ and so on, whereas current nodes occur at $\lambda/4$, $3\lambda/4$, $5\lambda/4$ and so on. For a resistive load greater than R_0, the voltage and current minimum occur at voltage and current nodal points for an open circuited line. For a resistive load less than R_0, the voltage and current minima occur at the voltage and current nodal points for a short circuited line. The voltage and current distribution for open circuit and short circuit are shown in Fig.2.2. It also shows that the distribution for proper matching $R_R = R_0$.

The equations for voltage and current on a transmission line from the receiving end are given below

$$V = \frac{V_R (Z_R + Z_0)}{2Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_0}{Z_R + Z_0} \right) e^{-\gamma x} \right]$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_0} \left[e^{\gamma x} - \left(\frac{Z_R - Z_0}{Z_R + Z_0} \right) e^{-\gamma x} \right]$$

For zero dissipation line, the attenuation constant α is zero.

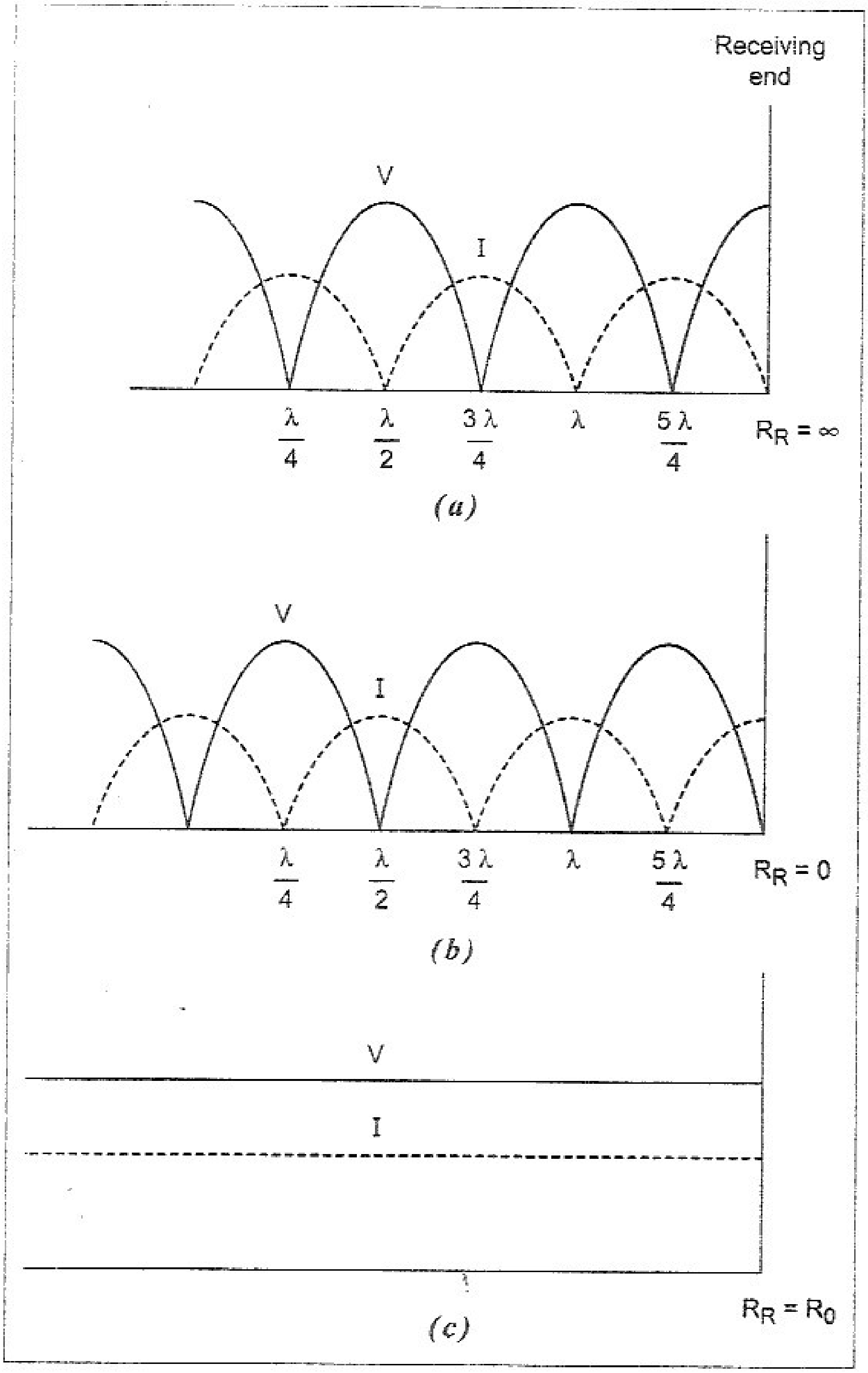

Fig 2.2 Voltage and currents on dissipationless line
(a) Open circuit (b) Short circuit (c) $R_R = R_0$

Since the reflection coefficient $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$, then

$$V = \frac{V_R\,(Z_R + Z_0)}{2Z_R}\,[e^{j\beta x} + K\,e^{-j\beta x}]$$

$$I = \frac{I_R\,(Z_R + Z_0)}{2Z_0}\,[e^{j\beta x} - K\,e^{-j\beta x}]$$

These two equations comprise of incident wave and reflected wave with definite maxima and minima along the line. The term involving $e^{j\beta x}$ is the incident wave whereas the term involving $e^{-j\beta x}$ is the reflected wave. The reflected wave depends upon the reflection coefficient.

2.3 STANDING WAVE RATIO

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio or voltage standing wave ratio (VSWR).

$$\boxed{S = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|}$$

Voltage equation is

$$V = \frac{V_R (Z_R + Z_0)}{2Z_R} [e^{j\beta x} + K\, e^{-j\beta x}]$$

Maxima of voltage along a line occurs at which the incident and reflected waves are in phase.

$$V_{max} = \frac{V_R (Z_R + Z_0)}{2Z_R} [1 + |K|]$$

Minima of voltage occurs at which the incident and reflected waves are out of phase.

$$V_{min} = \frac{V_R (Z_R + Z_0)}{2Z_R} [1 - |K|]$$

$$\frac{V_{max}}{V_{min}} = \frac{1 + |K|}{1 - |K|}$$

The standing wave ratio is defined in terms of the reflection coefficient.

$$S = \frac{1 + |K|}{1 - |K|}$$

$$\text{or } |K| = \frac{S - 1}{S + 1}$$

$$\text{or } |K| = \frac{\left|\frac{V_{max}}{V_{min}}\right| - 1}{\left|\frac{V_{max}}{V_{min}}\right| + 1} \qquad \left[\because S = \frac{V_{max}}{V_{min}}\right]$$

$$|K| = \frac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}$$

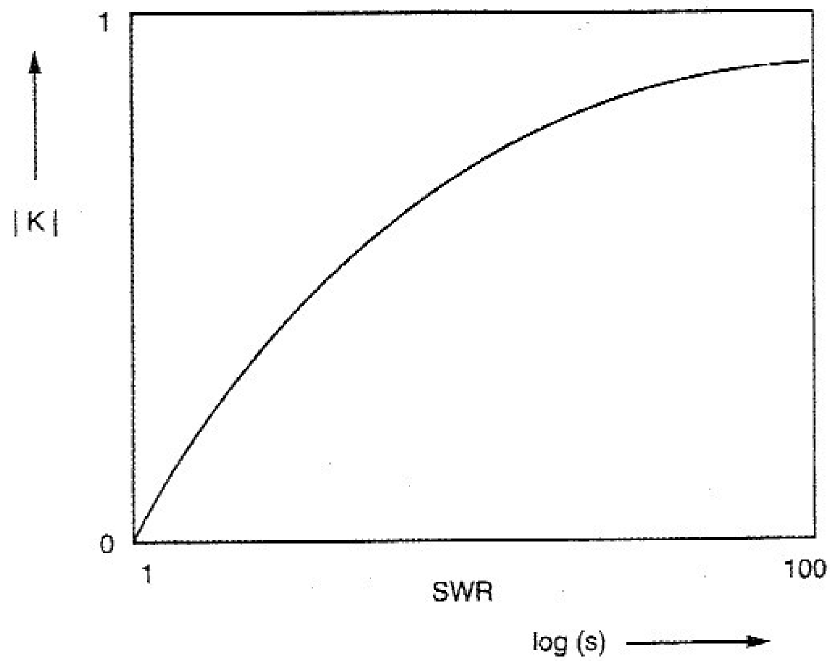

Fig 2.3 Relation between SWR(S) and reflection coefficient | K |

Fig.2.3 shows the relation between standing wave ratio S and reflection coefficient | K }.

2.4 ONE EIGHTH WAVELINE

For the transmission line the voltage and current at any point distant x from the receiving end of the transmission line is

$$V = \frac{V_R (Z_R + Z_0)}{2Z_R} (e^{\gamma x} + K e^{-\gamma x})$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_0} (e^{\gamma x} - K e^{-\gamma x})$$

The term with γx is identified as the incident wave progressing forward from the source to the load, where as the term involving $e^{-\gamma x}$ is the reflected wave travelling from load back towards the source.

For the line of zero dissipation, the attenuation constant α is zero.i.e $\gamma = j\beta$ and $Z_0 = R_0$.

$$V = \frac{V_R (Z_R + R_0)}{2Z_R} (e^{j\beta x} + Ke^{-j\beta x})$$

$$= \frac{V_R}{2Z_R} \left[Z_R \left(e^{j\beta x} + Ke^{-j\beta x}\right) + R_0 \left(e^{j\beta x} + Ke^{-j\beta x}\right)\right]$$

For standing wave $|K| = 1$

$$V = V_R \left[\frac{(e^{j\beta x} + e^{-j\beta x})}{2} + \frac{V_R R_0}{Z_R} \frac{(e^{j\beta x} - e^{-j\beta x})}{2} \right]$$

But $V_R = I_R Z_R$

$$V = V_R \left[\frac{(e^{j\beta x} + e^{-j\beta x})}{2} + jI_R R_0 \frac{(e^{j\beta x} - e^{-j\beta x})}{2j} \right]$$

$$= V_R \cos \beta x + jI_R R_0 \sin \beta x.$$

Similarly, for the current on the transmission line

$$I = I_R \cos \beta x + j \frac{V_R}{R_0} \sin \beta x$$

The input impedance of a dissipation line is

$$Z_s = \frac{V}{I}$$

$$= \frac{V_R \cos \beta x + jI_R R_0 \sin \beta x}{I_R \cos \beta x + j \frac{V_R}{R_0} \sin \beta x}$$

Since $V_R = I_R Z_R$

$$Z_s = \frac{I_R Z_R \cos \beta x + jI_R R_0 \sin \beta x}{I_R \cos \beta x + j \frac{I_R Z_R}{R_0} \sin \beta x}$$

$$= \frac{Z_R \cos \beta x + jR_0 \sin \beta x}{\cos \beta x + j \frac{Z_R}{R_0} \sin \beta x}$$

$$= R_0 \left[\frac{Z_R \cos \beta x + jR_0 \sin \beta x}{R_0 \cos \beta x + j\, Z_R \sin \beta x} \right]$$

or $$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan \beta x}{R_0 + j Z_R \tan \beta x} \right]$$

For an eighth wave line $x = \lambda/8$, $\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan (\pi/4)}{R_0 + j Z_R \tan(\pi/4)} \right]$$

$$\boxed{Z_s = R_0 \left[\frac{Z_R + jR_0}{R_0 + j Z_R} \right]}$$

If such a line is terminated with pure resistance R_R i.e $Z_R = R_R$

$$Z_s = R_0 \left[\frac{R_R + jR_0}{R_0 + j R_R} \right]$$

Since, both the numerator and denominator have identical magnitudes, then

$$\boxed{|Z_s| = R_0}$$

Thus an eighth – wave line may be used to transfer any resistance to an impedance with a magnitude equal to R_0 of the line, or to obtain a magnitude match between a resistance of any value and R_0, the internal resistance of the source.

2.5 QUARTER WAVELINE AND IMPEDANCE MATCHING

The input impedance of a dissipation transmission line is

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan \beta x}{R_0 + j Z_R \tan \beta x} \right]$$

$$Z_s = R_0 \left[\frac{\frac{Z_R}{\tan \beta x} + j R_0}{\frac{R_0}{\tan \beta x} + j Z_R} \right]$$

For a quarter wave line $x = \lambda/4$, $\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$$Z_S = R_0 \left[\frac{\dfrac{Z_R}{\tan \pi/2} + j\, R_0}{\dfrac{R_0}{\tan \pi/2} + j\, Z_R} \right]$$

$$= R_0 \left[\frac{j\, R_0}{j\, Z_R} \right]$$

$$\boxed{Z_S = \frac{R_0^2}{Z_R}}$$

A quarter wave section of line may be considered as a transformer to match a load of Z_R to a source of Z_S. Such a match can be obtained if the characteristic impedance R'_0 of the matching quarter wave section of the line is properly chosen

$$\text{i.e.,} \quad \boxed{R_0' = \left| \sqrt{Z_S\, Z_R} \right|}$$

The R_0' of the matching section should be equal to the geometric mean of the source and load impedance.

A quarter wave transformer may also be used if the load is not a pure resistance. It should then be connected between points corresponding to I_{max} or V_{min} at which places the transformation line has resistive impedances given by R_0/S or SR_0. For stepping down the impedance from the line value of R_0, the matching transformer characteristic impedance should be

$$R_0' = \sqrt{R_0 \frac{R_0}{S}}$$

$$= R_0 \sqrt{\frac{1}{S}}$$

The quarter wave transformer is also a single frequency or narrow band device. The bandwidth may be increased by using two or more quarter wave sections in series.

A quarter wave transformer may be considered as an impedance inverter in that it can transform a low impedance into a high impedance and vice versa. An important application of the quarter wave matching section is to couple a transmission line to a resistive load such as an antenna. This effect is illustrated by the action of $\lambda/4$ short circuited line in transforming the impedance as shown in Fig.2.4

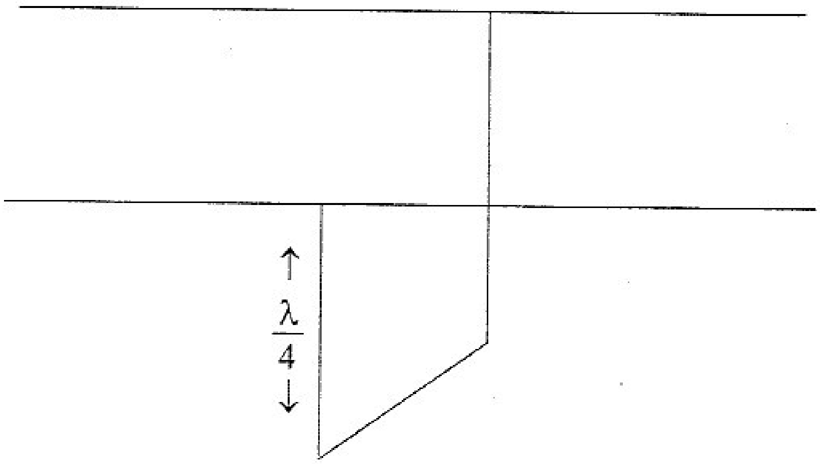

Fig.2.4 Quarter wave transformer

2.6 HALF–WAVE LINE

The input impedance of a dissipationless transmission line is

$$Z_s = \left[\frac{Z_R + jR_0 \tan \beta x}{R_0 + j Z_R \tan \beta x}\right]$$

For a half–wave line $x = \lambda/2$

$$\beta_x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z_s = R_0 \left[\frac{Z_R + j R_0 \tan \pi}{R_0 + j Z_R \tan \pi}\right]$$

$$= R_0 \frac{Z_R}{R_0}$$

$$Z_s = Z_R$$

A half wavelength of line may then be considered as one to one transformer. It has application in connecting a load to a source in cases when the load and source cannot be made adjacent.

2.7 STUB MATCHING

In general, the source or input impedance is a fixed one. By choosing the valve of load impedance to be equal to the input impedance, impedance matching is achieved. In certain cases (especially if the load is an antenna), the load impedance is also fixed. If the load impedance is not equal to the complex conjugate of the input impedance, the maximum power transfer will not take place. This is known as mismatching. So, it is necessary to introduce some form of an impedance transforming section between the source and load to achieve impedance matching. Such a section is called an impedance matching device. (e.g.. quarter wave transformer. Another means of accomplishing impedance matching is the use of an open or short circuited line of suitable length, called stub at a designated distance from the load. This is called stub matching. There are two types of stub matching. They are

(i) Single stub matching

(ii) Double stub matching.

2.7.1 Single Stub Matching

A transmission line having a characteristic admittance Y_0 terminated with load conductance Y_R (load resistance Z_R) is shown in Fig.2.5. Since Y_R is different from Y_S, standing waves are set up in between source and load.

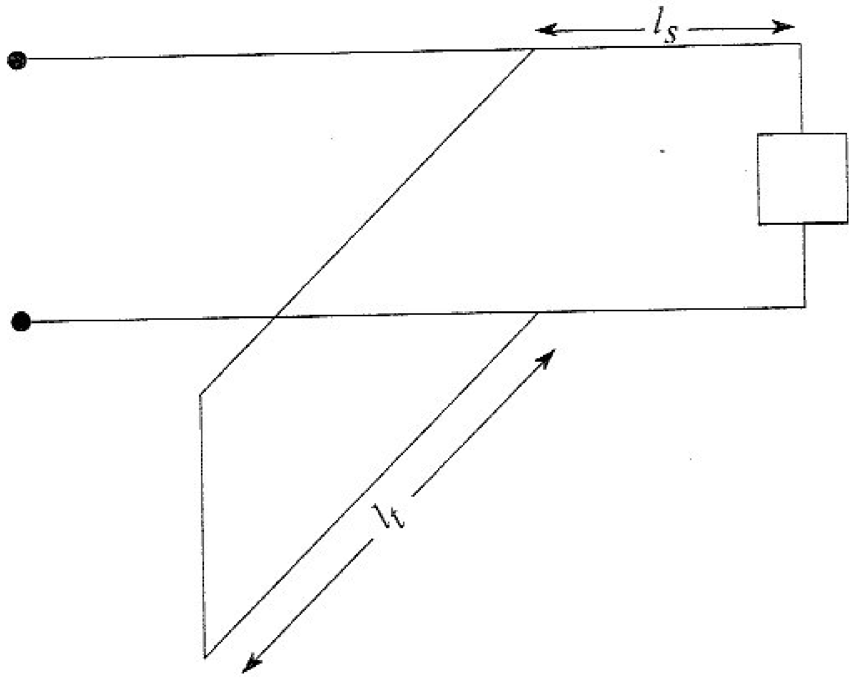

Fig.2.5 Single stub matching

The input admittance of a transmission line is changing from maximum to minimum conductance minimum to maximum conductance and so on and the cycle repeats for every $\lambda/2$. When the line is transversed from the point of maximum or minimum conductance to that of minimum or maximum conductance, there will be a point at which the real part of the admittance is equal to the characteristic admittance (i.e. $R = Z_S$). An appropriate length of a short circuited and/or open circuited line (stub) with the transmission line is shunted at this point to achieve the impedance matching. The input impedance of the transmission line at this point is $Z_S = R_0$. The line from the source to this point is then terminated with R_0 and is a smooth line. But there is a reflection between this point and load. Since the distance is less than the wavelength, the losses will be less.

The input impedance at any point of a transmission line is given by

$$Z_S = Z_0 \frac{Z_R + Z_0 \tan h\, \gamma l}{Z_0 + Z_R \tan h\, \gamma l}$$

The input admittance is

$$Y_S = Y_0 \frac{Y_R + Y_0 \tan h\, \gamma l}{Y_0 + Y_R \tan h\, \gamma l}$$

For propagation $\gamma = j\beta \quad (\alpha = 0)$

$$Y_S = Y_0 \frac{Y_R + jY_0 \tan \beta l}{Y_0 + jY_R \tan \beta l}$$

For normalization, the above expression is divided by Y_0

$$\frac{Y_S}{Y_0} = \frac{Y_R + jY_0 \tan \beta l}{Y_0 + jY_R \tan \beta l}$$

$$Y_{in} = \frac{\frac{Y_R}{Y_0} + j \tan \beta l}{1 + j\frac{Y_R}{Y_0} \tan \beta l}$$

where

$$\frac{Y_S}{Y_0} = Y_{in} \text{ normalised input admittance}$$

$$\frac{Y_R}{Y_0} = Y_r \text{ normalised load admittance}$$

$$Y_{in} = \frac{Y_r + j \tan \beta l}{1 + j\, Y_r \tan \beta l}$$

This can be written as

$$= \frac{Y_r + j \tan \beta l}{1 + j\, Y_r \tan \beta l} \; \frac{(1 - j\, Y_r \tan \beta l)}{(1 - j\, Y_r \tan \beta l)}$$

$$= \frac{Y_r \;(1 + \tan^2 \beta l) + j\,(1 - Y_r^2) \tan \beta l}{1 + Y_r^2 \tan^2 \beta l}$$

For perfect matching

$$Y_S = Y_0$$

$$\frac{Y_S}{Y_0} = 1$$

$$\therefore \quad Y_{in} = 1$$

The stub has to be located at a point where the real part of Y_{in} is equal to unity.

$$\therefore \quad \frac{Y_r\ (1+\tan^2 \beta l_s)}{1+Y_r^2 \tan^2 \beta l_s} = 1$$

$$Y_r + Y_r^2 \tan^2 \beta l_s = 1 + Y_r^2 \tan^2 \beta l_s$$

$$Y_r \tan^2 \beta l_s - Y_r^2 \tan^2 \beta l_s = 1 - Y_r$$

$$\tan^2 \beta l_s\, (Y_r - Y_r^2) = 1 - Y_r$$

$$Y_r (1 - Y_r) \tan^2 \beta l_s = 1 - Y_r$$

$$Y_r \tan^2 \beta l_s = 1$$

$$\tan^2 \beta l_s = \frac{1}{Y_r}$$

$$\tan \beta l_s = \frac{1}{\sqrt{Y_r}} \qquad \left(\because Y_r = \frac{Y_R}{Y_0}\right)$$

$$= \sqrt{\frac{Y_0}{Y_R}}$$

$$\beta l_s = \tan^{-1} \sqrt{\frac{Y_0}{Y_R}}$$

$$\frac{2\pi}{\lambda} l_s = \tan^{-1} \sqrt{\frac{Y_0}{Y_R}}$$

The location of the stub l_s is given by

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Z_R}{Z_0}}} \qquad \left[\because Z_R = \frac{1}{Y_R},\quad Z_0 = \frac{1}{Z_0}\right]$$

The succeptance at the location of the stub is

$$\frac{S_S}{Y_0} = \frac{(1-Y_r^2)\tan\beta l_s}{1-Y_r^2\tan^2\beta l_s}$$

$$= \frac{(1-Y_r^2)\sqrt{\dfrac{Y_0}{Y_R}}}{1+Y_r^2\dfrac{Y_0}{Y_R}}$$

$$= \frac{\left(1-\dfrac{Y_R^2}{Y_0^2}\right)\sqrt{\dfrac{Y_0}{Y_R}}}{1+\dfrac{Y_R^2}{Y_0^2}\;\dfrac{Y_0}{Y_R}}$$

$$= \frac{\left(1-\dfrac{Y_R^2}{Y_0^2}\right)\sqrt{\dfrac{Y_0}{Y_R}}}{1+\dfrac{Y_R}{Y_0}}$$

$$= \left(1-\frac{Y_R}{Y_0}\right)\sqrt{\frac{Y_0}{Y_R}}$$

$$= \frac{Y_0-Y_R}{Y_0}\sqrt{\frac{Y_0}{Y_R}}$$

The succeptance of the stub is

$$S_s = (Y_0-Y_R)\sqrt{\frac{Y_0}{Y_R}}$$

This can be obtained either by an open circuited or short circuited stub. But normally short circuited stub is preferred because of the following advantages.

(i) it radiates less power

(ii) its effective length may be varied by means of a shorting bar

The succeptance of a short circuited stub is equated to $Y_0 \cot \beta l_t$

$$(Y_0 - Y_R)\sqrt{\frac{Y_0}{Y_R}} = Y_0 \cot \beta l_t$$

$$\frac{Y_0 - Y_R}{Y_0}\sqrt{\frac{Y_0}{Y_R}} = \cot \beta l_t$$

$$\cot \beta l_t = Y_0 - Y_R \frac{1}{\sqrt{Y_0 Y_R}}$$

$$= \frac{Z_R - Z_0}{Z_R Z_0} \sqrt{Z_0 Z_R}$$

$$= \frac{Z_R - Z_0}{\sqrt{Z_R Z_0}}$$

$$\tan \beta l_t = \frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}$$

$$\beta l_t = \tan^{-1} \frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}$$

The length of the stub is given by

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \left[\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0} \right]$$

2.7.2 Location and length of the stub using reflection coefficient

The input impedance of the line is given by

$$Z_i = Z_0 \frac{1 + Ke^{-2\gamma l}}{1 - Ke^{-2\gamma l}}$$

For loss less line $\alpha = 0, \gamma = j\beta$

and $K = |K| e^{j\phi}$

where ϕ is the angle of reflection coefficient.

$$Z_i = Z_0 \frac{1 + |K| e^{j\phi} e^{-j2\beta l}}{1 - |K| e^{j(\phi - 2\beta l)}}$$

$$= Z_0 \frac{1 + |K| e^{j(\phi - 2\beta l)}}{1 - |K| e^{j\phi} e^{-j2\beta l}}$$

The input admittance is given by

$$Y_i = G_0 \frac{1 - |K| e^{j(\phi - 2\beta l)}}{1 + |K| e^{j(\phi - 2\beta l)}}$$

where the characteristic conductance is

$$G_0 = \frac{1}{Z_0} = \frac{1}{R_0} \qquad [\because Z_0 \text{ is resistive}]$$

$$Y_i = G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}$$

$$= G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j |K| \sin(\phi - 2\beta l)]}$$

Multiplying the numerator and denominator by

$$1 + |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)]$$

$$Y_i = G_0 \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Since $Y_i = G_i + jS_i$, then

$$\frac{Y_i}{G_0} = \frac{G_i}{G_0} + j\frac{S_i}{G_0} = \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the real parts

$$\frac{G_i}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the imaginary parts

$$\frac{S_i}{G_0} = \frac{-2 |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

At the location of stub $Z_i = Z_0$ for matching.

Since there is no reflection, $l = l_s$.

$$\therefore G_i = G_0$$

$$\frac{G_i}{G_0} = 1$$

$$\frac{1 - |K|^2}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l_s)} = 1$$

$$1-|K|^2 = 1+|K|^2+2|K|\cos(\phi-2\beta l_s)$$

$$2|K|\cos(\phi-2\beta l_s) = -2\ |K|^2$$

$$\cos(\phi-2\beta l_s) = -|K|$$

$$\phi-2\beta l_s = \cos^{-1}(-|K|)$$

$$\text{But } \cos^{-1}(-|K|) = -\pi+\cos^{-1}|K|$$

$$\therefore \quad \phi-2\beta l_s = -\pi+\cos^{-1}|K|$$

$$2\beta l_s = \phi+\pi-\cos^{-1}|K|$$

$$\boxed{l_s = \frac{\phi+\pi-\cos^{-1}|K|}{2\beta}}$$

$$\text{or} \quad \boxed{l_s = \frac{\lambda}{4\pi}[\phi+\pi-\cos^{-1}|K|]} \qquad (\because \beta = \frac{2\pi}{\lambda})$$

The normalised susceptance (imaginary part) equation is

$$\frac{S_i}{G_0} = \frac{-2|K|\sin(\phi-2\beta l)}{1+|K|^2+2|K|\cos(\phi-2\beta l)}$$

$$\text{But } \phi-2\beta l_s = -\pi+\cos^{-1}|K| \text{ and } \cos(\phi-2\beta l_s) = -|K|$$

$$\therefore \quad \frac{S_i}{G_0} = \frac{-2|K|\sin(-\pi+\cos^{-1}|K|)}{1+|K|^2+2|K|(-|K|)}$$

$$= \frac{2|K|\sin(\cos^{-1}|K|)}{1+|K|^2-2|K|^2}$$

Let $\cos^{-1}|K| = \theta$, then $|K| = \cos\theta$ and

$$\sin(\cos^{-1}|K|) = \sin\theta$$

$$= \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-|K|^2}$$

$$\therefore \quad \frac{S_i}{G_0} = \frac{2|K|\sqrt{1-|K|^2}}{1-|K|^2}$$

$$S_i = G_0 \frac{2|K|}{\sqrt{1-|K|^2}}$$

The susceptance of the stub is $G_0 \cot \beta l_s$

$$G_0 \cot \beta l_s = G_0 \frac{2\,|K|}{\sqrt{1-|K|^2}}$$

$$\frac{1}{\tan \beta l_s} = \frac{2\,|K|}{\sqrt{1-|K|^2}}$$

$$\tan \beta l_s = \frac{\sqrt{1-|K|^2}}{2\,|K|}$$

$$\beta\, l_s = \tan^{-1} \frac{\sqrt{1-|K|^2}}{2\,|K|}$$

$$l_s = \frac{1}{\beta} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2\,|K|}$$

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2\,|K|}}$$

The location of the stub 'l_s' and length of the stub 'l_t' can be determined, if the reflection coefficient and frequency are known.

A short circuited stub is normally preferred to an open circuited stub because of its simpler construction and the inability of the stub to remain open circuited. The short circuit can be easily established with a large metal plate and it also has a lower radiation loss of energy.

However the single stub matching has the following drawbacks.

i. Single stub matching is applicable for single frequency. For variable frequency the location of the stub is not fixed (i.e. changing)

ii. For final adjustment the stub has to be moved along the line slightly. So it is possible only in open wire lines.

To avoid the disadvantages of single matching, double stub matching is introduced. Double stub matching is one in which two short circuited stubs, whose lengths are adjustable independently are fixed as shown in Fig.2.6.

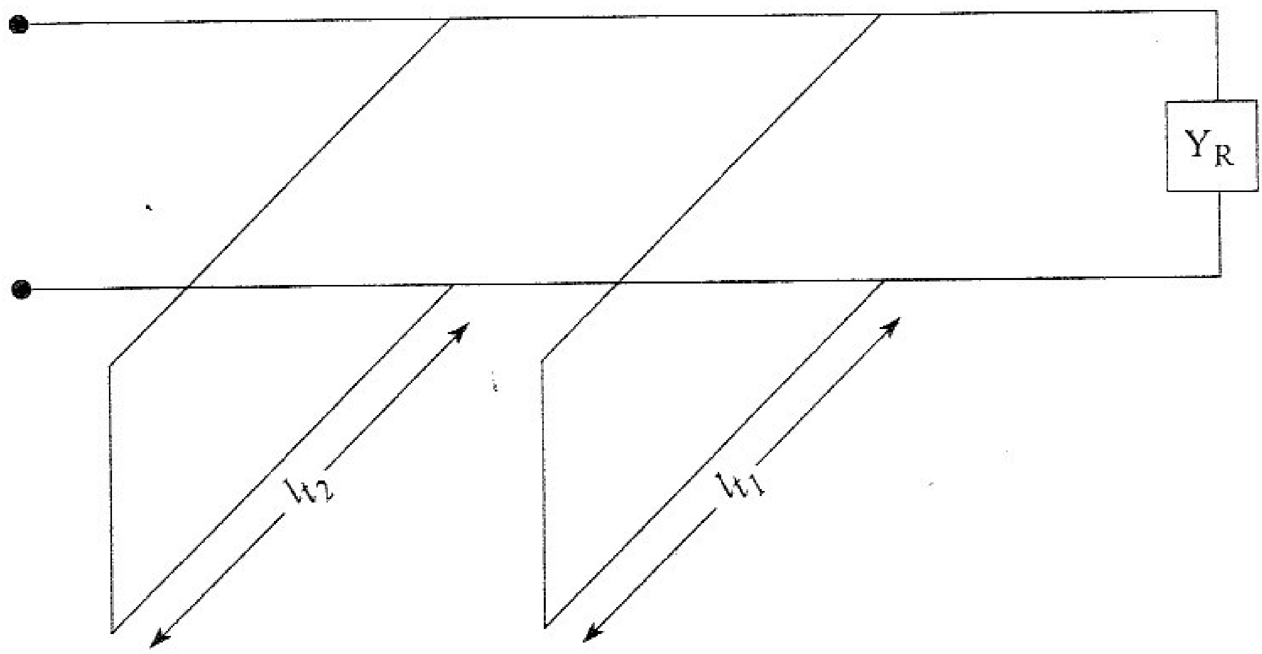

Fig 2.6 Double stub matching

2.8 CIRCLE DIAGRAM

The input impedance for the transmission line is given by

$$Z_S = \frac{V}{I} = \frac{\frac{V_R (Z_R + Z_0)}{2Z_R} [e^{\gamma x} + Ke^{-\gamma x}]}{\frac{I_R (Z_R + Z_0)}{2Z_0} [e^{\gamma x} - Ke^{-\gamma x}]}$$

$$= \frac{V_R Z_0 [e^{\gamma x} + Ke^{-\gamma x}]}{I_R Z_R [e^{\gamma x} - Ke^{-\gamma x}]}$$

$$= \frac{V_R Z_0 e^{\gamma x} [1 + Ke^{-2\gamma x}]}{V_R e^{\gamma x} [1 - Ke^{-2\gamma x}]} \qquad [\because I_R Z_R = V_R]$$

$$= Z_0 \frac{[1 + Ke^{-2\gamma x}]}{[1 - Ke^{-2\gamma x}]}$$

The input impedance of the transmission line is given by

$$Z_S = Z_0 \frac{1 + Ke^{-2\gamma x}}{1 - Ke^{-2\gamma x}}$$

For a loss less line $\gamma = j\beta$ ($\therefore \alpha = 0$)

The normalised input impedance is obtained by dividing Z_S by its characteristic impedance Z_0.

$$Z_{in} = \frac{Z_S}{Z_0}$$

$$= \frac{1 + Ke^{-j2\beta x}}{1 - Ke^{-j2\beta x}}$$

$$Z_{in}(1 - Ke^{-j2\beta x}) = 1 + Ke^{-j2\beta x}$$

$$Z_{in} - 1 = Ke^{-j2\beta x}(1 + Z_{in})$$

$$Ke^{-j2\beta x} = \frac{Z_{in} - 1}{Z_{in} + 1}$$

But Z_{in} is a complex quantity. It can be represented by

$$Z_{in} = R + jX$$

where R is the resistance, X is the reactance

$$Ke^{-j2\beta x} = \frac{R + jX - 1}{R + jX + 1}$$

$$= \frac{(R-1) + jX}{(R+1) + jX}$$

The above equation leads to two sets of circles. They are S circles and βx circle. S circles can be obtained by equating the magnitude and βx circles by equating the tangents of the angles.

$$Ke^{j2\beta x} = \left(\frac{(R-1) + jX}{(R+1) + jX}\right)\left(\frac{(R+1) - jX}{(R+1) - jX}\right)$$

$$= \frac{R^2 - 1 + jX(R+1) - jX(R-1) + X^2}{(R+1)^2 + X^2}$$

$$= \frac{R^2 - 1 + X^2}{(R+1)^2 + X^2} + j\frac{2X}{(R+1)^2 + X^2}$$

By converting rectangular co–ordinates in to polar co–ordinates.

$$Ke^{j2\beta x} = \sqrt{\frac{(R-1)^2+X^2}{(R+1)^2+X^2}} \quad \tan^{-1}\left[\frac{\dfrac{2X}{(R+1)^2+X^2}}{\dfrac{R^2-1+X^2}{(R+1)^2+X^2}}\right]$$

Constant S circles are obtained by equating the magnitude

$$K^2 = \frac{(R-1)^2+X^2}{(R+1)^2+X^2}$$

$$K^2(R+1)^2 + K^2X^2 = (R-1)^2 + X^2$$

$$K^2(R^2+2R+1) + K^2X^2 = R^2 + 1 - 2R + X^2$$

$$K^2(R^2+X^2+2R+1) = R^2 + X^2 - 2R + 1$$

$$R^2(K^2-1) + X^2(K^2-1) + 2R(K^2+1) + K^2-1 = 0$$

Divide by K^2-1

$$R^2 + X^2 + 2R\left(\frac{K^2+1}{K^2-1}\right) + 1 = 0$$

The reflection coefficient can be written in terms of the standing wave ratio

$$|K| = \frac{S-1}{S+1}$$

$$\frac{K^2+1}{K^2-1} = \frac{\left(\frac{S-1}{S+1}\right)^2+1}{\left(\frac{S-1}{S+1}\right)^2-1}$$

$$= \frac{(S-1)^2+(S+1)^2}{(S-1)^2-(S+1)^2}$$

$$= \frac{S^2-2S+1+S^2+2S+1}{S^2-2S+1-S^2-2S-1}$$

$$= \frac{2(S^2+1)}{-4S}$$

$$\frac{K^2+1}{K^2-1} = -\frac{(S^2+1)}{2S}$$

Substituting this value in the main equation

$$R^2 + X^2 - 2R\,\frac{(S^2+1)}{2S} + 1 = 0$$

Add $\left(\frac{S^2+1}{2S}\right)^2$ on both sides

$$R^2 - 2R\left(\frac{S^2+1}{2S}\right) + \left(\frac{S^2+1}{2S}\right)^2 + X^2 = -1 + \left(\frac{S^2+1}{2S}\right)^2$$

$$\left[R - \frac{S^2+1}{2S}\right]^2 + X^2 = \frac{-4S^2 + S^4 + 2S^2 + 1}{4S^2}$$

$$= \frac{S^4 - 2S^2 + 1}{4S^2}$$

$$= \left(\frac{S^2-1}{2S}\right)^2$$

$$\boxed{\left[R - \frac{S^2+1}{2S}\right]^2 + X^2 = \left(\frac{S^2-1}{2S}\right)^2}$$

This is the equation of the S circles whose radius is $\frac{S^2-1}{2S} = \frac{S - \frac{1}{S}}{2}$

and

centre is $\frac{S^2+1}{2S} = \frac{S + \frac{1}{S}}{2}$

A family of circles may be drawn for successive values of S as in Fig.2.7. Since the minimum value for S is unity, all the S circles must surrounded the (1, 0) point. The maximum value of S is infinity for the case of open circuit or short circuit line termination. As S increases, the radius of the S circle increases, and the centre moves to the right and the circle becomes the r axis for $S = \infty$. The interrupts of the circle on the r axis is $^1/_S$ when the circle is nearer to the origin and is S when the circle is far away from the origin.

The constant S circle represents all possible values of r and x for a given value of $\frac{Z_S}{R_0}$. The line from the origin tc a given point on the circle represents $\frac{Z_S}{R_0}$ in both magnitude and angle.

When $\frac{Z_S}{R_0}$ lies on the abscissa with magnitude S, the line impedance has a maximum value

$$\frac{Z_S}{R_0} = S$$

$$= \frac{1 + |K|}{1 - |K|}$$

Thus the point (0, S) X = 0, R = S on the S circle represents the resistive line impedance at a voltage maximum.

When $\frac{Z_S}{R_0}$ terminate at the circle intercept $\frac{1}{S}$, the line impedance has a minimum value

$$\frac{Z_S}{R_0} = \frac{1}{S}$$

$$= \frac{1 - |K|}{1 + |K|}$$

The constant βx circles are obtained by equating it to the tangent of angle.

$$-2\beta x = \tan^{-1}\left[\frac{\frac{2R}{(R+1)^2 + R^2}}{\frac{R^2-1+X^2}{(R+1)^2 + X^2}}\right]$$

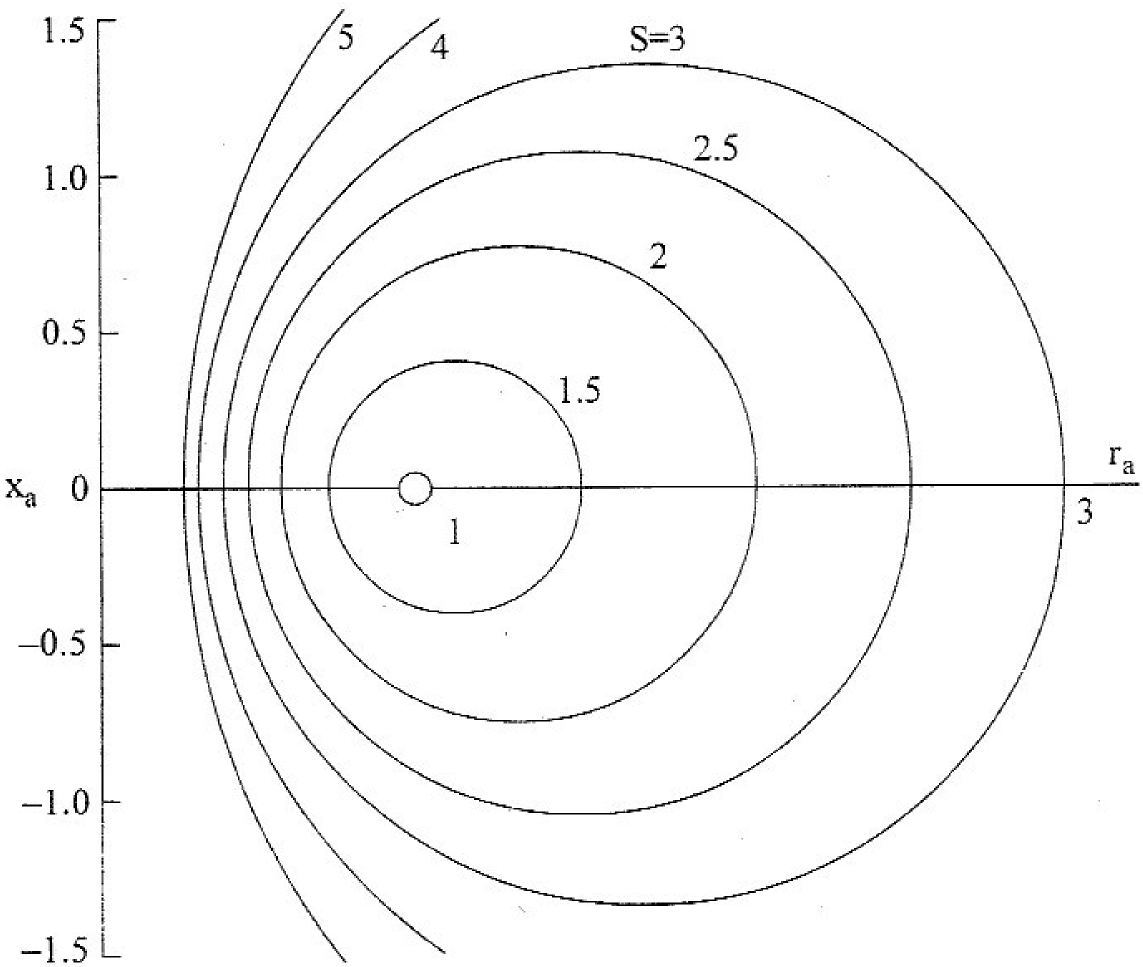

Fig 2.7 A family of constant–S circles

Taking tangent on both sides

$$\tan(-2\beta x) = \left[\frac{2X}{R^2 - 1 + X^2}\right]$$

$$-\tan(2\beta x) = \frac{2X}{R^2 - 1 + X^2}$$

$$R^2 + X^2 - 1 = -\frac{2X}{\tan(2\beta x)}$$

$$R^2 + X^2 - 1 + \frac{2X}{\tan(2\beta x)} = 0$$

Adding $\frac{1}{\tan^2 2\beta x}$ on both sides

$$R^2 + X^2 + \frac{1}{\tan^2 2\beta x} + \frac{2x}{\tan(2\beta x)} = 1 + \frac{1}{\tan^2 2\beta x}$$

$$R^2 + \left(X + \frac{1}{\tan^2 2\beta l}\right)^2 = 1 + \frac{1}{\tan^2 2\beta x}$$

But $1 + \frac{1}{\tan^2 2\beta x} = \frac{1}{\sin^2 2\beta x}$

$$R^2 + \left(X + \frac{1}{\tan 2\beta x}\right)^2 = \frac{1}{\sin^2 \beta x}$$

This is the equation of βx circle whose radius is $\frac{1}{\sin 2\beta x}$ and the centre is $\frac{1}{\tan 2\beta x}$

A family of such circles is drawn for successive values of βx as shown in Fig.2.8. All these circles pass through the point (1, 0). The superposition of βx circles on the S circles provides a scale of βx angles and results in the circle diagram as shown in Fig.2.9. The circle diagram may be used to find the input impedance of a line of any chosen length.

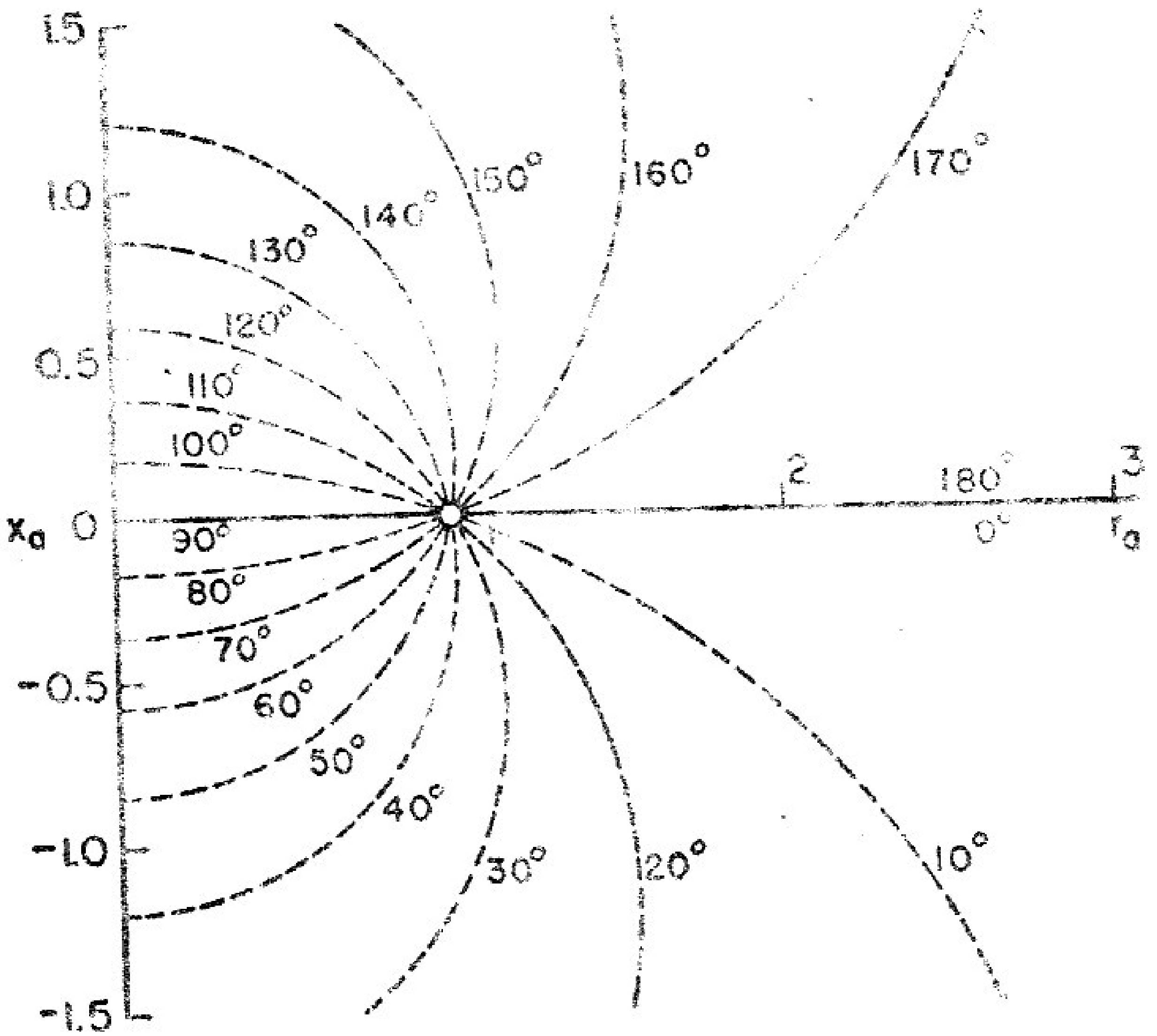

Fig 2.8 A family of constant $-\beta x$ circles

Though the circle diagram is very useful in calculating the line impedance and admittance it has the following drawbacks.

- S and βx circles are not concentric, making interpolation difficult.
- Only a limited range of impedance values can be accomodated in a chart of resonable size.

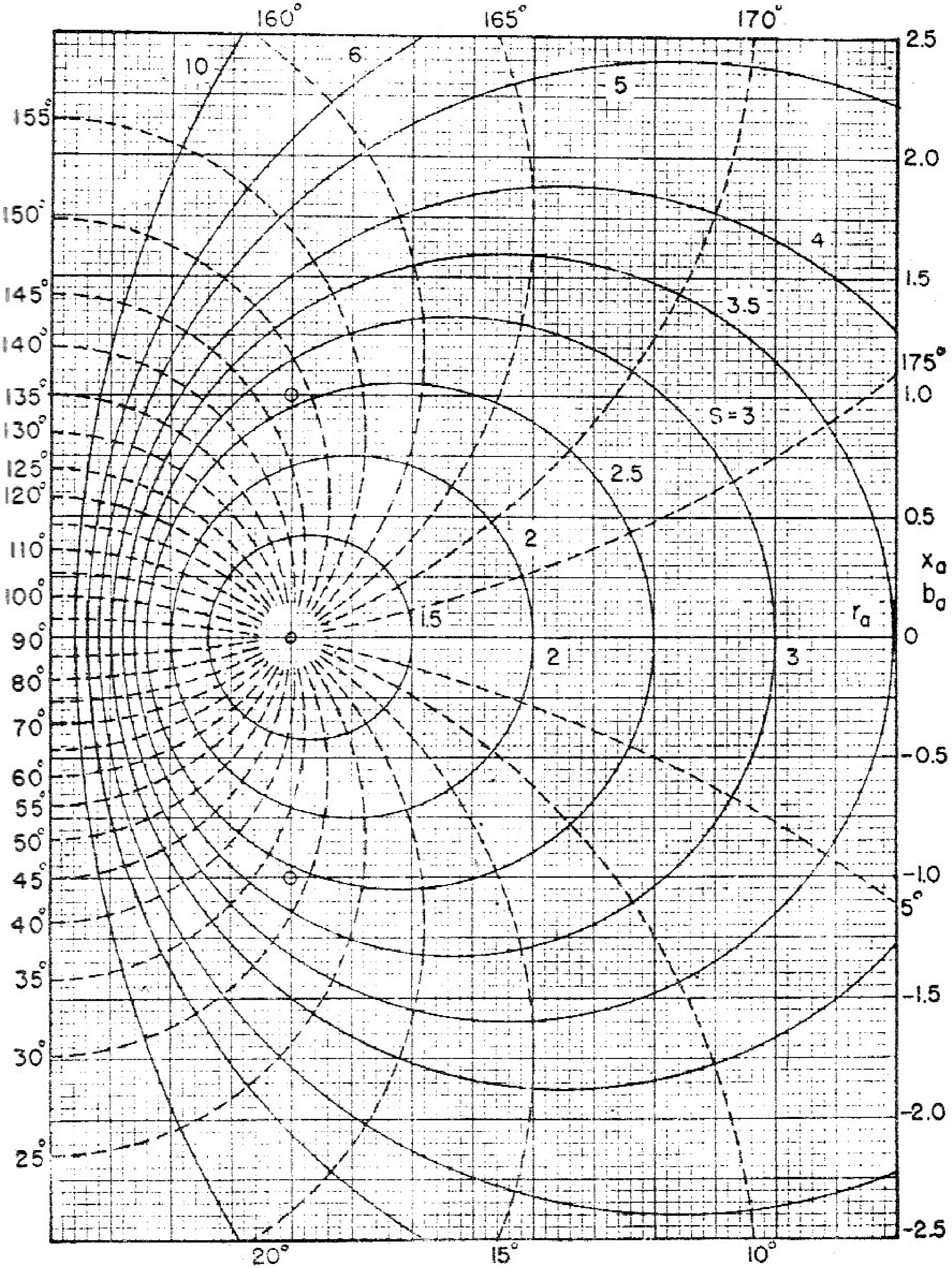

Fig 2.9 The transmission–line circle diagram

2.9 SMITH CHART

A modified form of the circle diagram for the dissipationless transmission line has been developed by Philip.H.Smith at Bell Laboratories. The various properties of the transmission lines may be represented graphically on a number of calculator charts. The most widely used such a calculator is the Smith Chart.

As defined by McGraw Hill dictionary of Electrical and Electronics Engineering.

"Smith Chart is a special polar diagram containing constant resistance circles, constant reactance circles, circles of constant standing wave ratio and radius lines representing line–angle loci; used in solving transmission line and waveguide problems".

The basic difference between circle diagram and Smith Chart is that in the circle diagram the impedance is represented in a rectangular form while in the Smith Chart the impedance is represented in a circular form.

Smith Chart is based on two sets of orthogonal circles. The tangents drawn at the points of intersection of two circles would be mutually perpendicular one set of circles represent the rate of the resistive component (R) of the line impedance to the characteristic impedance (Z_0) of the line, which for a lossless line is purely resistive. The second set of circles represents the ratio of the reactive component (X) of the line impedance to the characteristic impedance (Z_0) of the line.

The Smith Chart is obtained as follows.

To display the impedance of all possible passive networks the graph must extend in all three possible directions (R, +jX, –jX). The Smith Chart is commited to a bilinear transformation by plotting the complex reflection coefficient.

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

By normalizing the load impedance $z = \frac{Z_R}{Z_0}$

$$K = \frac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1}$$

$$= \frac{z-1}{z+1}$$

or $$(z+1)K = z-1$$

$$zK + K = z-1$$

$$1 + K = z(1-K)$$

$$z = \frac{1+K}{1-K}$$

Since the complex quantity $z = R + jX$ and

the complex quantity $K = K_R + jK_X$

$$z = \frac{1+K}{1-K} \text{ becomes}$$

$$R + jX = \frac{1 + K_R + jK_X}{1 - K_R - jK_X}$$

$$= \frac{(1 + K_R) + jK_X}{(1 - K_R) - jK_X}$$

$$= \frac{[(1 + K_R) + jK_X]\,[(1 - K_R) + jK_X]}{[(1 - K_R) - jK_X]\,[(1 - K_R) + jK_X]}$$

$$= \frac{1 + K_R + jK_X - K_R - K_R^2 - jK_RK_X + jK_X + jK_RK_X - K_X^2}{(1 - K_R)^2 + K_X^2}$$

$$R + jX = \frac{1 - K_R^2 - K_X^2 + 2jK_X}{(1 - K_R)^2 + K_X^2}$$

Equating the real parts on both sides

$$R = \frac{1 - K_R^2 - K_X^2}{(1 - K_R)^2 + K_X^2}$$

Equating the imaginary parts on both sides

$$X = \frac{2K_X}{(1 - K_R)^2 + K_X^2}$$

The real parts

$$R = \frac{1 - K_R^2 - K_X^2}{(1 - K_R)^2 + K_X^2}$$

$$R(1-K_R)^2 + RK_X^2 = 1 - K_R^2 - K_X^2$$

$$R(1-2K_R + K_R^2) + RK_X^2 = 1 - K_R^2 - K_X^2$$

$$R - 2RK_R + RK_R^2 + RK_X^2 = 1 - K_R^2 - K_X^2$$

(or) $$-2RK_R + RK_R^2 + RK_X^2 + K_R^2 + K_X^2 = 1 - R$$

$$K_R^2 + RK_R^2 + K_X^2 + RK_X^2 - 2RK_R = 1 - R$$

$$K_R^2(1+R) + K_X^2(1+R) - 2RK_R = 1 - R$$

$$K_R^2 + K_X^2 - \frac{2RK_R}{1+R} = \frac{1-R}{1+R}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 - \frac{R^2}{(1+R)^2} + K_X^2 = \frac{1-R}{1+R}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{1-R}{1+R} + \frac{R^2}{(1+R)^2}$$

$$= \frac{(1-R)(1+R) + R^2}{(1+R)^2}$$

$$= \frac{1 - R^2 + R^2}{(1+R)^2}$$

$$\left(K_R - \frac{R}{1+R}\right)^2 + K_X^2 = \frac{1}{(1+R)^2}$$

This equation represents a family of constant R circles having centres on the R axis at, $\left[\frac{R}{R+1}, 0\right]$ and radius of $\frac{1}{R+1}$. This is shown in Fig.2.10.

These circles have their centres on the positive K_R axis with the values of 0 to 1. A circle which corresponds to R = 0, with centre (0,0) and radius of 1 forms the periphery of the Smith Chart. All constant R circles touch the point (1,0).

Circles of Constant – R

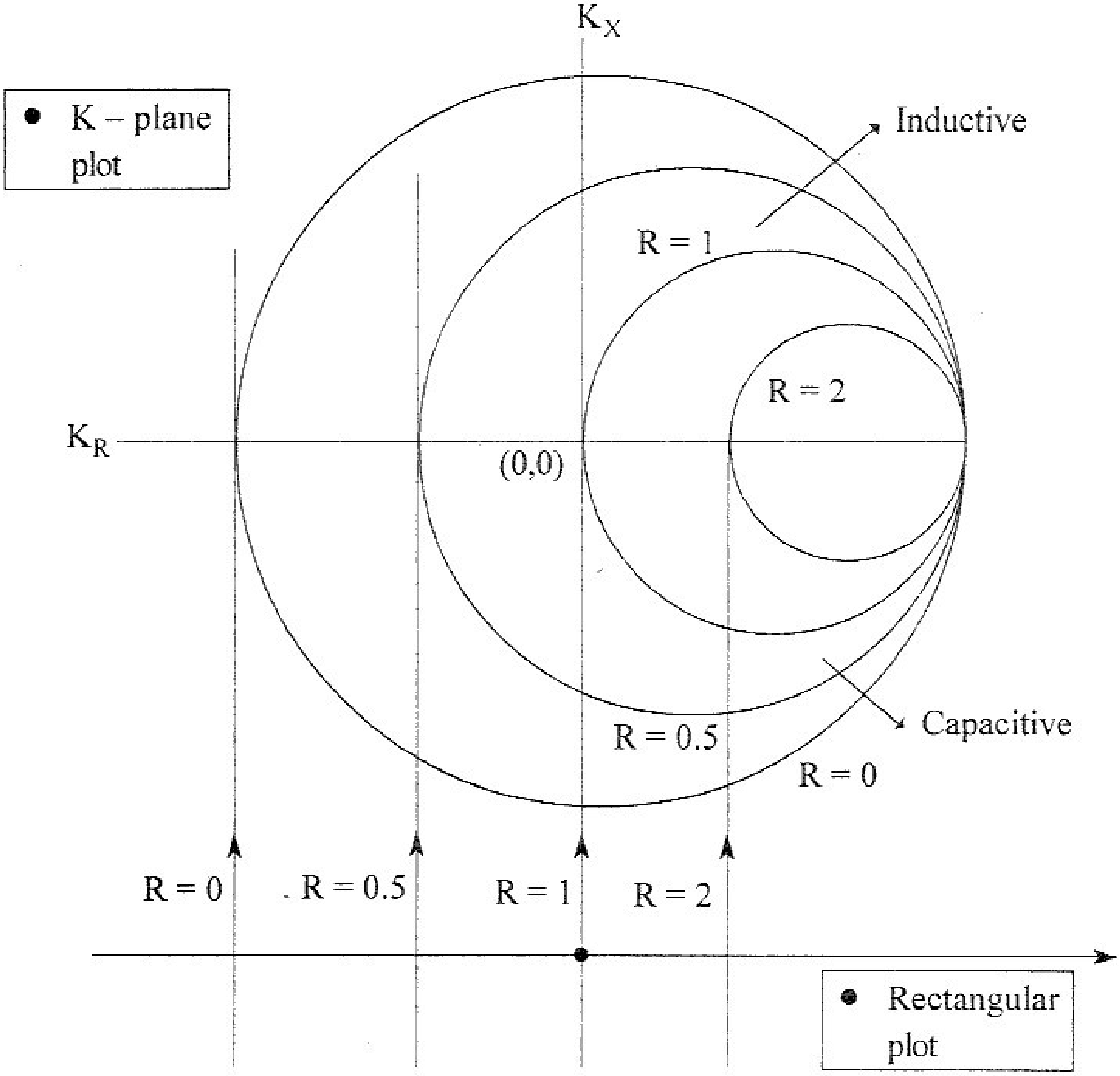

Fig. 2.10 Family of constant R circles

The imaginary parts

$$X = \frac{2K_X}{(1 - K_R)^2 + K_X^2}$$

$$X[1 + K_R^2 - 2K_R + K_X^2] = 2\,K_X$$

Dividing by X

$$1 + K_R^2 - 2K_R + K_X^2 - \frac{2K_X}{X} = 0$$

$$(K_R - 1)^2 + K_X^2 - \frac{2K_X}{X} = 0$$

Adding $\frac{1}{X^2}$ on both sides

$$(K_R - 1)^2 + K_X^2 - \frac{2K_X}{X} + \frac{1}{X^2} = \frac{1}{X^2}$$

$$(K_R - 1)^2 + \left[K_X - \frac{1}{X}\right]^2 = \frac{1}{X^2}$$

This equation represents a family of constant X circles having centres at $(1, \frac{1}{X})$ and radii of $\frac{1}{X}$. This is shown in Fig.2.11

Circles of Constant – X

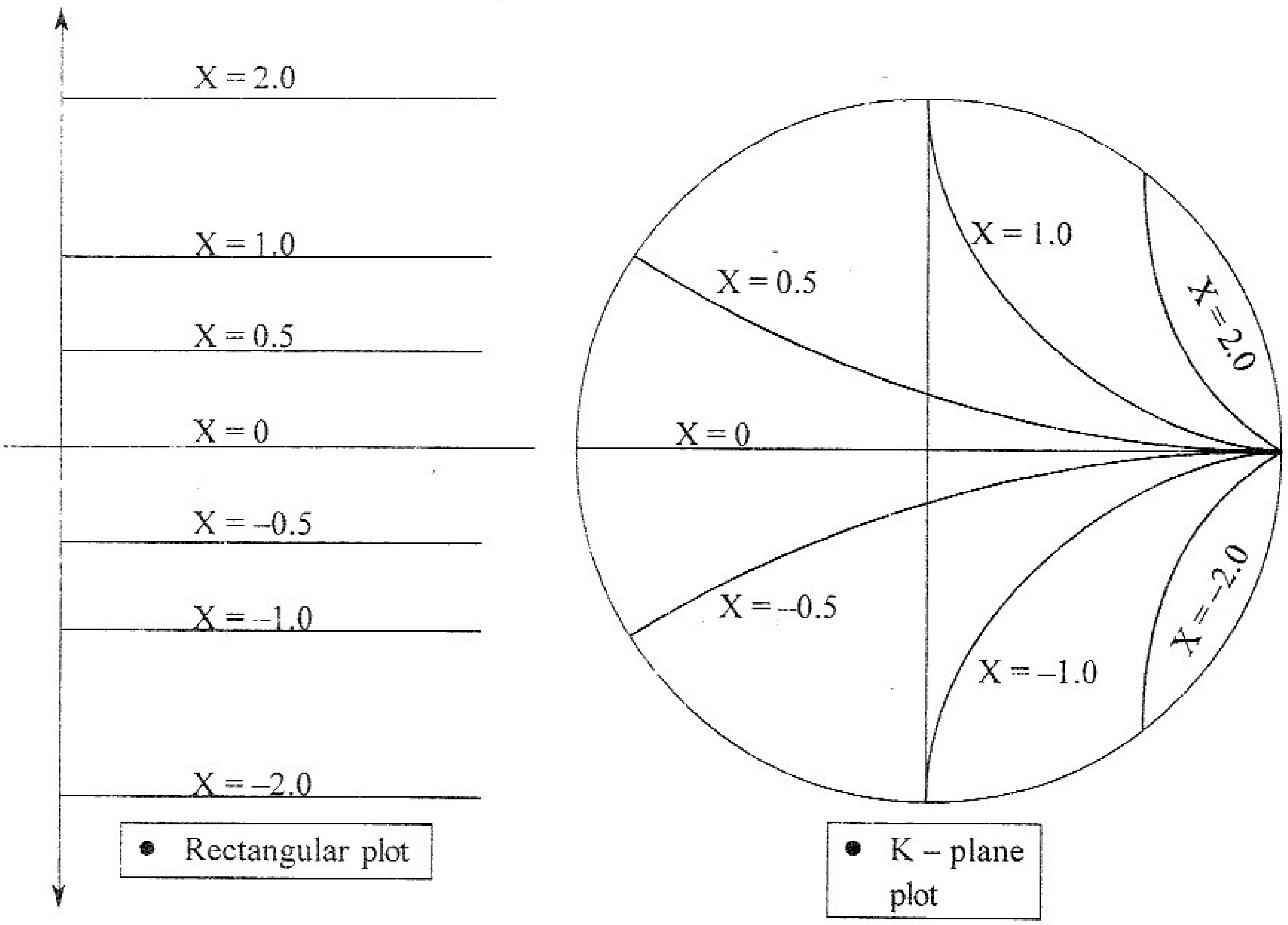

Fig.2.11 Family of constant X circles

When X is positive, the circle lies above the horizontal line(i.e., real axis)

when X is negative, the circle lies below the real axis

when X is zero, the circle becomes a straight line along the real axis. All the circles tough the point (1,0).

The Smith Chart is obtained by the superposition of two sets of constant X orthogonal circles and constant R orthogoanl circles. This is shown in Fig.2.12.

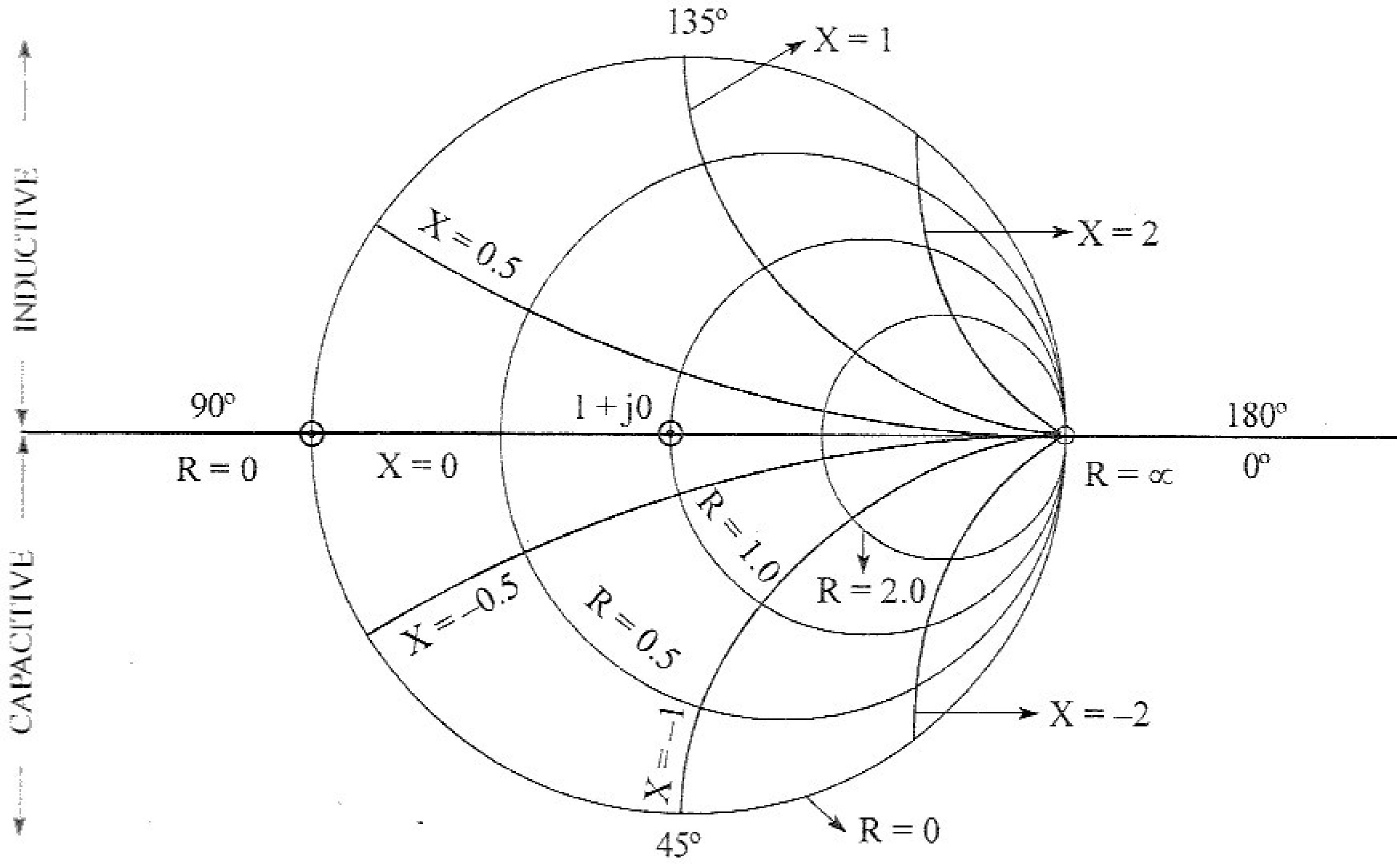

Fig.2.12 Smith circle diagram

The Commercial Smith Chart is shown in Fig.2.13.

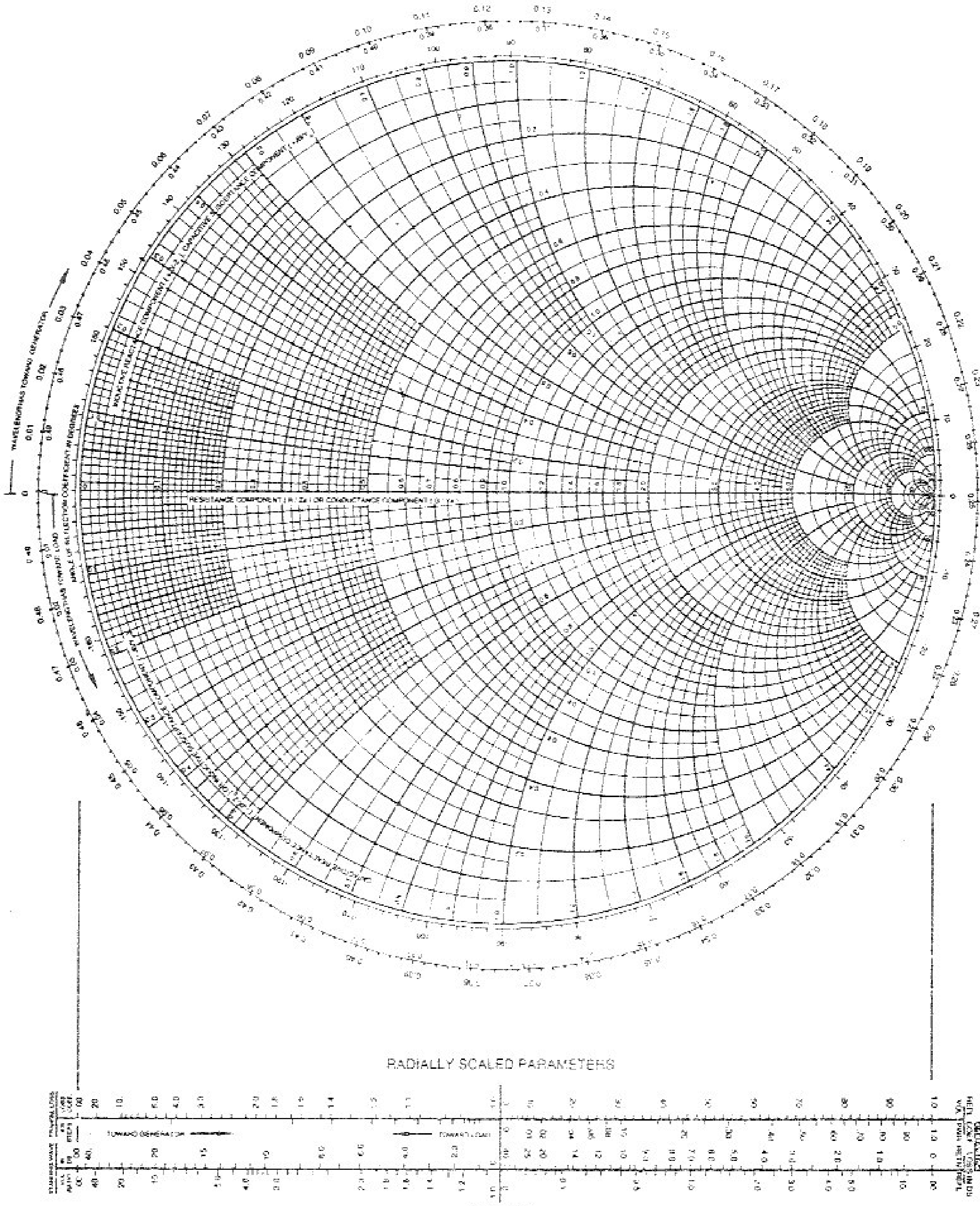

Fig.2.13 Smith Chart