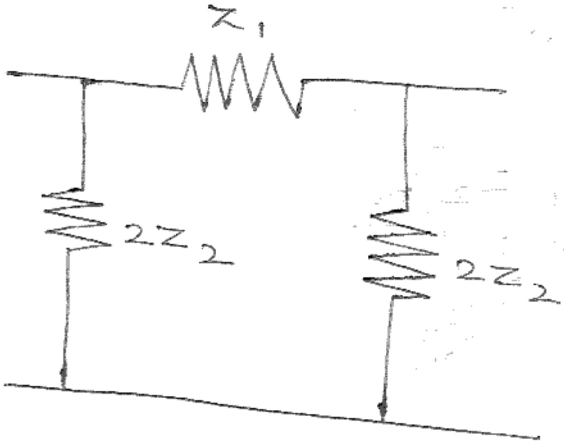
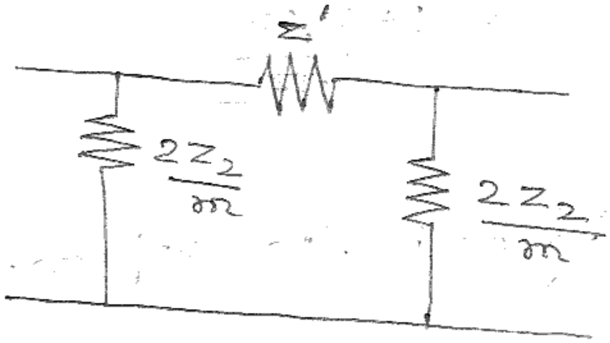
13.7.11

# M derived $\pi$ - section




$\Downarrow$




The characteristic impedance of constant K $\pi$ - section

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

$$= \frac{\sqrt{Z_1 Z_2 \cancel{Z_1} Z_2}}{\sqrt{\cancel{Z_1} Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

$$Z_1 = Z_1'$$

$$Z_2 = \frac{Z_2}{m}$$

$$Z_{o\pi} = \sqrt{\frac{Z_1' Z_2/m}{1 + \frac{Z_1'}{\frac{4Z_2}{m}}}}$$

$$= \sqrt{\frac{Z_1' Z_2/m}{1 + \frac{Z_1' m}{4Z_2}}}$$

equating both the impedance

$$\sqrt{\frac{Z_1 Z_2}{1 + Z_1/4}} = \sqrt{\frac{Z_1' Z_2/m}{1 + \frac{Z_1' m}{4Z_2}}}$$

Squaring on both side we get

$$\frac{Z_1 Z_2}{1 + Z_1/4Z_2} = \frac{Z_1' Z_2/m}{1 + \frac{Z_1' m}{4Z_2}}$$

$$(Z_1 \cancel{Z_2})\left(\frac{4Z_2 + Z_1' m}{4\cancel{Z_2}}\right) = \left(\frac{Z_1' \cancel{Z_2}}{m}\right)\left(\frac{4Z^2 + Z_1}{4\cancel{Z_2}}\right.$$

$$\frac{4z_1z_2 + z_1'z_1 m}{\cancel{4}} = \frac{4z_2z_1' + z_1'z_1}{\cancel{4}\, m}$$

$$4z_1z_2m + z_1'z_1m^2 = 4z_2z_1' + z_1'z_1$$

$$z_1'z_1m^2 - z_1'z_1 = 4z_2z_1' - 4z_1z_2m$$

$$z_1'z_1(m^2-1) = 4z_2(z_1' - z_1m)$$

$$z_1'z_1(m^2-1) - 4z_2z_1' = -4z_1z_2m$$

$$z_1'\left[z_1(m^2-1) - 4z_2\right] = -4z_1z_2m$$

$$z_1' = \frac{-4z_1z_2m}{z_1(m^2-1) - 4z_2}$$

$$= \frac{-\cancel{4}z_1z_2m}{\cancel{4}\left[z_1(m^2-1) - \frac{z_2}{4}\right]}$$

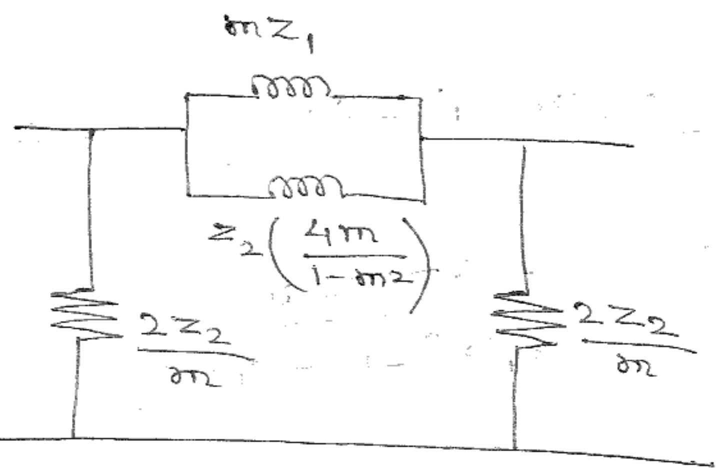
$$= \frac{z_1z_2m}{\frac{z_2}{4} - z_1(m^2-1)}$$

$$z_1' = \frac{z_1z_2}{\frac{z_2}{m} + \left(\frac{1-m^2}{4m}\right)z_1}$$

mult. num and deno by $\frac{4m^2}{1-m^2}$

$$Z_1' = \frac{Z_1 Z_2 \left(\frac{4m^2}{1-m^2}\right)}{Z_2\left(\frac{4m}{1-m^2}\right) + \left(\frac{\cancel{1-m^2}}{\cancel{4m}}\right) \times \frac{\cancel{4}m^{\cancel{2}}}{\cancel{1-m^2}} Z_1}$$

$$= \frac{Z_1 Z_2 \left(\frac{4m^2}{1-m^2}\right)}{Z_2\left(\frac{4m}{1-m^2}\right) + mZ_1}$$




- M derived low pass filter

The constant K low pass filter the series impedance

$$Z_1 = jWL$$

Shunt impedance $Z_2 = \frac{1}{jWC}$

For m-derived low pass filter T-sec

the series impedance of m-derived T section is

$$\frac{mZ_1}{2} = \frac{mj\omega L}{2} = j\omega\left(\frac{mL}{2}\right)$$

$$\frac{mL}{2} = L'$$

$$\therefore \frac{mZ_1}{2} = j\omega L'$$

Shunt impedance

$$Z_2' = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$$

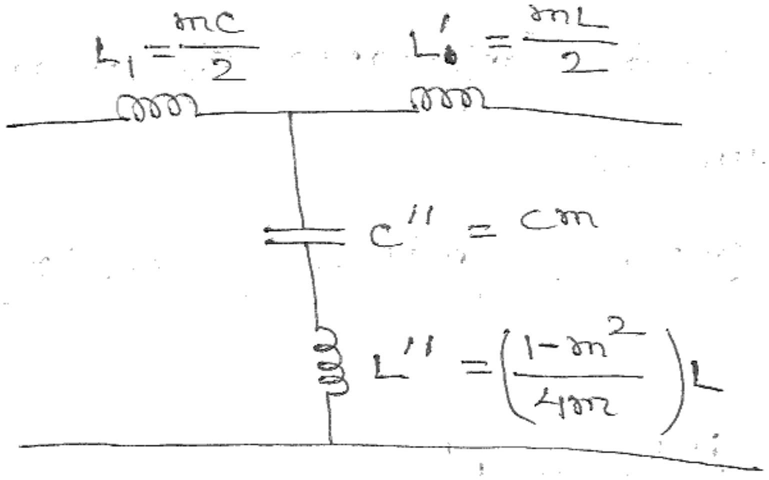
$$Z_2' = \frac{1}{j\omega Cm} + \left(\frac{1-m^2}{4m}\right)j\omega L$$

$$\cancel{\frac{1}{j\omega Cm} = C''} \quad Cm = C''$$

$$Z_2' = \frac{1}{j\omega C''} + \left(\frac{1-m^2}{4m}\right)j\omega L$$

$$Z_2' = \frac{1}{j\omega C''} + j\omega L''$$

Here, $\left(\frac{1-m^2}{4m}\right)L = L''$




- m derived LPF in $\pi$ section

~~series~~ Shunt imp $\frac{2Z_2}{m} = \frac{1}{j\omega \frac{mC}{2}}$

$$\frac{2Z_2}{m} = \frac{1}{j\omega C''}$$

Here,

$$\frac{mC}{2} = C''$$

series imp $Z_1' = mZ_1, \frac{4m}{1-m^2} Z_2$

$$Z_1' = j\omega(mL) = j\omega L'$$

Here,

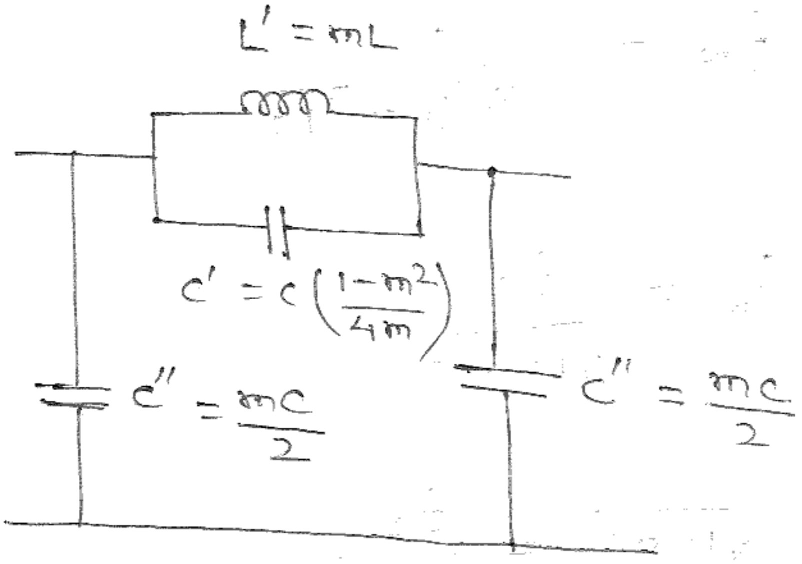
$$mL = L'$$

$$Z_1' = \frac{4m}{1-m^2} Z_2 = \frac{1}{j\omega C\left(\frac{1-m^2}{4m}\right)}$$

$$Z_1' = \frac{1}{jwC'}$$

Here,

$$\frac{1-m^2}{4m} = C'$$




In m-derived T-section filter it has finite impedance. If the condition is

$$X_L = X_C$$

$$wL'' = \frac{1}{wC''}$$

$$w^2 = \frac{1}{L''C''} = \frac{1}{(\cancel{m}C)\left(\frac{1-m^2}{4\cancel{m}}\right)L}$$

$$w^2 = \frac{4}{(1-m^2)LC}$$

$$4\pi^2 f_\infty^2 = \frac{4}{(1-m^2)LC}$$

$$f_\alpha^2 = \frac{1}{\pi^2(1-m^2)LC}$$

$$f_\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}}$$

$$f_\alpha = \frac{f_c}{\sqrt{1-m^2}}$$

$$\sqrt{1-m^2} = \frac{f_c}{f_\infty}$$

$$m^2 = 1-\left(\frac{f_c}{f_\infty}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

# M-derived high pass filter

The series impedance of constant K-filter

$$Z_1 = \frac{1}{j\omega C}$$

The shunt impedance of constant K-filter

$$Z_2 = j\omega L$$

The series impedance of m-derived T-sec

$$\frac{MZ_1}{2} = \frac{M}{2j\omega C} = \frac{1}{j\omega(2C/m)}$$

Assume

$$\frac{2C}{m} = C'$$

$$\therefore \frac{1}{j\omega C'} = \frac{Z_1 M}{2}$$

Shunt impedance of m-derived T-section

$$Z_2' = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$$

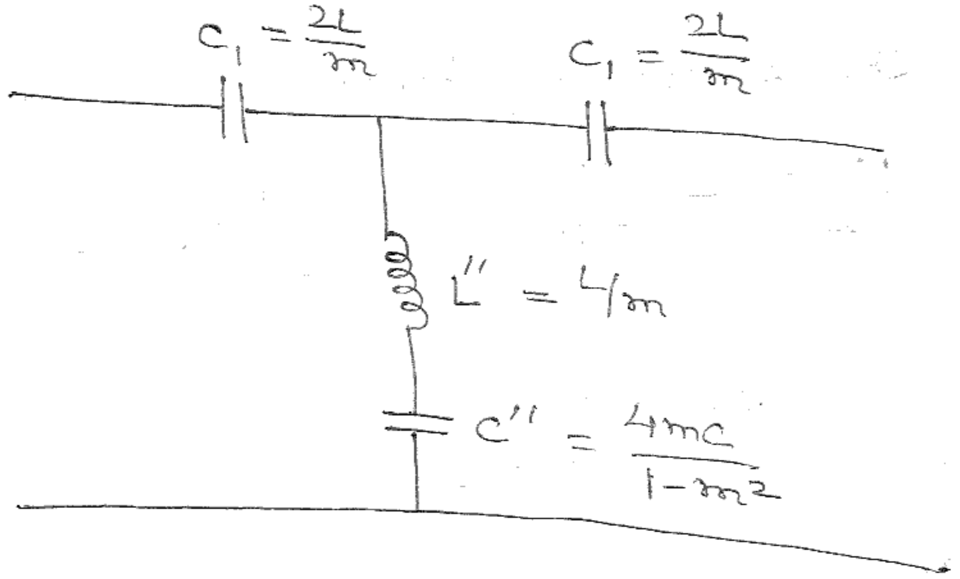
$$Z_2' = \frac{j\omega L}{m} + \left(\frac{1-m^2}{4m}\right)\frac{1}{j\omega C}$$

$$Z_2' = j\omega\left(L/m\right) + \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C}$$

Let

$$\frac{L}{m} = L'' \qquad \frac{4mC}{1-m^2} = C''$$

$$Z_2' = j\omega L'' + \frac{1}{j\omega C''}$$




**For π section**

Shunt impedance $\frac{2Z_2}{m} = \frac{2j\omega L}{m}$

$$\frac{2Z_2}{m} = j\omega\left(\frac{2L}{m}\right) \qquad \text{Here, } \frac{2L}{m} = L''$$

$$\frac{2Z_2}{m} = j\omega L''$$

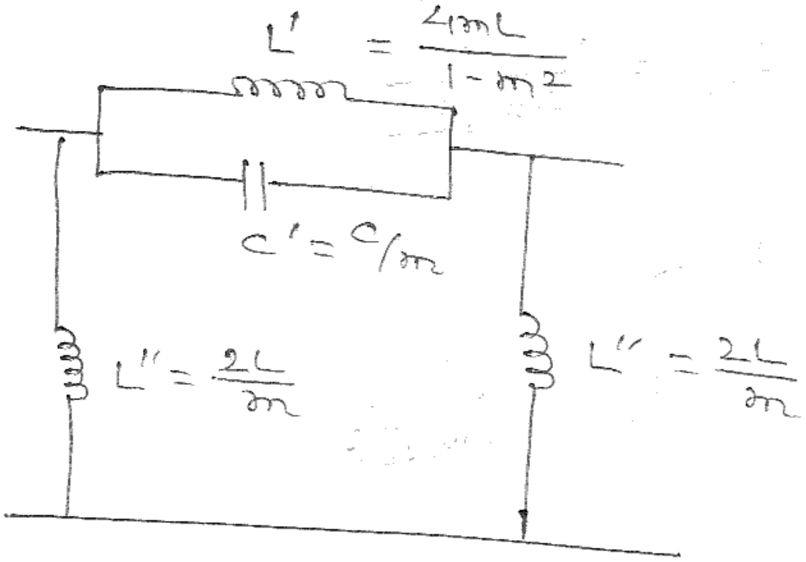
series impedance

$$Z' = mZ_1 = \frac{m}{j\omega C} = \frac{1}{j\omega\left(\frac{C}{m}\right)} = \frac{1}{j\omega C'}$$

Here, $\frac{C}{m} = C'$

$$Z_2\left(\frac{4m}{1-m^2}\right) = j\omega L\left(\frac{4m}{1-m^2}\right) = j\omega\left(\frac{L\,4m}{1-m^2}\right)$$

$$Z_1' = j\omega L'$$




In m-derived T-section filter it has finite impedance.

$$X_L = X_C$$

$$\omega L'' = \frac{1}{\omega C''}$$

$$\omega^2 = \frac{1}{L''C''}$$

$$\omega^2 = \frac{1}{\left(\frac{L}{m}\right)\left(\frac{4mC}{1-m^2}\right)}$$

$$\omega^2 = \frac{1-m^2}{4LC}$$

$$(2\pi f_\alpha)^2 = \frac{1-m^2}{4LC}$$

$$f_\alpha^2 = \frac{1-m^2}{16\pi^2 LC}$$

$$f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

Here,

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\alpha = \frac{\sqrt{1-m^2}}{1} \cdot f_c$$

$$\sqrt{1-m^2} = \frac{f_\alpha}{f_c}$$

squaring both side

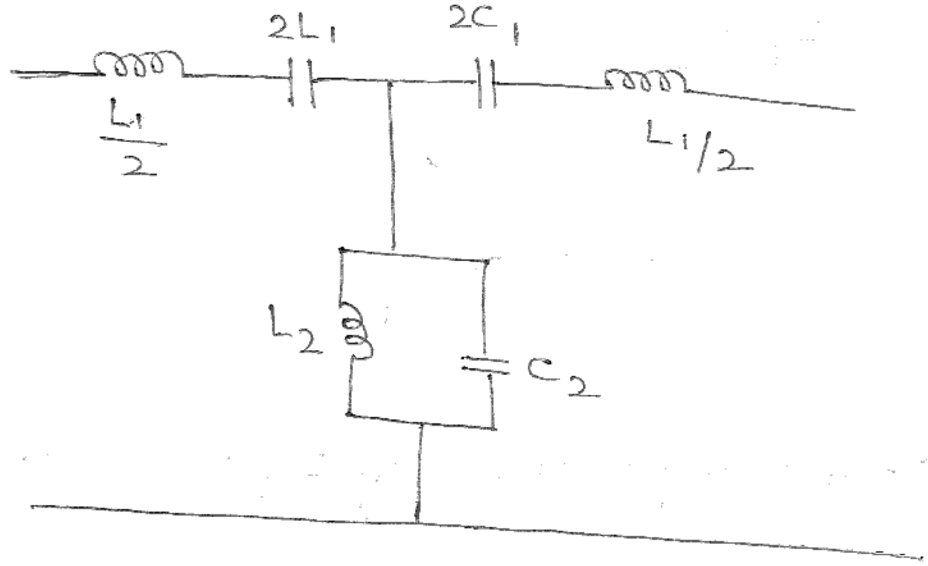
$$(1-m^2) = \left(\frac{f\alpha}{fc}\right)^2$$

$$-m^2 = \left(\frac{f\alpha}{fc}\right)^2 - 1$$

$$m^2 = 1 - \left(\frac{f\alpha}{fc}\right)^2$$

$$m = \sqrt{1-\left(\frac{f\alpha}{fc}\right)^2}$$

- m-derived band pass filter




The resonance of each series and shunt

$$w_0^2 L_1 C_1 = 1 \quad \text{--- (1)}$$

$$w_0^2 L_2 C_2 = 1 \quad \text{--- (2)}$$

equating both eq

$$L_1 C_1 = L_2 C_2$$

$$\cancel{\frac{L_1}{C_2}}$$

$$\frac{L_1}{C_2} = \frac{L_2}{C_1}$$

Consider the series impedance

$$\cancel{Z_1 = jX_L + jX_C} \quad Z_1 = Z_1' + Z_1^2$$

$$\cancel{Z_1 = j} \quad Z_1 = j\omega L + \frac{1}{j\omega C}$$

$$Z_1 = j\omega L - \frac{j}{\omega C}$$

$$= j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

Shunt impedance of $Z_2 = X_{L_2} \| X_{C_2}$

$$Z_2 = \frac{X_{L_2} X_{C_2}}{X_{L_2} + X_{C_2}} = \frac{j\omega L_2 \cdot 1/j\omega C_2}{j\omega L_2 + 1/j\omega C}$$

$$Z_2 = \frac{L_2/C_2}{j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)}$$

$$Z_2 = \frac{\omega L_2}{j(\omega^2 L_2 C_2 - 1)}$$

$$\boxed{Z_2 = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}}$$

we know that

$$Z_1 Z_2 = R_0^2$$

$$Z_1 Z_2 = \frac{j(\omega^2 L_1 C_1 - 1)}{\cancel{\omega} C_1} \times \frac{j \cancel{\omega} L_2}{(1-\omega^2 L_2 C_2)}$$

$$Z_1 Z_2 = \frac{(1-\omega^2 L_1 C_1) L_2}{(1-\omega^2 L_2 C_2) C_1}$$

~~If $L_1 = L_2$ and $L_2 = C_2$~~

$$L_1 C_1 = L_2 C_2$$

$$Z_1 Z_2 = \frac{L_2}{C_1}$$

$$\therefore R_0^2 = \frac{L_2}{C_1}$$

$$R_0 = \sqrt{\frac{L_2}{C_1}}$$

$$Z_1 + 4Z_2 = 0$$

$$Z_1 = -4Z_2$$

multiply both side by $Z_1$

$$Z_1^2 = -4Z_1Z_2$$

$$Z_1^2 = -4R_0^2$$

$$Z_1 = \pm j2R_0$$

$$Z_1 = \pm 2jR_0$$

To find lower cut off frequency the impedance

$-Z_1$ value is a lower cut off freq.

$-Z_1(f_1) = +ve\ Z_1$ of upper cut off frequency $f_2$.

$$Z_1 = -Z_1$$

$$\cancel{j\omega L_1 \cdot j\omega}$$

$$j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} = -\left(j\omega_2 L_1 + \frac{1}{j\omega_2 C_1}\right)$$

$$j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -\left(j\omega_2 L_1 - \frac{j}{\omega_2 C_1}\right)$$

$$\cancel{-j}\left(\frac{1}{\omega_1 C_1} - \omega_1 L_1\right) = \cancel{-j}\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\frac{1-\omega_1^2 L_1 C_1}{\omega_1 \cancel{C_1}} = \frac{\omega_2 L_1 C_1 - 1}{\omega_2 \cancel{C_1}}$$

$$\frac{1-\omega_1^2 L_1 C_1}{\omega_1} = \frac{\omega_2 L_1 C_1 - 1}{\omega_2}$$

$$1-\omega_1^2 L_1 C_1 = \frac{\omega_1}{\omega_2}\left(\omega_2 L_1 C_1 - 1\right)$$

$$\omega_0^2 L_1 C_1 = 1$$

$$L_1 C_1 = \frac{1}{\omega_0^2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$\omega_1 = 2\pi f_1$$

$$\omega_0 = 2\pi f_0$$

$$\omega_2 = 2\pi f_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = \frac{f_1}{f_2}\left[\left(\frac{f_2}{f_0}\right)^2 - 1\right]$$

$$1-\left(\frac{f_1}{f_0}\right)^2 = \frac{f_1 f_2}{f_0^2} - \frac{f_1}{f_2}$$

$$1+\frac{f_1}{f_2} = \frac{f_1 f_2}{f_0^2} + \frac{f_1^2}{f_0^2}$$

$$\frac{f_1+f_2}{f_2} = \frac{1}{f_0^2}\left(f_1 f_2 + f_1^2\right)$$

$$f_0^2 = \frac{(f_1 f_2 + f_1^2) f_2}{f_1+f_2}$$

$$f_0^2 = \frac{f_1 f_2 \cancel{(f_1+f_2)}}{\cancel{f_1+f_2}}$$

$$f_0^2 = f_1 \; f_2$$

$$f_0 = \sqrt{f_1 f_2}$$

<u>20.III.11</u>

At lower cut off freq

$$Z_1 = -2jR$$

$$j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} = -2jR$$

$$\Rightarrow j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -j2R$$

$$\cancel{-j}\left(\frac{1}{\omega_1 C_1} - \omega_1 L_1\right) = \cancel{-j}2R$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2R$$

$$\frac{1 - \omega_1^2 L_1 C_1}{\omega_1 C_1} = 2R$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2R\omega_1 C_1$$

Here,

$$L_1 C_1 = 1/\omega_0^2$$

$$1 - \frac{f_1^2}{f_0^2} = \cancel{2R}\, 4\pi R f_1 C_1$$

$$\frac{f_0^2 - f_1^2}{f_0^2} = 4\pi R f_1 C_1$$

$$\cancel{C_1 = \frac{f_0 - f_1^2}{f_0^2 \, 4\pi R f_1}}$$

$$1 - \frac{\cancel{f_1}^2}{\cancel{f_1} f_2} = 4\pi R f_1 C_1$$

$$\frac{f_2 - f_1}{f_2} = 4\pi R f_1 C_1$$

$$C_1 = \frac{f_2 - f_1}{f_1 \, 4\pi R f_2}$$

$$\boxed{C_1 = \frac{f_2 - f_1}{4\pi R f_1 f_2}}$$

We know that,

$$L_1 C_1 = \frac{1}{\omega_0^2}$$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{1}{\omega_0^2 \left( \frac{f_2 - f_1}{4\pi R f_1 f_2} \right)}$$

$$L_1 = \frac{1}{\cancel{4}\pi^2 f_0^2 \, \frac{(f_2 - f_1)}{\cancel{4}\cancel{\pi} R f_1 f_2}}$$

$$L_1 = \frac{R f_1 f_2}{\pi f_0^2 (f_2 - f_1)}$$

$$L_1 = \frac{R}{\pi (f_2 - f_1)}$$

We know that

$$\frac{L_2}{C_1} = R^2$$

$$L_2 = R^2 C_1$$

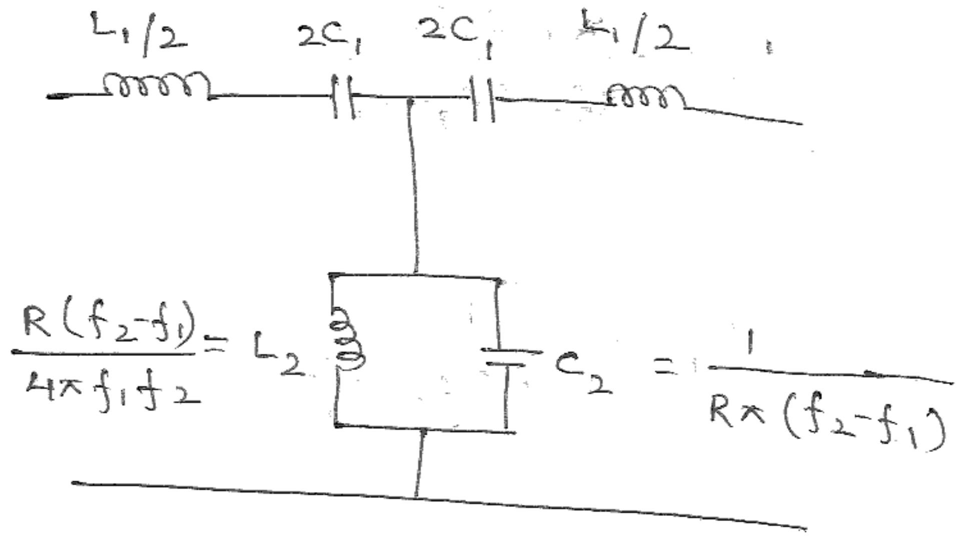
$$L_2 = R^2 \frac{(f_2 - f_1)}{4\pi R f_1 f_2}$$

$$L_2 = \frac{R(f_2 - f_1)}{4\pi f_1 f_2}$$

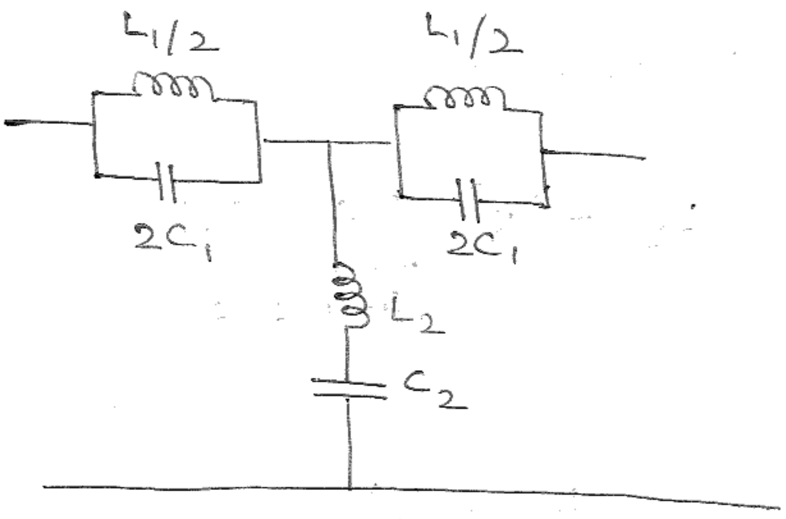
Also,

$$\frac{L_1}{C_2} = R^2$$

$$\Rightarrow C_2 = \frac{R^2}{L_1} = \frac{1}{R\pi(f_2 - f_1)}$$




★ Band elimination filter




The resonance freq of

$$\omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2 = 1$$

$$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0^2}$$

$$L_1 / C_2 = \frac{L_2}{C_1} = R_0^2$$

$$f_0^2 = f_1 f_2$$

$$f_0 = \sqrt{f_1 f_2}$$

For cut off freq

$$Z_1 + 4Z_2 = 0$$

$$Z_1 = -4Z_2$$

multi by $Z_2$ on both side

$$Z_2 Z_1 = -4 \cdot Z_2^2$$

$$R_0^2 = -Z_2^2 4$$

$$Z_2 = \frac{JR_0}{2}$$

$$Z_2 = \frac{-JR_0}{2}$$

$$j\omega_1 L_2 + \frac{1}{J\omega_1 C_2} = \frac{-JR_0}{2}$$

$$j\left(\omega_1 L_2 - \frac{1}{\omega_1 C_2}\right) = \cancel{j} R_0/2$$

$$1-\omega_1^2 L_2 C_2 = \frac{R_0}{2}\omega_1 C_2$$

$$1-\frac{\omega_1^2}{\omega_0^2} = R\pi f_1 C_2$$

$$1-\frac{f_1^2}{f_0^2} = R\pi f_1 C_2$$

$$1-\frac{f_1}{f_2} = R\pi f_1 C_2$$

$$\frac{f_2-f_1}{f_2} = R\pi f_1 C_2$$

$$C_2 = \frac{f_2-f_1}{R\pi f_1 f_2}$$

$$L_2 C_2 = \frac{1}{\omega_0^2}$$

$$L_2 = \frac{1}{C_2 \omega_0^2}$$

$$= \frac{1}{\left(\frac{f_2-f_1}{R\pi f_1 f_2}\right)(2\pi f_0)^2}$$

$$L_2 = \frac{R}{4\pi(f_2-f_1)}$$

$$\frac{L_2}{C_1} = \frac{L_1}{C_2} = R$$

$$\frac{L_2}{C_1} = R^2$$

$$C_1 = L_2/R^2$$

$$C_1 = \frac{R}{4\pi R^2 (f_2 - f_1)}$$

$$C_1 = \frac{1}{4\pi R (f_2 - f_1)}$$

$$\frac{L_1}{C_2} = R^2$$

$$L_1 = R^2 C_2$$

$$= \frac{R^2 (f_2 - f_1)}{R\pi f_1 f_2}$$

$$L_1 = \frac{R(f_2 - f_1)}{\pi f_1 f_2}$$

22.7.11

Q- Design a constant k bandpass filter terminate $R_1 = 600\Omega$ the cut off freq is 5KHz. Find $L_1$, $L_2$ and $C_1$ and $C_2$.

$f_1 = 2\,KHz$

Sol- $L_1 = \dfrac{R_0}{\pi(f_2 - f_1)}$

$= \cancel{\dfrac{600}{\pi(5-2)}} = \dfrac{600}{\pi(5-2)}$

$L_1 = 63.66$ mH

$L_2 = \dfrac{R_0(f_2 - f_1)}{4\pi f_1 f_2} = \dfrac{60\cancel{0}(5-2)}{4\pi\, 1\cancel{0}}$

$= \dfrac{60 \times 3}{4\pi} = 14.32$ mH

$C_1 = \dfrac{f_2 - f_1}{R_0 4\pi f_1 f_2} = \dfrac{5-2}{600 \times 4 \times \pi \times 10}$

$C_1 = 3.97 \times 10^{-5}$

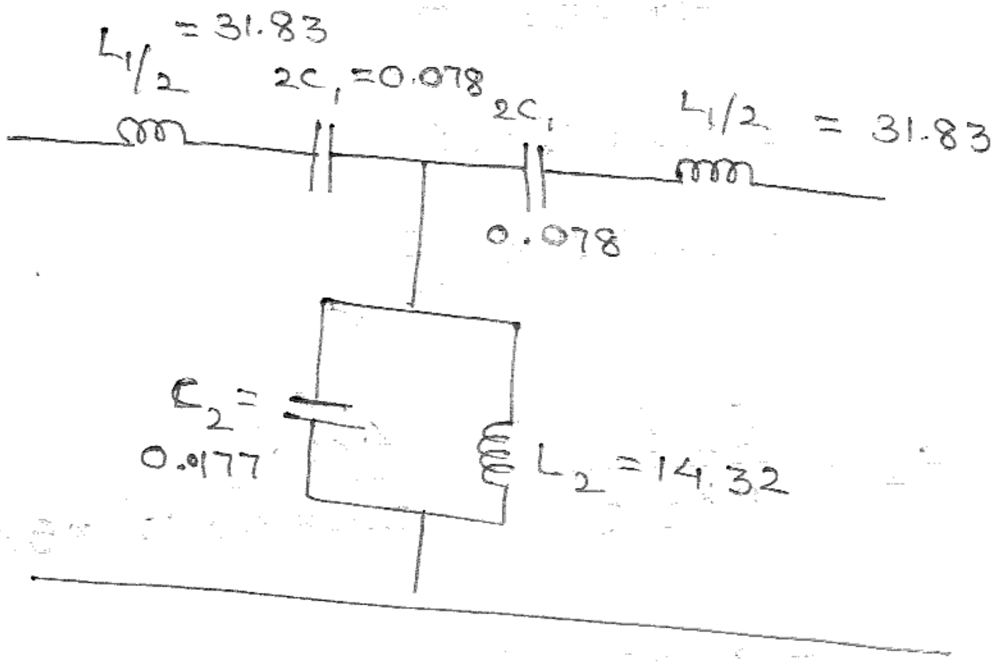
$= 0.039\,\mu f$

$$C_2 = \frac{1}{R_0 \pi (f_2 - f_1)} = \frac{1}{600 \times \pi \times 3}$$

$$= 1.76 \times 10^{-4}$$

$$= 0.0177\ \mu f$$

$$L_1 C_1 = L_2 C_2 = R^2$$




* Design a band elimination filter having a design impedance 500 Ω and cut off freq $f_1 = 1\,KHz$ and $f_2 = 5\,KHz$.

Sol- $$L_1 = \frac{R(f_2 - f_1)}{\pi f_1 f_2} = \frac{500(5\times10^3 - 1\times10^3)}{\pi \times 5\times10^3 \times 1\times10^3}$$

$$= 127.3\ H$$

$$L_2 = \frac{R}{4\pi(f_2-f_1)} = \frac{500}{4\times\pi\,(4\times10^3)}$$

$$L_2 = 9.95\ \text{MH}$$

$$C_1 = \frac{1}{4\pi R(f_2-f_1)}$$

$$= \frac{1}{4\pi\times500\,(4)}$$

$$= 3.978\times10^{-5}$$

$$= 0.03978\ \mu f$$

$$C_2 = \frac{f_2-f_1}{R\pi f_1 f_2} = \frac{4\times10^3}{500\times\pi\times10^3\times5\times10^3}$$

$$= 0.509\ \mu f$$