CHAPTER - 1

TRANSMISSION LINE THEORY

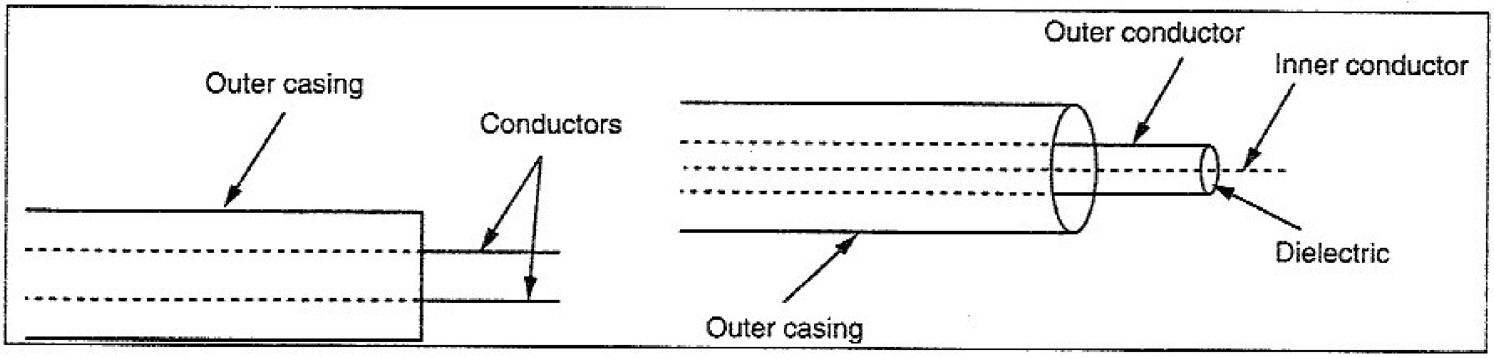
1.1. INTRODUCTION

The transfer of energy from one point to another takes place through either wave guides or transmission lines. Transmission lines always consist of atleast two separate conductors between which a voltage can exist, but the wave guides involve only one conductor; for example, a hollow rectangular or circular waveguide within which the wave propagates. Transmission lines are a means of conveying power from one point to another. There are two types of commonly used transmission lines.

1. Parallel wire (balanced) line
2. Coaxial (unbalanced) line

Parallel wire line : It is a common form of transmission line known as open wire line as shown in Fig. 1.1(a). It is employed where balanced properties are required. Telephone lines, line connecting between folded dipole antenna and TV receiver are good examples of parallel or balanced or open wire line. The parallel wire lines are not used for microwave transmission.

Coaxial line : Coaxial lines consist of inner and outer conductor spacers of dielectric as shown in Fig. 1.1(b). It is used when unbalanced properties are needed, as in the interconnection of a broadcast transmitter to its grounded antenna. It is employed at UHF and microwave frequencies.



(a) Parallel wire (balanced) line *(b) Coaxial (unbalanced) line*

Fig. 1.1. Transmission lines

1.2. TRANSMISSION LINE AS CASCADED T SECTIONS

To study the behaviour of transmission line, a transmission can be considered to be made up of a number of identical symmetrical T sections connected in series as in Fig.1.2. If the last section is terminated with its characteristic impedance, the input impedance at the first section is Z_0. Each section is terminated by the input impedance of the following section.

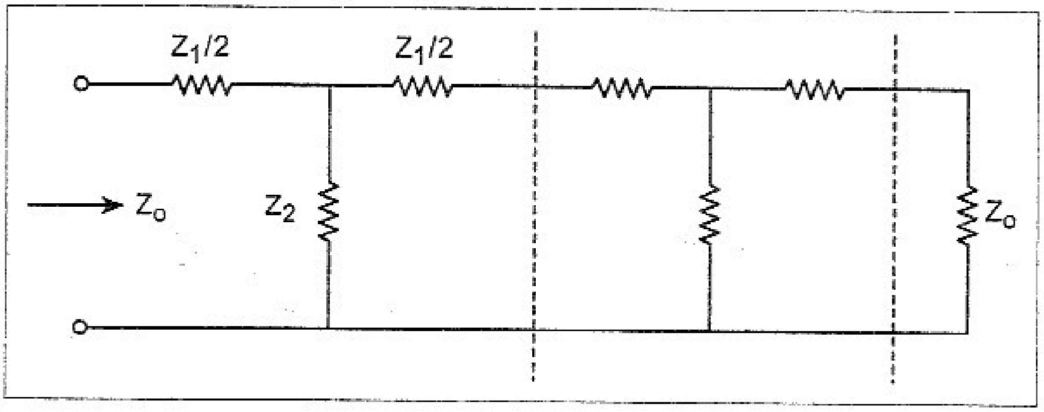


Fig. 1.2. A line of cascaded T sections

The characteristic impedance for a T section is

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4 Z_2}\right)}$$

If 'n' number of T sections are cascaded and if the sending and receiving currents are I_S and I_R respectively, then

$$I_S = I_R \, e^{n\gamma}$$

where γ is the propagation constant for one T section.

$$\gamma = \alpha + j\beta$$

$$e^{\gamma} = e^{\alpha + j\beta} = 1 + \frac{Z_1}{2 Z_2} + \sqrt{\frac{Z_1}{Z_2}\left(1 + \frac{Z_1}{4 Z_2}\right)}$$

One T section representing an incremental length Δx of the line has a series impedance $Z_1 = Z\,\Delta x$ and shunt impedance $Z_2 = \frac{1}{Y\,\Delta x}$. The characteristic impedance of any small T section is that of the line as a whole.

$$Z_0 = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4 Z_2}\right)}$$

Substituting the values of Z_1 and Z_2,

$$Z_0 = \sqrt{\frac{Z\,\Delta x}{Y\,\Delta x}\left(1 + \frac{Z\,\Delta x\, Y\,\Delta x}{4}\right)}$$

$$= \sqrt{\frac{Z}{Y}\left(1 + \frac{ZY\,(\Delta x)^2}{4}\right)}$$

If Δx tends to zero, then Z_0 becomes,

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{Z_1}{4Z_2}\right)^{\frac{1}{2}}$$

By the binomial theorem,

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left[1+\frac{1}{2}\left(\frac{Z_1}{4Z_2}\right)-\frac{1}{8}\left(\frac{Z_1}{4Z_2}\right)^2+\ldots\ldots\right]$$

Substituting this value in e^{γ} equation,

$$e^{\gamma} = 1+\frac{Z_1}{2Z_2}+\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$= 1+\frac{Z_1}{2Z_2}+\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{8}\left(\frac{Z_1}{Z_2}\right)\sqrt{\frac{Z_1}{Z_2}}-\frac{1}{128}\left(\frac{Z_1}{Z_2}\right)^2\sqrt{\frac{Z_1}{Z_2}}+\ldots\ldots$$

$$= 1+\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{2}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^2+\frac{1}{8}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^3-\frac{1}{128}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^5+\ldots\ldots$$

When applied to the incremental length of line Δx, then $Z_1 = Z\,\Delta x$, $Z_2 = \frac{1}{Y\,\Delta x}$ and propagation constant becomes $\gamma\,\Delta x$,

$$e^{\gamma\Delta x} = 1+\sqrt{ZY}\,\Delta x+\frac{1}{2}(\sqrt{ZY})^2(\Delta x)^2+\frac{1}{8}(\sqrt{ZY})^3(\Delta x)^3-128(\sqrt{ZY})^5(\Delta x)^5$$

Series expansion for an exponential $e^{\gamma\Delta x}$ is

$$e^{\gamma\Delta x} = 1+\gamma\Delta x+\frac{\gamma^2(\Delta x)^2}{2!}+\frac{\gamma^3(\Delta x)^3}{3!}+\ldots\ldots\ldots$$

Equating the above two expressions,

$$\sqrt{ZY}\,\Delta x+\frac{(\sqrt{ZY})^2(\Delta x)^2}{2}+\frac{(\sqrt{ZY})^2(\Delta x)^3}{8}+\ldots = \gamma\Delta x+\frac{\gamma^2(\Delta x)^2}{2}+\frac{\gamma^3(\Delta x)^3}{6}+\ldots$$

$$\gamma+\frac{\gamma^2\,\Delta x}{2}+\frac{\gamma^3(\Delta x)^2}{6}+\ldots\ldots = \sqrt{ZY}+\frac{(\sqrt{ZY})^2}{2}\Delta x+\frac{(\sqrt{ZY})^3(\Delta x)^2}{8}+\ldots\ldots$$

If Δx tends to zero then,

$$\gamma = \sqrt{ZY}$$

This is the value of propagation constant in terms of Z and Y.

Since each conductor of transmission line has a certain length and diameter, it must have resistance and inductance; moreover the two conductors are separated by a dielectric medium (say, air), therefore there must be a capacitance between them. This dielectric between the conducting wires may not be perfect, and hence a leakage current will flow creating leakage (shunt) capacitance between the conductors. These four parameters resistance (R), inductance (L), capacitance (C) and conductance (G), all distributed along the lines are known as distributed parameters. The equivalent circuit diagram of transmission line is shown in Fig. 1.3.

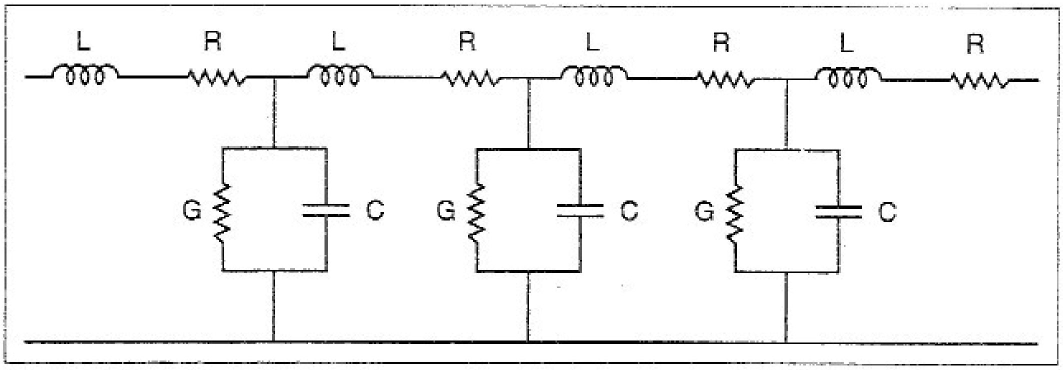


Fig. 1.3. Equivalent circuit diagram of transmission line

The four line parameters resistance (R), inductance (L), capacitance (C) and conductance (G) are also known as ***primary constants*** of the transmission line.

Resistance (R) is defined as the loop resistance per unit length of the transmission line. It is measured in ohms/km.

Inductance (L) is defined as the loop inductance per unit length of the transmission line. It is measured in Henries/km.

Capacitance (C) is defined as the shunt capacitance per unit length between the two transmission lines. It is measured in Farads/km.

Conductance (G) is defined as the shunt conductance per unit length between the two transmission lines. It is measured in mhos/km.

1.3. TRANSMISSION LINE EQUATION

Transmission line is a conductive method of guiding electrical energy from one place to another. A uniform transmission line can be considered to be made up of an infinite number of T sections, each of infinitesimal size dx. The equivalent circuit of T section of transmission line is shown in Fig. 1.4.

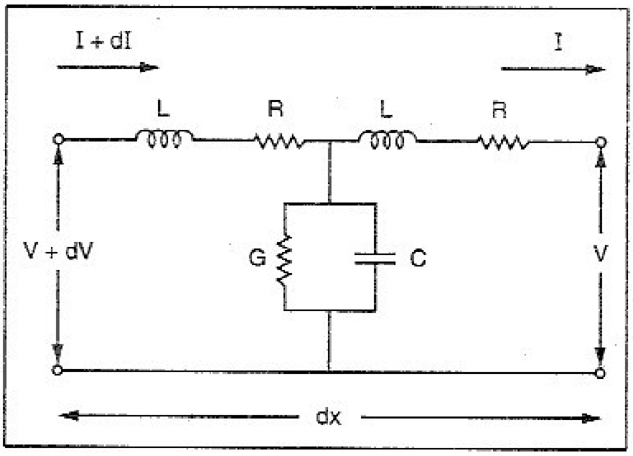


Fig. 1.4. Equivalent circuit of T section of Transmission line

The parameters R, L, G and C are distributed throughout the transmission line. The constants of an incremental length dx of a line are shown in Fig. 1.4. The series impedance per unit length and shunt admittance per unit length are given by

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Consider a T section of transmission line of length dx. Let V + dV be the voltage and I + dI be the current at one end of T section. Let V be the voltage and I be the current at the other end of this section.

The series impedance of a small section dx is $(R + jL\omega)\,dx$. The shunt admittance of this section dx is $(G + jC\omega)\,dx$.

The voltage drop across the series impedance of T sections *i.e.*, the potential difference between the two ends of T section is

$$V + dV - V = I\,(R + j\omega L)\,dx$$

$$dV = I\,(R + j\omega L)\,dx$$

$$\frac{dV}{dx} = I\,(R + j\omega L) \qquad \ldots (1.1)$$

$$\frac{dV}{dx} = IZ$$

The current difference between the two ends of T section is due to the voltage drop across the shunt admittance.

$$I + dI - I = V\,(G + j\omega C)\,dx$$

$$dI = V\,(G + j\omega C)\,dx$$

$$\frac{dI}{dx} = V\,(G + j\omega C) \qquad \ldots (1.2)$$

$$\frac{dI}{dx} = VY$$

Differentiating equation (1.1) w.r.t. 'x',

$$\frac{d^2V}{dx^2} = (R + j\omega L)\,\frac{dI}{dx}$$

Substituting the value of $\frac{dI}{dx}$ in the above equation

$$\frac{d^2V}{dx^2} = (R + j\omega L)\,(G + j\omega C)\,V \qquad \ldots (1.3)$$

Differentiating equation (1.2) w.r.t. 'x'

$$\frac{d^2I}{dx^2} = (G + j\omega C)\,\frac{dV}{dx}$$

Substituting the value of $\frac{dV}{dx}$ in the above equation

$$\frac{d^2I}{dx^2} = (R + j\omega L)\,(G + j\omega C)\,I \qquad \ldots (1.4)$$

But propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)\,(G + j\omega C)} = \sqrt{ZY}$$

Substituting the value of γ in equation (1.3) and (1.4),

$$\text{then } \frac{d^2V}{dx^2} = \gamma^2 V$$

$$\frac{d^2I}{dx^2} = \gamma^2 I$$

The solutions of the above linear differential equations are

$$V = A\,e^{\gamma x} + B\,e^{-\gamma x} \quad \dots (1.5)$$

$$I = C\,e^{\gamma x} + D\,e^{-\gamma x} \quad \dots (1.6)$$

where A, B, C and D are arbitrary constants.

Differentiating the equation (1.5), w.r.t. 'x'

$$\frac{dV}{dx} = A\gamma\,e^{\gamma x} - B\gamma\,e^{-\gamma x}$$

$$\text{But } \frac{dV}{dx} = IZ$$

$$IZ = A\gamma\,e^{\gamma x} - B\gamma\,e^{-\gamma x}$$

$$= A\sqrt{ZY}\,e^{\sqrt{ZY}\,x} - B\sqrt{ZY}\,e^{-\sqrt{ZY}\,x} \qquad [\because \gamma = \sqrt{ZY}\,]$$

$$I = A\sqrt{\frac{Y}{Z}}\,e^{\sqrt{ZY}\,x} - B\sqrt{\frac{Y}{Z}}\,e^{-\sqrt{ZY}\,x} \quad \dots (1.7)$$

Similarly, differentiating the equation (1.6) w.r.t. 'x'

$$\frac{dI}{dx} = C\gamma\,e^{\gamma x} - D\gamma\,e^{-\gamma x}$$

$$\text{But } \frac{dI}{dx} = VY$$

$$VY = C\gamma\,e^{\gamma x} - D\gamma\,e^{-\gamma x}$$

$$= C\sqrt{ZY}\,e^{\sqrt{ZY}\,x} - D\sqrt{ZY}\,e^{-\sqrt{ZY}\,x}$$

$$V = C\sqrt{\frac{Z}{Y}}\,e^{\sqrt{ZY}\,x} - D\sqrt{\frac{Z}{Y}}\,e^{-\sqrt{ZY}\,x} \quad \dots (1.8)$$

Since the distance x is measured from the receiving end of the transmission line,

$$x = 0, \qquad \therefore \quad I = I_R$$

$$V = V_R$$

$$V_R = I_R Z_R$$

where I_R is the current in the receiving end of line

V_R is the voltage across the receiving end of the lines

Z_R is the impedance of receiving end

Substituting this condition in equations (1.5), (1.6), (1.7) and (1.8).

$$V_R = A + B \quad \ldots (1.9)$$

$$I_R = C + D \quad \ldots (1.10)$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \quad \ldots (1.11)$$

$$V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \quad \ldots (1.12)$$

To solve these equations,

$$\text{Let } x = \sqrt{\frac{Z}{Y}} \quad \text{and} \quad \frac{1}{x} = \sqrt{\frac{Y}{Z}}$$

$$\text{Then } I_R = \frac{A}{x} - \frac{B}{x}$$

$$= \frac{1}{x}(A - B)$$

$$\text{But } I_R = C + D$$

$$C + D = \frac{1}{x}(A - B)$$

$$Cx + Dx = A - B$$

$$A - B = Cx + Dx \quad \ldots (1.13)$$

Similarly, equation (1.12) becomes,

$$V_R = Cx - Dx$$

$$\text{But } V_R = A + B$$

$$A + B = Cx - Dx \quad \ldots (1.14)$$

$$A - B = Cx + Dx \quad \ldots (1.13)$$

Adding the equations (1.13) and (1.14),

$$2A = 2Cx$$

$$A = Cx$$

Similarly subtracting the equations (1.13) and (1.14),

$$2B = -2x\,Dx$$

$$B = -Dx$$

Substituting the values of A and B in the following equations.

$$V_R = A + B$$

$$= Cx - Dx$$

$$\text{But } I_R = C + D$$

$$I_R x = Cx + Dx \quad \text{... (1.15)}$$

$$V_R = Cx - Dx \quad \text{... (1.16)}$$

Adding the equations (1.15) and (1.16),

$$2Cx = I_R x + V_R$$

$$C = \frac{I_R}{2} + \frac{V_R}{2x}$$

$$\boxed{\therefore C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \quad \text{... (1.17)} \qquad \left[\because x = \sqrt{\frac{Z}{Y}}\right]$$

Subtracting the equations (1.15) and (1.16),

$$2Dx = I_R x - V_R$$

$$D = \frac{I_R}{2} - \frac{V_R}{2x}$$

$$\boxed{\therefore D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \quad \text{... (1.18)}$$

$$\text{But } A = Cx$$

$$A = \frac{I_R}{2}x + \frac{V_R}{2}$$

$$\boxed{\therefore A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \quad \text{... (1.19)}$$

$$B = -Dx$$

$$B = -\frac{I_R}{2}x + \frac{V_R}{2}$$

$$\boxed{\therefore B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \quad \text{... (1.20)}$$

The characteristic impedance is defined as

$$Z_o = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \text{... (1.21)}$$

Substituting the value of Z_o in equations (1.19), (1.20), (1.17) and (1.18),

$$A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$A = \frac{V_R}{2} + \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{A = \frac{V_R}{2}\left[1 + \frac{Z_o}{Z_R}\right]} \qquad \ldots (1.22)$$

$$B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$= \frac{V_R}{2} - \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{B = \frac{V_R}{2}\left[1 - \frac{Z_o}{Z_R}\right]} \qquad \ldots (1.23)$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} + \frac{I_R\,Z_R}{2\,Z_o} \qquad [\because V_R = I_R\,Z_R\,]$$

$$\boxed{C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \ldots (1.24)$$

$$D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} - \frac{I_R\,Z_R}{2\,Z_o}$$

$$\boxed{D = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \ldots (1.25)$$

Substituting the values of A, B, C and D in equations (1.5) and (1.6), the solutions of the differential equations are

$$V = \frac{V_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}\,x} \qquad \ldots (1.26)$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right) e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right) e^{-\sqrt{ZY}\,x} \qquad \ldots (1.27)$$

$$V = \frac{V_R}{2}\left[\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}\,x} + \left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}\,x}\right] \qquad \ldots (1.28)$$

$$I = \frac{I_R}{2}\left[\left(1+\frac{Z_R}{Z_o}\right)e^{\sqrt{ZY}\,x} + \left(1-\frac{Z_R}{Z_o}\right)e^{-\sqrt{ZY}\,x}\right] \qquad \ldots (1.29)$$

After simplification,

$$V = \frac{V_R}{2}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\frac{Z_o}{Z_R}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}e^{-\sqrt{ZY}\,x} - \frac{V_R}{2}\frac{Z_o}{Z_R}e^{-\sqrt{ZY}\,x}$$

$$I = \frac{I_R}{2}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\frac{Z_R}{Z_o}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}e^{-\sqrt{ZY}\,x} - \frac{I_R}{2}\frac{Z_R}{Z_o}e^{-\sqrt{ZY}\,x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + I_R Z_o\left(\frac{e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}}{2}\right) \qquad [\because V_R = I_R Z_R]$$

$$I = I_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + \frac{V_R}{Z_o}\left(e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}\right) \qquad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

Then equations can be written in terms of hyperbolic functions.

$$V = V_R \cosh\sqrt{ZY}\;x + I_R Z_o \sinh\sqrt{ZY}\;x \qquad \ldots (1.30)$$

$$I = I_R \cosh\sqrt{ZY}\;x + \frac{V_R}{Z_o}\sinh\sqrt{ZY}\;x \qquad \ldots (1.31)$$

These are the equations for voltage and current of a transmission line at any distance 'x' from the receiving end of transmission line.

The equations for voltage and current at the sending send of a transmission line of length 'l' are given by

$$V_S = V_R \cosh\sqrt{ZY}\,l + \frac{V_R}{Z_R} Z_o \sinh\sqrt{ZY}\,l \qquad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

$$I_S = I_R \cosh\sqrt{ZY}\,l + \frac{I_R Z_R}{Z_o}\sinh\sqrt{ZY}\;l \qquad [\because V_R = I_R Z_R]$$

$$V_S = V_R\left[\cos\sqrt{ZY}\;l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\;l\right] \qquad \ldots (1.32)$$

$$I_S = I_R\left[\cos\sqrt{ZY}\;l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\;l\right] \qquad \ldots (1.33)$$

1.4. WAVELENGTH AND VELOCITY OF PROPAGATION

The propagation constant (γ) and characteristic impedance (Z_0) are called secondary constants of a transmission line.

Propagation constant is usually a complex quantity.

$$\gamma = \alpha + j\beta$$

where α is the attenuation constant.

β is the phase shift.

$$\gamma = \sqrt{ZY}$$

where $Z = R + j\omega L$

$$Y = G + j\omega C$$

The characteristic impedance of the transmission line is also a complex quantity.

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \text{... (1.34)}$$

Propagation constant is $\gamma = \alpha + i\beta$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\alpha + i\beta = \sqrt{RG - \omega^2 LC + j\omega(LG + RC)} \quad \text{... (1.35)}$$

Squaring on both sides,

$$(\alpha + j\beta)^2 = RG - \omega^2 LC + j\omega(LG + RC)$$

$$\alpha^2 - \beta^2 + 2j\,\alpha\beta = RG - \omega^2 LC + j\omega\,(LG + RC) \quad \text{... (1.36)}$$

Equating real parts,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad \text{... (1.37)}$$

Equating imaginary parts,

$$2\,\alpha\beta = \omega\,(LG + RC)$$

Squaring on both sides,

$$4\,\alpha^2\,\beta^2 = \omega^2\,(LG + RC)^2$$

$$\alpha^2\,\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

Substituting the value of α^2 [eqn. (1.37)] in the above equation,

$$(\beta^2 + RG - \omega^2 LC)\,\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

$$\beta^4 + \beta^2\,(RG - \omega^2 LC) - \frac{\omega^2}{4}\,(LG + RC)^2 = 0$$

The solution of the quadratic equation is

$$\beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + RC)^2}}{2}$$

By neglecting the negative values,

$$\therefore \quad \beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \quad \ldots (1.38)$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad \ldots (1.37)$$

Substituting the value of β [eqn. (1.38)] in the above equation,

$$\alpha^2 = \frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2} + RG - \omega^2 LC$$

$$= \frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}$$

$$\therefore \quad \alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \quad \ldots (1.39)$$

For a perfect transmission line $R = 0$ and $G = 0$,

$$\beta^2 = \omega^2 LC$$

$$\therefore \quad \beta = \omega \sqrt{LC} \quad \text{[only positive value]}$$

Velocity :

The velocity of propagation is given by,

$$v = \lambda f$$

$$= 2\pi f \frac{\lambda}{2\pi}$$

$$v = \frac{\omega}{\beta} \quad \left[\because \beta = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi f\right]$$

Substituting the value of $\beta = \omega \sqrt{LC}$

$$\therefore \quad v = \frac{\omega}{\omega \sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

This is the velocity of propagation for an ideal line.

Wavelength :

The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\beta\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{\beta} \quad \text{or} \quad \lambda = \frac{v}{f}$$

1.5. INPUT IMPEDANCE AND TRANSFER IMPEDANCE OF TRANSMISSION LINE

Input impedance :

The equations for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right) \quad \text{... (1.32)}$$

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right) \quad \text{... (1.33)}$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$= \frac{I_R Z_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)} \quad \text{... (1.40)}$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the line is

$$Z_S = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

or

$$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

In a different form, the equations for voltage and current at transmitting end of a line is given by equations (1.28) and (1.29),

$$V_S = \frac{V_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,l}\right] \quad \text{... (1.28)}$$

$$I_S = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,l}\right] \quad \text{... (1.29)}$$

or $$V_S = \frac{V_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_R}\right) e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R}\right) e^{-\sqrt{ZY}\,l}\right]$$

$$I_S = \frac{I_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_0}\right) e^{\sqrt{ZY}\,l} + \left(\frac{Z_0-Z_R}{Z_0}\right) e^{-\sqrt{ZY}\,l}\right]$$

or $$V_S = \left(\frac{V_R}{2}\right)\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}\right] \quad \text{... (1.41)}$$

$$I_S = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}\right] \quad \text{... (1.42)}$$

The input impedance of the transmission line is given by,

$$Z_S = \frac{V_S}{I_S} = Z_0\left[\frac{e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}}{e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}}\right] \quad [\because V_R = I_R Z_R] \quad \text{... (1.43)}$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the transmission line is,

$$Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\gamma l}}\right] \quad \text{... (1.44)}$$

If the line is terminated with its characteristic impedance *i.e.*, $Z_R = Z_0$, then the input impedance becomes equal to its characteristic impedance.

$$Z_S = Z_0$$

The input impedance of an infinite line is determined by letting $l \to \infty$.

$$\therefore \; Z_S = Z_0$$

It is found that a line of finite length, terminated with its characteristic impedance, appears to the transmitting end generator as an infinite line. A finite line terminated with Z_0 and an infinite line are same by measurements at the source.

If $K = \frac{Z_R-Z_0}{Z_R+Z_0}$, then

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K\, e^{-\gamma l}}{e^{\gamma l} - K\, e^{-\gamma l}}\right] \quad \text{... (1.45)}$$

Transfer impedance :

Transfer impedance is used to determine the current at the receiving end if voltage at transmitting end is known. Transfer impedance of a transmission line is defined as the ratio of voltage at the sending end (transmitted voltage) to the current at the receiving end (received current).

$$Z_T = \frac{V_S}{I_R}$$

Equation (1.41) becomes

$$V_S = \frac{V_R (Z_R + Z_0)}{2 Z_R} (e^{\gamma l} + K e^{-\gamma l})$$

$$V_S = \frac{I_R (Z_R + Z_0)}{2} (e^{\gamma l} + K e^{-\gamma l}) \qquad [\because V_R = I_R Z_R]$$

$$Z_T = \frac{V_S}{I_R} = \frac{Z_R + Z_0}{2} (e^{\gamma l} + K e^{-\gamma l})$$

$$= \frac{Z_R + Z_0}{2} \left(e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l} \right)$$

$$= \left(\frac{Z_R + Z_0}{2} \right) e^{\gamma l} + \left(\frac{Z_R - Z_0}{2} \right) e^{-\gamma l}$$

$$= Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right)$$

$$= Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

$$\left[\because \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l \text{ and } \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \right]$$

$$Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

1.6. LINE DISTORTION

Signal (*e.g.*, voice) transmitted over a transmission line is normally complex and consists of many frequency components. Such voice voltage will not have all frequencies transmitted with equal attenuation and equal time delay, the received waveform will not be identical with the input waveform at the sending end. This variation is known as ***distortion***. There are two types of line distortions. They are frequency distortion and delay distortion.

Frequency Distortion : A complex (voice) voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

The attenuation constant is given by

$$\alpha = \sqrt{\frac{RG - \omega^2LC + \sqrt{(RG - \omega^2LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

α is a function of frequency and therefore the line will introduce frequency distortion.

Delay or Phase Distortion : For an applied voice-voltage wave the received waveform may not be identical with the input waveform at the sending end, since some frequency components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion.

The phase constant is

$$\beta = \sqrt{\frac{RG - \omega^2LC + \sqrt{(RG - \omega^2LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

β is not a constant multiplied by ω and therefore the line will introduce delay distortion.

Frequency distortion is reduced in the transmission of high quality over wire lines by the use of equalizers at the line terminals.

Delay distortion is of relatively less importance to voice and music transmission. But it can be very serious for video transmission. This can be avoided by the use of co-axial cables.

1.7. THE DISTORTIONLESS LINE

If a line is to have neither frequency nor delay distortion, then attenuation factor α and the velocity of propagation v cannot be functions of frequency.

If $\quad v = \dfrac{\omega}{\beta}$

β must be a direct function of frequency.

$$\beta = \sqrt{\frac{\omega^2LC - RG + \sqrt{(RG - \omega^2LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

For β to be a direct function of frequency, the term

$(RG - \omega^2LC)^2 + \omega^2 (LG + CR)^2$ must be equal to $(RG + \omega^2LC)^2$

$$R^2G^2 + \omega^4L^2C^2 - 2\omega^2LCRG + \omega^2L^2G + \omega^2C^2R^2 + 2\omega^2LCRG$$

$$= R^2G^2 + \omega^4L^2C^2 + 2\omega^2LCRG$$

$$\omega^2L^2G^2 + \omega^2C^2R^2 = 2\,\omega^2LCRG$$

$$\omega^2L^2G^2 + \omega^2C^2R^2 - 2\omega^2LCRG = 0$$

$$(LG - CR)^2 = 0$$

$$LG = CR$$

$$\frac{R}{L} = \frac{G}{C}$$

This is the condition for distortionless line.

$$\text{Propagation constant } \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

$$= \sqrt{LC}\sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

$$\text{But } \frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$$

$$\text{Then} \quad \beta = \sqrt{\frac{\omega^2 LC - RG + RG + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2\omega^2 LC}{2}}$$

$$\beta = \omega\sqrt{LC}$$

Velocity of propagation is

$$v = \frac{\omega}{\beta}$$

$$v = \frac{1}{\sqrt{LC}}$$

This is the same velocity for all frequencies, thus eliminating delay distortion.

Attenuation factor

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

To make α is independent of frequency, the term $(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2$ is forced to be equal to $(RG + \omega^2 LC)^2$.

$$(LG - CR)^2 = 0$$

$$LG = CR$$

$$\boxed{\frac{L}{C} = \frac{R}{G}}$$

This will make α and the velocity independent of frequency simultaneously. To achieve this condition, it requires a very large value of L, since G is small.

The attenuation factor

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{RG - \omega^2 LC + RG + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2RG}{2}}$$

$$\alpha = \sqrt{RG}$$

It is independent of frequency, thus eliminating frequency distortion on the line.

The characteristic impedance Z_0 is given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{L\left(\frac{R}{L} + j\omega\right)}{C\left(\frac{G}{C} + j\omega\right)}}$$

But $\frac{R}{L} = \frac{G}{C}$ for distortionless line.

$$\therefore \; Z_0 = \sqrt{\frac{L}{C}}$$

It is purely real and is independent of frequency.

1.8. TELEPHONE CABLE

In the telephone cable the wires are insulated with paper and twisted in pairs. This construction results in negligible values of inductance and conductance. Therefore $L\omega \ll R$ and $G \ll C\omega$.

$$Z = R + j\omega L \approx R$$

$$Y = G + j\omega C \approx j\omega C$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \sqrt{j\omega RC}$$

$$= \sqrt{\frac{j2\omega RC}{2}}$$

But $\gamma = \alpha + j\beta$

$$\alpha + j\beta = (1 + j)\sqrt{\frac{\omega RC}{2}}$$

Equating real and imaginary parts $\alpha = \sqrt{\frac{\omega RC}{2}}$

$$\beta = \sqrt{\frac{\omega RC}{2}}$$

Velocity of propagation $v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega RC}{2}}} = \sqrt{\frac{2\omega}{RC}}$

The characteristic impedance $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}} \angle -45°$

It is found that the propagation constant α and velocity of propagation v are functions of frequency. Thus, the higher frequencies are attenuated more and travel faster than the lower frequencies resulting in considerable frequency and delay distortion.

1.9. LOADING OF LINES

It is necessary to increase L/C ratio to achieve distortionless condition in a transmission line. This can be done by increasing the inductance of a transmission line. Increasing inductance by inserting inductances in series with line is termed as ***loading*** and such lines are called ***loaded lines.*** The lumped inductors, known as ***loading coils*** are placed at suitable intervals along the transmission line to increase the effective distributed inductance.

The effect of loading can be realised by comparing the unloading of a transmission line in the attenuation Vs frequency graph. Fig.1.5 shows that the loaded line offers a low attenuation when compared to the unloaded line only for limited range of frequencies.

The important aspect of loading coil design is that saturation and stray fields should be avoided. It should have a low resistance and should be in small size. In general toroidal cores are used for loading coils.

Types of Loading

The open wire lines have more inductance of their own and so have much less distortion than cable. Therefore, the loading practice is not applicable to open wires but it is restricted to cables only. There are three types of loading in practice. They are

(a) Lumped loading

(b) Continuous loading

(c) Patch loading

(a) Lumped loading : The inductance of a transmission line can be increased by the introduction of loading coil at uniform intervals. This is called lumped loading. It acts as a low pass filter. So, it is applicable only for a limited range of frequency. The loading coils have an internal resistance R thus, increasing the total effective inductance increases R. Further hysteresis and eddy current losses which occur in the loading coils resulting in further apparent increase in R. Therefore, there is a practical limitation on the value of inductance that can be increased for the reduction of attenuation. Thus the loading coil should be carefully designed so that it will not introduce any distortion.

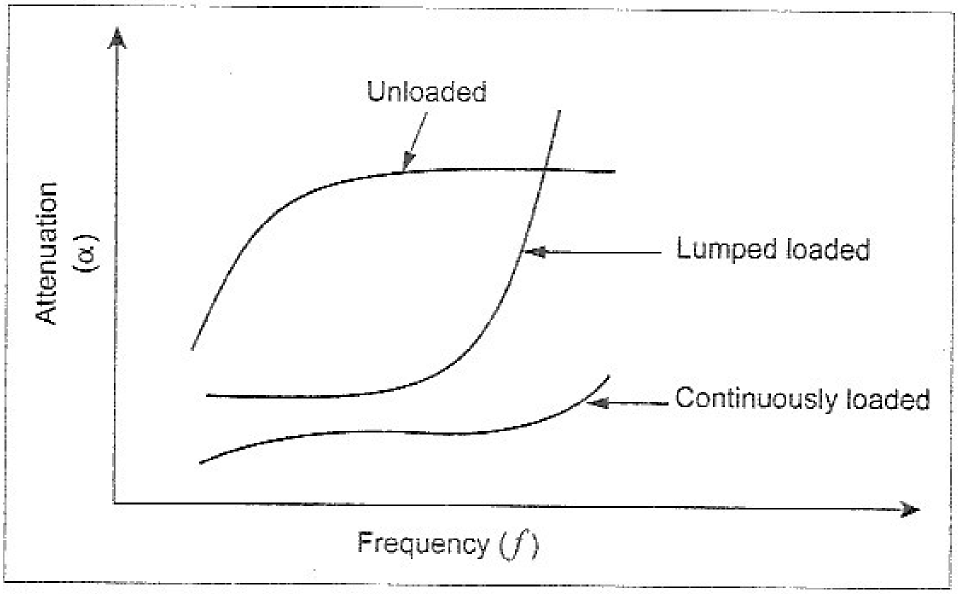


Fig. 1.5. Comparison of loaded and unloaded cable characteristics

(b) Continuous loading : A type of iron or some other magnetic material is wound on the transmission line (cable) to increase the permeability of the surrounding medium and thereby increase the inductance. It is a quite expensive method. Further eddy current and hysteresis losses in the magnetic material increases the primary constant R. Therefore, continuous loading is used only on ocean cables where lumped loading is difficult. The advantage of continuous loading over lumped loading is that attenuation factor α increases uniformly with increase in frequency.

(c) Patch loading : It employs sections of continuously loaded cable separated by sections of unloaded cable. The typical length for the section is normally a quarter kilometer. In this method the advantage of continuous loading is obtained and the cost is reduced considerably.

1.9.1. Inductance Loading of Telephone Cables

Distortionless line with distributed parameters is used to avoid the frequency and delay distortion experienced on telephone cables. It is necessary to increase the L/C to achieve distortionless condition $\frac{L}{C} = \frac{R}{G}$. Heaviside suggested that the inductance be increased and Pupin suggested that this increase in the inductance by lumped inductors spaced at intervals along the line. This use of inductance is called loading the line. The distributed loading is obtained by winding the cable with a high permeability steel tape such as permalloy in some submarine cables.

Consider an uniformly loaded cable with G = 0. Then,

$$Z = R + j\omega L$$

$$Y = j\omega C \qquad [\because G = 0]$$

$$Z = \sqrt{R^2 + (L\omega)^2} \;\Big|\; \tan^{-1}\left(\frac{L\omega}{R}\right)$$

$$= \sqrt{R^2 + (L\omega)^2} \angle \frac{\pi}{2} - \tan^{-1}\frac{R}{L\omega}$$

$$\text{Propagation constant } \gamma = \sqrt{ZY}$$

$$= \sqrt{\sqrt{R^2 + (L\omega)^2} \angle \frac{\pi}{2} - \tan^{-1}\frac{R}{L\omega} \left(\omega C \angle \frac{\pi}{2}\right)}$$

$$= \sqrt{\omega C \sqrt{R^2 + (L\omega)^2} \angle \pi - \tan^{-1}\frac{R}{L\omega}}$$

$$= \sqrt{(\omega C)(L\omega)\sqrt{1 + \frac{R^2}{(L\omega)^2}}} \angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

$$= \omega\sqrt{LC} \sqrt[4]{1 + \left(\frac{R}{L\omega}\right)^2} \angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

Since R is small with respect to $L\omega$, the term $\left(\frac{R}{L\omega}\right)$ is neglected.

$$\therefore \gamma = \omega\sqrt{LC} \angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

$$\text{If } \theta = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

$$\cos\theta = \cos\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

$$= \sin\left(\frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

For small angle,

$$\sin\theta \approx \tan\theta \approx \theta$$

so that
$$\cos\theta = \frac{R}{2L\omega}$$

Similarly,
$$\sin\theta = \sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right) = 1$$

$$\text{Propagation constant } \gamma = \omega\sqrt{LC}\,(\cos\theta + j\sin\theta)$$

$$= \omega\sqrt{LC}\left(\frac{R}{2L\omega} + j\right)$$

$$\gamma = \frac{R\sqrt{LC}}{2L} + j\omega\sqrt{LC}$$

$$= \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\therefore \text{ Attenuation constant } \alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\text{Phase-shift } \beta = \omega\sqrt{LC}$$

$$\text{Velocity of propagation } v = \frac{\omega}{\beta}$$

$$= \frac{1}{\sqrt{LC}}$$

It is noted that if G = 0 and Lω >> R, the attenuation and velocity are both independent of frequency and the loaded cable will be distortionless. Attenuation may be reduced by increasing L. Continuous (uniform) loading is expensive and achieves only a small increase in L per unit length. Lumped loading is preferred for cables.

Campbell's Equation

An analysis for the performance of a line loaded at uniform intervals can be obtained by considering a symmetrical section of line from the centre of one loading coil to the centre of the next coil. The section of line may be replaced with an equivalent T section having symmetrical series arms as shown in Fig.1.6. The series arm of T section including loading coil is given by

$$\frac{Z_1'}{2} = \frac{Z_c}{2} + \frac{Z_1}{2} \qquad \text{[From the fig.]}$$

where $\frac{Z_1}{2}$ is the series arm of T section.

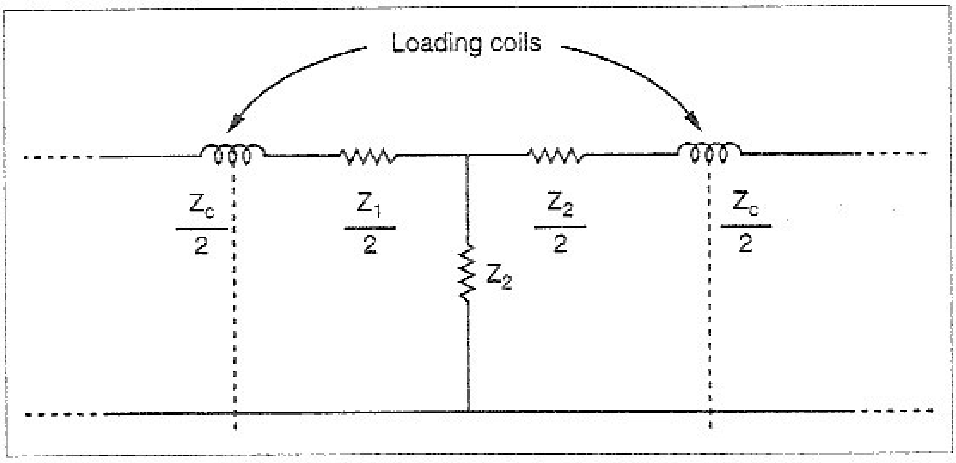


Fig. 1.6. Equivalent T section for part of a line between two lumped loading coils

$$\frac{Z_1}{2} = Z_o \tanh\frac{\gamma l}{2}$$

$$\therefore \frac{Z_1'}{2} = \frac{Z_c}{2} + Z_o \tanh\frac{\gamma l}{2}$$

where l is the distance between two loading coils.

The shunt Z_2 arm of the equivalent T section is

$$Z_2 = \frac{Z_o}{\sinh \gamma l}$$

For loaded T section

$$\cosh \gamma' l = 1 + \frac{Z_1'}{2 Z_2}$$

$$= 1 + \frac{\frac{Z_c}{2} + Z_o \tanh \frac{\gamma l}{2}}{\frac{Z_o}{\sinh \gamma l}}$$

$$\text{But } \tanh \frac{\gamma l}{2} = \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

Substituting this value in above equation

$$\therefore \cosh \gamma' l = 1 + \frac{\frac{Z_c}{2} + Z_o \frac{\cosh \gamma l - 1}{\sinh \gamma l}}{\frac{Z_o}{\sinh \gamma l}}$$

$$= 1 + \frac{\frac{Z_c}{2} \sinh \gamma l + Z_o (\cosh \gamma l - 1)}{Z_o}$$

$$= 1 + \frac{Z_c}{2 Z_o} \sinh \gamma l + \cosh \gamma l - 1$$

$$\cosh \gamma' l = \frac{Z_c}{2 Z_o} \sinh \gamma l + \cosh \gamma l$$

This equation is called as Campbell's equation and it is used to determine the value of γ' of a line section consisting of partially lumped and partially distributed elements. For a cable Z_2 is capacitance and the cable capacitance and lumped inductances appear similar to the circuit of the low pass filter. It is found that for frequencies below cutoff, the attenuation is reduced, but the cut-off attenuation is increased (as a result of filter action). In practice, pure distortionless line is not obtained by loading, because R and L are to some extent functions of frequency. Eddy current losses are more in these coils. However, there is a major improvement in the loaded cable over the unloaded cable for a reasonable frequency range.

1.10. OPEN CIRCUITED AND SHORT CIRCUITED LINES

The expressions for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left[\cosh\sqrt{ZY}\, l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\, l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\, l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\, l\right]$$

The input impedance of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{V_R\left[\cosh\sqrt{ZY}\, l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\, l\right]}{I_R\left[\cosh\sqrt{ZY}\, l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\, l\right]}$$

$$= \frac{V_R}{I_R}\,\frac{Z_o}{Z_R}\,\frac{(Z_R\cosh\gamma l + Z_o\sinh\gamma l)}{(Z_o\cosh\gamma l + Z_R\sinh\gamma l)}$$

$$= Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right) \qquad \left[\because Z_R = \frac{V_R}{I_R}\right]$$

$$Z_S = Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right)$$

If short circuited, the receiving end impedance is zero.

$$i.e., \quad Z_R = 0$$

$$\therefore Z_{sc} = Z_o\left(\frac{Z_o\sinh\gamma l}{Z_o\cosh\gamma l}\right)$$

Short circuited impedance

$$Z_{sc} = Z_o\tanh\gamma l$$

If open circuited, the receiving end impedance is infinite.

i.e., $$Z_R = \infty$$

Input impedance of transmission line can be written as

$$Z_S = Z_o\left[\frac{\cosh\gamma l + \frac{Z_o}{Z_R}\sinh\gamma l}{\frac{Z_o}{Z_R}\cosh\gamma l + \sinh\gamma l}\right]$$

Applying $Z_R = \infty$

$$\text{Then } Z_{oc} = Z_o \left[\frac{\cosh \gamma l}{\sinh \gamma l} \right]$$

The open circuited impedance

$$Z_{oc} = Z_o \coth \gamma l$$

By multiplying open circuited impedance and short circuited impedances

$$Z_{oc} Z_{sc} = Z_o^2 \tanh \gamma l \coth \gamma l$$

$$= Z_o^2$$

The characteristic impedance is given by

$$Z_o = \sqrt{Z_{oc} Z_{sc}}$$

By dividing short circuited impedance by open circuited impedance.

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_o \tanh \gamma l}{Z_o \coth \gamma l}$$

$$= \tan^2 h\, \gamma l$$

$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\gamma l = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

1.11. REFLECTION

When the load impedance is not equal to the characteristic impedance of transmission line, reflection takes place.

The expressions for voltage and current on the transmission line are

$$V = \frac{V_R}{2} \left[\left(1 + \frac{Z_o}{Z_R} \right) e^{\sqrt{ZY}\, x} + \left(1 - \frac{Z_o}{Z_R} \right) e^{-\sqrt{ZY}\, x} \right]$$

$$I = \frac{I_R}{2} \left[\left(1 + \frac{Z_R}{Z_o} \right) e^{\sqrt{ZY}\, x} + \left(1 - \frac{Z_R}{Z_o} \right) e^{-\sqrt{ZY}\, x} \right]$$

or

$$V = \frac{V_R}{2} \left[\frac{Z_R + Z_o}{Z_R} e^{\sqrt{ZY}\, x} + \frac{Z_R - Z_o}{Z_R} e^{-\sqrt{ZY}\, x} \right]$$

$$I = \frac{I_R}{2} \left[\frac{Z_R + Z_o}{Z_o} e^{\sqrt{ZY}\, x} - \frac{Z_R - Z_o}{Z_o} e^{-\sqrt{ZY}\, x} \right]$$

or

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o}\right) e^{-\gamma x} \right]$$

$$I = \frac{I_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} - \left(\frac{Z_R - Z_o}{Z_R + Z_o}\right) e^{-\gamma x} \right]$$

$$[\because \gamma = \sqrt{ZY}]$$

If the transmission line is not terminated with the characteristic impedance *i.e.*, $Z_R \neq Z_o$ (mismatch) the above expressions for voltage and current exist. It consists of two waves, one is moving in the forward (positive x) direction which is called incident wave and the other is moving in the opposite (negative x) direction which is called reflected ray. The term varying with $e^{\gamma x}$ represents a wave progressing from the sending end towards the receiving end and the amplitude decreasing with increased distance. The term varying with $e^{-\gamma x}$ represents a wave progressing from the receiving end towards the sending end, decreasing in amplitude with increased distance.

If the transmission line is terminated with characteristic impedance *i.e.*, $Z_R = Z_o$ (properly matched) then the voltage and current expressions are

$$V = V_R e^{\gamma x}$$

$$I = I_R e^{\gamma x}$$

The incident wave moves only in forward (positive x) direction. There is no reflected wave in the opposite direction.

1.11.1. Reflection Coefficient

Reflection coefficient is defined as the ratio of the reflected voltage to the incident voltage at the receiving end of the line.

$$K = \frac{\text{Reflected voltage at load}}{\text{Incident voltage at load}} = \frac{V_R}{V_S}$$

The equation for the voltage of a transmission line is

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o}\right) e^{-\gamma x} \right]$$

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} e^{\gamma x} + \frac{V_R (Z_R - Z_o)}{2 Z_R} e^{-\gamma x}$$

The first term ($e^{\gamma x}$) represents incident wave, whereas the second term ($e^{-\gamma x}$) represents the reflected wave. The ratio of amplitude of the reflected wave voltage to the amplitude of the incident wave voltage is nothing but reflection coefficient.

$$K = \frac{\dfrac{V_R (Z_R - Z_o)}{2 Z_R}}{\dfrac{V_R (Z_R + Z_o)}{2 Z_R}} = \frac{Z_R - Z_o}{Z_R + Z_o}$$

$$\boxed{K = \frac{Z_R - Z_o}{Z_R + Z_o}}$$

It is also defined as in terms of the ratio of the reflected current to the incident current. But it is negative.

$$-K = \frac{\text{Reflected current at load}}{\text{Incident current at load}} = \frac{I_R}{I_S}$$

If the transmission line is terminated by its characteristic impedance ($Z_R = Z_o$), the reflection coefficient becomes zero.

1.11.2. Reflection Factor and Reflection Loss

Consider a transmission line with a voltage source V_S and its impedance Z_1 and load impedance Z_2 as shown in Fig.1.7. If Z_2 is not equal to Z_1, reflection takes place. The power delivered to the load is less than that with impedance matching. Reflection results in power loss. This loss is known as reflection loss.

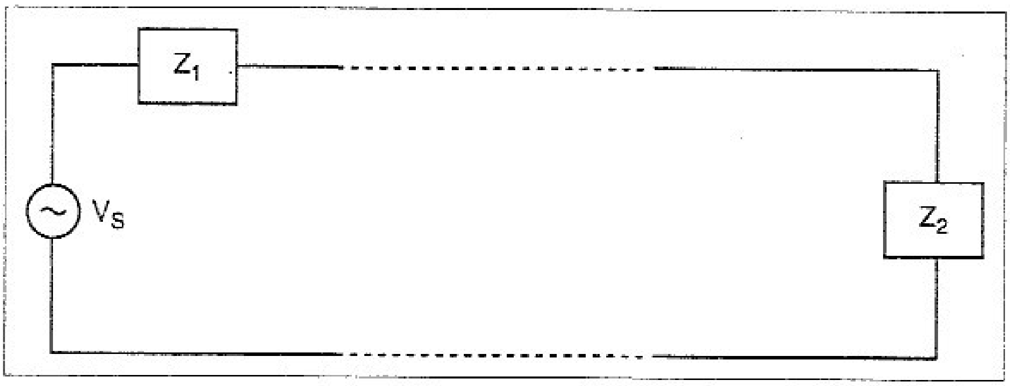


Fig. 1.7. Transmission line with voltage source V_S and impedance Z_1

Image matching between the impedances Z_1 and Z_2 can be obtained by inserting an ideal transformer and a phase shifting network between Z_1 and Z_2. If I_1 and I_2 be the currents in the primary and secondary of the transformer respectively, the current ratio of the transformer is given by

$$\frac{I_2}{I_1} = \sqrt{\frac{Z_1}{Z_2}}$$

Z_2 may be adjusted to that of Z_1 by choosing the proper transformation ratio and phase angle. Z_2 is the image impedance of Z_1. The current through the source is

$$I_1 = \frac{V_S}{2 Z_1}$$

The current flow in the secondary of the transformer under image impedance matching is

$$I_2' = I_1 \sqrt{\frac{Z_1}{Z_2}} = \frac{V}{2 Z_1} \sqrt{\frac{Z_1}{Z_2}} = \frac{V_S}{2\sqrt{Z_1 Z_2}}$$

The current in the load impedance Z_2 without image matching.

$$|I_2| = \frac{|V_S|}{|Z_1 + Z_2|}$$

The ratio of the current actually flowing in the load to that which might flow under matched condition is known as ***reflection factor.***

$$\left|\frac{I_2}{I_2'}\right| = \frac{\dfrac{|V_S|}{|Z_1 + Z_2|}}{\dfrac{|V_S|}{|2\sqrt{Z_1 Z_2}|}}$$

$$k = \left|\frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2}\right|$$

The reflection factor indicates the change in current in the load due to reflection at the mismatched junction.

The ***reflection loss*** is the reciprocal of the reflection factor in nepers or dB.

$$\text{Reflection loss} = ln \frac{1}{k}$$

$$= ln \left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right| \text{ nepers}$$

$$= 20 \log \left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right| \text{ dB}$$

1.12. T AND π SECTIONS EQUIVALENT TO LINES

A T section is shown in Fig.1.8 with two ports 1, 1 and 2, 2.

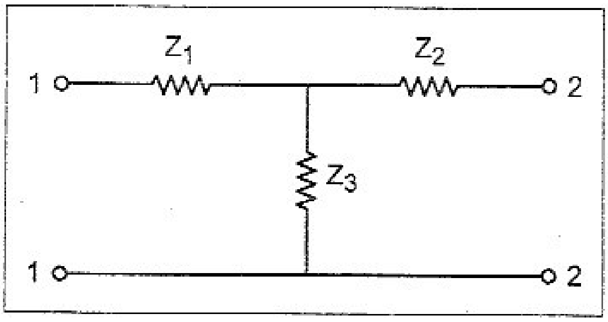


Fig. 1.8. T section network

Impedance measurements may be made at any port with the other port opened or shorted.

Let Z_{1OC} be the impedance at port 1 when port 2 is open circuited.

Z_{1SC} be the impedance at port 1 when port 2 is short circuited.

Z_{2OC} be the impedance at port 2 when port 1 is open circuited.

Z_{2SC} be the impedance at port 2 when port 1 is short circuited.

$$Z_{1OC} = Z_1 + Z_3$$

$$Z_{1SC} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$Z_{2OC} = Z_2 + Z_3$$

$$Z_{2SC} = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}$$

By solving these equations, the values of Z_1, Z_2 and Z_3 are determined.

$$Z_{1OC} - Z_{1SC} = Z_3 - \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{Z_3 Z_2 + Z_3^2 - Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{Z_3^2}{Z_2 + Z_3}$$

$$= \frac{Z_3^2}{Z_{2OC}} \qquad [\because Z_2 + Z_3 = Z_{2OC}]$$

$$Z_3^2 = Z_{2OC}(Z_{1OC} - Z_{1SC})$$

$$Z_3 = \pm\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

Taking the positive value,

$$Z_3 = \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

$$Z_1 = Z_{1OC} - Z_3 \qquad [\because Z_{1OC} = Z_1 + Z_3]$$

$$= Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

$$Z_2 = Z_{2OC} - Z_3 \qquad [\because Z_{2OC} = Z_2 + Z_3]$$

$$= Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

$$Z_1 = Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$

$$Z_2 = Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}$$