University Questions Solution

Part-B. Unit - II

AARUPADAI VEEDU INSTITUTE OF TECHNOLOGY TLWG Sheet No.

Q1 i) Explain physical Significance of a General Solution of transmission line

Soln From the Equations the Sending end current can be obtained by substituting $s = l$ measured from the receiving end.

$$E_S = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l) \quad \text{--- (1)}$$

and

$$I_S = I_R \cosh(\gamma l) + \frac{E_R}{Z_0} \sinh(\gamma l) \quad \text{--- (2)}$$

Now $Z_R = \frac{E_R}{I_R}$, $Z_R \cdot I_R = E_R$

Sub in (2)

$$\therefore I_S = I_R \cosh(\gamma l) + \frac{Z_R}{Z_0} \cdot I_R \sinh(\gamma l)$$

$$I_S = I_R \left[\cos(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right] \quad \text{--- (3)}$$

Now if the line is terminated in its Characteristic impedance Z_0 then,

$$I_S = I_R \left[\cosh(\gamma l) + \sinh(\gamma l)\right] \quad \text{as } Z_R = Z_0$$

$$\frac{I_S}{I_R} = \left[\cosh(\gamma l) + \sinh(\gamma l)\right] = e^{\gamma l} \quad \text{--- (4)}$$

This is the Equation which is already derived from the line terminated in Z_0, then,

~~...~~ $E_R = I_R Z_R$ in eqn (1)

$$E_S = Z_R I_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$E_S = I_R \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)\right] \quad \text{--- (5)}$$

Divide (5) by (3)

$$\frac{E_S}{I_S} = \frac{I_R \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)\right]}{I_R \left[\cosh(\gamma l) + Z_R/Z_0 \sinh(\gamma l)\right]}$$

But $\frac{E_S}{I_S} = Z_S$

$$Z_S = \frac{Z_0 \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)\right]}{\left[Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l)\right]} \quad \text{--- (6)}$$

We Know that

When the line is terminated in Z_0 then $Z_R = Z_0$. So substituting the equation ⑥ we get,

$$Z_s = Z_0$$

This shows that for a line terminated in its characteristic impedance its input impedance is also its characteristic impedance.

Now consider an infinite line with $l \to \infty$ using equ ⑥ we get

$$Z_s = \frac{Z_0 \left[Z_R + Z_0 \tanh(\gamma l) \right]}{\left[Z_0 + Z_R \tanh(\gamma l) \right]}$$

and $\tan(\gamma l) \to 1$

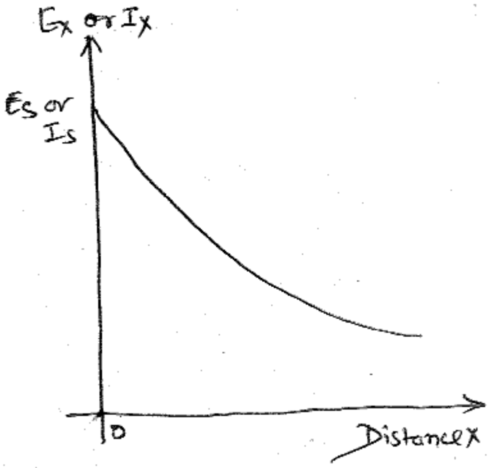
as $l \to \infty$

$$Z_s = Z_0.$$

This shows that finite line terminated in its characteristic impedance behaves as an infinite line, to the sending end generator.

Thus the equations for E_x and I_x are applicable for the finite line terminated in Z_0. The equations are reproduced here for the convenience of the reader.

$$E_x = E_s\, e^{-\gamma x} \quad \text{and} \quad I_x = I_s\, e^{-\gamma x}$$



If in practice instruments are connected along the line then the instruments will show the magnitudes $E_s e^{-\alpha x}$ and $I_s e^{-\alpha x}$ while the phase angles cannot be obtained. If the graph of E_x or I_x is plotted against x then it can be shown in the fig.

This is the physical significance of a general solution of a transmission line. Its use will be more clear by studying the various cases of the line.

1.25

Q1 ii) Describe the expression of a line not terminated in Z_0.

Sol^n If a line is not terminated in Z_0 then part of the wave is reflected back from the distance and or from the point of discontinuity, thus reflection occurs in a line which is not terminated in Z_0.

Reflection is max^m when the line is open ckt or short ckt $Z_R = \infty$, $Z_R = 0$ respectively.

No reflection occurs when $Z_R = Z_0$ from the general soln of a line we can write.

$$E = \frac{E_R (Z_R + Z_0)}{2Z_R} e^{\gamma s} + \frac{E_R (Z_R - Z_0)}{2Z_R} e^{-\gamma s} \quad \text{—— (1)}$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_R} e^{\gamma s} - \frac{I_R (Z_R - Z_0)}{2Z_R} e^{-\gamma s} \quad \text{—— (2)}$$

The first component of E or I which varies exponentially with positives is called incident wave which flow's from sending end to receiving end with decreasing amplitude.

$$E_1 = \frac{E_R (Z_R + Z_0)}{2Z_R} e^{\gamma s} \rightarrow \text{Incident voltage wave}$$

$$I_1 = \frac{I_R (Z_R + Z_0)}{2Z_R} e^{\gamma s} \rightarrow \text{Incident current wave.}$$

The Second component of E or I which varies exponentially with -ve 's' is called reflected wave.

The wave which flows from receiving end to the Sending end with decreasing amplitude is called reflected wave.

~~We know that~~

$$E_2 = \frac{E_R(Z_R - Z_0)}{2Z_R} e^{-\gamma s} \rightarrow \text{Reflected voltage wave.}$$

and $$I_2 = \frac{I_R(Z_R - Z_0)}{2Z_R} e^{-\gamma s} \rightarrow \text{Reflected current wave.}$$

Sh Consider.

$$Z_R = \infty, \quad s = 0$$

$$E_i = \frac{E_R\left(1 + \frac{Z_0}{Z_R}\right) e^{\gamma s}}{2} = \frac{E_R}{2} e^{\gamma s}$$

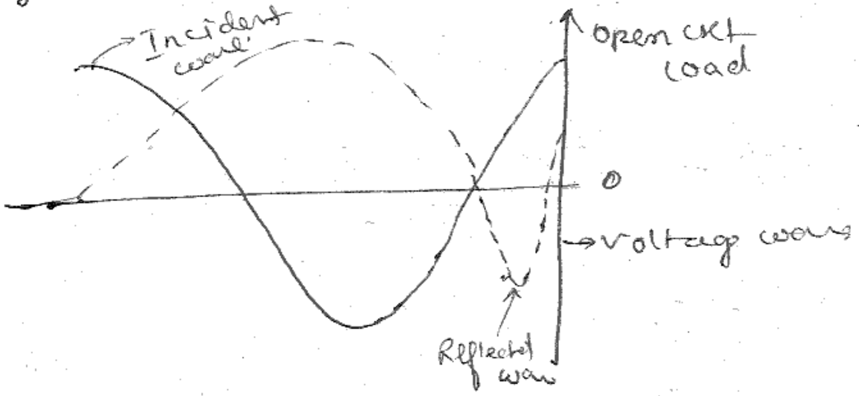
Su

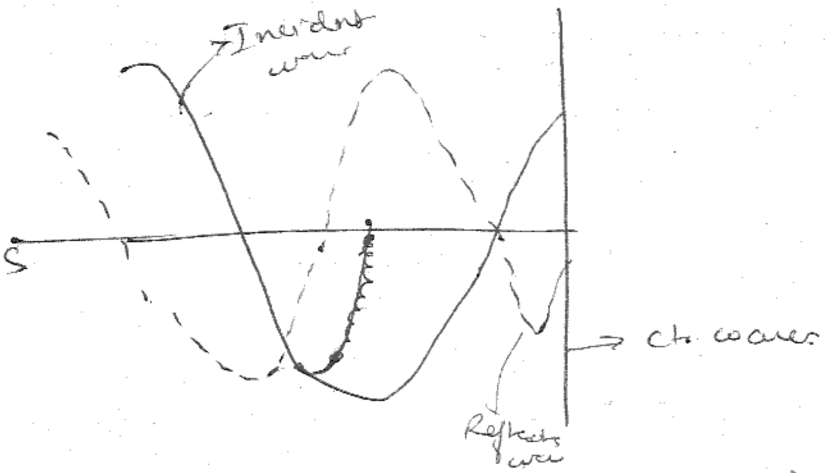
$$\boxed{E_1 = E_R/2}$$

Similarly

$$\boxed{E_2 = \frac{E_R}{2}}$$

Initially value of reflected wave is equal to incident voltage at load on open.ckt.





A fine line terminated in Z_0 without having any reflections is called smooth line.

Q2 A generator of 1 V. 1KHz supplies power to a 100 Km open wire line terminated in 200Ω resistance. The line parameters are R = 10 Ω/Km L = 3.8 mH/Km, G = 1×10⁻⁶ mho/Km, C = 0.0085 μF/Km. Calculate the input impedance reflection coefficient and sending end current.

Solⁿ The characteristics impedance is given by.

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{10+j(2\pi\times1\times10^3)3.8\times10^{-3}}{1\times10^{-6}+j(2\pi\times1\times10^3)0.0085\times10^{-6}}}$$

$$= \sqrt{\frac{10+j23.876}{1\times10^{-6}+j5.34\times10^{-5}}} = \sqrt{\frac{25.885\angle 67.27^\circ}{5.34\times10^{-5}\angle 88.92^\circ}}$$

$$= \sqrt{4.847\times10^{5}\angle -21.65^\circ} = 696.204\angle -10.825^\circ\ \Omega$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{25.885\times5.34\times10^{-5}\angle 67.27^\circ+88.92^\circ}$$

$$= 0.03717\angle 78.095^\circ = 0.0076 + j0.03637$$

$\alpha = 0.0076$ neper/km.

and $\beta = 0.03637$ rad/Km

$K = \frac{Z_R - Z_0}{Z_R + Z_0}$ where $Z_R = 200\angle 0^\circ\ \Omega$ given.

$$K = \frac{200\angle 0^\circ - 696.204\angle -10.825^\circ}{200\angle 0^\circ + 696.204\angle -10.825^\circ}$$

$$K = \frac{200+j0-683.8153+j130.754}{200+j0+683.8153-j130.754} \Rightarrow \frac{-483.8153+j130.754}{883.8153-j130.754}$$

$$\Rightarrow \frac{501.1724\angle 164.876^\circ}{893.435\angle -8.415^\circ} \Rightarrow \boxed{K = 0.5609\angle 173.291^\circ}$$

We

Now, $e^{\gamma l} = e^{\alpha l} \angle \beta l \text{ rad} = e^{0.00766 \times 100} \angle 0.03637 \times 100 \text{ rad}$.

$= 2.1511 \angle 3.637 \text{ rad} = 2.1511 \angle 208.4^\circ$

an

$$Z_s = Z_0 \left\{ \frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right\}$$

Si

$e^{-\gamma l} = e^{-\gamma l} \angle -\beta l \text{ rad} = e^{-0.00766 \times 100} \angle -0.03637 \times 100$ and

$= 0.4648 \angle -208.4^\circ$

Si

$$\therefore Z_s = 696.204 \angle -10.825^\circ \left\{ \frac{2.1511 \angle 208.4^\circ + 0.5609 \angle 173.29^\circ \times 0.4648 \angle -208.4^\circ}{2.1511 \angle 208.4^\circ - 0.5609 \angle 173.29^\circ \times 0.4648 \angle -208.4^\circ} \right\}$$

$$= 696.204 \angle -10.850 \left\{ \frac{2.1511 \angle 208.4^\circ + 0.2607 \angle -35.11^\circ}{2.1511 \angle 208.4^\circ - 0.2607 \angle -35.11^\circ} \right\}$$

$$= 696.204 \angle -10.85 \left\{ \frac{-1.8922 - j1.023 + 0.2132 - j0.1499}{-1.8922 - j1.023 - 0.2132 + j0.1499} \right\}$$

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$$= 696.204 \angle -10.85^\circ \left\{ \frac{-1.679 - j1.1729}{-2.1054 - j0.8731} \right\}$$

$$= 696.204 \angle -10.85^\circ \left\{ \frac{.20481 \angle -145.062^\circ}{2.2792 \angle -157.476^\circ} \right\}$$

$= 625.6122 \angle +1.564^\circ \ \Omega$

It is Know that.

$$I = \frac{I_R (Z_R + Z_0)}{2 Z_0} \cdot \left[e^{\gamma s} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma s} \right]$$

At sending end, $s = l$ as s is measured from the receiving end.

$$I_s = \frac{I_R (Z_R + Z_0)}{2 Z_0} \left[e^{-\gamma l} - K e^{-\gamma l} \right] \quad \text{——— (1)}$$

Now, $Z_s = \frac{E_s}{I_s}$ and $E_s = 1 \angle 0^\circ$ V

$$I_s = \frac{E_s}{Z_s} = \frac{1 \angle 0^\circ}{625.6122 \angle 1.564^\circ} = 1.5984 \times 10^{-3} \angle -1.1564^\circ \text{ A}$$

And. $Z_R + Z_o = 200\angle 0^\circ + 696.204 \angle -10.825^\circ$

$\Rightarrow 893.435 \angle -8.415^\circ$

$Ke^{-\gamma l} = 0.5609 \angle 173.291^\circ \times 0.4648 \angle -208.4^\circ$

$= 0.2607 \angle -35.11^\circ$

Sub in (1)

$\therefore 1.5984 \times 10^{-3} \angle -1.1564^\circ = \dfrac{I_R \times 893.435 \angle -8.415}{2 \times 696.204 \angle -10.825^\circ} \left[2.1511 \angle 208.4^\circ - 0.2607 \angle -35.11^\circ \right]$

$1.5984 \times 10^{-3} \angle -1.1564^\circ = I_R \; 0.6416 \angle +2.41^\circ \left[2.2792 \angle -157.476^\circ \right]$

$I_R = 1.0932 \times 10^{-3} \angle +153.909^\circ$ A

$\therefore E_R = I_R Z_R = 1.0932 \times 10^{-3} \angle 153.909^\circ \times 200 \angle 0^\circ$

$E_R = 0.2186 \angle +153.909^\circ$ V

So.

$P_S = E_S I_S \cos(E_S \hat{\ } I_S)$

$= 1\angle 0^\circ \times 1.5984 \times 10^{-3} \times \cos(-1.1564^\circ)$

$= 1.598 \times 10^{-3}$ W

And $P_R = E_R I_R \cos(E_R \hat{\ } I_R) \Rightarrow 0.2186 \times 1.0932 \times 10^{3} \times \cos(0^\circ)$

$= 2.3897 \times 10^{-4}$ W

Or $P_R = I_R^2 \times R = (1.0932 \times 10^{-3})^2 \times 200$

$= 2.3897 \times 10^{-4}$ W

$\% \eta = \dfrac{P_R}{P_S} \times 100$

$= \dfrac{2.3897 \times 10^{-4}}{1.598 \times 10^{-3}} \times 100$

$\% \eta = 14.954 \%$

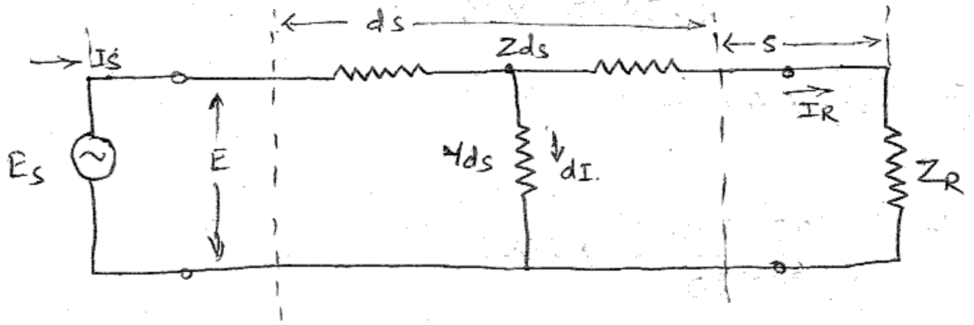
Q3 Derive the general solution of transmission lines.

Soln A transmission line is a circuit with distributed parameters hence the method of analysing such circuit is different than the method of analysis of a circuit with lumped parameters. It is seen that, the current and voltage varies from point to point along the transmission line. The general solution of a transmission line includes the expression for current and voltage at any point along a line of any length having uniformly distributed constants.

The various notations used in this derivation are,

R – Series resistance ohms per unit length,
L – Series inductance, henry per unit length.
C – Capacitance b/w the conductors, farads per unit length.
G – Shunt leakage conductance between the conductors mhos per unit length.
ωL = Series reactance per unit length.
ωC = Shunt susceptance in mhos per unit length.
$Z = R + j\omega L$ = series impedance in ohms per unit length.
$Y = G + j\omega C$ = Shunt admittance in mhos per unit length.
s – Distance upto point of consideration, measured from receiving end.
I – Current in the line at any point.
E – Voltage b/w the conductors at any point.
l = Length of the line.

The transmission line of length l can be considered to be made up of infinitesimal T sections. One such section of length ds is shown in the fig. It carries a current I.



Infinitesimal T section of a long line.

The point under consideration is at a distance s from the receiving end. The length of section is ds. hence its series impedance is Zds and shunt admittance is Yds. The current is I and voltage is E at this section.

The elemental voltage drop in the length ds is.

$$dE = IZ\,ds.$$

$$\frac{dE}{ds} = IZ \quad \text{——} (1)$$

The leakage current flowing through shunt admittance from one conductor to other is given by.

$$dI = EY\,ds$$

$$\frac{dI}{ds} = EY \quad \text{——} (2)$$

Differentiating Equation (1) & (2) with respect to s we get.

$$\frac{d^2E}{ds^2} = Z\frac{dI}{ds}$$

and.

$$\frac{d^2I}{ds^2} = Y\frac{dE}{ds}$$

This is because both E & I are functions of s.

$$\frac{d^2E}{ds^2} = ZEY \quad \text{——} (3)$$

and.

$$\frac{d^2I}{ds^2} = YIZ, \quad \text{——} (4)$$

The Equation (3) and (4) are the second order differential Eq^n. describing the transmission line having distributed constants, all along its length. It is necessary to solve these Equations to obtain the expressions of E and I.

Replace the operator d/ds by m hence we get

$$(m^2 - ZY)E = 0 \quad \text{but } E \neq 0.$$

$$m = \pm\sqrt{ZY} \quad \text{—— (5)}$$

Same result is true for the current Equation.

So there exists two solutions for positive sign of m and negative sign of m. The general solution of the Equations for E and I are

$$E = A e^{\sqrt{ZY}s} + B e^{-\sqrt{ZY}s} \quad \text{—— (6)}$$

$$I = C e^{\sqrt{ZY}s} + D e^{-\sqrt{ZY}s} \quad \text{—— (7)}$$

Where A, B, C and D are arbitary constants of integration.

It is now necessary to obtain the values of A, B, C and D.

As distance is measured from the receiving end $s = 0$ indicates the receiving end.

$$E = E_R \text{ and } I = I_R \quad \text{at } s = 0.$$

Substituting in the Solution,

$$E_R = A + B \quad \text{—— 8(a)}$$

$$I_R = C + D \quad \text{—— 8(b)}$$

Same condition can be used in the Equations obtained by differentiating the Equation (6) and (7) with respect to s.

$$\frac{dE}{ds} = A\sqrt{ZY}\, e^{\sqrt{ZY}s} + B(-\sqrt{ZY})\, e^{-\sqrt{ZY}s}$$

And.

$$\frac{dI}{ds} = C\sqrt{ZY}\, e^{\sqrt{ZY}s} + D(-\sqrt{ZY})\, e^{-\sqrt{ZY}s}$$

But $\frac{dE}{ds} = IZ$ and $\frac{dI}{ds} = EY$

$$IZ = A\sqrt{ZY}\cdot e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s} \quad \text{—— (9)}$$

and

$$EY = C\sqrt{ZY}\, e^{\sqrt{ZY}s} - D\sqrt{ZY}\, e^{-\sqrt{ZY}s} \quad \text{—— (10)}$$

$$I = \frac{A}{Z}\sqrt{ZY}\, e^{\sqrt{ZY}s} - \frac{B}{Z}\sqrt{ZY}\, e^{-\sqrt{ZY}s}$$

i.e.

$$I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{ZY}s} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}s} \quad \text{—— (11)}$$

And

$$E = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}s} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}s} \quad \text{—— (12)}$$

Now use $s = 0$, $E = E_R$ and $I = I_R$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \quad \text{—— 13 (a)}$$

and.

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \quad \text{—— 13 (b)}$$

The Equation: 8a, 8b, 13a and 13b are to be solved simultaneously to obtain the values of the constants A, B, C and D.

Now while solving these equations use the results.

$$Z_R = \frac{E_R}{I_R} \quad \text{and} \quad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}}$$

Hence the various constants obtained after solving the equations simultaneously are,

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) \quad (14)$$

$$B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} \Rightarrow \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) \quad (15)$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{Y}{Z}} \Rightarrow \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right) \quad (16)$$

$$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right) \quad (17)$$

Sub in Eqn (6) & (7)

Hence the general solution of the differential equation is

$$E = \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}\right] \quad (18)$$

$$I = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}s} + \left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}s}\right] \quad (19)$$

Taking LCM as Z_R and taking $\left(\frac{Z_R+Z_0}{Z_R}\right)$ out from Eqn (18)

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\sqrt{ZY}s} + \frac{(Z_R-Z_0)}{(Z_R+Z_0)}e^{-\sqrt{ZY}s}\right] \quad (20)$$

∴ Taking LCM as Z_0 and taking $\left(\frac{Z_R+Z_0}{Z_0}\right)$ out from Eqn (19)

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\cdot\left[e^{\sqrt{ZY}s} - \frac{(Z_R-Z_0)}{(Z_R+Z_0)}e^{-\sqrt{ZY}s}\right] \quad \text{—— (21)}$$

The negative sign is used to convert $Z_0 - Z_R$ to $Z_R - Z_0$

The Eqn (20) and (21) is the general solution of a transmission line.

Another way for representing the Equation is:

$$E = \frac{E_R}{2Z_R}\left[(Z_R+Z_0)e^{\sqrt{ZY}s} + (Z_R-Z_0)e^{-\sqrt{ZY}s}\right]$$

$$\therefore E = \frac{E_R}{2Z_R}\left[Z_R e^{\sqrt{ZY}s} + Z_0\sqrt{ZY}s + Z_R e^{-\sqrt{ZY}s} - Z_0 e^{-\sqrt{ZY}s}\right]$$

$$E = E_R\left(\frac{e^{\sqrt{ZY}s}+e^{-\sqrt{ZY}s}}{2}\right) + \frac{Z_0E_R}{Z_R}\left(\frac{e^{\sqrt{ZY}s}-e^{-\sqrt{ZY}s}}{2}\right)$$

But $\frac{E_R}{I_R} = Z_R$ hence $\frac{E_R}{Z_R} = I_R$.

$$\therefore E = E_R\left[\frac{e^{\sqrt{ZY}s}+e^{-\sqrt{ZY}s}}{2}\right] + I_R Z_0\left[\frac{e^{\sqrt{ZY}s}-e^{-\sqrt{ZY}s}}{2}\right] \quad \text{— (22)}$$

and

$$I = I_R\left[\frac{e^{\sqrt{ZY}s}+e^{-\sqrt{ZY}s}}{2}\right] + \frac{E_R}{Z_0}\left[\frac{e^{\sqrt{ZY}s}-e^{-\sqrt{ZY}s}}{2}\right] \quad \text{— (23)}$$

But $\frac{e^{\sqrt{ZY}s}+e^{-\sqrt{ZY}s}}{2} = \cosh\sqrt{ZY}s$ and $\frac{e^{\sqrt{ZY}s}-e^{-\sqrt{ZY}s}}{2} = \sinh\sqrt{ZY}s$

$$\therefore E = E_R\cosh(\sqrt{ZY}s) + I_R Z_0\sinh(\sqrt{ZY}s) \quad \text{—— (24)}$$

$$I = I_R\cosh(\sqrt{ZY}s) + \frac{E_R}{Z_0}\sinh(\sqrt{ZY}s) \quad \text{—— (25)}$$

The Equation (24) and (25) give the value of E and I at any point along the length of the line.

Important Note:- The similar Eqn can be ~~noted~~ obtained interms of sendingend ~~——~~ ~~——~~ Voltage E_s and I_s. If x is the distance measured down the line from the sending end then,

$$x = l - s$$

And the Eqn (24) and (25) get transfered interms of E_s and I_s are..

$$E = E_s \cosh(\sqrt{ZY}\,x) - I_s Z_0 \sinh(\sqrt{ZY}\,x) \quad (26)$$

$$I = I_s \cosh(\sqrt{ZY}\,x) - \frac{E_s}{Z_0} \sinh(\sqrt{ZY}\,x) \quad (27)$$

and $\sqrt{ZY} = \gamma$ as derived earlier and hence eqn can be written.

If receiving end parameters are known and s is distance measured from the receiving end then.,

$$E = E_R \cosh(\gamma s) + I_R Z_0 \sinh(\gamma s).$$

$$I = I_R \cosh(\gamma s) + \frac{E_R}{Z_0} \sin(\gamma s)$$

And if sending end parameters are known and x is distance measured from the sending end then,

$$E = E_s \cosh(\gamma x) - I_s Z_0 \sinh(\gamma x).$$

$$I = I_s \cosh(\gamma x) - \frac{E_s}{Z_0} \sinh(\gamma x)$$

Q4 i) A line will be distortion less if $CR = LG$.

If a line having neither freq. nor phase and is also correctly terminated is called a distortion less line. α and β cannot be the fn of fr. This type of line is called distortionless line.

$$v = \omega/\beta$$

β must be a direction fn of freq.

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

From above eqn.

$$(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2 = (RG + \omega^2 LC)^2$$

$$\Rightarrow \cancel{R^2G^2} - 2RG\omega^2 LC + \cancel{\omega^4 L^2 C^2} + \omega^2 [L^2G^2 + 2LGRC + C^2R^2]$$

$$= \cancel{R^2G^2} + 2RG\omega^2 LC \cancel{+\omega^4 L^2 C^2}$$

$$\Rightarrow -2RG\omega^2 LC + \omega^2 L^2 G^2 + \cancel{2\omega^2 LGRC} + \omega^2 C^2 R^2 = \cancel{2\omega^2 GRC}$$

$$\Rightarrow \omega^2 [L^2G^2 + R^2C^2 - 2RGLC] = 0$$

$$\omega^2 [LG - RC]^2 = 0$$

$$\therefore LG - RC = 0$$

$$\boxed{LG = RC} \quad \text{--- (2)}$$

or $$\boxed{\frac{R}{L} = \frac{G}{C}}$$

Q4(ii) $R = 52$ ohm/m

$L = 0.1$ μH/m

$C = 300$ pF/m

$G = 0.01$ mho/m.

$$\omega = 2\pi f = 2\pi \times 500 \times 10^6 \text{ rad/sec.}$$

$$= 2\pi \times 5 \times 10^8$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{52 + j2\pi \times 5\times 10^8 \times 0.1\times 10^{-6}}{0.01 + j2\pi\times 5\times 10^8 \times 300\times 10^{-12}}}$$

$$= \sqrt{\frac{52+314j}{0.01+0.942j}}$$

$$= \sqrt{\frac{318.27 \angle 80.59}{0.942 \angle 89.39}}$$

$$= 337.86 \angle 8.8$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(52+314j)(0.01+0.942j)}$$

$$= \sqrt{318.27 \angle 80.59 \times 0.942 \angle 89.39}$$

$$= 17.315 \angle 169.98$$

$$= -17.05 + 3.012i$$

$$= \alpha + j\beta$$

$\alpha = -17.05$ N/km , $\beta = 3.0126$ rad/m.

For the lossless line $R = G = 0$

$\therefore \alpha = 0$

$$\beta = \omega\sqrt{LC} = 2\pi\, 500\times 10^6 \sqrt{0.1\times 10^{-6} \times 300\times 10^{-12}}$$

$$= 17.2072 \text{ rad/km}$$

$$Z_0 = \sqrt{\frac{L}{C}} \angle 0^\circ = \sqrt{\frac{0.1\times 10^{-6}}{300\times 10^{-12}}} \angle 0^\circ$$

$$= 18.2574 \angle 0^\circ$$

We

Q4 i)

an A line in which there is no phase or freq distortion and also it is correctly terminated is called a distortionless line.

To derive the condition for distortionless line. consider

S $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\therefore \gamma^2 = (R+j\omega L)(G+j\omega C)$

$\therefore \gamma^2 = (RG-\omega^2 LC) + j\omega C(RC+LG)$ —— (1)

It is known that for min^m attenuation $L = \frac{CR}{G}$ $\therefore LG = CR$. Sub. this cond^n in eq^n (1) we get

$\gamma^2 = RG - \omega^2 LC + j2\omega RG$

But $RC = LG = \sqrt{RCLG}$

$\gamma^2 = RG - \omega^2 LC + j2\omega\sqrt{RCLG}$

$\gamma^2 = (\sqrt{RG} + j\omega\sqrt{LC})^2$

$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$ —— (2)

$\gamma = \alpha + j\beta$

17 But $\boxed{\alpha = \sqrt{RG}}$ —— (3)

and $\boxed{\beta = \omega\sqrt{LC}}$ —— (4)

It can be seen from Eq^n (3) that α does not vary with freq with elliminates the freq. distortion.

Now $\beta = \omega\sqrt{LC}$ for condition $LG = CR$

$V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$ km/sec.

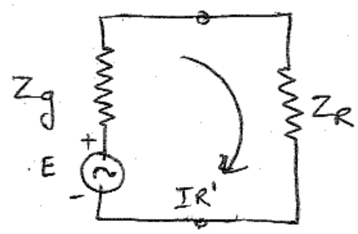
8 Then the condition $LG = CR$, the velocity becomes independent of freq. Thus elimanate the phase distor.

$RC = LG$

$\therefore \boxed{\frac{R}{G} = \frac{L}{C}}$

Q5) Derive the expression for the insertion loss of transmission line.

Soln Consider the circuit shown in the fig. with a load current of.



$$I_R' = \frac{E}{Z_g + Z_R} \quad \text{—— (1)}$$

The line is inserted b/w the load & the generator.

let Z_s be the input impedance of a line which is different than Z_g. hence,

$$I_s = \frac{E}{Z_g + Z_s} \quad \text{—— (2)}$$

It is known for the line that input impedance is given by.

$$Z_s = Z_0 \left[\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}} \right] \quad \text{—— (3)}$$

Sub in (2)

$$I_s = \frac{E}{Z_g + Z_0 \left(\frac{e^{\gamma l} + Ke^{-\gamma l}}{e^{\gamma l} - Ke^{-\gamma l}} \right)}$$

$$I_s = \frac{E(e^{\gamma l} - Ke^{-\gamma l})}{Z_g(e^{\gamma l} - Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})} \quad \text{—— (4)}$$

We want the current through the load ie. I_R due to the insertion of line. This can be obtained from the relation between I_s and I_R.

$$I_s = \frac{I_R(Z_R + Z_0)}{2Z_0} (e^{\gamma l} - Ke^{-\gamma l}) \quad \text{—— (5)}$$

The Equation is obtained from the general soln of a line Substituting $s = l$. i.e. $I = I_s$ at the sending end.

$$I_R = \frac{2Z_0 I_s}{(Z_R + Z_0)[e^{\gamma l} - Ke^{-\gamma l}]}$$

$$= \frac{\dfrac{2Z_0 E(e^{\gamma l} - Ke^{-\gamma l})}{Z_g(e^{\gamma l} - Ke^{-\gamma l}) + Z_0(e^{\gamma l} + Ke^{-\gamma l})}}{(Z_R + Z_0)(e^{\gamma l} - Ke^{-\gamma l})} \quad \text{Using (4)}$$

$$= \frac{2 Z_0 E}{(Z_R + Z_0)\left[Z_0(e^{\gamma l} + K e^{-\gamma l}) + Z_g(e^{\gamma l} - K e^{-\gamma l})\right]} \quad \text{——(6)}$$

Now introduce the value of reflection coefficent.

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} \quad \text{——(7)}$$

$$I_R = \frac{2 Z_0 E}{(Z_R + Z_0)\left\{Z_0\left(e^{\gamma l} + \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l}\right) + Z_g\left(e^{\gamma l} - \frac{(Z_R - Z_0)}{(Z_R + Z_0)} e^{-\gamma l}\right)\right\}}$$

$$= \frac{2 Z_0 E}{(Z_R + Z_0)\left\{Z_0 e^{\gamma l} + \left[\frac{Z_R - Z_0}{Z_R + Z_0}\right] Z_0 e^{-\gamma l} + Z_g e^{\gamma l} - \left[\frac{Z_R - Z_0}{Z_R + Z_0}\right] Z_g e^{-\gamma l}\right\}}$$

$$I_R = \frac{2 Z_0 E}{(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}} \quad \text{——(8)}$$

The insertion loss is the ratio of currents in the load without insertion and with insertion.

$$\frac{I'_R}{I_R} = \frac{E / Z_g + Z_R}{\dfrac{2 Z_0 E}{(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}}}$$

$$\frac{I'_R}{I_R} = \frac{\cancel{E}\,(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}}{\cancel{E}\, 2 Z_0 (Z_g + Z_R)} \quad \text{——(9)}$$

The current ratio is made up of two parts, one which is continiously increasing with line length and other decreasing with line length. The length of line isusually very large hence

We ... $e^{\gamma l} \to 0$ Hence the second term in the numerator can be neglected compared to first.

$$\frac{I'_R}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)\,e^{\gamma l}}{2Z_0(Z_g+Z_R)} \quad \text{——— (10)}$$

$$= \frac{(Z_R+Z_0)(Z_0+Z_g)\,e^{\alpha l}\,e^{j\beta l}}{2Z_0(Z_g+Z_R)}$$ use $\gamma = \alpha + j\beta$

But insertion loss is to be calculated as a function of ratio of current magnitudes and hence the term $e^{j\beta l}$ which gives phase angle can be neglected.

$$\left|\frac{I'_R}{I_R}\right| = \frac{|Z_R+Z_0|\,|Z_0+Z_g|\,e^{\alpha l}}{2|Z_0|\,|Z_g+Z_R|} \quad \text{——— (11)}$$

Now multiply numerator and denominator by $2\sqrt{Z_g \cdot Z_R}$.

$$\left|\frac{I'_R}{I_R}\right| = \frac{2\sqrt{Z_g Z_R}\,|Z_R+Z_0|\cdot|Z_0+Z_g|\,e^{\alpha l}}{4\sqrt{Z_g Z_R}\,|Z_0|\,|Z_g+Z_R|}$$

Using $|Z_0| = \sqrt{|Z_0|}\,\sqrt{|Z_0|}$ we get,

$$\left|\frac{I'_R}{I_R}\right| = \frac{|Z_g+Z_0|}{2\sqrt{Z_g \cdot Z_0}} \cdot \frac{|Z_R+Z_0|}{2\sqrt{Z_R Z_0}} \cdot \frac{2\sqrt{Z_g Z_R}}{|Z_g+Z_R|}\,e^{\alpha l} \quad \text{——— (12)}$$

All the term on right hand side indicates reflection factors.

let $K_S = \dfrac{2\sqrt{Z_g Z_0}}{|Z_g+Z_0|}$ = Reflection factor at Source side ——— (13)

This is the reflection factor at the terminal 1-1 when the generator is missmatched at its junction with the line

Then $K_R = \dfrac{2\sqrt{Z_R Z_0}}{|Z_R+Z_0|}$ = Reflection factor at load side ——— (14)

It is a factor at ~~which~~ the junction b/w line and load i.e. at the terminal 2-2.

while = $K_{SR} = \dfrac{2\sqrt{Z_g Z_R}}{|Z_g+Z_R|}$ = Reflection factor for direct connect ——— (15)

$= \left|\frac{I_R'}{I_R}\right| = \frac{1}{K_S}\cdot\frac{1}{K_R}\, K_{SR}\, e^{\alpha l}$

This is the reflection factor when the generator and load were directly Connected the last term $e^{\alpha l}$ indicates the loss in the line.

$$\left|\frac{I_R'}{I_R}\right| = \frac{K_{SR}}{K_S K_R}\, e^{\alpha l} \quad \text{—— (16)}$$

The insertion loss is defined in nepers or decibels hence can be expressed as.

$$\text{insertion loss} = \ln\left|\frac{I_R'}{I_R}\right| = \left[\ln\frac{1}{K_S} + \ln\frac{1}{K_R} + \ln\frac{1}{K_{SR}} + \alpha l\right] \text{nepers} \quad \text{—— 17(a)}$$

$$= 20\log\left|\frac{I_R'}{I_R}\right| = 20\left[\log\frac{1}{K_S} + \log\frac{1}{K_R} + \log\frac{1}{K_{SR}} + 0.4343\,\alpha l\right] dB \quad \text{—— 17(b)}$$

The $\boxed{\text{insertion loss} = 10\log\frac{P_R'}{P_R}\ dB}$

Qs (ii) A transmission line has $Z_0 = 700\angle -13.4°$ ohm is inserted b/w a generator of $200\,\Omega$ and a load of $400\,\Omega$. The attenuation and phase constant of the line is $\alpha = 0.00712$ nepers/km and $\beta = 0.0288$ rad/km. Calculate the insertion loss. If the length is 200 km.

Soln

$Z_0 = 700\angle -13.4°\,\Omega$, $Z_g = 200\angle 0°\,\Omega$, $Z_R = 400\angle 0°\,\Omega$.

$$K_S = \frac{2\sqrt{Z_g Z_0}}{|Z_g + Z_0|} = \frac{2\sqrt{200\times 700}}{|200 + j0 + 700\angle -13.4°|} \Rightarrow \frac{748.3314}{|200 + j0 + 680.94 - j162.2|}$$

$$= \frac{748.3314}{|880.94 - j162.22|} = \frac{748.3314}{895.7514\angle 10.43} = 0.83542$$

$$K_R = \frac{2\sqrt{Z_R Z_0}}{|Z_R + Z_0|} = \frac{2\sqrt{400 \times 700}}{|400 + j0 + 680.94 - j162.22|} = \frac{1058.3}{|1080.94 - j162.22|}$$

$$= \frac{1058.3}{1093.044} = 0.9682$$

$$K_{SR} = \frac{2\sqrt{Z_g Z_R}}{|Z_g + Z_R|} = \frac{2\sqrt{200 \times 400}}{|200 + j0 + 400 + j0|} = \frac{565.685}{600}$$

$$= 0.9428$$

$$\text{Insertion loss} = 20\left[\log\frac{1}{K_S} + \log\frac{1}{K_R} - \log\frac{1}{K_{SR}} + 0.4343\,\alpha l\right] \text{dB}$$

$$= 20\left[\log\frac{1}{0.83542} + \log\frac{1}{0.9682} - \log\frac{1}{0.9428} + 0.4343 \times 0.00712 \times 200\right]$$

$$= 13.7 \text{ dB} \quad \underline{\underline{\text{Ans}}}$$

Q6 A dipole antenna whose input impedance is 100Ω is to be matched at a frequency of 100 MHz to a transmission line having characteristic impedance of 600 be means of short circuit stub. Determine the location and length of the stub.

Soln

$Z_R = 100\Omega$, $Z_0 = 600\Omega$, $f = 100\,MHz$.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{100\times10^6} = 3m$$

location of the stub is.

$$l_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_R}{Z_0}}$$

$$l_s = \frac{3}{2\pi}\tan^{-1}\sqrt{\frac{100}{600}} \Rightarrow \frac{3}{2\pi}\times\frac{22.2^\circ}{180}\times\pi$$

(∵ To convert 22.2° in to radians, multiply $\frac{\pi}{180}$)

$$l_s = 0.185\,m.$$

length of the stub is.

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left[\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}\right]$$

$$= \frac{3}{2\pi}\tan^{-1}\left[\frac{\sqrt{100\times600}}{100-600}\right]$$

$$= \frac{3}{2\pi}\tan^{-1}(-0.4899)$$

$$= \frac{3}{2\pi}\times(-26.1^\circ)$$

$$= \frac{3}{2\pi}(180^\circ - 26.1^\circ)$$

(Because of negative, angle will be subtracted from 180°)

$$l_t = \frac{3}{2\pi} \times \frac{153.9^\circ}{180} \times \pi$$

$$l_t = 1.28\ m.$$

Q7. Determine the SWR, Characteristics Impedance of the quarter wave transformer and the distance the transformer must be placed from the load to match a 75Ω transmission line to a load $Z_L = 25 - j50\Omega$.

Soln

Characteristics impedance $Z_0 = 75\Omega$.

Load Impedance $Z_L = 25 - j50\Omega$.

Normalized load Impedance $\frac{Z_L}{Z_0} = \frac{25 - j50}{75}$

$= 0.33 - j0.66$

The normalised load impedance is plotted at Pon. Smith chart and normalized impedance circle is drawn. It cuts the X = 0 line on the righ side at S.

SWR = 4.6 (∵ OS = 4.6)

∴ The distance from the load to a point where the i/p impedance is purely resistive is calculated in wavelength to the shortest of two resistive input impedance i/p

$d = 0.098\lambda$.

$Z_i = 0.22 = \frac{Z_i}{Z_0}$

$Z_i = 0.22 \times 75$ [∵ $Z_0 = 75$]

$= 16.5\Omega$.

The characteristics impedance of quarter wave transformer is found from the formula.

$$Z_0' = \sqrt{Z_0 \cdot Z_i}$$

$$= \sqrt{75 \times 16.5}$$

$$= 35.2\ \Omega$$

Q8 Determine the input impendance of open and Short circuited Dissipationless transmission line.

Soln Input Impedance of open and Short Circuited Lines

$$Z_s = Z_{in} = R_o\left[\frac{Z_R + jR_o \tan\beta s}{R_o + jZ_R \tan\beta s}\right]$$

let us find the i/p impedance of a line, open circuited & short circuited at the receiving end separately.

* Input impedance of Short Circuited line.

If a line is Short circuited at the receiving end.

then $Z_R = 0$

then the i/p impedance is given by.

$$Z_{sc} = Z_{sc} = R_o\left[\frac{jR_o \tan\beta s}{R_o}\right] = jR_o \tan\beta s \quad \text{--- (1)}$$

But $\beta = 2\pi/\lambda$ then

$$Z_s = jR_o \tan\left(\frac{2\pi s}{\lambda}\right) \quad \text{--- (2)}$$

As Z_s is purely reactive, let it be denoted by X_s

$$Z_s = jX_s = jR_o \tan\left(\frac{2\pi s}{\lambda}\right)$$

$$\frac{X_s}{R_o} = \tan\left(\frac{2\pi s}{\lambda}\right) \quad \text{--- (3)}$$

Above ratio gives normalized value of reactance for a short circuited line.

Let us consider the ratio $\frac{X_s}{R_o}$ for different values of s.

when $s = 0$, $\tan\left(\frac{2\pi \cdot 0}{\lambda}\right) = 0 \quad \therefore \frac{X_s}{R_o} = 0$

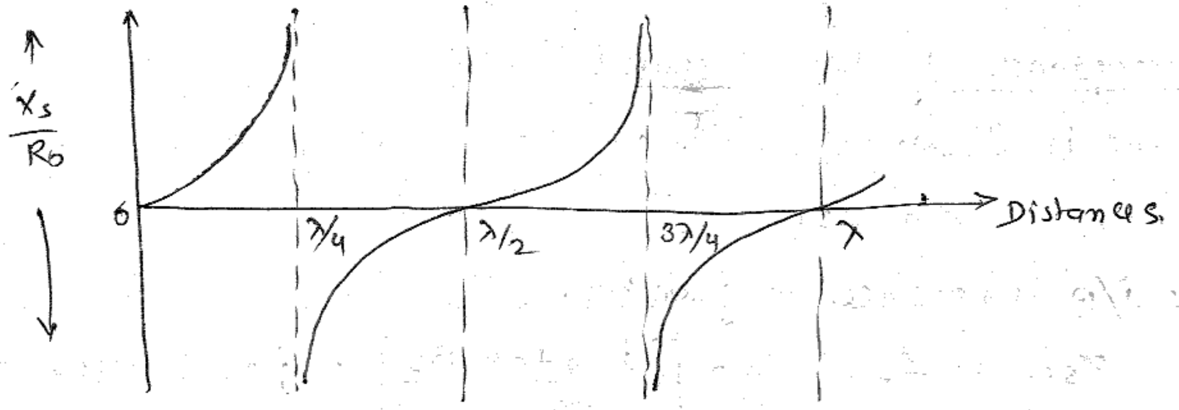
$$s = \lambda/4, \quad \tan\left(\frac{2\pi \cdot \lambda/4}{\lambda}\right) = \tan\left(\frac{\pi}{2}\right) = \infty \qquad \therefore \frac{X_s}{R_0} = \infty$$

$$s = \lambda/2; \quad \tan\left(\frac{2\pi \cdot \lambda/2}{\lambda}\right) = \tan(\pi) = 0 \qquad \therefore \frac{X_s}{R_0} = 0$$

$$s = \frac{3\lambda}{4}, \quad \tan\left(\frac{2\pi \cdot \frac{3\lambda}{4}}{\lambda}\right) = \tan\left(\frac{3\pi}{2}\right) = \infty \qquad \therefore \frac{X_s}{R_0} = \infty \quad \text{and}$$

$$s = \lambda, \quad \tan\left(\frac{2\pi \cdot \lambda}{\lambda}\right) = \tan(2\pi) \qquad \therefore \frac{X_s}{R_0} = 0$$

The graph of variation of $\frac{X_s}{R_0}$ for a short circuited line with various lengths of lines is as shown in the fig.



* Input impedance of open circuited line

Rearranging the expression for input impedance as follows

$$Z_s = R_0 \left[\frac{1 + j\frac{R_0}{Z_R} \tan\beta s}{\frac{R_0}{Z_R} + j\tan\beta s} \right]$$

when a line is open circuited at the receiving end,

$$Z_R = \infty$$

Then the input impedance is given by.

$$Z_s = R_0 \left[\frac{1}{j\tan\beta s} \right] = \frac{-jR_0}{\tan\beta s} = -jR_0 \cot\beta s$$

But $\quad \beta = \frac{2\pi}{\lambda}$

$$Z_{oc} = Z_s = -jR_0 \cot\left(\frac{2\pi s}{\lambda}\right) \quad \text{—— (2)}$$

$\therefore$ Again $Z_{oc} = Z_s$ is purely reactive, let it be denoted by X_s.

$$\therefore \quad jX_s = -jR_o \cot\left(\frac{2\pi s}{\lambda}\right)$$

$$\therefore \quad \frac{X_s}{R_o} = -\cot\left(\frac{2\pi s}{\lambda}\right) \quad \text{—— (3)}$$

Let us calculate the value of ratio $\frac{X_s}{R_o}$ for different value of s.

when $s = 0$ $\cdot$ $\cot\left(\frac{2\pi \cdot 0}{\lambda}\right) = \cot(0) = \infty$ $\therefore \frac{X_s}{R_o} = -\infty$

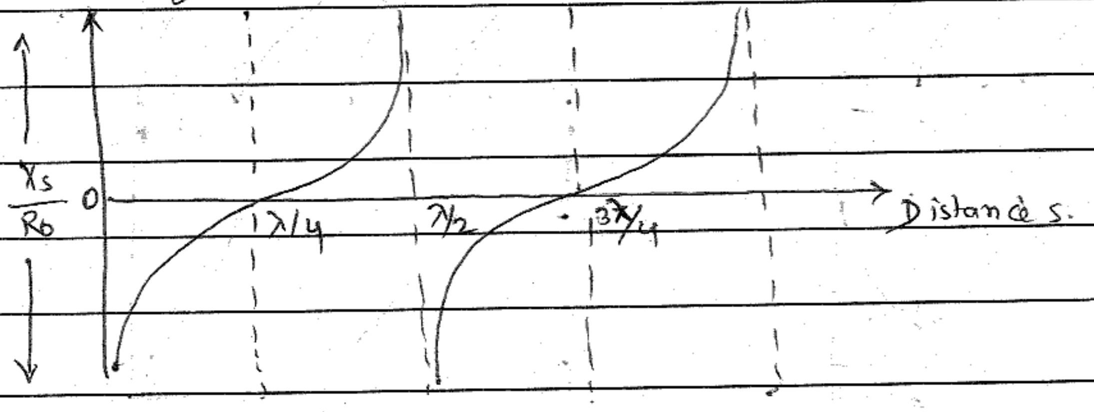
when $s = \frac{\lambda}{4}$, $\cot\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \cot\left(\frac{\pi}{2}\right) = 0$, $\therefore \frac{X_s}{R_o} = 0$

when $s = \frac{\lambda}{2}$, $\cot\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = \cot(\pi) = \infty$ $\therefore \frac{X_s}{R_o} = \infty$

$s = \frac{3\lambda}{4}$, $\cot\left(\frac{2\pi}{\lambda} \cdot \frac{3\lambda}{4}\right) = \cot\left(\frac{3\pi}{2}\right) = 0$ $\therefore \frac{X_s}{R_o} = 0$, and.

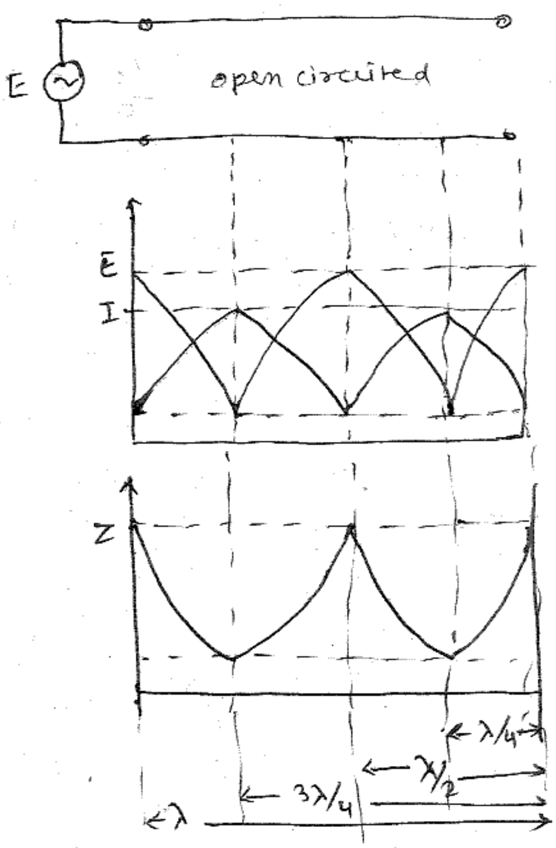
$s = \lambda$, $\cot\left(\frac{2\pi}{\lambda} \cdot \lambda\right) = \cot(2\pi) = \infty$ $\therefore \frac{X_s}{R_o} = \infty$.

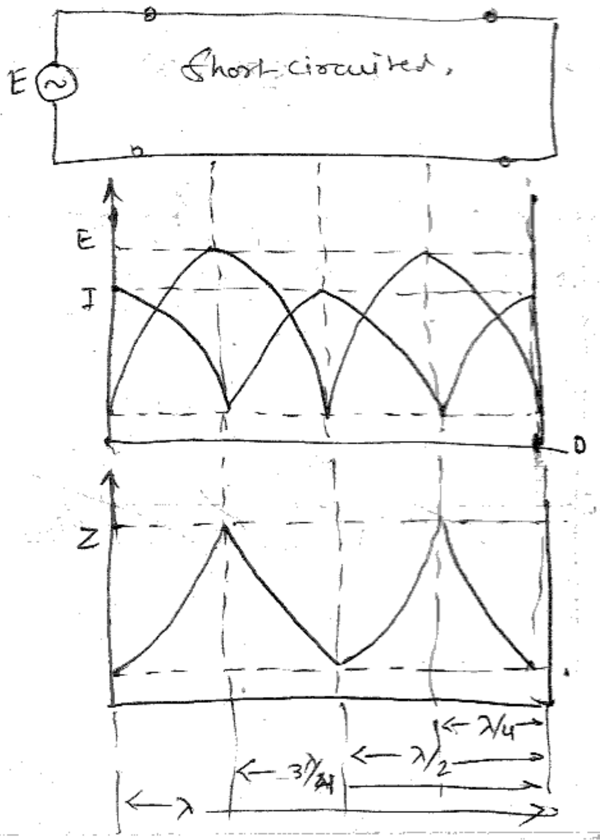
The graph of variation of X/R_o for an open circuited line with various lengths of lines is as shown in the fig.



According to the discussion in the last two sections it is clear that the input impedance of a line either open circuited or short circuited is pure reactive in nature. It is also observed that the value of reactance is repeated after every $s = \lambda/2$ period. For first quarter wavelength, short circuited line acts as an inductance while the open circuited line acts as a capacitance. After each quarter wavelength, the nature of reactances reverses. These curves as shown in fig 1&2 are of ideal dissipationless line. In practical line, zero or infinite impedance cannot be achieved because of small resistive component indicating some power loss. Thus practically the value of impedances tends to maxima and minima.

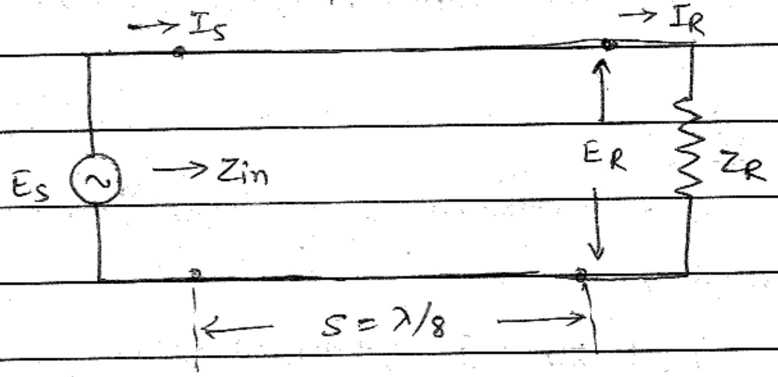
When transmission line is open circuited at its end, the current is always zero and voltage is maximum at the open end. After every half wavelength ($\lambda/2$) distance, these conditions of voltage and current will repeat themselves. When there is a voltage maxima, there is current minima and at voltage minima we get current maxima. This repeats at every $\lambda/4$ distance from open end. From this it is clear that the input impedance varies all along the length of a line. The nature of the input impedance is also varying such as low resistance, high resistance, inductive reactance or capacitive reactance. These characteristics are similar to those of resonant circuit. Hence mismatched lines are called resonant lines.





Q9 Explain about one Eight wave line and quarter Wave line.

Soln Let λ be the wavelength of the transmitted frequency 'f'. Consider a transmission line of length $\lambda/8$ as shown in the fig.



The generalised expression for the input impedance is given by

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\beta s)}{R_0 + jZ_R \tan(\beta s)} \right]$$

But $\beta = 2\pi/\lambda$

$$\therefore Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda} s\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} s\right)} \right]$$

But for a line, $s = \lambda/8$.

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \cdot \lambda/8\right)} \right]$$

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\pi/4)}{R_0 + jZ_R \tan(\pi/4)} \right]$$

$$\boxed{Z_{in} = R_0 \left[\frac{Z_R + jR_0}{R_0 + jZ_R} \right]} \quad \text{——— (2)}$$

If a line is terminated in pure resistance $Z_R = R_R$, then

$$Z_{in} = R_0 \left[\frac{R_R + jR_0}{R_0 + jR_R} \right] \quad \text{——— (3)}$$

From Equation (3) it is clear that Z_{in} is a complex quantity. Thus the magnitude of the input impedance is given by,

$$\boxed{|Z_{in}| = R_o\left[\frac{\sqrt{R_R^2 + R_o^2}}{\sqrt{R_o^2 + R_R^2}}\right] = R_o}$$

Thus, the eight wave line is generally used to transform any resistance R_R to an impedance Z_{in} having its magnitude equal to the characteristic resistance R_o of the line.

* Quarter Wave Line :

The generalised expression for the input impedance of the line is given by.

$$Z_{in} = R_o\left[\frac{Z_R + jR_o \tan(\beta s)}{R_o + jZ_R \tan(\beta s)}\right] \quad \text{——} (1)$$

Rearranging interms in R.H.S of the eqn (1)

$$Z_{in} = R_o\left[\frac{\frac{Z_R}{\tan(\beta s)} + jR_o}{\frac{R_o}{\tan(\beta s)} + jZ_R}\right]$$

But.

$$\beta = \frac{2\pi}{\lambda},$$

$$\therefore Z_{in} = R_o\left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda}\cdot s\right)} + jR_o}{\frac{R_o}{\tan\left(\frac{2\pi}{\lambda} s\right)} + jZ_R}\right] \quad \text{——} (2)$$

The quarter-wave line, $s = \lambda/4$

$$\therefore Z_{in} = R_o\left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda}\cdot\lambda/4\right)} + jR_o}{\frac{R_o}{\tan\left(\frac{2\pi}{\lambda}\cdot\lambda/4\right)} + jZ_R}\right]$$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\pi/2)} + j R_0}{\frac{R_0}{\tan(\pi/2)} + j Z_R} \right]$$

$$Z_{in} = R_0 \left[\frac{j R_0}{j Z_R} \right]$$

$$\therefore \boxed{Z_{in} = \frac{R_0^2}{Z_R}} \quad \text{—— (3)}$$

Thus Eqn (3) is similar to the Equation for impedance matching using transformer.

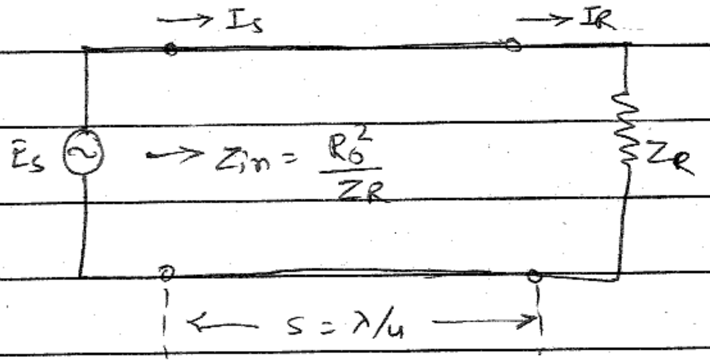


Fig The quarter wave line.

~~Thus a quarter wave line may be used as a transform a low impedance into a high impedance and vice-versa thus it can be considered as an impedance inverter Hence~~

Thus a quarter wave line may be used as a transformer for impedance matching of load Z_R with input impedance $Z_{in} = Z_R$.

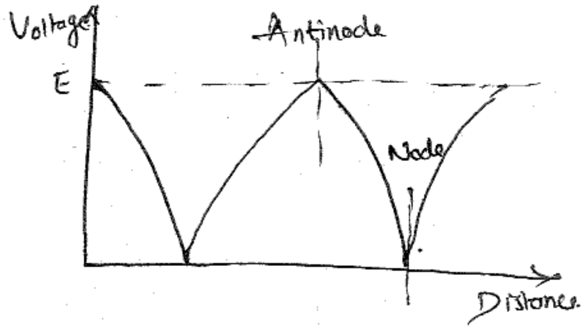
For matching impedance Z_R and Z_{in} the line with characteristic impedance R_0 may be selected such that condition given in Eqn (4)

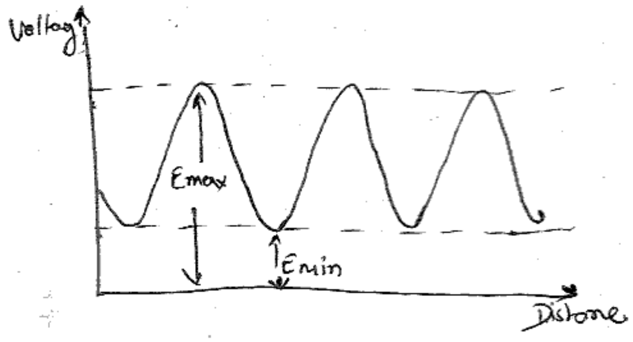
$$R_0 = \left| \sqrt{Z_R \cdot Z_{in}} \right| \quad \text{—— (4)}$$

Q 10 Define and explain the following.
i) Standing wave (ii) Standing Wave Ratio (iii) Relation b/w SWR & K

Sol^n i) If a line is either open circuited or short circuited at the receiving end, we get nodes and antinodes in voltage distribution as shown in the fig (a)

If a line is terminated in a load other than R_0, the distribution of voltage at a point along the length of the line consists max^m and min^m values of voltage as shown in the fig (b)





We know that voltage and current are in quadrature; thus it is obvious that the magnitude of the current along the line would be same except for a $\lambda/4$ shift in a position of maxima & minima.

The points along the line where magnitude of voltage or current is zero are called Nodes while the points along the lines where magnitude of voltage or current is maximum are called Antinodes or loops. The nodes and antinodes are as shown in fig.

When a line is terminated in R_0, the standing wave are absent, such a line is called smooth line.

ii) Standing wave Ratio (S)

The ratio of maximum to minimum magnitude of voltage or current on a line having standing wave ratio and it is denoted by S.

The standing wave ratio (S) is given by,

$$\boxed{S = \frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|}}$$

When line is not terminated properly, standing waves are produced. Then the total power absorption is not possible in such case. The standing wave ratio S is measured using RF Voltmeter across the line at a point. Then the ratio of E_{max} to E_{min} is referred as Voltage Standing wave ratio (VSWR). Similarly the ratio of I_{max} to I_{min} can be measured using RF Ammeter in series with the line at a point. Then such ratio is referred as current Standing wave ratio (ISWR). But in practice, ISWR calculation is very impractical because for this one has to cut the line, insert RF ammeter and then rejoin the line. Hence practically only VSWR measurement is done. So it is understood that VSWR is nothing but SWR. Theoritically the value of S lies between 1 and ∞.

* iii) Relation b/w SWR (S) & Magnitude of Reflection Coefficient (K)

The Standing wave Ratio bears a simple relationship with the magnitude of the reflection coefficient i.e $|K|$.

Along the line, at a point, if the incident and reflected waves are in phase and added directly, we get voltage maxima at the point.

Let E^+ = Magnitude of the incident wave, E^- = Magnitude of Reflected wave.

Then the magnitude of voltage maxima is given by,

$$|E_{max}| = |E^+| + |E^-| \quad \text{--- (1)}$$

Similarly, along the line, at a point if the incident and reflected wave are out of phase and subtracted directly, We get voltage minima at that point.

The magnitude of voltage minima is given by.

$$|E_{min}| = |E^+| - |E^-| \quad \text{—— (2)}$$

The Standing wave ratio is given by.

$$S = \frac{|E_{max}|}{|E_{min}|} = \frac{|E^+| + |E^-|}{|E^+| - |E^-|}$$

$$S = \frac{1 + \frac{|E^-|}{|E^+|}}{1 - \frac{|E^-|}{|E^+|}} \quad \text{(3)}$$

Divid by $|E^+|$

But the ratio $\frac{|E^-|}{|E^+|}$ is nothing but the magnitude of the reflection coefficient,

$$|K| = \frac{|E^-|}{|E^+|}.$$

Then

$$\boxed{S = \frac{1 + |K|}{1 - |K|}} \quad \text{—— (4)}$$

or.

$$\boxed{|K| = \frac{S-1}{S+1} = \frac{|E_{max}| - |E_{min}|}{|E_{max}| + |E_{min}|}} \quad \text{—— (5)}$$

$$|K| = \frac{R_R - R_o}{R_R + R_o}$$

$$S = \frac{1 + |K|}{1 - |K|} = \frac{1 + \left[\frac{R_R - R_o}{R_R + R_o}\right]}{1 - \left[\frac{R_R - R_o}{R_R + R_o}\right]} \quad \text{—— (6)}$$

Then if $R_R > R_o$

$$S = \frac{R_R}{R_o}$$

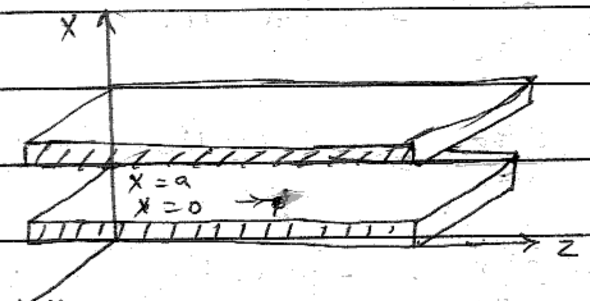
And if $R_R < R_o$ $\quad S = \frac{R_o}{R_R}$

In general, for resistive load R_R,

$$\boxed{S = \frac{1 + |K|}{1 + |K|} = \frac{1 + \left[\frac{R_R - R_o}{R_R + R_o}\right]}{1 + \left[\frac{R_R - R_o}{R_R + R_o}\right]} = \frac{R_R}{R_o}}$$

Q1 Derive the electromagnetic field expressions for wave guided by a parallel conducting planes?

Soln Consider an electromagnetic wave, propagating b/w a pair of perfectly conducting planes as shown in fig. The planes considered are such that both extend infinitely in y and z directions.



The conducting planes are placed at x = 0 and x = a.
To study the behaviour of the electromagnetic field between the two conducting plate, the procedure is to solve the Maxwell's Equation using proper boundary conditions.

The Maxwell's curl Equation is phasor from are as given below.

$$\boxed{\nabla \times \bar{H} = (\sigma + j\omega\epsilon)\bar{E}} \quad \text{--- (1)}$$

and

$$\boxed{\nabla \times \bar{E} = -j\omega\mu\bar{H}} \quad \text{--- (2)}$$

Similarly the wave Equation in phasor form are as given below

$$\boxed{\nabla^2 \bar{E} = \gamma^2 \bar{E}} \quad \text{--- (3)}$$

and,

$$\boxed{\nabla^2 \bar{H} = \gamma^2 \bar{H}} \quad \text{--- (4)}$$

In Eqn (3) and (4) γ is the propagation constant and is given by.

$$\boxed{\gamma = \alpha + j\beta = \sqrt{(\sigma + j\omega\epsilon)(j\omega\mu)}} \quad \text{--- (5)}$$

Between the two conducting planes the region is non conducting with $\sigma = 0$.

Simplifying eqn ① in rectangular co-ordinate system, we can write.

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \quad \text{—— (6)}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad \text{—— (7)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad \text{—— (8)}$$

Simplifying eqn ② in the similar way, we can write,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \text{—— (9)}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad \text{—— (10)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \text{—— (11)}$$

For the non-conducting medium $(\sigma = 0)$ the propagation constant is given by.

$$\gamma = \pm j\omega\sqrt{\mu\varepsilon} \quad \text{—— (12)}$$

Using the value of the propagation constant expressed in equation (12) and simplifying eqn ③ we can write,

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = -\omega^2\mu\varepsilon\bar{E} \quad \text{—— (13)}$$

In the similar way simplifying eqn ④ using rectangular co-ordinate system.

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \frac{\partial^2 \bar{H}}{\partial y^2} + \frac{\partial^2 \bar{H}}{\partial z^2} = -\omega^2\mu\varepsilon\bar{H} \quad \text{—— (14)}$$

Assume that the electromagnetic wave is propagating through z-direction. So in the phasor form $e^{-\gamma z}$ represents that the wave propagates in +ve z-direction. This factor also represents the variation of the field components. The term $\bar{\gamma}$ in the factor $e^{-\bar{\gamma} z}$ is called complex propagation constant. The value of the complex propagation constant is given by.

$$\boxed{\bar{\gamma} = \bar{\alpha} + j\bar{\beta}} \quad \text{—— (15)}$$

In general, the fields along the line obey the exponential law

$$e^{j\omega t} \cdot e^{-\bar{\gamma} z} = e^{j\omega t} \cdot e^{-(\bar{\alpha}+j\bar{\beta})z} \Rightarrow e^{-\bar{\alpha} z} \cdot e^{j(\omega t - \bar{\beta} z)} \quad \text{—— (16)}$$

For the variation field component in z-direction, we can express each field component in following fashion.

$$H_y = H_{y0}\, e^{-\bar{\gamma} z} \quad \text{—— (17)}$$

Here H_y is the field component in Y-direction and H_{y0} is the amplitude of H_y.

Differentiating H_y with respect to z, we get.

$$\frac{\partial^2 H_y}{\partial z} = -\bar{\gamma} H_{y0}\, e^{-\bar{\gamma} z} = -\bar{\gamma} H_y \quad \text{—— (18)}$$

Using all above results, eqn (6 to 8) can be written as.

$$\bar{\gamma} H_y = j\omega \epsilon E_x \quad \text{—— (19)}$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y \quad \text{—— (20)}$$

$$\frac{\partial H_y}{\partial x} = j\omega \epsilon E_z \quad \text{—— (21)}$$

Similarly using all above eqn (9 to 11) can be written as

$$\bar{\gamma} E_y = -j\omega \mu H_x \quad \text{—— (22)}$$

$$-\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega \mu H_y \quad \text{—— (23)}$$

$$\frac{\partial E_y}{\partial x} = -j\omega \mu H_z \quad \text{—— (24)}$$

Equation (13) and (14) can be rearranged using the same results as give below:

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \bar{\gamma}^2 \bar{E} = -\omega^2 \mu \epsilon \bar{E} \quad \text{—— (25)}$$

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \bar{\gamma}^2 \bar{H} = -\omega^2 \mu \epsilon \bar{H} \quad \text{—— (26)}$$

To obtain the values of the field components E_x, E_y, H_x and H_y eqn. (19 to 21) and (22 to 24) must be solved simultaneously.

From Eqn (19) we can write.

$$E_x = \frac{\bar{\gamma}}{j\omega\varepsilon} H_y \quad \text{——— (27)}$$

But from Eqn (23) Hy can be expressed as,

$$H_y = \frac{\bar{\gamma}}{j\omega\mu} E_x + \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial x} \quad \text{——— (28)}$$

Substituting. the value of Hy in eqn (27)

$$E_x = \frac{\bar{\gamma}}{j\omega\varepsilon}\left[\frac{\bar{\gamma}}{j\omega\mu} E_x + \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}\right]$$

$$E_x = \frac{-\bar{\gamma}^2}{-\omega^2\mu\varepsilon} E_x + \frac{\bar{\gamma}}{-\omega^2\mu\varepsilon} \frac{\partial E_z}{\partial x}$$

$$E_x = -\frac{\bar{\gamma}^2}{\omega^2\mu\varepsilon} E_x - \frac{\bar{\gamma}^2}{\omega^2\mu\varepsilon} \frac{\partial E_z}{\partial x}$$

$$\therefore E_x\left(1 + \frac{\bar{\gamma}^2}{\omega^2\mu\varepsilon}\right) = \frac{-\bar{\gamma}^2}{\omega^2\mu\varepsilon} \frac{\partial E_z}{\partial x}$$

$$\therefore E_x(\omega^2\mu\varepsilon + \bar{\gamma}^2) = -\bar{\gamma}^2 \frac{\partial E_z}{\partial x}$$

$$\therefore E_x = \frac{-\bar{\gamma}^2}{-\bar{\gamma}^2 + \omega^2\mu\varepsilon} \frac{\partial E_z}{\partial x}$$

Let $\boxed{h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon}$ ——— (29)

Sub. value of the term $(\bar{\gamma}^2 + \omega^2\mu\varepsilon)$ as h^2 in above eqn we get,

$$\boxed{E_x = -\frac{\bar{\gamma}^2}{h^2} \frac{\partial E_z}{\partial x}} \quad \text{——— (30)}$$

Exactly one the similar lines other field components can be obtaind. The remaining field component are given by.

$$\boxed{\begin{aligned} E_y &= \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \\ H_x &= -\frac{\bar{\gamma}}{h^2} \frac{\partial H_z}{\partial x} \\ H_y &= -\frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x} \end{aligned}}$$

Unit IV (2)

Q3 Define wave impedance. Obtain the expression for wave impedance of TE, TM and TEM waves in two parallel conducting planes.

Soln The ~~wave~~ wave impedance is defined as the ratio of components of electric field to the magnetic field. Thus in the positive direction the ratio of electric field to the magnetic field strength are defined as.

$$Z_{xy} = \frac{E_x}{H_y}$$

For the positive directions of the co-ordinate.

$$Z^+_{xy} = \frac{E_x}{H_y}, \quad Z^+_{yz} = \frac{E_y}{H_z}, \quad Z^+_{zx} = \frac{E_z}{H_x}$$

$$Z^+_{yn} = -\frac{E_y}{E_x}, \quad Z^+_{zy} = -\frac{E_z}{H_y}, \quad Z^+_{xz} = -\frac{E_x}{H_z}$$

For the negative direction of the co-ordinates.

$$Z^-_{xy} = -\frac{E_x}{H_y}, \quad Z^-_{yz} = -\frac{E_y}{H_z}, \quad Z^-_{zx} = -\frac{E_x}{H_x}$$

$$Z^-_{yx} = \frac{E_y}{H_x}, \quad Z^-_{zy} = \frac{E_z}{H_y}, \quad Z^-_{xz} = \frac{E_x}{H_z}$$

* For TE waves, the wave impedance is given by.

$$Z^+_{yx} = -\frac{E_y}{H_x} = \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta}$$

$$Z^+_{yx} = \frac{\omega\mu}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}}$$

$$= \frac{\cancel{\omega}\mu}{\cancel{\omega}\sqrt{\mu\epsilon}\sqrt{1-(m\pi/a\,/\,\omega\sqrt{\mu\epsilon})^2}}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad \left(\because \omega_c = \frac{m\pi}{a\sqrt{\mu\epsilon}}\right)$$

$$\boxed{Z^+_{yx} = \frac{\eta}{\sqrt{1-(\beta_c/\beta)^2}}}$$ For TE waves.

$$(\because \eta = \sqrt{\mu/\epsilon})$$

At cut-off freq. $\omega^2\mu\epsilon = (m\pi/a)^2$ wave impedance Z^+_{yx} becomes infinity. At every high freq. (greater that cut-off freq.) wave impedance becomes.

$$Z^+_{yx} = \frac{\omega\mu}{\sqrt{\omega^2\mu\epsilon}} = \frac{\mu}{\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

At $\omega \gg \omega_c$, the wave impedance becomes intrinsic impedance.

$$\boxed{Z^+_{yx} = \eta} \quad \text{for } \beta \gg \beta_c.$$

For TM waves, the wave impedance is given by

$$Z^+_{xy} = \frac{E_x}{H_y} = \frac{\beta}{\omega\epsilon} = \frac{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}}{\omega\epsilon}$$

$$= \frac{\cancel{\omega}\sqrt{\mu\epsilon}\cdot\sqrt{1-\left(\frac{m\pi}{a}\right)^2 \omega\sqrt{\mu\epsilon}}}{\cancel{\omega}\epsilon}$$

$$\sqrt{\mu/\epsilon}\cdot\sqrt{1-\left(\frac{m\pi}{a}\right)^2/\omega\sqrt{\mu\epsilon}}$$

$$= \eta\sqrt{1-(\beta_c/\beta)^2}$$

$$\therefore \boxed{Z^+_{xy} = \eta\sqrt{1-(\beta_c/\beta)^2}}$$ for TM waves

At cut-off freq. $\omega^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)$; the wave impedance becomes zero. At very high freq. (greater than cut-off freq). the wave impedance becom

$$Z^+_{xy} = \frac{\sqrt{\omega^2 \mu \epsilon}}{\omega \epsilon} = \frac{\omega \sqrt{\mu \epsilon}}{\omega \epsilon} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

$$\therefore \boxed{Z^+_{xy} = \eta} \quad (\text{for } f >> f_c)$$

For TEM waves, the wave impedance is.

$$Z^+_{xy} = \frac{E_x}{H_y} = \frac{\beta}{\omega \epsilon} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \eta_0 = \text{intrinsic impedan}$$

$$\therefore \boxed{Z^+_{xy} = \eta_0}$$

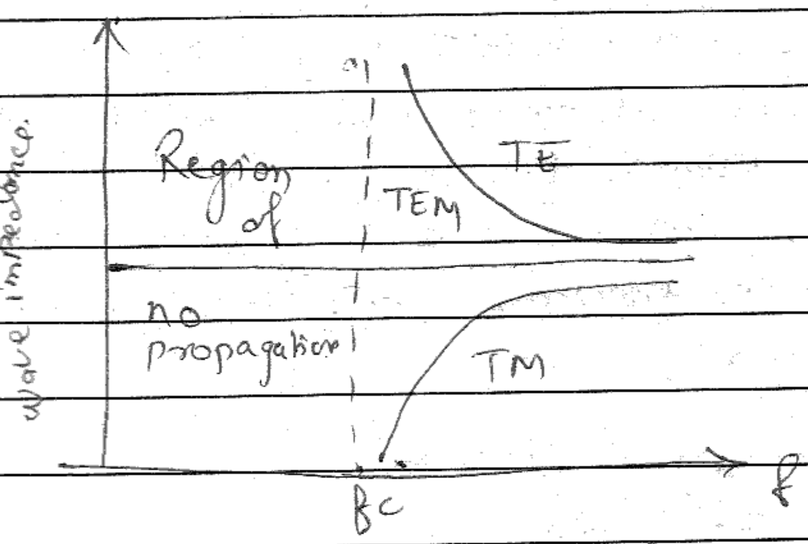
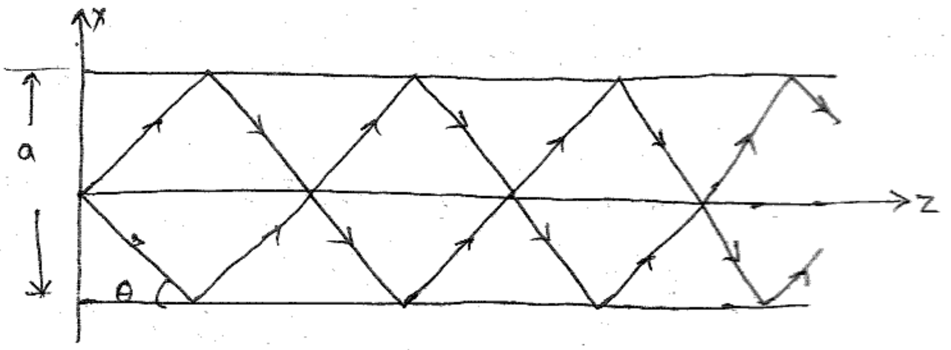


fig. wave impedance versus freq. Characteristics of wave b/w parallel conducting planes

Q4. Explain about the Velocity propagation of guided waves.

Sol^n Consider an electromagnetic wave travelling in positive z direction, propagates through a waveguide of width 'a' as shown in the fig.



The angle θ is the angle made by the wave with the walls of the guide. This angle made by the wave, with the walls and the direction of the wave, depends on the frequency and the distance b/w two walls i.e a for each of the component waves, the component of electric field $\bar{E}$ wall be in y-direction, while that of the magnetic field $\bar{H}$ will be in x-z plane which is perpendicular to the direction of travel of the wave considered.

To satisfy the boundary conditions at the walls of the guide, the total electric field due to two component wave must be equal to zero. So it is possible only if E_y is zero at the walls and non-zero at points b/w the walls. This condition demands that the distance of seperation b/w two walls i.e. a must be some multiple of a half wavelength measured in the perpendicular direction. Thus the condition is given by,

$$a = \frac{m\lambda}{2} \quad \text{--- (1)}$$

Hence the wavelength is given by,

$$\lambda_c = \frac{2a}{m} \quad \text{--- (2)}$$

This wavelength is called 'cut off' wavelength. with the value of λ equal to cut-off wavelength, the waves bounce back and fourth b/w the wall of the guided This ~~is called~~ clearly indicates that there is no wave motion parallel to the axis. But if the value of λ is smaller as compared to the cut-off wavelength, then the waves travel almost parallel to the axis of the guide. This wavelength is that wavelength which is measured as guided wavelength λ_g. Hence the guided wavelength is given by,

$$\boxed{\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{m\lambda}{2a}\right)^2}}}$$

But $\lambda_c = 2a/m$, hence we can rewrite expression for guided wavelength as:

$$\boxed{\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}} \longrightarrow (3)$$

The phase velocity is defined as the rate at which wave changes its phase as the wave propagates inside the region b/w parallel planes. It is denoted by V_p and is given

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi\sqrt{\mu\epsilon}\sqrt{f^2-f_c^2}}$$

$$V_p = \frac{1/\sqrt{\mu\epsilon}}{1/f\sqrt{f^2-f_c^2}}$$

But $\frac{1}{\sqrt{\mu\epsilon}} - v$ = Velocity of propagation of wave along medium b/w parallel planes.

$$\therefore \boxed{V_p = \frac{v}{\sqrt{1-(f_c/f)^2}}} \quad \text{——— (4)}$$

The group velocity is defined as the actual velocity with which the wave propagates inside the region b/w two parallel planes. It is denoted by V_g and is given by.

$$V_g = \frac{d\omega}{d\beta} \quad \text{——— (5)}$$

But $\beta = 2\pi\sqrt{\mu\epsilon}\,\sqrt{f^2-f_c^2}$

$$\therefore \beta = \sqrt{\mu\epsilon}\,\sqrt{\omega^2-\omega_c^2} \quad \text{———} \quad \omega = 2\pi f$$

Differentiating β with respect to ω, we get,

$$\frac{d\beta}{d\omega} = \sqrt{\mu\epsilon}\left[\frac{1}{2\sqrt{\omega^2-\omega_c^2}}\cdot 2\omega\right]$$

$$\frac{d\beta}{d\omega} = \sqrt{\mu\epsilon}\left[\frac{\omega}{\sqrt{\omega^2-\omega_c^2}}\right]$$

$$\therefore \frac{d\beta}{d\omega} = \sqrt{\mu\epsilon}\left[\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}\right] = \sqrt{\mu\epsilon}\left[\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\right] \text{ — (6)}$$

Hence the group velocity is given by,

$$V_g = \frac{d\omega}{d\beta} = \frac{1}{\sqrt{\mu\epsilon}}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

But $\frac{1}{\sqrt{\mu\epsilon}} = v$ = velocity of propagation of waves along the medium b/w the parallel planes.

$$\boxed{V_g = v\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad \text{——— (7)}$$

The relation b/w the velocity of propagation (v), the phase velocity (V_p) and the group velocity (V_g) is given by multiplying eqn (4) with (7). Hence we get.

$$V_p \cdot V_g = \left(\frac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\right)\left(v\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right)$$

$\because \quad V^2 = V_p \cdot V_g$

i.e. $\boxed{V = \sqrt{V_p \cdot V_g}}$ ——— (8)

from Eqn (8) it is clear that the velocity of propagation of wave propagating along the dielectric medium b/w the parallel plane is the geometric mean of the phase and group Velocity.

Q5 Derive the expression for the fields components of TM wave in a parallel plane waveguide.

Soln The transverse magnetic (TM) wave has the electric field in the direction of propagation, but no component of the magnetic field in the same direction. Hence the TM waves are also called E-waves. The equations for the TM wave field components are as follows.

$$\bar{\gamma} H_y = j\omega\varepsilon E_x \quad \text{——— (a)}$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \quad \text{——— (b)}$$

$$\bar{\gamma} E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad \text{——— (c)}$$

(1)

Simplifying eqn (1)-(c) we get,

$$\frac{\partial E_z}{\partial x} = -\bar{\gamma} E_x + j\omega\mu H_y \quad \text{——— (2)}$$

from Eqn 1-(a) we can write.

$$E_x = \frac{\bar{\gamma}}{j\omega\varepsilon} H_y \quad \text{——— (3)}$$

Substituting value of Ex in Eqn ③ we get,

$$\frac{\partial E_z}{\partial x} = -\bar{\gamma}\left[\frac{\bar{\gamma}}{j\omega\varepsilon} H_y\right] + j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} = \left(-\frac{\bar{\gamma}^2}{j\omega\varepsilon} + j\omega\mu\right) H_y \quad \text{—— (4)}$$

Differentiating Eqn 1-(b) with respect to x, we get.

$$\frac{\partial^2 H_y}{\partial x^2} = j\omega\varepsilon \frac{\partial E_z}{\partial x} \quad \text{—— (5)}$$

Substituting value of $\frac{\partial E_z}{\partial x}$ from Eqn ④ in above Eqn, we get

$$\frac{\partial^2 H_y}{\partial x^2} = j\omega\varepsilon \left\{\left(\frac{-\bar{\gamma}^2}{j\omega\varepsilon} + j\omega\mu\right) H_y\right\}$$

$$\therefore \frac{\partial^2 H_y}{\partial x^2} = (-\bar{\gamma}^2 - \omega^2\mu\varepsilon) H_y$$

$$\therefore \frac{\partial^2 H_y}{\partial x^2} = -(\bar{\gamma}^2 + \omega^2\mu\varepsilon) H_y$$

But we know that, $h^2 = (\bar{\gamma}^2 + \omega^2\mu\varepsilon)$. Hence we can rewrite above Eqn as follows.

$$\boxed{\frac{\partial H_y}{\partial x^2} = -h^2 H_y} \quad \text{—— (6)}$$

Similar to the previous case, the Eqn ⑥ written above is a second order differential Eqn representing Simple harmonic motion. Then the Solution of such Eqn can be written as.

$$H_{y0} = C_3 \sinh x + C_4 \cosh x \quad \text{—— (2)}$$

where C_3 and C_4 are the arbitrary Constants

Encorporating the factors representing variations of the field component Hy with respect to time and z-direction, the Eqn can be written as.

$$H_y = (C_3 \sinh x + C_4 \cosh x)\, e^{-\bar{\gamma} z} \quad \text{—— (3)}$$

Similar to the previous case, for the transverse magnetic wave, we cannot directly obtain the values of arbitrary constants by applying the boundary conditions. The reason behind this is that the tangential component of $\bar{H}$ is not zero at the boundary or the surface of the conductor. To overcome this difficulty, obtain expression for E_z and equate to zero to get arbitary constants.

let us consider expression for E_z interms of H_y.

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

Substituting value of H_y from Eqn (3), we get.

$$\frac{\partial}{\partial x}\left[(C_3 \sin hx + C_4 \cos hx)\, e^{-\bar{\gamma} z}\right] = j\omega\varepsilon E_z$$

$$\therefore h\left[(C_3 \cos hx - C_4 \sin hx)\, e^{-\bar{\gamma} z}\right] = j\omega\varepsilon E_z$$

$$\therefore E_z = \frac{h}{j\omega\varepsilon}\left[C_3 \cos hx - C_4 \sin hx\right] e^{-\bar{\gamma} z} \quad \text{— (4)}$$

We Know that the ~~tag~~ tangential Component of $\bar{E}$ is Zero at the boundary for any values of z. and t. Now to have $E_z = 0$ at $x = 0$ from Eqn (4) it is clear that C_3 must be Zero. Hence the expression is given by

$$E_z = -\frac{h}{j\omega\varepsilon} C_4 \sin x\, e^{-\bar{\gamma} z} \quad \text{— (5)}$$

When the second boundary condition is applied i.e at $x = a$ E_z must be zero, again one can obtain a value of h as.

$$\boxed{h = \frac{m\pi}{a} \quad \text{where } m = 1, 2, \ldots} \quad \text{— (6)}$$

Then the field Component E_y is given by.

$$E_z = -\frac{m\pi}{j\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right)\bar{e}^{\bar{\gamma}z} \text{ --}$$

$$\therefore \quad E_z = \frac{jm\pi}{\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right)\bar{e}^{\bar{\gamma}z} \quad \text{--- (7)}$$

let us obtain remaining non-zero field components. Consider the expression for H_y in term of field component E_z given by.

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

Substituting value of E_z we can write.

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon\left[\frac{jm\pi}{\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right)\bar{e}^{\bar{\gamma}z}\right]$$

$$\therefore \quad \frac{\partial H_y}{\partial x} = -\frac{m\pi}{a} C_4 \sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z}$$

Integrating both sides with respect to x, we get

$$H_y = C_4 \cos\left(\frac{m\pi}{a}x\right)\bar{e}^{\bar{\gamma}z} \quad \text{--- (8)}$$

Similary the remaining field component E_x is given by.

$$E_x = \frac{\bar{\gamma}}{\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \quad \text{--- (9)}$$

Collecting equation (7) (8) and (9) together, the field strengths for the transverse magnetic waves b/w the two parallel plates are given by.

$$\boxed{\begin{aligned} E_z &= \frac{jm\pi}{\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \\ E_x &= \frac{\bar{\gamma}}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \\ H_y &= C_4 \cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \end{aligned}} \quad \text{--- (10)}$$

The Same expressions can be written in the zero subscript notation, without writing $e^{-\bar{\gamma}z}$ factor throughout the analysis. The expressions for the field components using zero subscript notation are as follows.

$$\left.\begin{aligned} E_{z0} &= \frac{jm\pi}{\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right) \\ E_{x0} &= \frac{\bar{\gamma}}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) \end{aligned}\right\}$$

$$H_{y0} = C_4 \cos\left(\frac{m\pi}{a}x\right) \quad \text{--- (11)}$$

For the wave guide propagation analysis the expression for TM_{m0} waves over the range of frequencies in which $\bar{\gamma} = j\bar{\beta}$ are given by,

$$\left.\begin{aligned} E_z &= \frac{jm\pi}{\omega\varepsilon a} C_4 \sin\left(\frac{m\pi}{a}x\right) e^{-j\bar{\beta}z} \\ E_x &= \frac{\beta}{\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\bar{\beta}z} \\ H_y &= C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\bar{\beta}z} \end{aligned}\right\} \quad \text{--- (12)}$$

(ii) freq. $= 8000\,MHz = 8\times10^3\times10^6 = 8\times10^9\,Hz$

In TE_1 mode. $m=1$, $n=0$ $\quad d = 10\,cm = a = \frac{10}{100} = 10^{-1}\,m$

Cut off freq $= m/2a\sqrt{\mu\varepsilon}$

$= \frac{m}{2a}\times V$

$= \frac{1}{2\times10^{-1}}\times 3\times10^8$

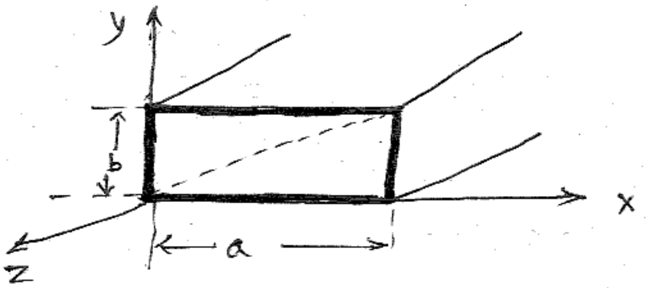
$= \frac{3}{2}\times10^9$

$= 1.5\times10^8\,Hz.$

Q1. Explain about application of Maxwells equations to the rectangular wave guides?

Soln The analysis of the electromagnetic waves within the parallel plate. Conductors extendend infinitely in y and z directions, practically, the conducting plates are always infinite in width, with fringing field at the edges. Hence the electromagnetic energy leaks through the sides of the parallel plate. conductor. Hence the practical waveguides are uniform structures with rectangular or circular cross-section The rectangular cross-section is simpler as it is to analyze as well as manufacture.

As rectangular waveguided is a hollow metal pipe with rectangular cross-section of width 'a' and height 'b' as shown in the fig. According to the standard convention the longest side of the waveguide is considered along x-axis with condition $a > b$.



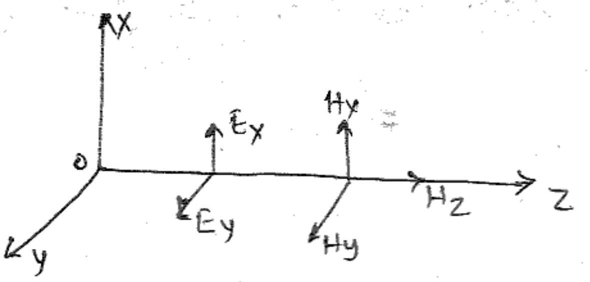
Assume that four conducting boundaries of the waveguide enclose a dielectric which extends in z-direction axially. let ε be the permittivity and μ be the permeability of the dielectric enclosed by the waveguided boundaries. Also assume that both, the conductor and the dielectric are loss-free in the ideal guide.

As discussed earlier, to analyze a waveguide means to find the electric and magnetic field configurations by solving the Maxwell's equations with the boundary conditions applied.

Assume that the four Boundaries of the guide are of perfect Conducting material and the dielectric within the guided is also perfect dielectric (i.e $\sigma = 0$). As the walls of the guide are assumed to be perfect conductor, we can use the boundary Conditions such as the tangential component of the electric field (E_{tan}) and the normal component of the magnetic field (H_{Norm}) are Zero at the surface of the Conductor.

Q2 Derive the expressions in the field components of TE waves in a rectangular waveguide.

Soln Similar to the TM_{mn} waves in the rectangular waveguide, for the transverse electric TE_{mn} wave also one field exists along the transverse direction to the direction of propagation i.e. along the Z-direction. ~~of~~ : Thus for TE_{mn} wave, the electric field can exist only along the transverse direction of the wave propagation. No field component of the electric field can exist along z-direction i.e. $E_z = 0$. But the magnetic field component can exists in all three directions as shown in the fig. Hence for TE_{mn} wave, $E_z = 0$, but $H_z \neq 0$.



For the a TE wave, a partial differential equation is given by,

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \qquad (1)$$

Above eqn (1) is the second order partial differential equation in H_z. This equation is very much similar to the second order partial differential equation in E_z obtained for TM waves. As the equations are of similar type, the method of solving for the solution remains identical. Moreover the solutions obtained are also identical in both the cases. Thus using the variable. ~~Seperation method described in last section~~ ~~we~~ ~~and~~

$$\boxed{H_z = (K_1 \cos Bx + K_2 \sin Bx)\,(K_3 \cos Ay + K_4 \sin Ay)} \qquad (2)$$

Note that K_1, K_2, K_3 and K_4 are the arbitrary constants. To evaluate these constants, we should apply the appropriate boundary conditions.

Boundary Conditions :-

For the rectangular waveguide, the geometry using the rectangular Co-ordinate system is as shown in the fig. 2. Note that no Component of electric field can exist along the side walls of the rectangular waveguide. In case of the TE wave the boundary Conditions are slightly different than those for the TM wave. Because in TE wave the component in the direction of propagation ie. Z-direction can not exist. So in case of the the TE wave the Boundary conditions are specified for the electric field components E_x and E_y in X and Y directions respectively. Thus the boundary conditions for the TE wave are given as.

$E_y = 0$ at $x = 0$, for all values of y from 0 to b.
$E_y = 0$ at $x = a$, for all values of y from 0 to b.
$E_x = 0$ at $y = 0$, for all values of x from 0 to a and.
$E_x = 0$ at $y = b$, for all values of x from 0 to a

With the help of four boundary conditions, the four arbitary constants can be easily evaluated as follows. As the field component E_z can not exist, we must apply the boundary Conditions with the field components E_x and E_y only. consider eq^n 20(a) & 20(b) ~~derived~~ ...

$$E_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad \text{——} (3)$$

$$E_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \quad \text{——} (4)$$

For the TE wave the boundary conditions are applied with Eq^n (2) and (3) and (4) with $E_z = 0$.

Differentiating Eq^n (2) with respect to y, we get.

$$\frac{\partial H_y}{\partial y} = (K_1 \cos Bx + K_2 \sin Bx)(-A K_3 \sin Ay + A K_4 \cos Ay)$$

Substituting these values in Eq^n (3) and applying third boundary condition, get,

$$E_x = 0 ; = -\frac{\bar{\gamma}}{h^2}\frac{\partial}{\partial x}(0) - \frac{j\omega\mu}{h^2}\left[(K_1 \cos Bx + K_2 \sin Bx)\cdot(A K_3 \sin 0 + K_4 \cos 0)\right]$$

$\therefore E_x = 0 = \frac{-j\omega\mu}{h^2}(K_1 \cos Bx + K_2 \sin Bx)(AK_4)$ --- for all x varying from 0 to a.

Note that for all x, term $(K_1 \cos Bx + K_2 \sin Bx) \neq 0$ Also ω, μ, h are constant, hence $\frac{-j\omega\mu}{h^2} \neq 0$. A being constant with squared negative value equal to the coefficient of the double differential equation can't be zero.

Thus the condition is given by

$$\boxed{K_4 = 0}$$

Substituting this value in eq^n (2) we get.

$$\boxed{H_z = (K_1 \cos Bx + K_2 \sin Bx)(K_3 \cos Ay)} \quad \text{——} (5)$$

Now differentiating eq^n (5) with respect to x, we get.

$$\frac{\partial H_z}{\partial x} = (-K_1 \cdot B \sin Bx + K_2 \cdot B \cos Bx)(K_3 \cos Ay)$$

Substituting this value in eq^n (4) and applying first boundary condition, we get,

$$E_y = 0 = -\frac{\bar{\gamma}}{h^2}\frac{\partial}{\partial y}(0) + \frac{j\omega\mu}{h^2}\left[(-BK_1 \sin 0 + BK_2 \cos 0)(K_3 \cos Ay)\right]$$

$\therefore E_y = 0 = \frac{j\omega\mu}{h^2}\left[(K_2 B)(K_3 \cos Ay)\right]$ for all y varying from 0 to b.

Examining above Equation, we observe that for all y $\cos Ay \neq 0$. Also B Can't be zero as per the reason given for constant A. Note that K_3 can't be zero. Because if $K_3 = 0$ and $K_4 = 0$ there is no Solution in existence. Hence K_3 also can't be zero. Thus the Condition is given by,

$$\boxed{K_2 = 0}$$

Substituting the Value of K_2 in Eq^n (5) we get.

$$\boxed{H_z = K_1 K_3 \cos Bx \cos Ay} \quad \text{—— (6)}$$

Differentiating Eq (6) with respect to y, we get.

$$\frac{\partial H_z}{\partial y} = -A K_1 K_3 \cos Bx \sin Ay$$

Sub. value of $\frac{\partial H_z}{\partial y}$ in eq^n (3) and using fourth boundary Condition in Eq^n (3) we get,

$$E_x = 0 = -\frac{\bar{\gamma}}{h^2}\frac{\partial}{\partial x}(0) - \frac{j\omega\mu}{h^2}\left[-A K_1 K_3 \cos Bx \sin Ab\right]$$

$$E_x = 0 = \left(\frac{j\omega\mu}{h^2}\right)(A)(K_1 K_3)\cos Bx \sin Ab.$$

Now for all x varying from 0 to A, $\cos Bx \neq 0$, Being constant as explained previously. A also can't be zero. As ω, μ, h all are constant the term consisting these constants can't be zero. Also $K_1 \neq 0$ and $K_3 \neq 0$ to avoid the condition of no sol^n in existance. Thus the condition is given by

$\sin Ab = 0$ i.e. $Ab = \sin^{-1}(0)$ multiple of π

$Ab = n\pi$

$$\boxed{A = \frac{n\pi}{b}} \quad \text{—— (7)}$$

Now diff. eqⁿ (6) w.r.t x we get

$$\frac{\partial H_z}{\partial x} = -BK_1K_3 \sin Bx \cos Ay.$$

Substituting value of $\frac{\partial H_z}{\partial x}$ in eqⁿ (4) and using second boundary condition in eqⁿ (a) we get,

$$E_y = 0 = \frac{-\bar{\gamma}}{h^2}\frac{\partial}{\partial y}(0) + \frac{j\omega\mu}{h^2}\left[-BK_1K_3 \sin Ba \cos Ay\right]$$

$$\therefore E_y = 0 = \left(-\frac{j\omega\mu}{h^2}\right)(B)(K_1K_3) \sin Ba \cos Ay.$$

Now for any value of y varing from 0 to b, $\cos Ay \neq 0$.

$$\sin Ba = 0$$

$$Ba = \sin^{-1}(0)$$ where $m = 0, 1, 2 - - -$

$$Ba = m\pi$$

$$\boxed{B = m\pi/a} \quad \text{—— (8)}$$

Sub. value of A and B from eqⁿ (7) & (8) resp. in Eqⁿ (6)

$$\boxed{H_z = K_1K_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y} \quad \text{—— (9)}$$

let $K_1, K_2 = K'$ be arbitary const. ∴ eqⁿ (9) rewrite as

$$\boxed{H_z = K' \underset{\cos}{\sin}\left(\frac{m\pi}{a}\right)x \underset{\sin}{\cos}\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\cdot e^{-\bar{\gamma}z} \quad A/m} \quad \text{—— (10)}$$

Now the remaining field components can obtained by using eqⁿ

$$E_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} = \frac{-j\omega\mu}{h^2}\left[K'\left(\frac{n\pi}{b}\right)\cdot\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\cdot e^{-\bar{\gamma}z}\right]$$

$$\therefore \boxed{E_x = K'\left(\frac{j\omega\mu}{h^2}\right)\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\cdot e^{j\omega t}\cdot e^{-\bar{\gamma}z} \quad V/m} \quad \text{—— (11)}$$

Using Eqⁿ, we get

$$E_y = -\frac{\bar{\gamma}}{h^2}\cdot\frac{\partial E_z}{\partial x} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} = \frac{j\omega\mu}{h^2}\left[-K'\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\, e^{-\bar{\gamma}z}\right]$$

$$\therefore \boxed{E_y = K'\left(\frac{j\omega\mu}{h^2}\right)\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\cdot e^{j\omega t}\cdot e^{-\bar{\gamma}z} \quad V/m} \quad \text{—— (12)}$$

Using Eqⁿ

$$H_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} = \frac{-\bar{\gamma}}{h^2}\left[-K'\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\cdot e^{-\bar{\gamma}z}\right]$$

$$\boxed{H_x = K'\left(\frac{\bar{\gamma}}{h^2}\right)\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\cdot e^{-\bar{\gamma}z} \quad A/m} \quad \text{—— (13)}$$

Sim.

$$H_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} + j\omega\varepsilon\frac{\partial E_z}{\partial y} = -\frac{\bar{\gamma}}{h^2}\left[-K'\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\cdot e^{-\bar{\gamma}z}\right]$$

$$\boxed{H_y = K'\left(\frac{\bar{\gamma}}{h^2}\right)\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t}\, e^{-\bar{\gamma}z} \quad A/m} \quad \text{—— (14)}$$

Q3. Explain about various TE_{mn} modes and dominate TE_{mn} mode in rectangular wave guide?

Sol^n Similar to TM_{mn} wave, the mode of the TE_{mn} wave in the rectangular waveguide depends on the value of m and n. To check which mode of TE_{mn} wave exists in the guide; different values of m and n in eq^n let us consider value of m & n.

a) TE_{00} mode (with m=0, n=0) :- If we sub. these value of m & n in eq^n (11) & (14) it is observed that all the field components inside the guide becomes zero. In other word TE_{00} mode cannot exist inside the rectangular waveguide.

b) TE_{01} mode (with m=0, n=1) :- If we sub. these value of m and n in eq^n (11) & (14) the field component of E_y and H_x inside the guide becomes zero. But the field component E_x & H_y can exist inside the guide. TE_{01} mode can exist inside the Rectangular wave guide.

c) TE_{10} mode (with m=1, n=0) :- If we sub. these value of m & n in eq^n (11) & (14) the field component E_x and H_y inside the guide becomes zero. But the field components E_y & H_x exist inside the guide. Thus TE_{10} mode can exist inside the rectangular waveguide.

d) TE_{11} mode (with m=1, n=1) :- It is clear from the second and third conditions for either m or n non-zero. The mode exists in the guide. Thus TE_{11} mode and all highest mode can exist inside the rect. waveguide.

The mode of the TE_{mn} wave for which the value of λ_{cmn} is highest is known as the dominant mode of the TE_{mn} wave.

Consider expr. for the cut-off wavelength given by,

$$\lambda_{cmn} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

i) For TE_{01} mode $m=0, n=1$

$$\lambda_{c01} = \frac{2ab}{\sqrt{0^2+a^2}} = 2b.$$

ii) For TE_{10} mode : $m=1, n=0$

$$\lambda_{c10} = \frac{2ab}{\sqrt{b^2+0}} = 2a.$$

iii) For TE_{11} mode $m=1, n=1$

$$\lambda_{c11} = \frac{2ab}{\sqrt{a^2+b^2}}.$$

iv) For TE_{12} mode, $m=1, n=2$

$$\lambda_{c12} = \frac{2ab}{\sqrt{a^2+4b^2}}$$

From above value of λ_{cmn} for different values of m, n, it is clear that for the higher mode, the term in the denominator increases, which reduces λ_{cmn} effectively. Hence clearly for the TE_{10} mode, the value of λ_{cmn} is max^m i.e. 2a. Hence the TE_{10} mode is the dominate mode inside the rectangular waveguide for the TE wave.

The value of cut-off wavelength for the dominant mode TM_{11} for TM waves is given by.

$$\lambda_{c11} = \frac{2ab}{\sqrt{a^2+4b^2}}$$

Similarly the value of the cut-off wavelength for the dominant mode i.e. TE_{10} for TE waves is given by.

$$\lambda_{c10} = 2a.$$

Thus $\lambda_{c10} > \lambda_{c11}$. Hence out of these two dominant modes the ~~most~~ dominant mode is obviously TE_{10}.

* Guide wavelength (λ_g), Group velocity (V_g) & Phase velocity (V_p)

The distance travelled by the wave to have phase shift of 2π radian inside the guide is called guide wavelength. The guide wavelength is given by.

$$\boxed{\lambda_g = \frac{2\pi}{\beta}}$$

b) Group velocity: The group velocity of the wave is defined as the rate at which the wave actually propagates - It is denoted by V_g.

$$\boxed{V_g = \frac{d\lambda_g}{dt} = \frac{d\omega}{d\beta}}$$

Phase Velocity (V_p):- The phase velocity is defined as the rate at which the wave changes its phase with respect to the guided wavelength λg.

$$V_p = \frac{\lambda g}{t} = \lambda g \cdot f = \frac{(2\pi f)\lambda g}{2\pi} = \frac{2\pi f}{\frac{2\pi}{\lambda g}}$$

$$\boxed{V_p = \omega \lambda g / \beta} \quad \Rightarrow \boxed{V_p = \lambda g \cdot f = \omega/\beta}$$

TLWG. Unit - I

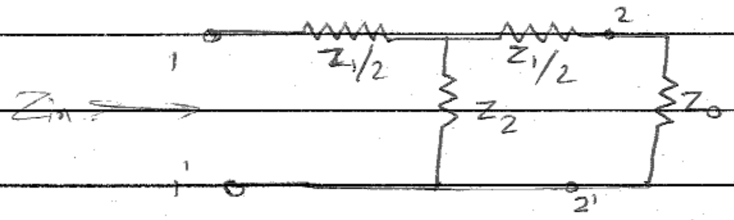
AARUPADAI VEEDU INSTITUTE OF TECHNOLOGY Sheet No.

Q1. Characteristics Impedance of Symmetrical N/w.

The N/w is said to be symmetrical if the electrical properties of the n/w are unaffected even after interchanging i/p and o/p terminal symmetrical n/w has two important electrical properties are:-

i) Characteristics impedance (Z_0) (ii) propagation contant (γ)

Characteristic impedance of 'T' section symmetrical N/w :-



When $Z_1 = Z_2$, then the N/w are said to be symmetrical. The value of Z_0 for a symmetrical N/w can be easily determined. For the T-N/w of above fig. terminate in an impedance Z_0, then i/p impedance

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2(Z_1/2 + Z_0)}{Z_2 + Z_1/2 + Z_0} \quad \text{—— (1)}$$

for symmetrical n/w $Z_{in} = Z_0$, sub in eqn (1) we get

$$Z_0 = \frac{(Z_1/2)}{1} + \frac{Z_2(Z_1/2 + Z_0)}{Z_2 + Z_1/2 + Z_0}$$

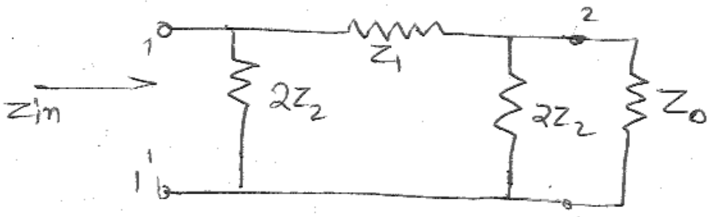
$$Z_0\left(Z_1/2 + Z_2 + Z_0\right) = Z_1/2\left(Z_1/2 + Z_2 + Z_0\right) + Z_2\left(Z_1/2 + Z_0\right)$$

$$\Rightarrow \frac{Z_0 Z_1}{2} + Z_0 Z_2 + Z_0^2 = \frac{Z_1^2}{4} + \frac{Z_1 Z_2}{2} + \frac{Z_1 Z_0}{2} + Z_2 Z_0 + \frac{Z_2 Z_1}{2}$$

$$\Rightarrow Z_0^2 = Z_1^2/4 + Z_1 Z_2$$

$$\therefore \quad \boxed{\begin{aligned} Z_0 &= \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \\ Z_{0T} &= \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} \end{aligned}}$$

Similarly. For π- Section, terminated in an impedance Z_0 then the impedance is.



For symmetrical π-n/w : $Z_{in} = Z_0$ therefore.

$$Z_{in} = Z_0 = 2Z_2 \,||\, \left[Z_1 + (2Z_2 \,||\, Z_0)\right]$$

$$Z_0 = 2Z_2 \,||\, \left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]$$

$$Z_0 = \frac{2Z_2\left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}}$$

$$Z_0 = \frac{2Z_2\left[\frac{2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}{\cancel{2Z_2 + Z_0}}\right]}{\frac{2Z_2(2Z_2 + Z_0) + Z_1(2Z_2 + Z_0) + 2Z_2 Z_0}{\cancel{2Z_2 + Z_0}}}$$

$$Z_0 = \frac{4Z_1 Z_2^2 + 2Z_2 Z_0 Z_1 + 4Z_2^2 Z_0}{4Z_2^2 + 2Z_2 Z_0 + 2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}$$

$$\cancel{4Z_2^2 Z_0} + 4Z_2 Z_0^2 + \cancel{2Z_2 Z_1 Z_0} + Z_1 Z_0^2 = 4Z_1 Z_2^2 + \cancel{2Z_2 Z_0 Z_1} + \cancel{4Z_2^2 Z_0}$$

$$\Rightarrow 4Z_2 Z_0^2 + Z_1 Z_0^2 = 4Z_1 Z_2^2$$

$$\Rightarrow Z_0^2\left[4Z_2 + Z_1\right] = 4Z_1 Z_2^2$$

$$\Rightarrow Z_0^2 = \frac{4Z_1 Z_2^2}{4Z_2 + Z_1}$$

Multiply by $Z_1/4$ in num & denom

$$Z_0^2 = \frac{Z_1^2 Z_2^2}{Z_1 Z_2 + Z_1^2/4} = \frac{Z_1 Z_2 \cancel{(Z_1 Z_2)}}{\cancel{Z_1 Z_2}(1 + Z_1/4Z_2)}$$

$$Z_0^2 = \frac{Z_1 Z_2}{1 + Z_1/4Z_2}$$

$$\boxed{Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}}}$$

In terms of open and short ckt impedance.

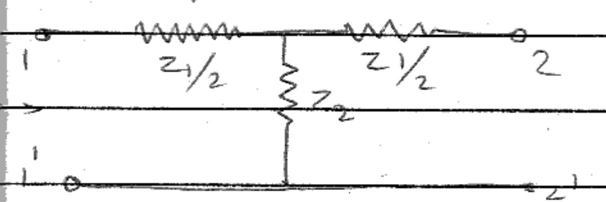
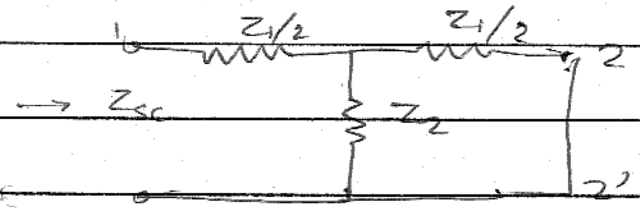


Fig (a)



(b)

From fig (a)

$$Z_{oc} = Z_{1oc} = Z_{2oc} = Z_1/2 + Z_2 \quad \text{—— (1)}$$

From the fig (b)

$$Z_{sc} = \frac{Z_1/2}{1} + \frac{Z_1 Z_2/2}{Z_2 + Z_1/2}$$

$$Z_{sc} = \frac{Z_1^2/4 + Z_1 Z_2/2 + Z_1 Z_2/2}{Z_1/2 + Z_2} \quad \text{—— (2)}$$

Multiplying eqn (2) & eqn (1)

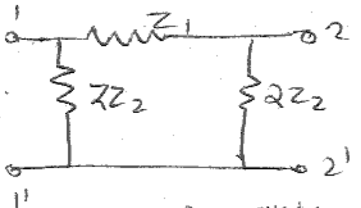
$$Z_{oc} \cdot Z_{sc} = \left(\cancel{\frac{Z_1}{2} + Z_2}\right) \cdot \left(\frac{Z_1^2/4 + Z_1 Z_2/2 + Z_1 Z_2/2}{\cancel{Z_1/2 + Z_2}}\right)$$

$$Z_{oc} \cdot Z_{sc} = \frac{Z_1^2}{4} + Z_1 Z_2$$

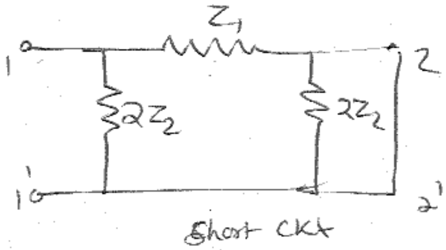
$$Z_{oc} \cdot Z_{sc} = Z_0^2 \cdot \left(\because \; Z_0^2 = Z_1^2/4 + Z_1 Z_0\right)$$

$$\boxed{Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}}$$

For π/N/W



open ckt.



Short ckt

From fig (a)

$$Z_{oc} = \frac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2} \quad \text{——— (1)}$$

From fig (b)

$$Z_{sc} = \frac{2Z_1 Z_2}{Z_1 + 2Z_2} \quad \text{——— (2)}$$

Multiplying eqn (1) × (2)

$$\therefore Z_{oc} \cdot Z_{sc} = \frac{2Z_2 \cancel{(Z_1 + 2Z_2)}}{Z_1 + 4Z_2} \times \frac{2Z_1 Z_2}{\cancel{(Z_1 + 2Z_2)}}$$

$$Z_{oc} \cdot Z_{sc} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

∴ Multiply $Z_1/4$ in num & Denom.

$$\therefore Z_{oc} \cdot Z_{sc} = \frac{Z_1^2 Z_2^2}{Z_1^2/4 + Z_1 Z_2}$$

$$Z_{oc} \cdot Z_{sc} = Z_0^2$$

$$\therefore \boxed{Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}}$$

Q3 Design a LPF (both T & π) having cut-off freq 1KHz to.

i) Operate with a terminate load resistance of 200.

ii) Find the freq at which this filter offers attenuation of 19.1 dB.

Ans Cut-off freq. = 1KHz
= 1000 Hz.

$R_0 = 200$

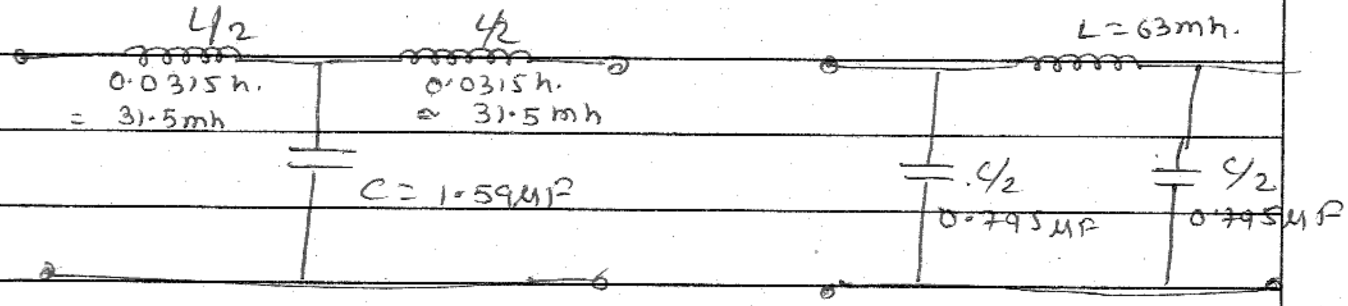
$$L = \frac{R_0}{\pi f_c} = \frac{\cancel{200}}{\pi \times \cancel{1000}_{5}} = \frac{1}{5\pi} = 0.063$$

$L = 0.063$ Henery. $L/2 = 0.0315$ henery.

$C = \dfrac{1}{\pi f_c R_0} = \dfrac{1}{1000\pi \times 200} = \dfrac{10^{-5}}{2\pi}$

$= 1.59 \times 10^{-6}$ Farad.

$= 1.59\,\mu F$ $\qquad C/2 = 0.795\,\mu F$



T - Section $\qquad$ π - Section.

Attenuation = 19.1 db.

WKT. 1 db = 0.115 nepher.

∴ $19.1\,db = 19.1 \times 0.115$ nepher. $= 2.1965$ nepher.

$\alpha = 2\cosh^{-1}\left(\dfrac{f_\infty}{f_c}\right)$

$2.1965 = 2\cosh^{-1}\left(\dfrac{f_\infty}{1000}\right)$

$\cosh^{-1}\left(\dfrac{f_\infty}{1000}\right) = \dfrac{2.1965}{2}$

$\cosh^{-1}\left[\dfrac{f_\infty}{1000}\right] = 1.09825$

$\cosh\left[\cosh^{-1}\left(\dfrac{f_\infty}{1000}\right)\right] = \cosh\left[1.09825\right]$

$f_\infty = f_c \times \cosh(19.1/2)$

$= 1000 \times \cosh(9.55)$

$= 7022.34 \times 10^3$

$= 7022.34$ KHz

For π/N $\Rightarrow$ $\frac{f_\infty}{1000} = 0.4551$

$\Rightarrow f_\infty = 0.4551 \times 1000 = 455.1$ KHz.

$= 0.4551$ KHz.

Q4 Filter Fundamental (pass & stop band)

Soln It is desired that a filter n/w transmitt or pass a desired frequency band without loss, where it should stop or completely attenuate all undesired frequency.

The ability of any filter n/w to work as per requirement is decided by the propagation constant (γ) of the n/w.

$\Rightarrow$ The propagation cont. is given by.

$$\gamma = \alpha + j\beta$$

α = attenuation const.

β = phase constant.

If $\alpha = 0$ or $I_1 = I_2$, then there is no attenuation only a phase shift. this is the operation of filter in the pass band of frequencies.

when α has a positive value & I_2 is smaller in magnitude then I_1, attenuation occured. This is the operation of filter in the stop band of frequencies.

The propagation constant γ for symmetrical 'T' section given by.

$$\sinh \gamma/2 = \sqrt{Z_1/4Z_2} \quad \text{——— (3)}$$

$$\gamma = \alpha + j\beta$$

$$\therefore \sinh \gamma/2 = \sinh(\alpha/2 + j\beta/2)$$

$$\therefore \sinh \gamma/2 = \sinh \alpha/2 \cdot \cos \beta/2 + j \cosh \alpha/2 \cdot \sin \beta/2 \quad \text{——— (1)}$$

$$\therefore \sinh \alpha/2 \cos \beta/2 + j \cosh \alpha/2 \sin \beta/2 = \sqrt{Z_1/4Z_2} \quad \text{——— (2)}$$

from the above Eqn α & β also depends on the two impedance i.e. Z_1 & Z_2.

Case I: If Z_1 and Z_2 are the same type of reactance then $Z_1/4Z_2 > 0$ or the ratio is positive & reac

This requires that $\sinh \gamma/2$ be real which means that the imaginary term must be equal to zero.

$\therefore \sin \beta/2 \cdot \cosh \alpha/2 = 0$ — (3)

$\sinh \alpha/2 \cdot \cos \beta/2 = \sqrt{Z_1/4Z_2}$ — (4)

From eqn (3)

$\sin \beta/2 = 0; \quad \beta = n\pi$ where $n = 0, 2, 4 \ldots$

From eqn (4), $\sin \beta/2 = 1$, then

$\sinh \alpha/2 = \sqrt{Z_1/4Z_2}$

Then the attenuation will be given by

$$\boxed{\alpha = 2\sinh^{-1}\sqrt{Z_1/4Z_2}}$$

Thus the ~~attenuation~~ condition that $Z_1/4Z_2 > 0$ implies a stop or attenuation band of freq.

Case II: If Z_1 & Z_2 are opposite type of reactances then $Z_1/4Z_2$ is negative $|Z_1/4Z_2| < 0$, then the real part must be zero.

So that, $\sinh \alpha/2 \cdot \cos \beta/2 = 0$ — (5)

$\cosh \alpha/2 \cdot \sin \beta/2 = \sqrt{Z_1/4Z_2}$ — (6)

Two conditions are possible from the above eqn.

i) $\sinh \alpha/2 = 0$

$\therefore \alpha = 0, \beta \neq 0$; $\sin \beta/2 = \sqrt{Z_1/4Z_2}$

ii) $\cos \beta/2 = 0$

$\therefore \sin \beta/2 = \pm 1$ & $\alpha \neq 0$

$\beta (2n-1)\pi$

$\therefore \cosh \alpha/2 \sqrt{Z_1/4Z_2}$

Condition (i) gives a passband as $\alpha = 0$ i.e zero attenuation than phase angle in the passband is given by,

$$\boxed{\beta = 2\sinh^{-1} \sqrt{Z_1/4Z_2}} \quad \text{--- (7)}$$

Condition (ii) gives a stop band as $\alpha = 0$, then the phase angle is π, then the attenuation is given by,

$$\boxed{\alpha = 2\cosh^{-1} \sqrt{Z_1/4Z_2}} \quad \text{--- (8)}$$

From eqn (7) & eqn (8) we can easily calculate α in the stop band in which $\beta = \pi$ & the phase shift β is phase band in which $\alpha = 0$.

The freq. at which the N/w changes from a pass N/w to stop N/w or vice versa are called cut-off freq. f_i these freq. occur when

$\frac{Z_1}{4Z_2} = 0$ (or) $Z_1 = 0$

$\frac{Z_1}{4Z_2} = -1$ or $Z_1 = -4Z_2$

where Z_1 and Z_2 are opposite types of reactances.

Qs Behaviour of Characteristics eqn ? –

In the sys. T n/w the char. imp. is given by,

$$Z_0 = \sqrt{Z_1 Z_2 \times \left(1 + \frac{Z_1}{4Z_2}\right)}$$

For pure reactance this exp. for char. imp. becomes

$$Z_0 = \sqrt{-x_1 x_2 \times \left(1 + x_1/4x_2\right)}$$

Const. K- Filter:-

1) A T or π Section in which Series and Shunt impedance Z_1 and Z_2 Satisfy the relationship.

$$\boxed{Z_1 \cdot Z_2 = R_0^2}$$

where R_0 = Real const. it is denoted as

R_K = Real const independent of freq.

$$\boxed{Z_1 Z_2 = K^2} \quad \text{or.} \quad \boxed{Z_1 Z_2 = R_K^2}$$

Networks or filter section for which this relation holds are called const. K- filters.

2) Constant K. LPF

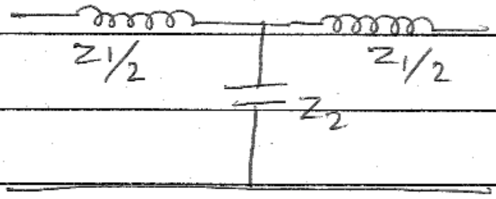
In LPF the series impedance $Z_1 = j\omega L$, & $Z_2 = -j/\omega C$

$$\boxed{Z_1 \times Z_2 = j\omega L \times -j/\omega C = \frac{L}{C} = R_K^2}$$

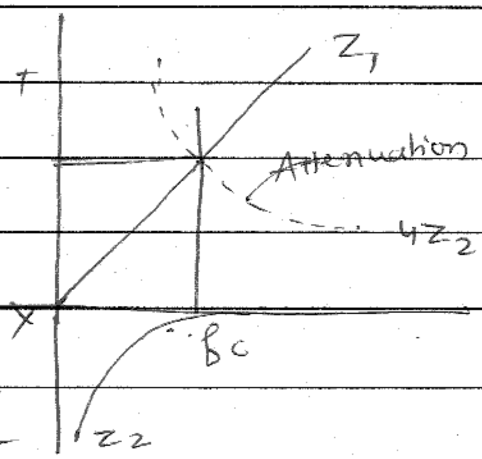
The term R_K is used since K is real and const. if Z_1 & Z_2 are oposite type.

$$R_0^2 = L/C$$

$$R_0 = \sqrt{L/C}$$



a) LPF)



We Know that,

$Z_1 = -4Z_2$.

Does the reactance show that a pass band starts at $f=0$ and continues for f_c.

All freq. above f_c lies is attenuation or Stop band does the n/w is called LPF.

The cut-off f_c may be obtained by using.

$Z_1 = -4Z_2$ —— (1)

Sub the $Z_1 = j\omega L$, $Z_2 = -j/\omega_c C$

$$j\omega_c L = -4 \times 1/j\omega_c C$$

$$\omega_c L = \frac{4}{\omega C}$$

$$\omega_c = 2\pi f_c.$$

$$L \cdot 2\pi f_c = 4/2\pi f_c C$$

$$f_c^2 = \frac{4}{4\pi^2 LC}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

17 Const. β with freq:

For pass band $\alpha = 0$, $\beta = \pi$.

then $\sin h\frac{\gamma}{2} = \sqrt{Z_1/4Z_2}$

$$\sin h\,\gamma/2 = \sqrt{\frac{j\omega L}{4\times -j/\omega C}} = \sqrt{\frac{-\omega^2 LC}{4}}$$

$$= \frac{j\omega}{2}\sqrt{LC}$$

We Know that $f_c = 1/\pi\sqrt{LC}$, $\omega = 2\pi f$.

$$\sin h\,\gamma/2 = j\,\frac{2\pi f}{2\pi f_c} = j\,f/f_c.$$

∴ For pass band $\alpha = 0$

$$\sin \beta/2 = f/f_c$$

$$\beta = 2\sin^{-1} f/f_c.$$

The characteristics impedance of T-section can be calculated by

$$Z_0 \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{(j\omega L)^2}{4} - \frac{L}{j\omega C} \times j\omega L}$$

$$Z_0 = \sqrt{\frac{j^2\omega^2L^2}{4} + (j\omega L)\left(\frac{-j}{\omega C}\right)}$$

$$Z_0 = \sqrt{\frac{-\omega^2L^2}{4} + \frac{L}{C}} = Z_0 = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{\omega^2 LC}{4}}$$

w.k.T. $R_0 = \sqrt{L/C}$

$$Z_0 = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}$$

from the above exp. it is clear that R_0 is Real if $\frac{\omega^2 LC}{4} < 1$ and imaginary if $\frac{\omega LC}{4} > 1$

Hence condn $\frac{\omega^2 LC}{4} - 1 = 0$ which gives

$$\frac{\omega^2 LC}{4} - 1 = 0$$

$$\omega^2 = \frac{4}{LC} \qquad \omega = \frac{2}{\sqrt{LC}} \quad \text{thus.}$$

the above exp. shows it pass all freq. below $\omega = 2\pi/\sqrt{LC}$ while attenuates above this value

∴ cut-off freq. of low pass filter is given by.

$$\omega_c = 2/\sqrt{LC}$$

$$2\pi f_c = 2/\sqrt{LC}$$

$$\boxed{f_c = 1/\pi\sqrt{LC}}$$

* Variation of Z_0 with freq.

Consider $Z_0 = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}$ —(1)

WKT. $\omega_c = \frac{2}{\sqrt{LC}}$ or $\omega_c^2 = \frac{4}{LC}$ sub in (1)

$$Z_0 = \sqrt{\frac{1 - 4/LC \times L}{4}} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c L}}$$

$$Z_0 = \sqrt{1 - \left(\frac{2\pi f}{2\pi f_c}\right)^2}$$

$$\boxed{Z_{0T} = R_0 \sqrt{1 - (f/f_c)^2}}$$

WKT. that

Illly. $Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$

$$Z_{0\pi} = \frac{Z_1 Z_2}{R_0\sqrt{1-(f/f_c)^2}}$$

WKT. $Z_1 \cdot Z_2 = R_0^2$

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1-(f/f_c)^2}}}$$

Q2 General Solution.