$$Z_3 = \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

The input impedances of open and short circuited lines are

$$Z_{1OC} = Z_0 \coth \gamma l$$

$$= \frac{Z_0}{\tanh \gamma l}$$

$$Z_{1OC} = Z_0 \left(\frac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} \right)$$

and $$Z_{1SC} = Z_0 * \tanh \gamma l$$

$$= Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} \right)$$

Since a line is a symmetrical network,

$$Z_{1OC} = Z_{2OC} \qquad [Z_1 = Z_2]$$

The shunt element $Z_3 = \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$

$$= \sqrt{\frac{Z_0}{\tanh \gamma l}\left(\frac{Z_0}{\tanh \gamma l} - Z_0 \tanh \gamma l\right)}$$

$$= \sqrt{\frac{Z_0^2}{\tanh^2 \gamma l} - Z_0^2}$$

$$= Z_0 \sqrt{\coth^2 \gamma l - 1}$$

$$= \frac{Z_0}{\sinh \gamma l}$$

The series elements for the equivalent T section are

$$Z_1 = Z_2 = Z_{1OC} - Z_3$$

$$= Z_0 \coth \gamma l - \frac{Z_0}{\sinh \gamma l}$$

$$= Z_0 \left(\frac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} - \frac{2}{e^{\gamma l} - e^{-\gamma l}} \right)$$

$$= Z_0 \left[\frac{e^{\gamma l} + e^{-\gamma l} - 2}{e^{\gamma l} - e^{-\gamma l}} \right]$$

$$= Z_0 \left[\frac{(e^{\gamma l/2} - e^{-\gamma l/2})^2}{(e^{\gamma l/2} - e^{-\gamma l/2})(e^{\gamma l/2} + e^{-\gamma l/2})} \right]$$

$$= Z_0 \left[\frac{e^{\gamma l/2} - e^{-\gamma l/2}}{e^{\gamma l/2} + e^{-\gamma l/2}} \right]$$

$$Z_1 = Z_2 = Z_0 \tanh \frac{\gamma l}{2}$$

The T section equivalent for the transmission line is shown in Fig.1.9.

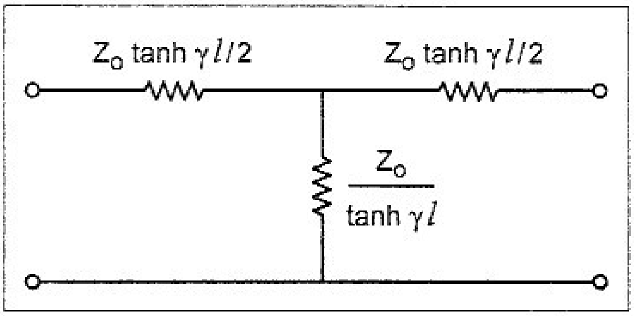


Fig. 1.9. T section equivalent for a transmission line

A π section network is shown in Fig.1.10.

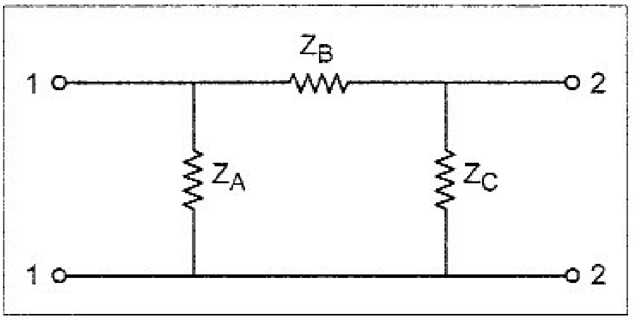


Fig. 1.10. π section network

The input impedance may be measured at either ports while the other port may be shorted or opened.

$$Z_{1OC} = \frac{Z_A (Z_B + Z_C)}{Z_A + Z_B + Z_C}$$

$$Z_{1SC} = \frac{Z_A Z_B}{Z_A + Z_B}$$

$$Z_{2OC} = \frac{Z_C (Z_A + Z_B)}{Z_A + Z_B + Z_C}$$

$$Z_{2SC} = \frac{Z_B Z_C}{Z_B + Z_C}$$

$$Z_{1SC} Z_{2OC} = \left(\frac{Z_A Z_B}{Z_A + Z_B} \right) \left(\frac{Z_C (Z_A + Z_B)}{Z_A + Z_B + Z_C} \right)$$

$$Z_{1SC} Z_{2OC} = \frac{Z_A Z_B Z_C}{Z_A + Z_B + Z_C}$$

and
$$Z_{1OC} - Z_{1SC} = \frac{Z_A(Z_B + Z_C)}{Z_A + Z_B + Z_C} - \frac{Z_A Z_B}{(Z_A + Z_B)}$$

$$= \frac{Z_A(Z_A + Z_B)(Z_B + Z_C) - Z_A Z_B (Z_A + Z_B + Z_C)}{(Z_A + Z_B + Z_C)(Z_A + Z_B)}$$

$$= \frac{Z_A^2 Z_B + Z_A Z_B^2 + Z_A^2 Z_C + Z_A Z_B Z_C - Z_A^2 Z_B - Z_A Z_B^2 - Z_A Z_B Z_C}{(Z_A + Z_B + Z_C)(Z_A + Z_B)}$$

$$Z_{1OC} - Z_{1SC} = \frac{Z_A^2 Z_C}{(Z_A + Z_B + Z_C)(Z_A + Z_B)}$$

Multiplying with Z_{2OC},

$$Z_{2OC}(Z_{1OC} - Z_{1SC}) = \left(\frac{Z_C(Z_A + Z_B)}{Z_A + Z_B + Z_C}\right)\left(\frac{Z_A^2 Z_C}{(Z_A + Z_B + Z_C)(Z_A + Z_B)}\right)$$

$$= \frac{Z_A^2 Z_C^2}{(Z_A + Z_B + Z_C)^2}$$

$$\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$\frac{Z_{1SC} Z_{2OC}}{\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}} = \frac{(Z_A Z_B Z_C)/(Z_A + Z_B + Z_C)}{Z_A Z_C/(Z_A + Z_B + Z_C)}$$

$$= Z_B$$

$$\therefore \quad Z_B = \frac{Z_{1SC} Z_{2OC}}{\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}}$$

$$Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} = \frac{Z_C(Z_A + Z_B)}{(Z_A + Z_B + Z_C)} - \frac{Z_A Z_C}{(Z_A + Z_B + Z_C)}$$

$$= \frac{Z_B Z_C}{(Z_A + Z_B + Z_C)}$$

$$\frac{Z_{1SC} Z_{2OC}}{Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}} = \frac{Z_A Z_B Z_C/(Z_A + Z_B + Z_C)}{Z_B Z_C/(Z_A + Z_B + Z_C)}$$

$$= Z_A$$

$$\therefore \quad Z_A = \frac{Z_{1SC} Z_{2OC}}{Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}}$$

$$Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} = \frac{Z_A(Z_B + Z_C)}{(Z_A + Z_B + Z_C)} - \frac{Z_A Z_C}{(Z_A + Z_B + Z_C)}$$

$$= \frac{Z_A Z_B}{(Z_A + Z_B + Z_C)}$$

$$\frac{Z_{1SC} Z_{2OC}}{Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}} = \frac{Z_A Z_B Z_C / (Z_A + Z_B + Z_C)}{Z_A Z_B / (Z_A + Z_B + Z_C)}$$

$$= Z_C$$

$$\therefore\ Z_C = \frac{Z_{1SC} Z_{2OC}}{Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}}$$

Since a line is symmetrical network,

$$Z_A = Z_C$$

$$\sqrt{Z_{OC} Z_{SC}} = Z_0$$

$$Z_{1OC} = Z_{2OC} = \frac{Z_0}{\tanh \gamma l}$$

$$Z_{1SC} = Z_o \tanh \gamma l$$

$$\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} = \frac{Z_0}{\sinh \gamma l}$$

$$Z_A = Z_C = \frac{Z_{1SC} Z_{2OC}}{Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}}$$

$$= \frac{Z_0^2}{\dfrac{Z_0}{\tanh \gamma l} - \dfrac{Z_0}{\sinh \gamma l}} = \frac{Z_0}{\left(\dfrac{1}{\tanh \gamma l} - \dfrac{1}{\sinh \gamma l}\right)}$$

$$= \frac{Z_0}{\left(\dfrac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}}\right) - \left(\dfrac{2}{e^{\gamma l} - e^{-\gamma l}}\right)}$$

$$= \frac{Z_0 (e^{\gamma l} - e^{-\gamma l})}{(e^{\gamma l} + e^{-\gamma l} - 2)}$$

$$= \frac{Z_0 (e^{\gamma l/2} - e^{-\gamma l/2})(e^{\gamma l/2} + e^{-\gamma l/2})}{(e^{\gamma l/2} - e^{-\gamma l/2})^2}$$

$$= \frac{Z_0 (e^{\gamma l/2} + e^{-\gamma l/2})}{(e^{\gamma l/2} - e^{-\gamma l/2})}$$

$$Z_A = Z_C = \frac{Z_0}{\tanh \frac{\gamma l}{2}}$$

$$Z_B = \frac{Z_{2OC} Z_{1SC}}{\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}}$$

$$= \frac{Z_0^2}{Z_0 / \sinh \gamma l}$$

$$Z_B = Z_0 \sinh \gamma l$$

The equivalent π section for a line is shown in Fig.1.11.

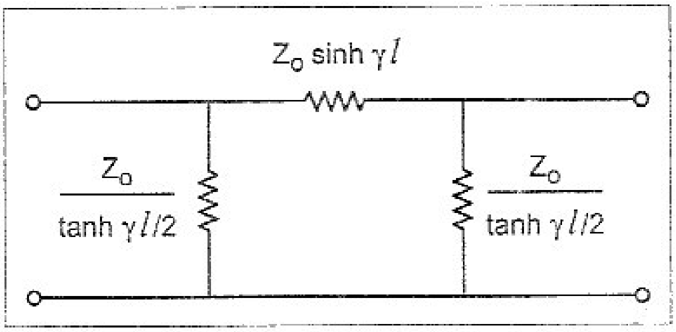


Fig. 1.11. An equivalent π section

The equations for voltage and current on a transmission line are given below.

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

$$I = \frac{I_R (Z_R + Z_o)}{2 Z_o} \left[e^{\gamma x} - \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

But $\quad K = \dfrac{Z_R - Z_o}{Z_R + Z_o}$

Then $\quad V = \dfrac{V_R (Z_R + Z_o)}{2 Z_R} [e^{\gamma x} + K e^{-\gamma x}]$

$$I = \frac{I_R (Z_R + Z_o)}{2 Z_o} [e^{\gamma x} - K e^{-\gamma x}]$$

These two equations comprise of incident wave and reflected wave with definite maxima and minima along the line. The term involving $e^{\gamma x}$ is the incident wave whereas the term involving $e^{-\gamma x}$ is the reflected wave. The reflected wave depends upon the reflection coefficient. The voltage and current distributions for open circuit and short circuit conditions are shown in Fig.1.12. It also shows the distribution for proper matching $R_R = R_o$.

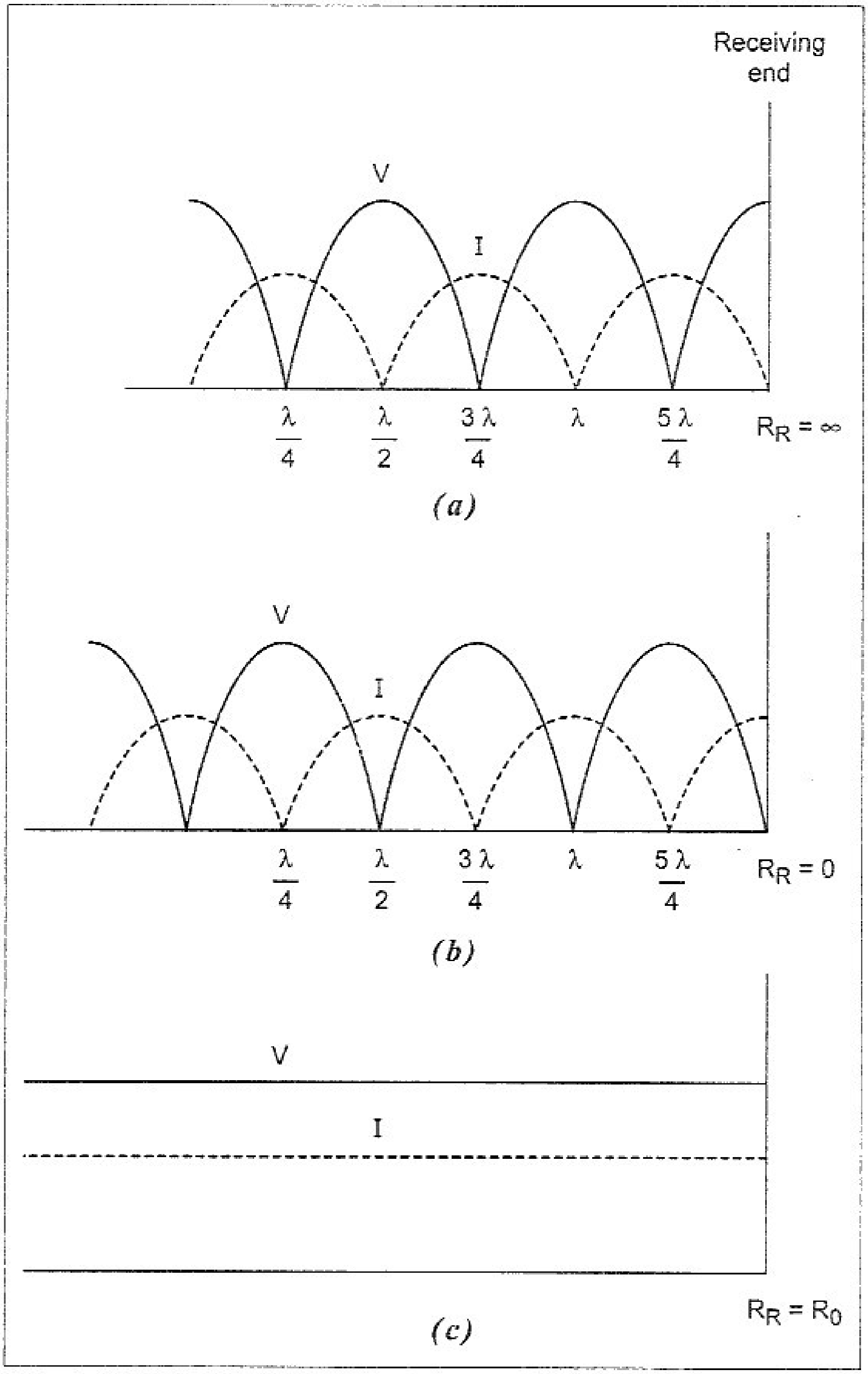


Fig. 1.12. Voltages and currents on dissipationless line
(a) Open circuit (b) Short circuit (c) $R_R = R_0$

SOLVED PROBLEMS

Example 1.1 ***An open wire telephone line has R = 10 ohm / km, L = 0.004 H/km, C = 0.008 × 10⁻⁶ F/km, and G = 0.4 × 10⁻⁶ ohm/km. Determine its Z_o, α and β at 1 KHz.***

Solution : Series impedance,

$$Z = R + j\omega L$$
$$= 10 + j2 \times 3.14 \times 1000 \times 0.004$$
$$= 10 + j25.12$$
$$Z = 27.04 \angle 68.3° \ \Omega$$

Similarly, shunt admittance

$$Y = G + j\omega C$$
$$= 0.4 \times 10^{-6} + j2 \times 3.14 \times 1000 \times 0.008 \times 10^{-6}$$
$$= [0.4 + j50.24] \times 10^{-6}$$
$$Y = 50.24 \times 10^{-6} \angle 89.54° \ ℧$$

Characteristic impedance, $Z_0 = \sqrt{\dfrac{Z}{Y}}$

$$= \sqrt{\frac{27.04 \angle 68.3°}{50.24 \times 10^{-6} \angle 89.54°}}$$
$$= \sqrt{\frac{27.04}{50.24 \times 10^{-6}}} \ \angle \frac{68.3° - 89.54°}{2}$$
$$= 0.7336 \times 10^{3} \ \angle -10.62°$$
$$= 733.6 \angle -10.62°$$
$$Z_0 = 721.03 - j135.2 \text{ ohms}$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \sqrt{27.04 \angle 68.3° \times 50.24 \times 10^{-6} \angle 89.54°}$$
$$= \sqrt{27.04 \times 50.24 \times 10^{-6}} \ \angle \frac{68.3° + 89.54°}{2}$$
$$= 36.85 \times 10^{-3} \ \angle 78.92°$$
$$= 0.0368 \angle 78.92°$$
$$\gamma = 0.007 + j0.0361 \text{ per km}$$

Therefore, the attenuation constant is

$$\alpha = 0.007 \text{ neper/km} \qquad [\because \gamma = \alpha + i\beta]$$

and the phase constant is $\beta = 0.0361$ radians/km

Example 1.2 ***The characteristic impedance of a uniform transmission line is 2000 ohm at frequency of 1 KHz. At this frequency the propagation constant was found to be 0.054 ∠ 60°. Determine the values of line constants R, L, G and C.***

Solution : It is given that $Z_o = 2000$ ohms, $\gamma = 0.054 \angle 60°$

$$\text{and } \omega = 2\pi f$$

$$= 2 \times 3.14 \times 1000 = 6280$$

$$\omega = 6280$$

It is known that, $R + j\omega L = \gamma \times Z_o$

$$= 0.054 \angle 60° \times 2000 = 108 \angle 60°$$

$$= 54 + j93.53 \text{ ohms / km}$$

Equating real and imaginary parts, we have

$$R = 54 \text{ ohms/km}$$

$$\omega L = 93.53$$

$$L = \frac{93.53}{6280} \text{ H/km}$$

$$L = 14.89 \text{ mH/km}$$

Also, $G + j\omega C = \dfrac{\gamma}{Z_o} = \dfrac{0.054 \angle 60°}{2000} = 27 \times 10^{-6} \angle 60°$

$$= (13.5 + j23.38) \times 10^{-6} \text{ mhos / km}$$

Equating real and imaginary parts,

$$G = 13.5 \times 10^{-6} \text{ mhos/km}$$

$$\omega C = 23.38 \times 10^{-6}$$

$$C = \frac{23.38 \times 10^{-6}}{6280} = 3.723 \times 10^{-9} \text{ F/km}$$

$$C = 3.723 \text{ mF/km}$$

Example 1.3 ***The constant of a L.F transmission line per km are R = 6 ohms, L = 2.2 mH, C = 0.005 mF, G = 0.25 × 10⁻⁶ mhos. Calculate at the frequency of 1 KHz, (i) the terminating impedance for which no reflection will be setup in the line, (ii) the attenuation in db suffered by signal at 1 KHz, while travelling a distance of 100 km when the line is properly terminated and the phase velocity with which the signal would travel.***

Solution : (i) When the transmission line is terminated by its characteristic impedance, there is no reflection. Therefore, the terminating impedance will be Z_o, which has to be calculated.

$$\omega = 2\pi f$$

$$= 2 \times 3.14 \times 1000$$

$$\omega = 6.28 \times 10^3$$

Series impedance $Z = R + j\omega L$

$$= 6 + j6.28 \times 10^3 \times 2.2 \times 10^{-3}$$

$$= 6 + j13.8$$

$$Z = 15.04 \angle 66.5^\circ \ \Omega$$

Similarly shunt admittance,

$$Y = G + j\omega C$$

$$= 0.25 \times 10^{-6} + j6.28 \times 10^3 \times 0.005 \times 10^{-6}$$

$$= (0.25 + j31.4) \times 10^{-6}$$

$$Y = 31.42 \times 10^{-6} \angle 89.5^\circ \ \mho$$

Characteristic impedance, $Z_o = \sqrt{\frac{Z}{Y}}$

$$= \sqrt{\frac{15.04 \angle 66.5^\circ}{31.4 \times 10^{-6} \angle 89.5^\circ}}$$

$$= \sqrt{\frac{15.04}{31.4 \times 10^{-6}}} \angle \frac{66.5^\circ - 89.5^\circ}{2}$$

$$Z_o = 0.692 \times 10^3 \angle -11.5^\circ$$

$$Z_o = 692 \angle -11.5^\circ \ \text{ohms}$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \sqrt{15.04 \angle 66.5^\circ \times 31.42 \times 10^{-6} \angle 89.5^\circ}$$

$$= \sqrt{15.04 \times 31.42 \times 10^{-6}} \angle \frac{66.5 + 89.5}{2}$$

$$\gamma = 21.73 \times 10^{-3} \angle 78^\circ$$

$$\gamma = 0.0046 + j0.0215$$

Since $\gamma = \alpha + j\beta$

$$\alpha = 0.0046 \text{ neper/km}$$

$$\beta = 0.0215 \text{ radians/km}$$

(a) Attenuation suffered while travelling, 100 km

$$= 100 \times \alpha = 100 \times 0.0046 \text{ nepers}$$

$$= 0.46 \times 8.06 \text{ db}$$

$$= 3.99 \text{ db}$$

(b) Phase velocity V_P, by which the signal would travel

$$= \frac{\omega}{\beta} = \frac{6.28 \times 10^3}{0.0215}$$

$$= 2.9 \times 10^5 \text{ km/sec}$$

Example 1.4 *A 10% voltage drop across in 3 km of a uniformly loaded transmission line terminated by its characteristic impedance and there is a phase change of 30° over the same distance at a frequency of 800 Hz. Find the value of (i) the line attenuation in db/km, (ii) the velocity of propagation.*

Solution : (i) Attenuation in $\text{db} = 20\log_{10}\left[\frac{V_S}{V_R}\right]$ for 3 km

Since voltage drop is 10%

$$V_R = \frac{90}{100} \times V_S = 0.9\,V_S$$

$$\therefore \text{Attenuation in db} = 20\log_{10}\left[\frac{V_S}{0.9\,V_S}\right]$$

$$= 20\log_{10}\frac{10}{9}$$

$$= 20 \times [1 - 0.942] = 20 \times 0.458$$

$$= 0.916 \text{ db for 3 km}$$

For line of 1 km,

$$\alpha = \frac{0.916}{3} \text{ db/km}$$

$$= 0.3053 \text{ db/km}$$

(ii) Phase change for 3 km = 30°

$$\therefore \text{Phase change/km} = \frac{30}{3} = 10°$$

$$\beta = 10°$$

$$= 10 \times \frac{\pi}{180°} \text{ radian} = \frac{\pi}{18} \text{ radian}$$

$$\therefore V_P = \frac{\omega}{\beta} = \frac{2\pi \times 800}{\frac{\pi}{18}}$$

$$= 36 \times 800$$

$$V_P = 28800 \text{ km/sec}$$

Example 1.5 *An open wire line which is 200 km long is properly terminated. The generator at the sending end has V = 10V, f = 1 KHz and internal impedance of 500 ohms. At that frequency Z_o of the line is (700 – j100) and γ = 0.007 + j0.04 per km. Determine the sending end voltage, current and power and the receiving end voltage, current and power.*

$$V_g = 10\text{V}, \quad Z_g = 500 + j0 \text{ ohm}$$

$$Z_S = Z_o = (700 - j100) \text{ ohms}$$

$$|I_S| = \left| \frac{V_g}{Z_g + Z_s} \right|$$

$$= \left| \frac{10}{500 + 700 - j100} \right|$$

$$= \frac{10}{\sqrt{(1200)^2 + (100)^2}} = \frac{10}{1204.2}$$

$$|I_S| = 8.3 \times 10^{-3} \text{ A} = 8.3 \text{ mA}$$

$$|V_S| = |I_S Z_S| = 8.3 \times 10^{-3} \sqrt{(700)^2 + (100)^2}$$

$$|V_S| = 5.869 \text{ V}$$

i.e., Average power entering the line,

$$P_S = |I_S|^2 \cdot R_S$$

$$= (8.3 \times 10^{-3})^2 \times 700$$

$$= 48223 \times 10^{-6}$$

$$P_S = 48.22 \text{ mW}$$

$$l = 200 \text{ km}$$

It is known that,

$$V_R = V_S e^{-\gamma l} = V_S e^{-(\alpha + j\beta) l}$$

$$= V_S e^{-\alpha l} e^{-j\beta l}$$

$$= 5.869 \times e^{-0.007 \times 200} e^{-j0.04 \times 200}$$

$$= 1.45 e^{-j8}$$

$$= 1.45 \angle -8 \text{ radians}$$

Thus the magnitude of V_R is 1.45 V rms and $\beta = -8$ radians.

The line is $\frac{8}{2\pi} = 1.273\lambda$ long, and V_R lags V_S by 1273 Hz. In so far as the position of sending voltage is concerned, 2π radians or 360° can be subtracted without changing the result. Therefore

$$V_R = 1.45 \angle -1.72 \text{ V}$$

$$= 1.45 \angle -98.5° \text{ V}$$

$$Z_R = 700 - j100 \qquad [\because Z_R = Z_0]$$

$$= \sqrt{700^2 + 100^2}$$

$$= 707\ \Omega$$

$$|I_R| = \left| \frac{V_R}{Z_R} \right| = \frac{1.45}{707}$$

$$|I_R| = 2.05 \text{ mA}$$

The average power absorbed by the terminating load is

$$P_R = |I_R|^2 \cdot R_R = (2.05 \times 10^{-3})^2 \times 700$$

$$P_R = 2.94 \text{ mW}$$

Example 1.6 ***A telephone line has resistance of 20 ohms, inductance of 10 mH, capacitance of 0.1µF and insulation resistance of 100 K ohm/km. Find the input impedance at angular frequency 5000 radian/sec., if the line is very long.***

Given that,

$$R = 20 \text{ ohms}$$

$$\alpha = 10 \text{ mH} = 10^{-2} \text{ H}$$

$$G = \frac{1}{100 \times 10^3} = 10^{-5} \text{ ohm}$$

$$C = 0.1 \ \mu\text{F} = 0.1 \times 10^{-6} = 10^{-7} \text{ F}$$

$$\omega = 5000 \text{ radians/sec}$$

Since the line is very long, it approximates to an infinite line. The input impedance of an infinite line as explained is the characteristic impedance of the line. Hence Z_o must be found.

$$\therefore \text{ Series impedance, } Z = R + j\omega L$$

$$= 20 + j5000 \times 10^{-2}$$

$$= 20 + j50$$

$$Z = 53.85 \angle 68.2^\circ \ \Omega$$

$$\text{Shunt admittance, } Y = G + j\omega C$$

$$= 10^{-5} + j5000 \times 10^{-7} = (10 + j500) \times 10^{-6}$$

$$Y = 500 \times 10^{-6} \angle 88.9^\circ \ \mho$$

$$\text{Using this in } Z_o = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{59.85 \angle 68.2^\circ}{500 \times 10^{-6} \angle 88.9^\circ}}$$

$$= \sqrt{10.77 \times 10^4} \ \angle \frac{68.2^\circ - 88.9^\circ}{2}$$

$$Z_o = 328.17 \angle -10.4^\circ \text{ ohms}$$

Example 1.7 ***A 12 km line is terminated by its characteristic impedance. At a certain frequency the voltage at 1 km from the sending end is 10% below that at the sending end. Find the voltage across the load impedance interms of percentage of the sending end voltage.***

Since the line is terminated in its Z_o, it can be considered as equivalent to an infinite line. Therefore,

$$V = V_S \, e^{-\gamma x}$$

where V is the voltage at a distance x km from the sending end.

It is given that, $x = 1$ km, V = 10% less than V_S. That is

$$V = \frac{90}{100} V_S = 0.9 V_S$$

Using these values, $$0.9 V_S = V_S \times e^{-\gamma \times 1}$$

$$0.9 = e^{-\gamma}$$

when $x = 12$ km, it follows from the same equation

$$V_L = V_S \, e^{-\gamma \times 12}$$

Since the line is 12 km long V_L will be the voltage across the load impedance. Substituting the value of $e^{-\gamma} = 0.9$ in the above expression,

$$V_L = V_S \, (9.9)^{12}$$

$$V_L = V_S \times 0.2821$$

$$V_L = 28.21\% \text{ of } V_S$$

This V_L is 28.21% of the sending voltage V_S.

Example 1.8 ***At 8 MHz the characteristic impedance of a transmission line is (40 – j2)Ω and the propagation constant is (0.01 + j0.18) per meter. Find the primary constants.***

$$\omega = 2 \times 3.14 \times 8 \times 10^6$$

$$\omega = 50.24 \times 10^6$$

Given : $$Z_0 = 40 - j2, \quad \gamma = 0.01 + j0.18$$

$$R + j\omega L = Z_0 \times \gamma$$

$$R + j\omega L = (40 - j2) \times (0.01 + j0.18)$$

$$R + j\omega L = 0.76 + j7.18$$

$$\therefore \quad R = 0.76 \quad \text{ohm/m}$$

$$\omega L = 7.18$$

$$50.24 \times 10^6 \times L = 7.18$$

$$L = \frac{7.18}{50.24 \times 10^6}$$

$$L = 0.1429 \ \mu\text{H/m}$$

$$G + j\omega C = \frac{\gamma}{Z_0} = \frac{0.18 \angle 86.82}{40 \angle -2.86}$$

$$= 4.5 \times 10^{-3} \angle 89.06$$

$$G + j\omega C = 7.38 \times 10^{-4} + j449.9 \times 10^{-4}$$

$$G = 7.38 \times 10^{-4} \text{ mho/meter}$$

$$\omega C = 449.9 \times 10^{-4}$$

$$C = \frac{449.9 \times 10^{-4}}{50.24 \times 10^6}$$

$$C = 8.95 \times 10^{-10} \text{ F/m}$$

TWO MARK QUESTIONS AND ANSWERS

1. What are the primary constants of a transmission line ?

The four line parameters resistance (R), inductance (L), capacitance (C) and conductance (G) are termed as primary constants of a transmission line.

2. What are the secondary constants of a transmission line ?

Propagation constant (γ) and characteristic impedance (Z_0) are the secondary constants of a transmission line.

3. When will a transmission line deliver maximum power to a load ?

A transmission line will deliver maximum power to a load when the load resistance is equal to the characteristic resistance.

4. Name the types of line distortion.

Line distortion is usually of two types :

1. Frequency distortion
2. Delay distortion

5. Write the condition for a distortionless line.

The condition for a distortionless line is

$$\frac{R}{L} = \frac{G}{C}$$

6. What are loaded lines ?

To achieve distortionless condition in transmission line, inductance L has to be increased. Increasing the value by inserting inductances in series with line is termed as loading and such lines are called loaded lines.

7. Define wavelength and velocity of wave.

The distance the wave travels along the line while the phase angle is changing through 2π radians is called a wavelength.

$$\lambda = \frac{2\pi}{\beta}$$

Velocity of propagation is defined as

$$v = \frac{\omega}{\beta}$$

8. What is phase distortion or delay distortion ?

All the frequencies applied to a transmission line will not travel uniformly, some of them may be delayed more than others. This phenomenon is known as delay or phase distortion.

9. What is frequency distortion ?

A complex voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

10. How can distortion be reduced in a transmission line ?

Frequency distortion is reduced by the use of equalisers. Delay distortion is avoided by the use of a coaxial cable.

11. Write an expression for characteristic impedance.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

12. What is called an infinite line ?

It is to a hypothetical line which has input impedance equals to the characteristic impedance.

A finite line terminated in a load equivalent to the characteristic impedance appears to the sending end as an infinite line.

13. Name the types of waveform distortions.

Waveform distortion is usually of two types.

1. Frequency distortion
2. Delay distortion

14. Define propagation constant.

Propagation constant per unit length may be defined as the natural logarithmic of ratio of the sending end current or voltage to the receiving end current or voltage.

$$\gamma = ln\left(\frac{I_S}{I_R}\right) = ln\left(\frac{V_S}{V_R}\right)$$

It is a complex quantity

$$\gamma = \alpha + j\beta$$

where α is attenuation constant

β is phase shift

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

15. List the different methods of loading.

The different methods of loading are

1. Lumped loading
2. Continuous loading
3. Patch loading

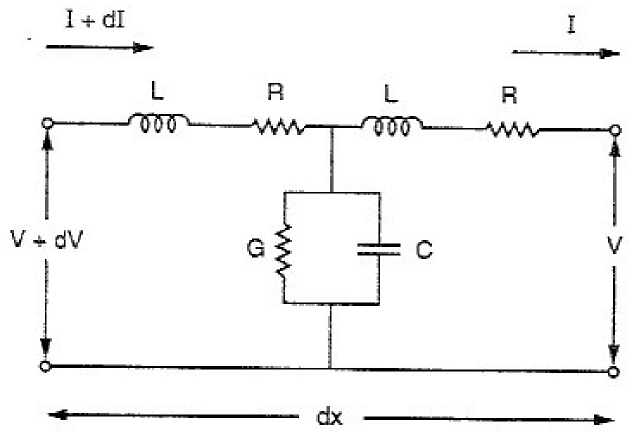
16. Mention the relation between characteristic impedance and primary constant of a transmission line.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

17. Write the formula for velocity of propagation.

$$v = \frac{\omega}{\beta}$$

18. Draw the equivalent circuit of a transmission line.



19. Write the Campbell's formula for propagation constant of a loaded line.

$$\cosh \gamma' l = \frac{Z_c}{2 Z_0} \sinh \gamma l + \cosh \gamma l$$

where Z_c is the impedance of loading coil

Z_0 is the characteristic impedance

γ is the propagation constant

l is the distance between two loading coils

20. When does reflection take place on a transmission line ?

When the load impedance (Z_R) is not equal to characteristic impedance (Z_0) of the transmission line, (*i.e.*, $Z_R \neq Z_0$) reflection takes place.

21. Define reflection coefficient.

Reflection coefficient is defined as the ratio of the reflected voltage to the incident voltage at the receiving end of the line.

$$K = \frac{V_R}{V_S}$$

$$= \frac{Z_R - Z_0}{Z_R + Z_0}$$

22. Define SWR.

The ratio of maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio (SWR).

$$\text{SWR} = \left| \frac{V_{max}}{V_{min}} \right| = \left| \frac{I_{max}}{I_{min}} \right|$$

23. How are practical lines made appear as infinite lines ?

A finite line terminated in a load equivalent to the characteristic impedance appears at the sending end as an infinite line.

24. Define reflection factor.

Reflection factor indicates the change in current in the load due to reflection at the mismatched junction.

$$k = \left| \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2} \right|$$

25. Define reflection loss.

Reflection loss is the reciprocal of reflection factor in nepers or dB.

$$\text{Reflection loss} = \ln \frac{1}{k}$$

$$= \ln \left| \frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}} \right| \text{ nepers}$$

$$= 20 \log \left| \frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}} \right| \text{ dB}$$

26. What is meant by waveform distortion ?

If the received waveform on a transmission line is not identical with the input waveform at the sending end, it is called waveform distortion. This is due to the fact that all frequencies applied on the transmission line are not equally attenuated and are not delayed equally.

27. What is inductive loading ?

The inductance of a transmission line can be increased by the introduction of loading coil at uniform intervals so that the transmission line is distortionless. This is known as inductive or lumped loading.

28. The open circuit and short circuit impedances of a transmission line at 1500 Hz are 800 ∠ – 30 ° Ω and 400 ∠ – 10 °Ω respectively. Calculate its propagation constant.

$$Z_{OC} = 800 \angle -30° \; \Omega$$

$$Z_{SC} = 400 \angle -10° \; \Omega$$

Propagation constant/unit length

$$\gamma = \tanh^{-1}\sqrt{\frac{Z_{SC}}{Z_{OC}}}$$

$$= \tanh^{-1}\sqrt{\frac{400\angle -10°}{800\angle -30°}}$$

$$= \tanh^{-1}\sqrt{0.5\angle 20°}$$

29. Give the general equation for the input impedance of a dissipation line.

$$Z_S = Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right)$$

where Z_o is the characteristic impedance

Z_R is the receiving end impedance

γ is the propagation constant

l is the length of the transmission line from the sending end

30. Write the expressions for the characterisitc impedance and propagation constant for the dissipationless line.

Propagation constant $\gamma = \sqrt{LC}\left(\frac{G}{C}+j\omega\right)$

Characteristic impedance $Z_o = \sqrt{\frac{L}{C}}$

31. How will you find out the propagation constant if the values of open and short circuited impedances are given ?

Propagation constant can be determined from the following expression.

$$\tanh\gamma l = \sqrt{\frac{Z_{SC}}{Z_{OC}}}$$

32. Determine the values of VSWR in the case of (a) $Z_R = 0$ and (b) $Z_R = Z_o$.

(a) $Z_R = 0$, $|K| = 1$, $SWR = \infty$

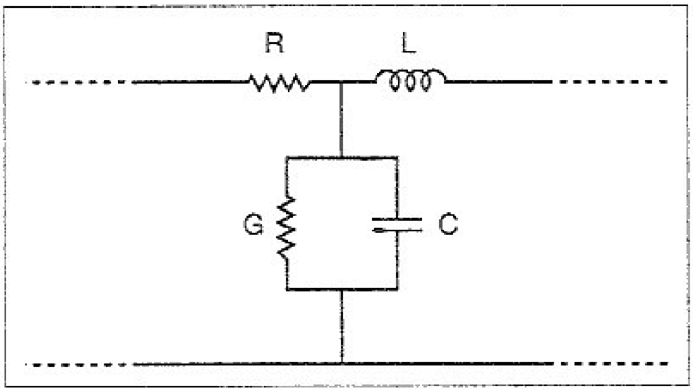
(b) $Z_R = Z_o$, $K = 0$, $SWR = 1$

33. Give the relationship between the input impedance and characteristic impedance of an infinite line.

$$Z_S = \frac{V_S}{I_S} = Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right)$$

SUMMARY

Equivalent circuit of a transmission line



$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Primary Constant	Secondary Constant
Resistance (R)	Characteristic Impedance (Z_0)
Inductance (L)	Propagation constant (γ)
Capacitance (C)	
Conductance (G)	

Voltage and current equations at any point on a transmission line

$$V = V_R \cosh \gamma x + I_R Z_o \sinh \gamma x$$

$$I = I_R \cosh \gamma x + \frac{V_R}{Z_o} \sinh \gamma x$$

$$\text{Propagation constant } \gamma = \sqrt{ZY}$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\text{Characteristic impedance } Z_o = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\text{Propagation constant } \gamma = 20 \log \left(\frac{V_S}{V_R}\right)$$

$$= 20 \log \left(\frac{I_S}{I_R}\right)$$

$$\text{Attenuation constant} \quad \alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

Phase shift $$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

Condition for distortionless line

$$LG = CR$$

$$\frac{L}{C} = \frac{R}{G}$$

Para meters	Distortion line	Distortionless line	Telephone cable
γ	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{LC}\,\sqrt{\left(\frac{R}{L} + j\omega\right)}$	$\sqrt{j\omega RC}$
α	$\sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$	$\sqrt{RG}$	$\sqrt{\frac{\omega RC}{2}}$
β	$\sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$	$\omega\sqrt{LC}$	$\sqrt{\frac{\omega RC}{2}}$
v	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{LC}}$	$\sqrt{\frac{2\omega}{RC}}$
Z_o	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$	$\sqrt{\frac{L}{C}}$	$\frac{R}{\omega C} \angle -45°$

Campbell's equation

$$\cosh \gamma' l = \frac{Z_c}{2 Z_o} \sinh \gamma l + \cosh \gamma l$$

$$Z_{sc} = Z_o \tanh \gamma l$$

$$Z_{oc} = Z_o \coth \gamma l$$

$$Z_o = \sqrt{Z_{oc} Z_{sc}}$$

$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

Reflection coefficient $$K = \frac{Z_R - Z_o}{Z_R + Z_o}$$

Reflection factor $$k = \left| \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2} \right|$$

$$\text{Reflection loss} = ln \frac{1}{k} \text{ neper}$$

$$= 2 \log \left| \frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}} \right| \text{ dB}$$

Voltage and current equations on transmission line in terms of reflection coefficient.

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

$$I = \frac{I_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} - \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

Standing wave ratio $$S = \left| \frac{V_{max}}{V_{min}} \right|$$

$$S = \frac{1 + |K|}{1 - |K|}$$

$$|K| = \frac{S - 1}{S + 1}$$

$$|K| = \frac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}$$

EXERCISE

1. Derive expressions for attenuation and phase constants after obtaining an expression for the characteristic impedance.
2. State and explain Campbell's formula for the loading cables.
3. Explain the term SWR and derive its expression in terms of reflection coefficient for a lossless line.
4. Derive an expression for the propagation constant and the velocity of propagation for an ordinary telephone cable.
5. Show that a line will be distortionless if CR = LG.
6. Explain the purpose of loading a telephone cable.
7. Briefly explain about waveform distortion.
8. Derive the condition to be satisfied for a distortionless line.
9. Develop the differential equations governing the voltage and current at any point on a uniform transmission line. Solve these to obtain the voltage and current in terms of the load current and voltage.
10. Describe how inductive loading helps to achieve a distortionless condition for a transmission line.

11. Explain the wave propagation in the zero dissipation line with waveforms of voltage and current for various loads.

12. Derive an expression for the reflection co-efficient in terms of characteristic impedance Z_0 and terminal impedance Z_R.

13. Derive an expression for the input impedance of a transmission line. Assume the length of the line as 'l', characteristics impedance Z_0 terminated with Z_R.

14. Derive an expression for the reflection coefficient in terms of characteristics impedance Z_0 and terminal impedance Z_R.

15. Derive the equations for α and β. Obtain the conditions for distortionless lines.

16. Show that for any uniform transmission line the following relations are valid.

$$Z_0 = \sqrt{Z_{oc} \times Z_{sc}} \text{ and } \tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

□□