Behaviour of The characteristic Impedance.

For Symmetric T network, the characteristic impedance is

$$Z_{oT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

If the impedance Z_1 and Z_2 are purely reactive

$Z_1 = jX_1$ and $Z_2 = jX_2$

$$\therefore Z_{oT} = \sqrt{-X_1 X_2 \left(1 + \frac{X_1}{4X_2}\right)}$$

If X_1 and X_2 are the same type of reactance, a stop band exists. The ratio $\frac{X_1}{4X_2}$ will be real and positive and the characteristic impedance will be pure reactance in attenuation region

If X_1 and X_2 are the opposite type of reactance and $-1 < \frac{X_1}{4X_2} < 0$, a pass band exists. The characteristic impedance will be real and positive in pass band region.

If X_1 and X_2 are the opposite type of reactance and $\frac{X_1}{4X_2} < -1$, a stop band exists.

The characteristic impedance will be pure reactance in stop region.

CONSTANT K-FILTER:

If Z_1 is series impedance and Z_2 is the shunt impedance of T or π network, then

$$Z_1 Z_2 = K^2$$

Where, K is a constant independent of frequency. This network is called as constant K-filter.

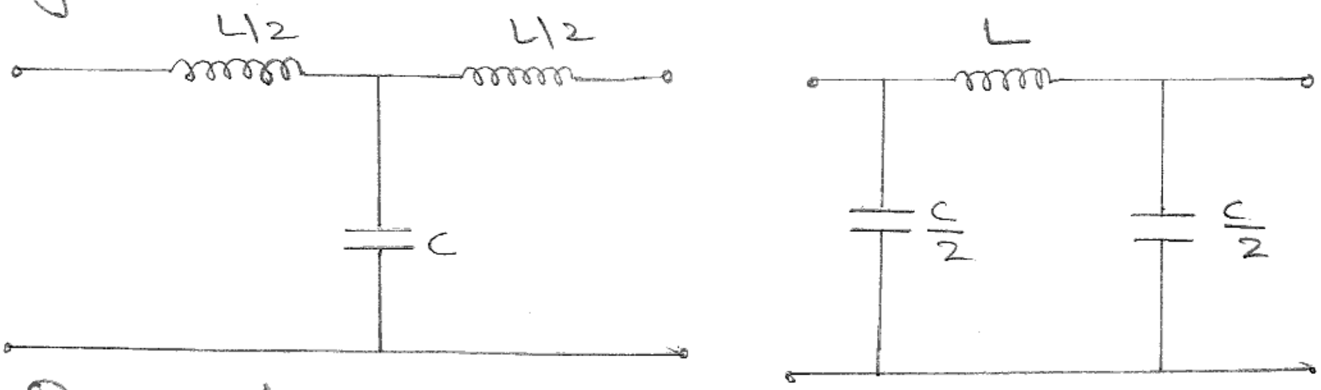
If $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ then

$$Z_1 Z_2 = (j\omega L)\left(\frac{-1}{\omega C}\right) = \frac{L}{C}$$

$$= \frac{L}{C} = R_K^2 = K$$

Constant K low pass filter:

The constant K low pass filter in T and π type is shown.



(a) T-network

(b) π-network

Constant K low pass filter.

Pass band starts at the frequency at which $Z_1 = 0$ and continues to the frequency (f_c) at which $Z_1 = -4Z_2$. This frequency f_c is called cut-off frequency.

At cut-off frequency

$$Z_1 = -4Z_2$$

$$j\omega_c L = \frac{4j}{\omega_c C}$$

$$\omega_c^2 = \frac{4}{LC}$$

$$\omega_c = \frac{2}{\sqrt{LC}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

All frequency above cut-off frequencies lie in a stop band. This network is known as low pass filter

The characteristic impedance of T-network.

$$Z_{OT} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$Z_1 = jL\omega \text{ and } Z_2 = \frac{-j}{C\omega}$$

$$Z_{OT} = \sqrt{jL\omega\left(\frac{-j}{C\omega}\right)\left(1 + \frac{jL\omega}{4\left(\frac{-j}{C\omega}\right)}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)} \qquad \left\{\because \omega_c^2 = \frac{LC}{4}\right\}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2}{\omega_c^2}\right)}$$

$$\sqrt{\frac{L}{C}\left[1-\left(\frac{f}{f_c}\right)^2\right]}$$

$$= \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$$= R_K\sqrt{1-\left(\frac{f}{f_c}\right)^2} \qquad \left\{\because R_K = \sqrt{\frac{L}{C}}\right\}$$

If $f < f_c$, Z_{OT} becomes real indicating the pass band. If $f > f_c$, Z_{OT} becomes imaginary indicating the stop band

At $f = f_c$, $Z_{OT} = R_K$

Design: At cut-off

$$Z_1 = -4Z_2$$

$$L\omega_c = \frac{4}{C\omega_c}$$

$$\omega_c^2 = \frac{4}{LC}$$

$$f_c^2 = \frac{1}{\pi^2 LC}$$

But $Z_1 Z_2 = R_K^2$

$$L\omega\left(\frac{1}{C\omega}\right) = R_K^2$$

$$\frac{L}{C} = R_K^2$$

$$L = R_K^2 C$$

Substituting the value of L in f_c^2 equation

$$f_c^2 = \frac{1}{\pi^2 R_K^2 C^2}$$

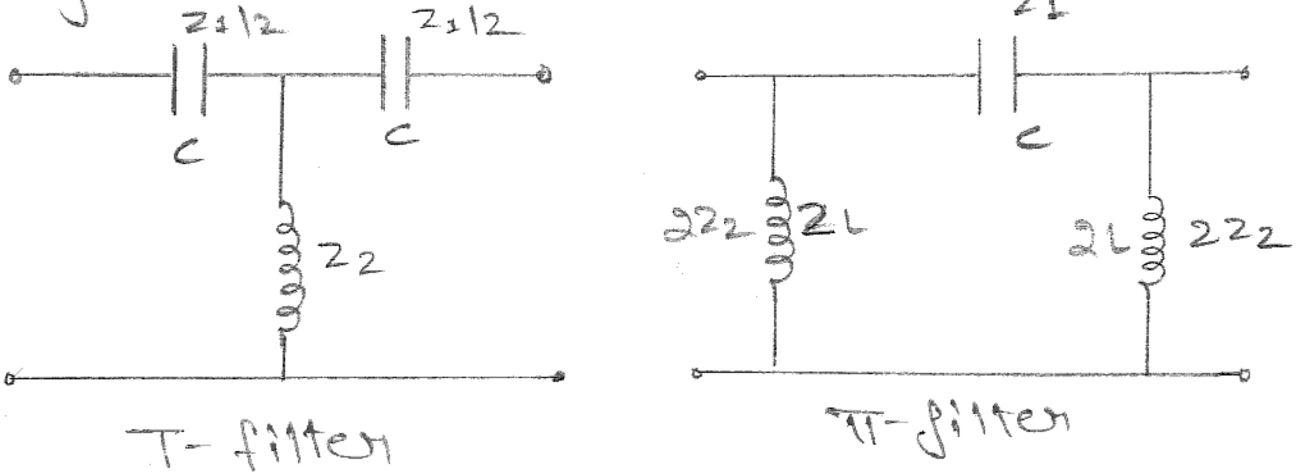
$$C^2 = \frac{1}{\pi^2 f_c^2 R_K^2}$$

$$\therefore C = \frac{1}{\pi f_c R_K}$$

$$\therefore L = \frac{R_K}{\pi f_c} \qquad \left\{ \because L = R_K^2 C \right\}$$

Constant K - High Pass filter

The constant K high pass filter in T and π type is shown



Constant K high pass filter.

If $Z_1 = \frac{-j}{\omega_c C}$ and $Z_2 = j\omega_c L$; then

$$Z_1 Z_2 = \left(\frac{-j}{\omega_c C}\right)\left(j\omega_c L\right) = \frac{L}{C} = K = R_K^2$$

Pass band starts at cut-off frequency (f_c) at which $Z_1 = -4Z_2$ to infinity at which $Z_1 = 0$

$$Z_1 = -4Z_2$$

$$\frac{-j}{\omega_c C} = -j4L\omega_c$$

$$4LC\omega_c^2 = 1$$

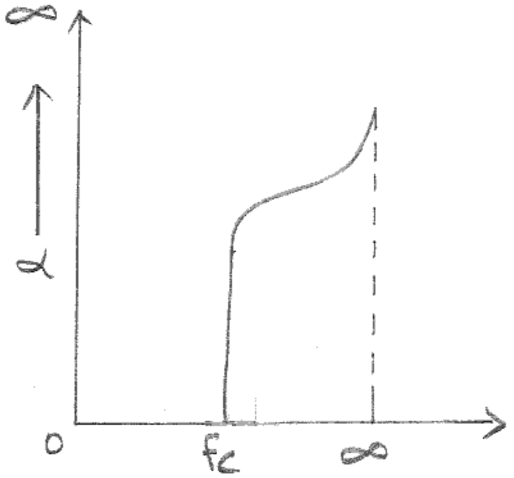
$$\omega_c^2 = \frac{1}{4LC}$$

$$\omega_c = \frac{1}{2\sqrt{LC}}$$

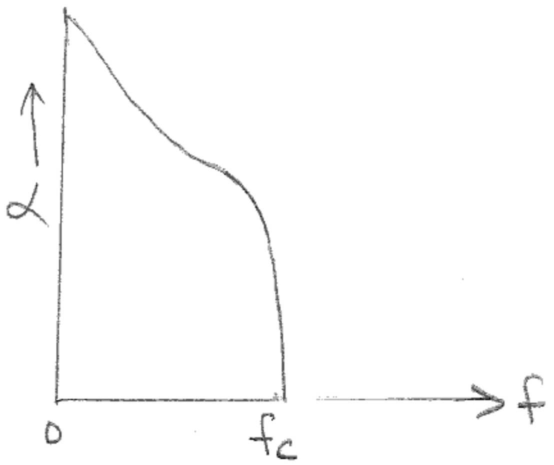
$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

All the frequency below cut-off frequency (f_c) lie in stop band. This network is said to be high pass filter.

The frequency response of constant K low pass and high pass filter are shown in fig.



Low pass filter



High pass filter.

Frequency response of constant K low pass and high pass filter.

The characteristic impedance of T network

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{Z_2}\right)}$$

$$= \sqrt{\left(\frac{-j}{C\omega}\right)(jL\omega)\left(1 + \frac{-j/C\omega}{4jL\omega}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{1}{4LC\omega}\right.}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega_c^2}{\omega^2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{f_c^2}{f^2}\right)} \quad \left\{ \because \omega_c^2 = \frac{1}{4LC} \right.$$

$$= \sqrt{\frac{L}{C}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$= R_K \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

If $f < f_c$, Z_0 becomes imaginary stop band

If $f > f_c$, Z_0 become real indicating pass band

At $f = f_c$, $Z_{OT} = R_K$

Design: At cut-off

$$Z_1 = -4 \cdot Z_2$$

$$\frac{-j}{C\omega_c} = -j4L\omega_c$$

$$\omega_c^2 = \frac{1}{4LC}$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

But $Z_1 Z_2 = R_K^2$

$$\left(\frac{-j}{C\omega}\right)(jL\omega) = R_K^2$$

$$\frac{L}{C} = R_K^2$$

$$L = R_K^2 C$$

Substituting the value of L in f_c^2 equation

$$f_c^2 = \frac{1}{16\pi^2 R_K^2 C^2}$$

$$f_c = \frac{1}{4\pi R_K C}$$

$$\therefore C = \frac{1}{4\pi f_c R_K}$$

$$\therefore L = \frac{R_K}{4\pi f_c} \quad \left\{ \because L = R_K^2 C \right\}$$