Physical Significance of Tx Line

Consider the general solution of a Tx line in a receiving End.

$$E = E_R \cosh(\gamma s) + I_R Z_0 \sinh(\gamma s)$$

$$I = I_R \cosh(\gamma s) + \frac{E_R}{Z_0} \sinh(\gamma s)$$

Sending End current can be obtained by Substituting $s = l$

$$E = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l) \quad \text{—} (1)$$

$$I = I_R \cosh(\gamma l) + \frac{E_R}{Z_0} \sinh(\gamma l) \quad \text{—} (2)$$

But, $Z_R = \frac{E_R}{I_R}$

Sub the value of Z_R in equ(2)

$$I_S = I_R \cosh(\gamma l) + \frac{Z_R I_R}{Z_0} \sinh(\gamma l).$$

$$I_S = I_R \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right] \quad \text{—} (3)$$

$$\frac{I_S}{I_R} = \cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l) \quad \text{—} (3)$$

$$\frac{I_S}{I_R} = \left[\cosh(\gamma l) + \sinh(\gamma l)\right] \quad \{\because Z_R = Z_0\}$$

$$\frac{I_S}{I_R} = e^{\gamma l} \quad \text{—} (4)$$

If the line is terminated in Z_0 and using, $E_R = I_R Z_R$ in equ^n ①

we get

$$E_s = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$E_s = I_R Z_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$E_s = I_R \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right] \quad -⑤$$

Divide equ^n ③ & equ^n ⑤

$$\frac{E_s}{I_s} = \frac{Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)}$$

$$Z_s = Z_0 \frac{\left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}{\left[Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l) \right]} \quad -⑥$$

$$Z_s = Z_0 \cancel{\frac{\left[Z_0 \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}{\left[Z_0 \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}} \quad (Z_R = Z_0)$$

$$\boxed{Z_s = Z_0}$$

when the ~~line is terminated in~~ Z_0, $Z_R = Z_0$

This shows that line terminated in Z_0, its input impedence is characteristic impedence.

Consider an infinite line with $l \to \infty$

Sub in (6)

multiplying $\frac{1}{\cosh(\gamma l)}$ in numerator & denominator in equn (6)

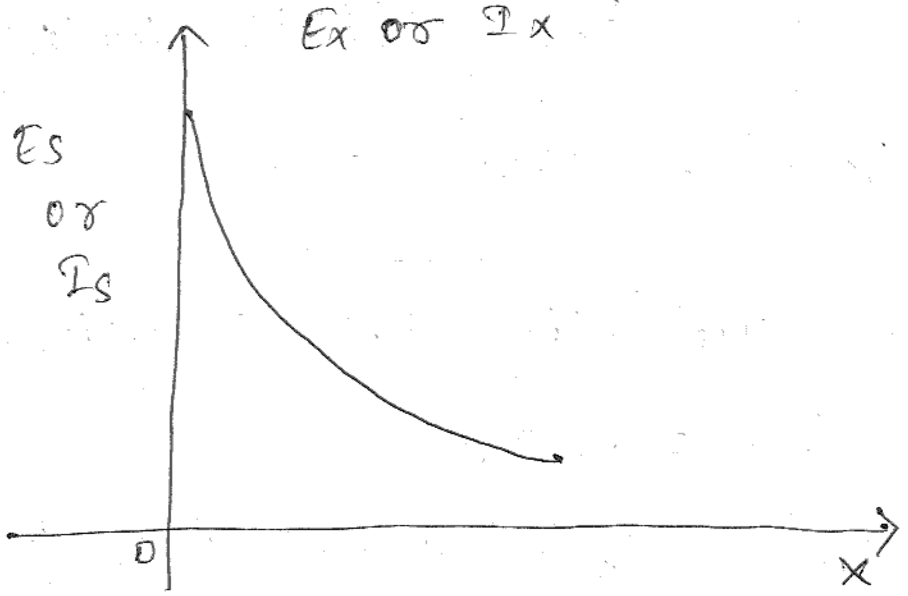
~~cos α = 1~~

$$Z_s = \frac{Z_0 \left[Z_R \cancel{\cosh l} + Z_0 \tanh(\gamma \alpha) \right]}{\left[Z_0 + Z_R \tanh(\gamma d) \right]}$$

$$Z_s = Z_0 \frac{\left[Z_R + Z_0 \right]}{\left[Z_0 + Z_R \right]} \qquad \{\tan \alpha = 1\}$$

$$\boxed{Z_s = Z_0}$$

This shows that finite line terminated in its characteristics impedence, behaves like an infinite line.



decreases Exponentially.

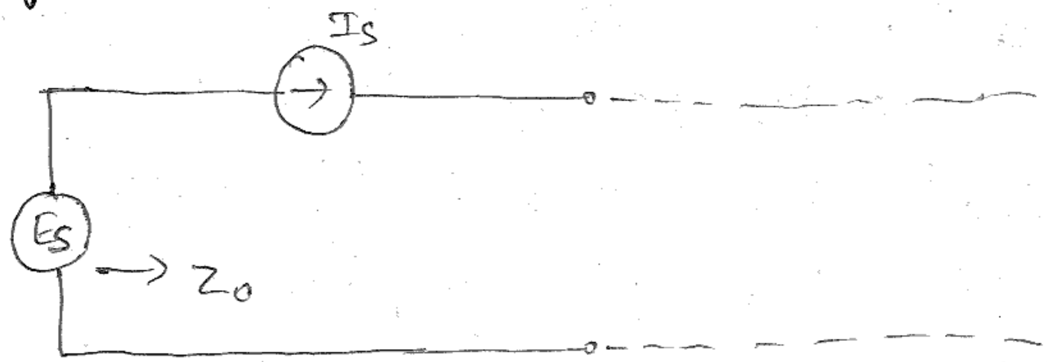
$$E_x = E_s e^{-\gamma x}$$

or

$$I_x = I_s e^{-\gamma x}$$

If instruments are connected along the line, then the instruments will show the magnitude while the phase angle cannot be obtained.

Infinite line :- (only sender no receiver)



I_s depends upon capacity & leakage conductance.

$$Z_s = \frac{E_s}{I_s}$$

Input impedence of infinite line is Z_0 of transmission line and it is denoted by Z_0.

$\therefore$ Z_0 plays an imp role in transmission line.

→ Z_0 is a phasor quantity having magnitude $|Z_0|$ & ϕ.

→ Magnitude and angle of Z_0 vary with frequency.

Important properties of Infinite line

1. As the line has infinite length, no waves will reach the receiving End and hence there is no possibility of reflection at receiving end. Thus there cannot be any reflected waves and the complete power is absorbed by the transmission line.

2. As reflected waves are absent, Z_0 decides the flow of current, when a Voltage is applied to the Sending End. The current will not be Effected by terminating impedence (Z_R) at receiving End. This condition is fulfilled by long lines in practice.

Distortion less line

A line in which there is no phase or frequency distortion and also it is correctly terminated called as Distortion less line.

Consider $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\gamma^2 = RG + j\omega RC + jG\omega L - \omega^2 LC$$

$$\gamma^2 = RG - \omega^2 LC + j(\omega RC + \omega LG)$$

$$\gamma^2 = (RG - \omega^2 LC) + j\omega(RC + LG) \quad \text{--- (1)}$$

For minimum attenuation one condition is there

$$L = \frac{CR}{G}$$ { attenuation controls the power level }

$$\Rightarrow \boxed{LG = CR} \rightarrow \text{Sub in (1)}$$

$$\gamma^2 = (RG - \omega^2 LC) + j\omega(RC + RC)$$

$$\gamma^2 = (RG - \omega^2 LC) + j2\omega RC$$

But $RC = LG = \sqrt{RCLG}$

$$\gamma^2 = (RG - \omega^2 LC) + j2\omega\sqrt{RCLG}$$

$$\gamma^2 = \left(\sqrt{RG} + j\omega\sqrt{LC}\right)^2$$

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC} \quad - (2)$$

But $\gamma = \alpha + j\beta$

where $\boxed{\alpha = \sqrt{RG} \quad \text{and} \quad \beta = \omega\sqrt{LC}}$

$\rightarrow (3)$ $\qquad \rightarrow (4)$

It can be seen from eqn (3) that α doesnot vary with frequency which Eliminates the frequency distortion.

$$V = \frac{\omega}{\beta} \quad , V = \text{velocity}$$

$$V = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

Thus for condition $LG = RC$ the velocity becames independent of frequency, this Eliminates the phase distortion.

It is already proved that for $RC = LG$, Z_0 became resistive.

and line can be correctly terminated to eliminate distortion due to Zo varying with frequency.

Thus the overall distortion are eliminated

$$\boxed{RC = LG} \Rightarrow \boxed{\frac{R}{G} = \frac{L}{C}}$$

Telephone cable

Since inductance & conductance is negligible it can be neglected and hence impedence and admittance of such a cable can be written by

$$Z = R \quad , \quad Y = j\omega C$$

and $\gamma = \sqrt{ZY} = \sqrt{j\omega RC}$

divide 2 in numerator & denominator.

$$\gamma = \sqrt{\frac{j2\omega RC}{2}} \qquad \{ j = 1, \ \phi = 90^\circ \}$$

$$\sqrt{j} = \sqrt{1 \angle 90^\circ} = 1 \angle 45^\circ \qquad \{ \sqrt{\angle 90/2} = \angle 45^\circ$$

$$\sqrt{2j} = (1 + j \cdot 1)$$

sub in above equn

$$\gamma = (1+j\cdot 1)\sqrt{\frac{\omega RC}{2}}$$

$$\gamma = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}} = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{\omega RC}{2}}, \quad \beta = \sqrt{\frac{\omega RC}{2}}$$

Sub $v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega RC}{2}}}$

$$= \sqrt{\frac{\omega^2}{\frac{\omega RC}{2}}} = \sqrt{\frac{2\omega}{RC}}$$

$$\boxed{v = \sqrt{\frac{2\omega}{RC}}}$$

Both α and v are functions of frequency ω.
Hence for high frequencies there is large attenuation and also velocity. Hence wave travel very fast then lower frequencies when frequency is high.

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open and Short circuited lines

The Expressions for voltage and current at the sending end of a transmission line of length 'l' are given by

$$E_S = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l) \quad (1)$$

$$I_S = I_R \cosh(\gamma l) + \frac{E_R}{Z_0} \sinh(\gamma l) \quad (2)$$

$$V_S = V_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$V_S = V_R \cosh(\gamma l) + \frac{V_R}{Z_R} Z_0 \sinh(\gamma l)$$

$$V_S = V_R \left[\cosh(\gamma l) + \frac{Z_0}{Z_R} \sinh(\gamma l)\right] \quad (3)$$

$$I_S = I_R \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right] \quad (4)$$

The input impedence of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

divide (3) & (4)

$$\frac{V_S}{I_S} = \frac{V_R}{I_R} \frac{\left[\cosh(\gamma l) + \sinh(\gamma l) \frac{Z_0}{Z_R}\right]}{\left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right]}$$

$$Z_S = \frac{V_R}{I_R} \frac{\left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)\right]}{\left[Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l)\right]} \times \frac{Z_0}{Z_R}$$

$$\cancel{Z_S = \frac{V_R}{I_R}}$$

$$Z_S = \cancel{Z_R}\frac{Z_0}{\cancel{Z_R}}\left[\frac{Z_R\cosh(\gamma l)+Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l)+Z_R\sinh(\gamma l)}\right]$$

$$Z_S = Z_0\left[\frac{Z_R\cosh(\gamma l)+Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l)+Z_R\sinh(\gamma l)}\right] \quad \because Z_R = \frac{V_R}{I_R}$$

$$Z_S = Z_0\left[\frac{Z_R\cosh(\gamma l)+Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l)+Z_R\sinh(\gamma l)}\right]$$

If short circuited, the receiving end impedend is zero

$$Z_S = Z_0\left[\frac{\cancel{Z_0}\sinh(\gamma l)}{\cancel{Z_0}\cosh(\gamma l)}\right]$$

$$Z_{SC} = Z_0\frac{\sinh(\gamma l)}{\cosh(\gamma l)}$$

$$\therefore \boxed{Z_{SC} = Z_0\tanh(\gamma l)}$$

This is the short circuit imp.

If open circuited, the receiving end impedence is infinite.

i.e. $Z_R = \infty$

$$Z_S = Z_0 \left[\frac{\cosh(\gamma l) + \frac{Z_0}{Z_R}\sinh(\gamma l)}{\frac{Z_0}{Z_R}\cosh(\gamma l) + \sinh(\gamma l)} \right]$$

$$Z_{OC} = Z_0 \left[\frac{\cosh(\gamma l)}{\sinh(\gamma l)} \right]$$

$$\therefore \boxed{Z_{OC} = Z_0 \coth(\gamma l)}$$

By multiplying open circuited impedence & short circuited impedence

$$\boxed{Z_{SC} \times Z_{OC} = Z_0^2}$$

The characteristics impedence is given by

$$Z_0 = \sqrt{Z_{OC} \cdot Z_{SC}}$$

By dividing short circuited impedence by open circuit impedence

$$\frac{Z_{sc}}{Z_{oc}} = \frac{\cancel{Z_0} \tanh(\gamma l)}{\cancel{Z_0} \coth(\gamma l)}$$

$$\frac{Z_{sc}}{Z_{oc}} = \tanh^2(\gamma l)$$

$$\tanh(\gamma l) = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\boxed{\gamma l = \tanh^{-1}\sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

Reflection

When the load impedence is not Equal to the characteristics impedence of transmission line, reflection takes Place. The Expression for voltage & current on the transmission line are

$$V = \frac{V_R}{2}\left[\frac{Z_R + Z_0}{Z_R} e^{\sqrt{ZY}x} + \frac{Z_R - Z_0}{Z_R} e^{-\sqrt{ZY}x}\right]$$

$\because S = a$

$$\boxed{I = \frac{I_R}{2}\left[\frac{Z_R+Z_0}{Z_0}e^{\sqrt{ZY}x} - \frac{Z_R-Z_0}{Z_0}e^{-\sqrt{ZY}x}\right]}$$

$$\boxed{V = \frac{V_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\gamma x} + \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\gamma x}\right]}$$

$\because \gamma = \sqrt{ZY}$

$$I = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\gamma x} - \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\gamma x}\right]$$

$Z_R \neq Z_0$ and this condition is called mismatched condition.

If the transmission line is not terminated with characteristics impedence, then the above Exp exist and this condition is called mismatched condition ($Z_R \neq Z_0$).

It consists of two waves one is moving in the forward (+ve) direction called incident wave, other one is moving in backward (-ve) direction called reflected wave.

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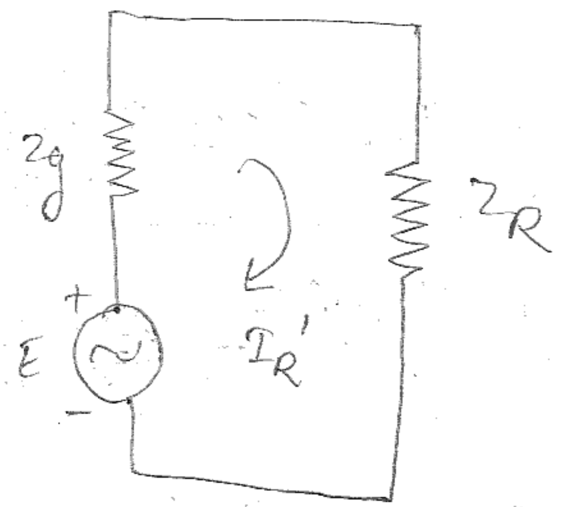
Insertion loss

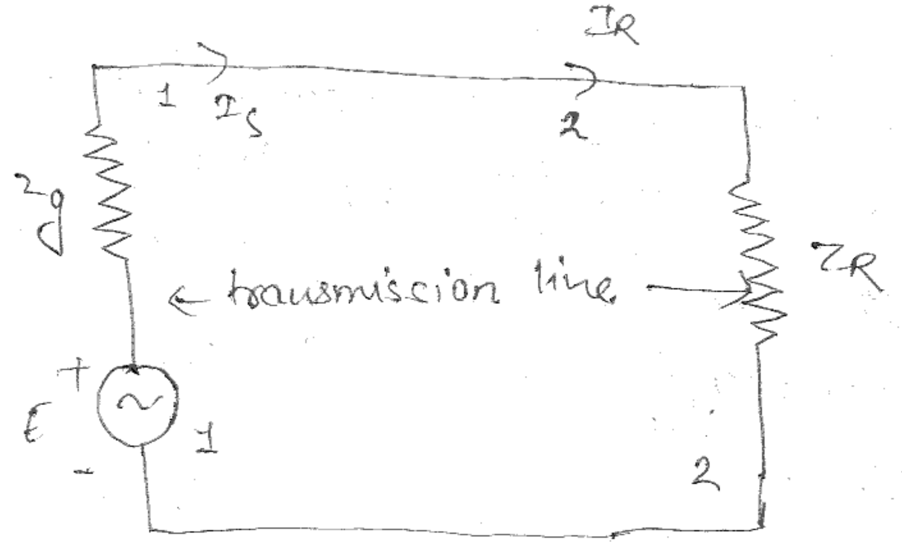


The generator is directly connected to impedence ZR, the current flowing through the load is given by

$$E = Z_g I_R' + Z_R I_R'$$

$$E = (Z_g + Z_R) I_R'$$

$$I_R' = \frac{E}{Z_g + Z_R} \quad \text{--- (1)}$$



$$Z_s \neq Z_g \,, \quad Z_R \neq Z_0$$

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Thus insertion loss of a line or a network is defined as the no of decibel by which the current in the load is changed by the insertion of a line or a network in between the load and the source.

Let Z_S be the i/p impedence of a line which is different than Z_g and it can be given by

$$I_S = \frac{E}{Z_S + Z_g} \quad \text{—} \quad (2)$$

It is known for the line that i/p impedence is given by

$$Z_S = Z_0 \left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right] \quad \text{—} \quad (3)$$

Sub the value of equ^n (3) in (2)

$$I_S = \frac{E}{Z_0 \left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right] + Z_g}$$

$$I_S = \frac{E \left(e^{\gamma l} - K e^{-\gamma l} \right)}{Z_0 \left(e^{\gamma l} + K e^{-\gamma l} \right) + Z_g \left(e^{\gamma l} - K e^{-\gamma l} \right)} \quad (4)$$

We want the current through the load i.e. I_R due to insertion of a line. This can be obtained from the relation b/w I_s and I_R.

$$I_s = \frac{I_R(Z_R + Z_0)}{2Z_0}\left(e^{\gamma l} - ke^{-\gamma l}\right) \quad - (5)$$

The eqn is obtained from the general solution of line substituting $s = l$, i.e. $I = I_s$ at the sending End.

from (5)

$$I_R = \frac{2Z_0 I_s}{(Z_R + Z_0)(e^{\gamma l} - ke^{-\gamma l})}$$

Sub I_s value in the above eqn

$$I_R = \frac{2Z_0 E\,\cancel{(e^{\gamma l} - ke^{-\gamma l})}}{\left[Z_g(e^{\gamma l} - ke^{-\gamma l}) + Z_0(e^{\gamma l} + ke^{-\gamma l})\right] \times [Z_0 + Z_R]\,\cancel{(e^{\gamma l} - ke^{-\gamma l})}}$$

$$I_R = \frac{2Z_0 E}{\left[Z_g(e^{\gamma l} - ke^{-\gamma l}) + Z_0(e^{\gamma l} + ke^{-\gamma l})\right][Z_0 + Z_R]} \quad - (6)$$

Reflection coefficient can be given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} \quad — (7)$$

Sub eqn (7) in (6)

$$I_R = \frac{2Z_0E}{\left[Z_R\left(e^{\gamma l} - \frac{Z_R - Z_0}{Z_R + Z_0}e^{-\gamma l}\right) + Z_0\left(e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0}e^{-\gamma l}\right)\right]\left[Z_0 + Z_R\right]}$$

$$= \frac{2Z_0E\,\cancel{(Z_R + Z_0)}}{\left\{Z_R\left(e^{\gamma l}(Z_R + Z_0) - (Z_R - Z_0)e^{-\gamma l}\right) + Z_0\left[e^{\gamma l}(Z_R + Z_0) + e^{-\gamma l}(Z_R - Z_0)\right]\right\} \times \cancel{(Z_0 + Z_R)}}$$

$$I_R = \frac{2Z_0E}{\left[Z_R\left((Z_R + Z_0)e^{\gamma l} - (Z_R - Z_0)e^{-\gamma l}\right) + Z_0\left((Z_R + Z_0)e^{\gamma l} + (Z_R - Z_0)e^{-\gamma l}\right)\right]}$$

$$= \frac{2Z_0E}{Z_R^2 e^{\gamma l} + Z_R Z_0 e^{\gamma l} - Z_R^2 e^{-\gamma l} + Z_0 Z_R e^{-\gamma l} + Z_0 Z_R e^{\gamma l} + Z_0^2 e^{\gamma l} + Z_0 Z_R e^{-\gamma l} - Z_0^2 e^{-\gamma l}}$$

$$= \frac{2 Z_0 E}{(Z_R^2 + 2Z_R Z_0 + Z_0^2) e^{\gamma l} - (Z_R^2 - 2Z_R Z_0 + Z_0^2) e^{-\gamma l}}$$

$$= \frac{2 Z_0 E}{(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}}$$

The insertion loss is the ratio of currents in the load without insertion and with insertion.

$$\frac{I_R'}{I_R} = \frac{\dfrac{E}{Z_g + Z_R}}{\dfrac{2 Z_0 E}{(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}}}$$

$$\frac{I_R'}{I_R} = \frac{(Z_R + Z_0)(Z_0 + Z_g) e^{\gamma l} + (Z_R - Z_0)(Z_0 - Z_g) e^{-\gamma l}}{2(Z_R + Z_g) Z_0} \quad - (9)$$

$$\gamma = \alpha + j\beta$$

~~$\frac{I_R'}{I_R} = \quad Z_R +$~~

The current ratio is made up of two Parts, one is continuously increasing towards length and other is decreasing towards length. Since the length of line is very large

$e^{-\gamma l} \to 0$

$$\frac{I_R'}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)\,e^{\gamma l}}{2Z_0(Z_g+Z_R)} \quad \text{— (10)}$$

Sub in (10) $\gamma = \alpha + j\beta$

$$\frac{I_R'}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)\,e^{(\alpha+j\beta)l}}{2Z_0(Z_g+Z_R)}$$

$$\frac{I_R'}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)\,e^{\alpha l}\cdot e^{j\beta l}}{2Z_0(Z_g+Z_R)}$$

Insertion loss is to be calculated as a function of ratio of current magnitudes and hence the term $e^{j\beta l}$ which gives phase angle can be neglected.

$$\left|\frac{I_R'}{I_R}\right| = \frac{|Z_R+Z_0|\,|Z_0+Z_g|\,e^{\alpha l}}{2|Z_0|\,|Z_g+Z_R|}$$

Multiply $2\sqrt{Z_g Z_R}$ in num & deno.

$$\left|\frac{I'_R}{I_R}\right| = \frac{2\sqrt{Z_g Z_R}\,|Z_R+Z_0|\,|Z_0+Z_g|\,e^{\alpha l}}{2\,|Z_0|\,|Z_g+Z_R|\,\sqrt{Z_g Z_R}}$$

Sub $|Z_0| = \sqrt{|Z_R|\,|Z_0|} = \sqrt{1} = 1$

$$\left|\frac{I'_R}{I_R}\right| = \frac{2\sqrt{Z_g Z_R}\,|Z_R+Z_0|\,|Z_0+Z_g|\,e^{\alpha l}}{\underbrace{\left(2\sqrt{Z_R Z_g}\cancel{+2\sqrt{Z_g Z_R}}\right)|Z_g+Z_R|}_{\text{Reflection factors}}\;|Z_0|(\sqrt{Z_0})}$$

Let $K_S = \dfrac{2\sqrt{Z_g Z_0}}{|Z_g+Z_0|}$ → reflection at Sending End

$K_R = \dfrac{2\sqrt{Z_R Z_0}}{|Z_R+Z_0|}$ → Reflection factor at load.

$K_{SR} = \dfrac{2\sqrt{Z_g Z_R}}{|Z_g+Z_R|}$ → Reflection factor for direct connection.

$e^{\alpha l}$ → loss

$$\left|\frac{I'_R}{I_R}\right| = \frac{K_{SR}}{K_R \cdot K_S}\cdot e^{\alpha l}$$

loss $e^{\alpha l}$

αl $\log R$

$$\ln\left|\frac{I_R'}{I_R}\right| = \left[\ln\left(\frac{1}{K_S}\right) + \ln\left(\frac{1}{K_R}\right) - \ln\left(\frac{1}{K_{SR}}\right) + \alpha l\right] \text{ nepers.}$$

$$\text{Insertion loss} = 20\log\left|\frac{I_R'}{I_R}\right| = 20\left[\log\left(\frac{1}{K_S}\right) + \log\left(\frac{1}{K_R}\right) - \log\left(\frac{1}{K_{SR}}\right) + 0.2\underbrace{\alpha l}_{\text{loss}}\right]$$

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Reflection on a line not terminated in Z_0

The Reflection phenomenon exist for a line which is not terminated in Z_0. Such a reflection is maximum when a line is open circuit.

$Z_R = \infty$ for an open circuit

$Z_R = 0$ " short circuit.

The general solution of a transmission line can be given by

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\sqrt{ZY}s} + \frac{Z_R-Z_0}{Z_R+Z_0}\cdot e^{-\sqrt{ZY}s}\right] \quad (1)$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\sqrt{ZY}s} - \frac{Z_R-Z_0}{Z_R+Z_0}\, e^{-\sqrt{ZY}s}\right] \quad (2)$$

If the value of $Z_R \neq Z_0$ of the line, then E and I consists of two parts

i) one part vary Exponentially with Positive S.

ii) another part vary exponentially with -ve S.

Now sub $\sqrt{ZY} = \gamma$ in ① & ②

$$E = \left[\frac{E_R(Z_R+Z_0)}{2Z_R} e^{\gamma S} + \frac{E_R(Z_R-Z_0)}{2Z_R} e^{-\gamma S}\right] \quad -③$$

$$I = \left[\frac{I_R(Z_R+Z_0)}{2Z_0} e^{\gamma S} \mp \frac{I_R(Z_R-Z_0)}{2Z_0} e^{-\gamma S}\right] \quad -④$$

S → R (incident wave) → amp ↓es

R → S (Reflected wave) → amp increases.

Incident	Reflected.
$E_1 = \frac{E_R(Z_R+Z_0)}{2Z_R} e^{\gamma S}$ incident voltage wave	$E_2 = \frac{E_R}{2Z_R}(Z_R-Z_0)e^{-\gamma S}$
$I_1 = \frac{I_R(Z_R+Z_0)}{2Z_0} e^{\gamma S}$ incident current wave	$I_2 = \frac{I_R}{2Z_0}(Z_R-Z_0)e^{-\gamma S}$

The wave which flows from sending End to the receiving End with decreasing I amplitude.

Consider open circuit with $Z_R = \infty$ then the incident component of voltage becomes

$$E_1 = \frac{E_R (Z_R + Z_0)}{2 Z_R} e^{\gamma s}$$

$$= \frac{E_R \left(1 + \frac{Z_0}{Z_R}\right)}{2} e^{\gamma s}$$

Put $Z_R = \infty$

(3) $$E_1 = \frac{E_R (1+0)}{2} e^{\gamma s}$$

$$\boxed{E_1 = \frac{E_R e^{\gamma s}}{2}}$$

(4) $$E_2 = \frac{E_R (Z_R - Z_0)}{2 Z_R} e^{-\gamma s}$$

$$= \frac{E_R (1-0)}{2} e^{-\gamma s}$$

$$\boxed{E_2 = \frac{E_R e^{-\gamma s}}{2}}$$

At $s = 0$, at the receiving end, an open circuit E_1 & E_2

$$\boxed{E_1 = \frac{E_R}{2} \quad \& \quad E_2 = \frac{E_R}{2}}$$

Reflection phenomenon

The quantity which is actually transmitted along the line is not the current or voltage but the Energy. Such Energy is transmitted through Electric & magnetic fields along the line.

The Energy conveyed in Electric field is given by

$$W_e = \frac{1}{2} C E^2 \quad J/m^3 \quad \text{--- (1)}$$

The Energy conveyed in magnetic field is given by.

$$W_m = \frac{1}{2} L I^2 \quad J/m^3 \quad \text{--- (2)}$$

For an ideal line which is terminated in Z_0, the ratio of E and I is fixed along the line which is Z_0.

$$Z_0 = \frac{E}{I} \quad -(3)$$

For such a ideal line $R = G = 0$ &

Z_0 is given by

$$Z_0 = \sqrt{\frac{L}{C}} \quad -(4)$$ (terminating by Z_0)

from (3) $\quad$ from (4)

$$E = I Z_0 \quad \& \quad Z_0^2 = \frac{L}{C}$$

$$C = \frac{L}{Z_0^2}$$

$$W_e = \frac{1}{2} \cdot \frac{L}{Z_0^2} \cdot (I Z_0)^2$$

$$= \frac{1}{2} \frac{L}{\cancel{Z_0^2}} \times I^2 \cancel{Z_0^2}$$

$$W_e = \frac{1}{2} L I^2$$

$W_e = W_m$ (magnetic field is equal to electric field)

15/4/07

Disadvantages of Reflection

1. There is reduction in efficiency, ~~the~~

2. The part of the received energy is rejected by the load, hence output reduces.

Reflection Coefficient

The ratio of the amplitudes of the Reflected and incident ~~voltage~~ wave at the receiving end of the line is called Reflection coefficient.

$$K = \frac{\text{reflected wave at load}}{\text{incident wave at load}}$$

The reflected voltage at Load is E_2 at receiving End ~~and incident~~ i.e. with $S = 0$

$$E_1 = \frac{E_R(Z_R + Z_0)}{2Z_R} \cdot e^{\gamma S}$$

$$E_2 = \frac{E_R(Z_R - Z_0)}{2Z_R} e^{-\gamma S}.$$

$$E_2 \big|_{s=0} = \frac{E_R (Z_R - Z_0)}{2 Z_R} \quad \text{--- (1)}$$

$$E_1 \big|_{s=0} = \frac{E_R (Z_R + Z_0)}{2 Z_R} \quad \text{--- (2)}$$

The incident voltage at load is E_1 at the receiving end with $S = 0$.

$$K = \frac{\frac{\cancel{E_R}(Z_R - Z_0)}{\cancel{2 Z_R}}}{\frac{\cancel{E_R}(Z_R + Z_0)}{\cancel{2 Z_R}}}$$

$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

The following observations can be made with respect to the reflection coefficient

1) when $Z_R = Z_0$, $K = 0$, there will be no reflection.

2) $Z_R = 0$, means when the line is short circuited

$$K = -\frac{Z_0}{Z_0} = -1$$

$$\Rightarrow \boxed{K = -1} \qquad K = 1 \angle 180^\circ$$

Reflection is maximum

3) $Z_R = \infty$ i.e. open circuit

$$K = \frac{1 - \frac{Z_0}{Z_R}}{1 + \frac{Z_0}{Z_R}} = 1 = 1\angle 0^\circ$$

Reflection is max.

4) K ranges in magnitude from 0 to 1 and phase ranges from 0 to 180°.

Reflection loss

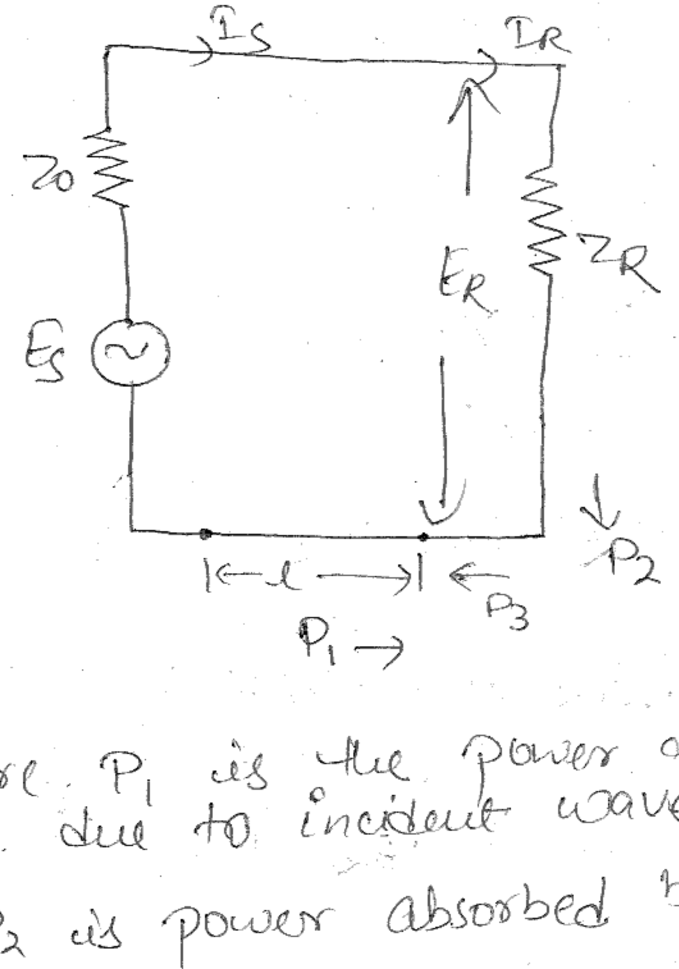
The Reflection loss is defined as the number of nepers or decibel by which current in the load under mismatched conditions would exceed the current, actually flowing in the load.

$$\text{Reflection loss} = \ln\left[\frac{|I_2'|}{|I_2|}\right] \text{ nepers.} \quad - 1(a)$$

$$\text{Reflection loss} = 20 \log\left[\frac{|I_2'|}{|I_2|}\right] \text{ decibel} - 1(b)$$

where I_2' is the load current under mismatched condition.

I_2 is actual load current under mismatched condition.



where P_1 is the power at the receiving end due to incident wave.

P_2 is power absorbed by the load.

P_3 is power reflected back down the line.

$$P_1 = P_2 + P_3 \quad , \quad P \propto I^2$$

$$I \propto P^{1/2}$$

1(b)

$$\text{Reflection loss} = 20 \log \left[\frac{|I_2'|}{|I_2|} \right]$$

$$\left[\because I_2' = P_1 \, , \; I_2 = P_2' \right]$$

$$= 20 \log \left[\frac{|P_1^{1/2}|}{|P_2^{1/2}|} \right]$$

$$\text{Reflection loss} = 10 \log \left[\frac{|P_1|}{|P_2|} \right] \ dB \quad -(2)$$

The Reflection coefficient is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

If E_R and I_R are the values of the voltage & current at the receiving end due to incident waves, then the values of voltage & current due to the reflected wave are : $K \cdot E_R$ and $K \cdot I_R$

So P_1 = received power = $E_R I_R$

P_2 = absorbed power by load $= P_1 - P_3$

P_3 = Reflected power $= (|K| E_R)(I_R |K|)$

$= |K|^2 P_1$

$$P_2 = P_1 - P_3$$

$$P_2 = P_1 - |K|^2 P_1$$

$\Rightarrow P_2 = (1-|K|^2) P_1$

Sub all the values in equn ②

$$\text{Reflection loss} = 10 \log \left[\frac{P_1}{(1-|K|^2) P_1} \right]$$

$$\boxed{\text{Reflection loss} = 10 \log \left[\frac{1}{1-|K|^2} \right]} \quad -③$$

$$\frac{1}{1-K^2} = \frac{1}{1-\left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)^2} = \frac{Z_R+Z_0}{\cancel{Z_R+Z_0} - \cancel{Z_R+Z_0}}$$

$$= \frac{\left(\frac{\cancel{Z_R+Z_0}}{Z_R+Z_0}\right)^2}{(Z_R+Z_0)^2 - (Z_R-Z_0)^2}$$

$$= \frac{Z_R^2 + Z_0^2 + 2Z_R Z_0}{\cancel{Z_R^2} + \cancel{Z_0^2} + 2Z_R Z_0 - \cancel{Z_R^2} - \cancel{Z_0^2} + 2Z_R Z_0}$$

$$\frac{1}{1-K^2} = \frac{(Z_R+Z_0)^2}{[\cancel{2}\,2 Z_R Z_0]}$$

$$\frac{1}{1-|K|^2} = \frac{|Z_R+Z_0|^2}{|4 Z_R Z_0|} \quad \{\text{sub in ③}\}$$

$$RL = 10 \log \left[\frac{|Z_R+Z_0|^2}{|4 Z_R Z_0|} \right]$$

$$\boxed{RL = 20 \log \left| \frac{|Z_R+Z_0|}{|2\sqrt{Z_R Z_0}|} \right|}$$

$R_f = \frac{2\sqrt{Z_R+Z_0}}{\cancel{2}\sqrt{Z_R Z_0}}$

The ratio which indicates the change in current in the load due to reflection at mismatched junction

Return loss

Return loss is defined as to

$$RL = 10 \log \frac{P_1}{P_3} \ dB.$$

It is the ratio of power at the receiving end due to incident wave to the power reflected by the wave.

$$RL = 10 \log \frac{P_1}{|K|^2 \cancel{P_1}}$$

$$= 10 \log \frac{1}{|K|^2}$$

$$= 10 \log \left[\frac{1}{|K|}\right]^2$$

$$= 20 \log \left[\frac{1}{|K|}\right]$$

$$\boxed{RL = 20 \log \left[\frac{Z_R + Z_0}{Z_R - Z_0}\right] \ dB}$$