nstant

Determination of ... Constant :-

$$e^{\gamma l} = 1 + \gamma l + \frac{\gamma^2 l^2}{2!} + \frac{\gamma^3 l^3}{3!} + \dots$$

$$\therefore e^{\gamma l} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{2Z_2}$$

$$= 1 + \frac{(R+j\omega L)l}{\frac{\cancel{2l\times}\, 2}{(G+j\omega C)l}} + \frac{Z_0}{\frac{\cancel{2}\,1}{(G+j\omega C)l}}$$

$$= 1 + (R+j\omega L)l \times \frac{(G+j\omega C)l}{2} + Z_0(G+j\omega C)l$$

$$= 1 + \frac{(R+j\omega L)(G+j\omega C)l^2}{2} + Z_0(G+j\omega C)l.$$

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$$= 1 + \frac{(R+j\omega L)(G+j\omega C)l^2}{2} + \sqrt{\frac{R+j\omega L}{G+j\omega C}}\,(G+j\omega C)l$$

$$= 1 + \frac{(R+j\omega L)(G+j\omega C)l^2}{2} + (R+j\omega L)^{1/2}(G+j\omega C)^{1/2} l$$

$$e^{\gamma l} = 1 + \frac{(R+j\omega L)(G+j\omega C)l^2}{2} + \sqrt{(R+j\omega L)(G+j\omega C)}\; l \qquad \text{—(1)}$$

$$e^{\gamma l} = 1 + \gamma l + \frac{\gamma^2 l^2}{2!} + \frac{\gamma^3 l^3}{3!} + \cdots$$

when $l \to 0$.

$$e^{\gamma l} = 1 + \gamma l + \frac{\gamma^2 l^2}{2!} \quad \text{--- (2)}$$

Comparing (1) & (2),

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\therefore \boxed{\gamma = \sqrt{ZY}}$$

<u>Determination of α & β in terms of Primary Constant :-</u>

$$\because \gamma = \alpha + j\beta$$

$$= \sqrt{\alpha^2 + \beta^2} \angle \tan^{-1} \frac{\beta}{\alpha}$$

$\because \gamma = \sqrt{ZY}$

$= \sqrt{(R+j\omega L)(G+j\omega C)}$

$= \sqrt{\cancel{RG + jR\omega C + Gj\omega L + j^2\omega^2 LC}}$

$= \sqrt{\sqrt{R^2+\omega^2L^2}\ \tan^{-1}\frac{\omega L}{R} \cdot \sqrt{G^2+\omega^2C^2}\ \tan^{-1}\frac{\omega C}{G}}$

$= \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \angle \frac{1}{2}\left[\tan^{-1}\frac{\omega L}{R} + \tan^{-1}\frac{\omega C}{G}\right]$

$\because |\gamma| = \sqrt{\alpha^2+\beta^2}$

$\Rightarrow \sqrt{\alpha^2+\beta^2} = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$

& taking squaring on both sides,

$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$

~~$\Rightarrow (\alpha^2+\beta^2)^2 = R$~~

$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$

$\alpha^2-\beta^2+2j\alpha\beta = (R+j\omega L)(G+j\omega C)$

$= RG + Rj\omega L + Gj\omega C - \omega^2 LC$

$\boxed{\alpha^2-\beta^2 = RG - \omega^2 LC}$

& $\boxed{2j\alpha\beta = Rj\omega L + Gj\omega C}$

$$\boxed{\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

$$2\alpha^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + RG - \omega^2LC$$

$$\Rightarrow \boxed{\alpha = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + RG - \omega^2LC\right\}}}$$

$$2\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - RG + \omega^2LC$$

$$\Rightarrow \boxed{\beta = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - RG + \omega^2LC\right\}}}$$

General solution of Transmission line

or, Derivatⁿ of voltage & current in Transmission line

The general solⁿ of transmission line includes the expression for current & voltage at any point along the line of any length having uniformly distributed constants

The various notation used in the derivation are:-

R = Series resistance (Ω).

L = Series Inductance (Henry, H) / length.

C = Series Capacitance betn two Conductors. (farad per unit length)

G = Shunt leakage Conductance (mho)

ωL = Angular frequency series reactance per unit length.

ωC = Shunt Succeptance per unit length

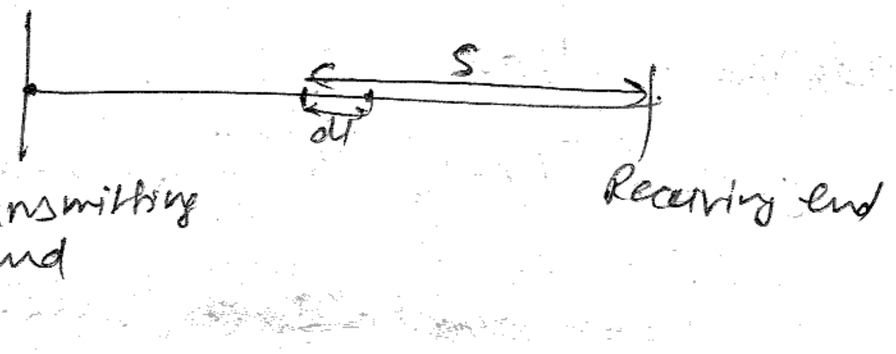
Y = shunt admittance (mho)

Z = series Impedance in Ω / length (Ω).

$$\boxed{Z = R + j\omega L}$$

$$\boxed{Y = \frac{1}{Z}} \qquad \boxed{Y = G + j\omega C}$$

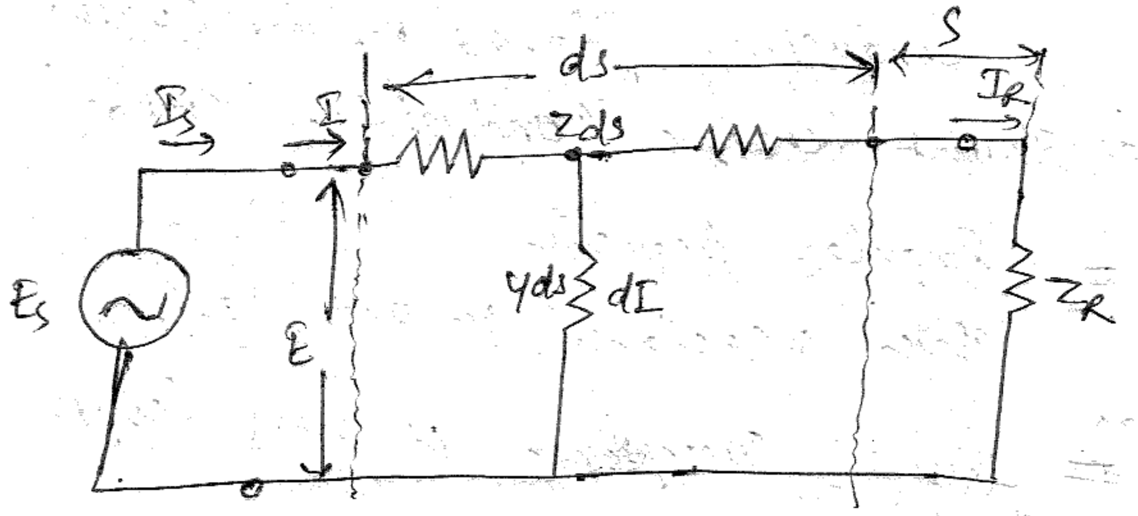
S = Distance upto point of Consideration measured from receiving



I = Current in a line at any point.

V = voltage between conductors, at any point.

l = length of transmission line.



Transmission line of length l can be considered to be made up of infinitely small T-section

At the point of consideration the length of the section is ds. Hence its ~~sh~~ series impedance is Zds, and shunt admittance Yds

Current passing is I & voltage is V

dE = Elemental voltage drop. = Izds

dI = E Yds

leakage current flows through shunt amp resistance

$$\frac{dE}{ds} = IZ \quad \text{—(1)}$$

$$\frac{dI}{ds} = EY \quad \text{—(2)}$$

Diff (1) & (2) w.r.to s,

$$\frac{d^2E}{ds^2} = Z\frac{dI}{ds} \leftarrow (2) = ZEY \quad \text{—(3)}$$

$$\frac{d^2I}{ds^2} = Y\frac{dE}{ds} \leftarrow (1) = YIZ \quad \text{—(4)}$$

Taking LCM as Z_R and taking $\frac{Z_R+Z_0}{Z_R}$

out from eqn (18),

$$\boxed{E = \frac{E_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}s}\right]} \quad -(20)$$

Taking LCM as $\frac{Z_R+Z_0}{Z_R}$ out from eqn (19)

$$\boxed{I = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}s}\right]} \quad -(21)$$

Eqn (20) & (21) is the general solⁿ for the transmission line.

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Physical Significance of general solⁿ of Transmission line :-

Take the Expression E, as:-

$$E = \frac{E_R}{2Z_R}\left[(Z_R+Z_0)\,e^{\sqrt{ZY}s} + (Z_R-Z_0)\,e^{-\sqrt{ZY}s}\right]$$

$$E = \frac{E_R}{2Z_R}\left[Z_R e^{\sqrt{ZY}s} + Z_0 e^{\sqrt{ZY}s} + Z_R e^{-\sqrt{ZY}s} - Z_0 e^{-\sqrt{ZY}s}\right]$$

$$= \frac{E_R}{2Z_R}\left[Z_R\left(e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}\right) + Z_0\left(e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}\right)\right]$$

$$= \frac{E_R}{Z_R}\left[Z_R\left(\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right) + Z_0\left(\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right)\right]$$

$$= \frac{\cancel{E_R}}{Z_R}\Big[$$

$$= \cancel{E_R}\Big[Z_R$$

$$\left[\because \frac{E_R}{Z_R} = I_R\right]$$

$$\boxed{E = E_R \cosh\sqrt{ZY}s + I_R Z_0 \sinh\sqrt{ZY}s} \quad -(24)$$

Similarly,

$$\boxed{I = I_R \cosh\sqrt{ZY}s + \frac{E_R}{Z_0}\sinh\sqrt{ZY}s} \quad -(25) \text{ eqn.}$$

(24) & (25),

from that eqn, Sending end Current is obtained by $S = l$, i.e measured from receiving end,

$$\boxed{E_S = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)} \quad -(1)$$

$\because \sqrt{ZY} = \gamma$

$S = l$

$$\boxed{I_S = I_R \cosh(\gamma l) + \frac{E_R}{Z_0} \sinh(\gamma l)} \quad -(2) \text{ eq}^n$$

$$\because Z_R = \frac{E_R}{I_R}$$

$$I_S = I_R \cosh(\gamma l) + \frac{Z_R I_R}{Z_0} \sinh(\gamma l)$$

$$\boxed{I_S = I_R \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right]}$$

If the line is terminated to Characteristic Impedance,

$$\boxed{I_S = I_R \left[\cosh(\gamma l) + \sinh(\gamma l)\right]} \quad -(3)$$

$$\boxed{\frac{I_S}{I_R} = \cosh(\gamma l) + \sinh(\gamma l)}$$

But $\frac{I_S}{I_R} = e^{\gamma l}$

$$\Rightarrow \boxed{e^{\gamma l} = \cosh(\gamma l) + \sinh(\gamma l)}$$

— (4) eqⁿ.

I_S = Sending Current

I_R = Receiving Current

~~Now taking E~~

~~$E_S = E_R \cos h$~~

$$E_S = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$E_S = I_R \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]$$

— (5) eqⁿ.

divide eqⁿ (5) by (3),

$$\frac{E_S}{I_R} = \frac{\cancel{I_R} \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}{\cancel{I_R} \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l) \right]}$$

$$= \frac{Z_0 Z_R \cosh(\gamma l) + Z_0^2 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l)}$$

$Z_0 = Z_R$ (Consider for Symmetrical N/work)

$\Rightarrow$

$$\frac{E_S}{I_R} = \frac{Z_0^2 [\cosh(\gamma l) + \cancel{\sinh(\gamma l)}]}{Z_0 [\cosh(\gamma l) + \cancel{\sinh(\gamma l)}]}$$

$$= Z_0$$

$$\boxed{\frac{E_S}{I_R} = Z_0}$$

This shows that a line transmitted its characteristic impedance. Its i/p impedance (Z_S) = o/p impedance Z_R.

WAVELENGTH AND VELOCITY OF PROPAGATION CONSTANT

$\rightarrow$

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{ZY}$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\alpha + j\beta = \sqrt{(R+j\omega L)\cdot(G+j\omega C}$

$\Rightarrow (\alpha + j\beta)^2 = (R+j\omega L)(G+j\omega C)$

$= RG + Rj\omega C + Gj\omega L - \omega^2 LC$

$\Rightarrow$ ~~$\alpha^2 + \beta$~~

$\alpha^2 - \beta^2 + 2j\alpha\beta = RG - \omega^2 LC + j\omega(RC + GL)$

So, eqn real & img parts,

$\alpha^2 - \beta^2 = RG - \omega^2 LC$

$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad -(1)$

img parts

$2\alpha\beta = \omega(RC + GL)$

$\Rightarrow 4\alpha^2\beta^2 = \omega^2(LG + RC)^2$

$\Rightarrow \alpha^2\beta^2 = \dfrac{\omega^2(LG+RC)^2}{4} \quad -(2)$

Sub (1) in (2).

$\Rightarrow (\beta^2 + RG - \omega^2 LC)\beta^2 = \dfrac{\omega^2(LG+RC)^2}{4}$

$$\Rightarrow \left(\beta^4 + \beta^2 RG - \beta^2\omega^2 LC\right) - \frac{\omega^2}{4}(LG+RC)^2 = 0$$

$$\Rightarrow \beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(LG+RC)^2 = 0$$

$ax^2 +$

$$\beta^2 = \frac{-(RG-\omega^2 LC) \pm \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+RC)^2}}{2}$$

$$\beta = \sqrt{\frac{\omega^2 LC - RG \pm \sqrt{(RG-\omega^2 LC)^2 + \omega^2(LG+RC)^2}}{2}}$$

Substitute the value of β in eq^n ②

$$\therefore \alpha^2\beta^2 = \frac{\omega^2(LG+RC)^2}{4}$$

$\Rightarrow$

for Perfect transmission, $R=0$, $G=0$, in

$$\beta^2 = \frac{-(-\omega^2 LC) \pm \sqrt{(-\omega^2 LC)^2 + 0}}{2}$$

$$= \frac{\omega^2 LC + \omega^2 LC}{2}$$

$$\Rightarrow \frac{2\omega^2 LC}{2} = \omega^2 LC.$$

$\beta^2 = \omega^2 LC.$

$$\Rightarrow \boxed{\beta = \omega\sqrt{LC}}$$

Velocity, $V = \lambda f$

$$= \frac{2\pi \lambda f}{2\pi}$$

$$= 2\pi f \cdot \frac{\lambda}{2\pi}$$

$\therefore \beta = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$ in terms of freq.

$$\therefore \boxed{V = \frac{\omega}{\beta}}$$

$$V = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{V = \frac{1}{\sqrt{LC}}}$$

velocity of transmission line.

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

TELEPHONE CABLES :-

In telephone Cables the wires are insulated with paper twisted in pairs, these Construction results in neglegible values of inductance and Conductance for perfect transmission, ie $R=0$ & $G=0$.

Conditions :- $L\omega <<< R$ and $G << C\omega$.

Impedance, $Z = R + j\omega L = R$ ← for transmission line; for telephone lines.

Admittance, $Y = G + j\omega C = j\omega C$

$$\gamma = \sqrt{ZY} = \sqrt{j\omega RC}$$

$\sqrt{2j}$

$$= \sqrt{\frac{2j\omega RC}{2}}$$

$$\alpha + j\beta = \sqrt{\frac{j2\omega RC}{2}} \qquad \because 2j = (1+j)^2$$

$$= (1+j)\sqrt{\frac{\omega RC}{2}}$$

$$= \sqrt{\frac{\omega RC}{2}} + \sqrt{\frac{\omega RC}{2}}\, j$$

$$\boxed{\alpha = \sqrt{\frac{\omega RC}{2}} \qquad \beta = \sqrt{\frac{\omega RC}{2}}}$$

$$V = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega RC}{2}}} = \sqrt{\frac{2\omega}{RC}}$$

$$V = \sqrt{\frac{2\omega}{RC}}$$

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R}{j\omega C}}}$$

Line Distortion :-

Signal transmitted over a transmission line is a Complex and Consist of many frequency Component.
Such ~~as~~ voice Signal voltage, will not have equal attenuation and equal time delay. The Received signal will not be identical with I/P signal. These variation is known as distortion.

Types :- (i) Frequence Distortions
ii) Delay Time Distortion

Phe rdi (6 marks)

Condition for distortion less line :-

If a line is to have neither frequency nor delay distortion, then attenuation factor and velocity of propagation 'v' cannot be funcⁿ of frequency..

$V = \frac{\omega}{\beta}$, and β must be the direct func^ns of frequency.

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}}$$

$$(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2 = (RG + \omega^2 LC)^2$$

$$\Rightarrow \cancel{R^2 G^2} + \cancel{\omega^4 L^2 C^2} - \cancel{2RG\omega^2 LC} + \omega^2 L^2 G^2 + \omega^2 C^2 R^2 + 2\omega^2 \cancel{LG}\, CR$$

$$= \cancel{R^2 G^2} + \cancel{\omega^4 L^2 C^2} + 2RG\omega^2 LC$$

$$\Rightarrow \omega^2 L^2 G^2 + \omega^2 C^2 R^2 = 2RG\omega^2 LC$$

$$\Rightarrow \omega^2 L^2 G^2 + \omega^2 C^2 R^2 - 2RG\omega^2 LC = 0$$

$$\Rightarrow \omega^2 (L^2 G^2 + C^2 R^2 - 2RGLC) = 0$$

$$\Rightarrow \omega^2 (LG - RC)^2 = 0$$

$\Rightarrow$ Either $\boxed{\omega = 0,}$

& ~~LR = R·C~~ C

$\boxed{LG = CR}$

$\Rightarrow \boxed{\frac{R}{L} = \frac{G}{C}}$

17/8/19 UNIT 3

Line at Radio-frequency

~~1st Assumption~~

For a radio frequency line of either open wire type or coaxial line type, the standard assumptions made for the analysis of the performance of the line.

Assumptions :-

(1). At very high frequency the skin effect is afectable.

~~Skin effect:- Current flow over the cond~~

It is assumed that the current may flow ~~over~~ surface of conducter, then thetorney conductance becomes zero.

(2) Due to skin effect, the resistance R increases with frequency. ⇒ $\omega L >> R$

Hence ωL (reactance) also increased with frequency.

(3) The line at radio frequency is constructed such that the leakage ~~conductor~~ conductance $G \rightarrow$ zero.

for analysis of line performance

Consideration :- (i)

(i) R is slightly small with respect to ωL.

(ii) R is completly neglegible, when $\omega L >>> R$

Case 2
If R is completly neglegible, then such a line is called as Zero dissipation line.

Case 1
If R is small then such a line is called small dissipation line.

PARAMETERS OF LINE AT HIGH FREQUENCY

Basic parameter, R, L, C, G.

For open wire line :-

$$\boxed{L = 10^{-7}\left(\frac{\mu}{\mu_r} + 4 \ln\frac{d}{a}\right) \text{ H/m}}$$

for closed wire :

$$L = 10^{-7}\left[2 \ln\frac{b}{a} + \frac{2c^4 \ln\frac{c}{a}}{(c^2-b^2)^2} - \frac{c^2}{c^2-b^2}\right] \text{ H/m.}$$

where,

$$C = \frac{\pi \varepsilon}{\ln\frac{d}{a}} \text{ F/m} \leftarrow \text{for open line.}$$

for closed line, $C = \frac{2\pi\varepsilon}{}$

Line Constant for zero dissipation line

for general, $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

But for high frequency,

$j\omega L >> R$, & $j\omega C >> R$

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} \Rightarrow \boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Propagation Constant $\gamma = \sqrt{ZY}$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j^2\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

$\because \gamma = \alpha + j\beta$

$= 0 + j\omega\sqrt{LC}$

$$\Rightarrow \left.\boxed{\begin{array}{l}\alpha = 0 \\ \beta = \cancel{j}\,\omega\sqrt{LC}\end{array}}\right\} \leftarrow \text{for high freq.}$$

Velocity of ~~prog~~ propagation

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \boxed{V_p = \frac{1}{\sqrt{LC}}}$$

Voltage n Current on dissipation less line (zero dissipation line) :-

$$E = E_R \cosh \gamma(l-x) + I_R Z_0 \sinh\gamma(l-x)$$

Put $l-x = s$

$$\Rightarrow E = E_R \cosh\gamma s + I_R Z_0 \sinh\gamma s$$

at high freq, $Z_0 = R_0$, $\gamma = j\beta$

$$E = E_R \cosh j\beta s + I_R R_0 \sinh j\beta \cdot s$$

$$\boxed{E = E_R \cos\beta s + I_R j \sin\beta s \cdot R_0}$$

Ily, $\boxed{I = I_R \cos(\beta s) + j \frac{E_R}{R_0} \sin(\beta s) \cdot R_0}$

$$\because \beta = \frac{2\pi}{\lambda}$$

$$\therefore E = E_R \cos\left(\frac{2\pi s}{\lambda}\right) + I_R j \sin\left(\frac{2\pi s}{\lambda}\right) R_0$$

$$I = I_R \cos\left(\frac{2\pi s}{\lambda}\right) + j\,\frac{E_R}{R_0}\sin\left(\frac{2\pi s}{\lambda}\right) R_0$$

Assumptions :-

(i) open loop cor. $\Rightarrow I_R = 0.$

$$\boxed{E_{oc} = E_R \cos\left(\frac{2\pi s}{\lambda}\right)}$$

(ii) line is short circuit, $\Rightarrow E_R = 0$

$$\Rightarrow \boxed{E_{sc} = j\, I_R R_0 \sin\left(\frac{2\pi s}{\lambda}\right)}$$