$$Z_{sc} \times Z_{oc} = \frac{Z_1 Z_2}{Z_1 + Y Z_2}$$

~~5/06/11~~

28/6/11

Q. Calculate Characteristic impedance & propagatⁿ Constant, if each series impedance is 50 Ω ($\frac{Z_1}{2}$) & Shunt impedance is 500 Ω.

Q2. Design symmetric T-section, to have $Z_0 = 600\,\Omega$ & $Y = 0 + j\frac{\pi}{4}$.

Solⁿ 2

$$Z_2 = \frac{Z_0}{\sinh Y} = \frac{600}{\sin\left(\frac{j\pi}{4}\right)}$$

$\frac{Z_1^2}{\left(\frac{Z_1^2}{4}\right)}$

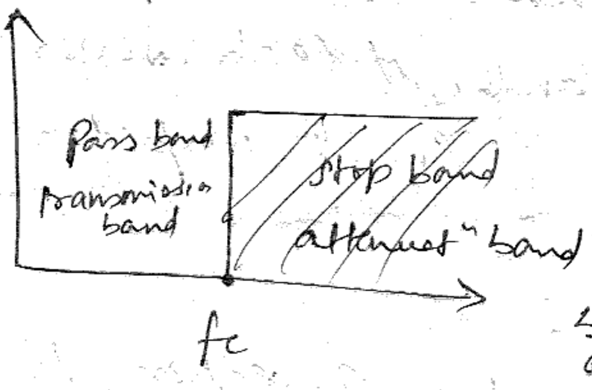
28/9/1?

Ideal filter.

0 attenuatⁿ in P.b.

∞ " in S.B.

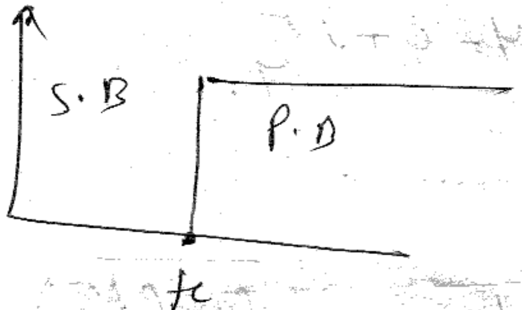
L.P.F



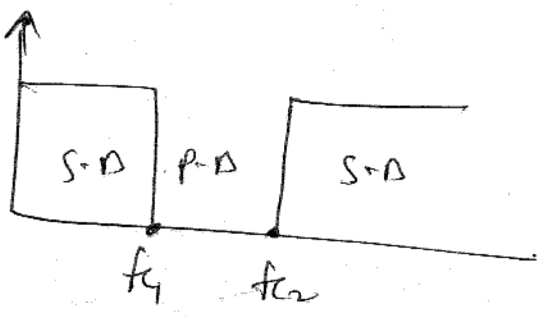
fc = cut off freq.

L.P.F passes all freq. greater than cut off freq.

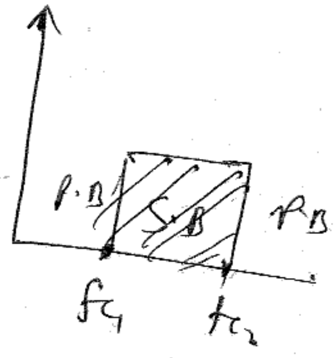
H.P.F



B.P.F



B.S.F



Ideal filter :- An ideal filter ~~to~~ could have zero attenuation in Pass Band & infinite attenuation in Stop Band.

But practically in the Stop Band attenuation gradually changes.

It cannot change either from zero to infinite ~~an~~ or infinite to zero.

Filter fundamentals:- The complete study of behaviour of any filter section needs the calculation of Z_0, γ, α, β.

→ An important consideration of all filter is that they are constructed from purely reactive element (Z_1, Z_2); otherwise the attenuation could never become zero.

→ From the expression of characteristics of T & π netk, the Z_0 depends on the reactance i.e on Z_1 & Z_2 offered by purely reactive elements.

$$\boxed{Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

Hence the characteristic impedance Z_0 varies with frequency. as like Z_1 & Z_2. Both varies with frequency.

⇒ The theorem connects Z_0 & α, under the assumptions :- that filter is currently terminated in its characteristic impedance ; The theorem states that

" over the range of frequency for which the characteristic impedance Z_0 of filter is purely resistive (real). The attenuation α is zero.

⇒ ~~OV~~

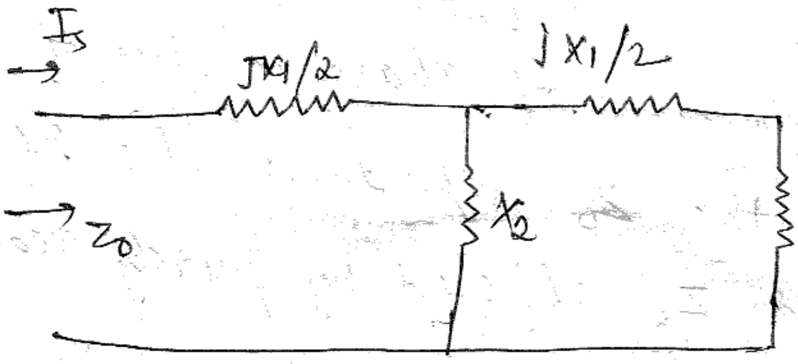


Fig 1

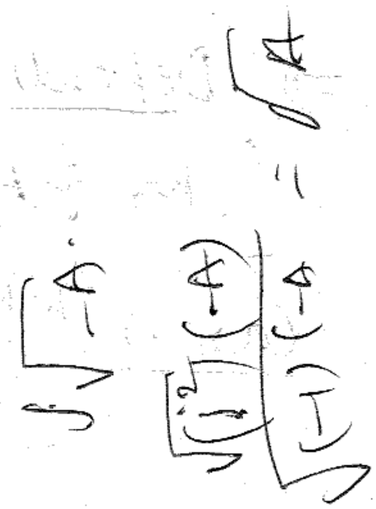
⇒ over the range of frequencies for which Z_0 is purely reactive, than the attenuation is greater than zero.

⇒ All the elements in the T- section are reactive hence all are represented in the form of jX, here X is a real. , ~~by~~ but it may be +ve or -ve

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$e^{\gamma} = \frac{I_S}{I_R} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$\alpha = 20 \log_{10} \left| \frac{I_S}{I_R} \right| \text{ in dB.}$$



From fig 1

$$Z_1 = jX_1 \quad \& \quad Z_2 = jX_2.$$

$$Z_0 = \sqrt{\frac{j^2 X_1^2}{4} + j^2 X_1 X_2}$$

$$= \sqrt{-\frac{X_1^2}{4} - X_1 X_2} = \sqrt{-\left(\frac{X_1^2}{4} + X_1 X_2\right)}$$

$$= \sqrt{\left(\frac{X_1^2}{4} + X_1 X_2\right) j^2}$$

$$\boxed{Z_0 = j\sqrt{\frac{X_1^2}{4} + X_1 X_2}} \text{ —— (1) eq}^n$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$= 1 + \frac{jX_1}{2jX_2} + \frac{Z_0}{jX_2}$$

$$= 1 + \frac{X_1}{2X_2} - J\frac{Z_0}{X_2}$$

$$\boxed{e^{\gamma} = \left[1 + \frac{X_1}{2X_2}\right] - J\left[\frac{Z_0}{X_2}\right]}$$

$\Rightarrow$ Depending on the sign of X_1 & X_2 we get two cases ..

Case i) when $\frac{X_1^2}{4} + X_1X_2$ is negative, then it is '$-A$'

Case ii) when $\frac{X_1^2}{4} + X_1X_2$ is +ve, then it is '$+B$'.

where A and real and constant values.

then for case (i),

$$Z_0 = J\sqrt{-A}$$

$$= J\sqrt{A(J^2)}$$

$$= J^2\sqrt{A}$$

$$\boxed{Z_0 = -\sqrt{A}}$$

$$Z_0 = J\sqrt{-A}$$

$$\frac{x_1^2}{4} + x_1 x_2 = -A$$

$$= \sqrt{J^2(-A)}$$

$$\boxed{Z_0 = \sqrt{A}}$$

$\Rightarrow$ Z_0 is +ve, it is purely resistive.

29/6/14

then

$$e^{\gamma} = \frac{I_s}{I_R}$$

$$e^{\gamma} = \left[1 + \frac{x_1}{2x_2}\right] - J\left[\frac{Z_0}{x_2}\right]$$

$$= \left[1 + \frac{x_1}{2x_2}\right] - J\left[\frac{\sqrt{A}}{x_2}\right]$$

attenuation in dB $= 20 \log_{10} \left|\frac{I_s}{I_R}\right|$

$$\alpha = 20\log_{10} \left| \left[1 + \frac{x_1}{2x_2}\right] - J\left[\frac{\sqrt{A}}{x_2}\right] \right|$$

$$= 20\log_{10} \sqrt{\left(1 + \frac{x_1}{2x_2}\right)^2 + \left(\frac{\sqrt{A}}{x_2}\right)^2}$$

$$= 20\log_{10} \sqrt{\left(1 + \frac{x_1}{2x_2}\right)^2 + \frac{A}{x_2^2}}$$

$$= 20\log_{10} \sqrt{\left(1 + \frac{x_1}{2x_2}\right)^2 + \frac{1}{x_2^2}\left(-\frac{x_1^2}{4} - x_1 x_2\right)}$$

$$= 20\log_{10} \sqrt{}$$

$$= 20\log_{10}\sqrt{1+\cancel{\frac{X_1^2}{4X_2^2}}+\cancel{\frac{X_1}{X_2}}-\cancel{\frac{X_1^2}{4X_2^2}}-\cancel{\frac{X_1}{X_2}}}$$

$$= 20\log_{10}\sqrt{1}$$

$$\alpha = 20\log_{10}\sqrt{1}$$

$$\alpha = 20\log_{10} 1$$

$$\boxed{\alpha = 0}$$

for case ii)

$$Z_0 = j\sqrt{B}$$

$$e^{\gamma} = \left[1+\frac{X_1}{2X_2}\right] - j\left[\frac{j\sqrt{B}}{X_2}\right]$$

$$e^{\gamma} = 1+\frac{X_1}{2X_2} + \frac{\sqrt{B}}{X_2}$$

$$\alpha = 20\log_{10}\left|\frac{I_s}{I_R}\right|$$

$$= 20\log_{10}\left|1+\frac{X_1}{2X_2}+\frac{\sqrt{B}}{X_2}\right| \Rightarrow \alpha \geq 1.$$

⇒ The ratio is real and greater than 1. Thus α is not zero.

If Z_0 is not real, i.e. it is imaginary

$$\alpha = 20\log_{10}\left| 1 + \frac{X_1}{2X_2} + \frac{\sqrt{\frac{X_1^2}{4} + X_1X_2}}{X_2} \right|$$

<u>Cutoff frequency</u> :- The frequency at which separates the pass band and stop band.

or,

the frequency at which Z_0 changes from real to imaginary.

The Cutoff freq at which Z_0 changes from being real to being imaginary.

<u>For T-secn</u>

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

<u>for purely ~~real~~ Imaginary.</u>

$Z_1 = JX_1$ $\qquad Z_0 = \sqrt{-\frac{X_1^2}{4} - X_1X_2}$

$Z_2 = JX_2$

$$Z_0 = \sqrt{-X_1\left(\frac{X_1}{4} + X_2\right)}$$

The Z_0 is purely imaginary, if X_1 & $\frac{X_1}{4}+X_2$ have the same sign, this gives the stopband

when Z_0 is purely real :-

If X_1 & $\left(\frac{X_1}{4}+X_2\right)$ has opposite sign, and it gives ~~stop~~ pass band.

Case iii) when $X_1\left(\frac{X_1}{4}+X_2\right) = 0$, we get cut off frequency.

$$\boxed{X_1 = -4X_2}$$ ← Condⁿ of Cutoff frequency

Pass Band & stop Band :-

for P.B

$$\boxed{\gamma = \alpha + j\beta}$$

γ = propagatⁿ const.
α = attenuation

The ability of n/wᵏ to work as filter is decided by propagation Constant γ.

When $\alpha = 0$, the o/p current = i/p current

$I_1 = I_R$

Indicating no attenuation in the o/p but only the phase shift in o/p.

$\therefore \alpha = 0$

$$\boxed{\gamma = j\beta}$$

~~The~~

The magnitude of o/p current becomes very less than the i/p current indicating the attenuation in the o/p current

we know that:-

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\left(\frac{\alpha+j\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\left(\frac{\alpha}{2}+j\frac{\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\alpha}{2}\cosh\left(\frac{j\beta}{2}\right)+\cosh\frac{\alpha}{2}\sinh\left(\frac{j\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}+\cosh\frac{\alpha}{2}\cdot j\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Condition

If Z_1 and Z_2 are reactances of opposite

then

$$\sinh\left(\frac{\alpha}{2}\right)\cos\frac{\beta}{2} = 0 \quad \text{— ① eq}$$

$$\cosh\left(\frac{\alpha}{2}\right)\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{— ② eq}$$

from ①,

$\sinh\left(\frac{\alpha}{2}\right) = 0$

$\Rightarrow \boxed{\alpha = 0}$

from eqⁿ ②,

$\cosh\left(\frac{\alpha}{2}\right) \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ — ①'

$\Rightarrow \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

$\Rightarrow \boxed{\beta \neq 0}$ ie $\beta = (2n-1)\pi$

~~$\cosh\frac{\alpha}{2}$~~

$\cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ — ②'

$$\boxed{\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}}$$ from ①'

from ②'

$$\boxed{\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}}$$

Constant K Section Filter :-

A - T or π - Section in which series & shunt arm impedance Z_1 & Z_2 satisfy the relationship $\boxed{Z_1 \cdot Z_2 = R_0^2}$, then it is a Constant K section.

Where,

$R_0 \rightarrow$ Real Constant, also called as design impedance.

$$\boxed{Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}}$$ ← characterstic impedance for π sectn

for Constant T - section

$$\boxed{Z_{0\pi} = \frac{R_0^2}{Z_{0T}}}$$

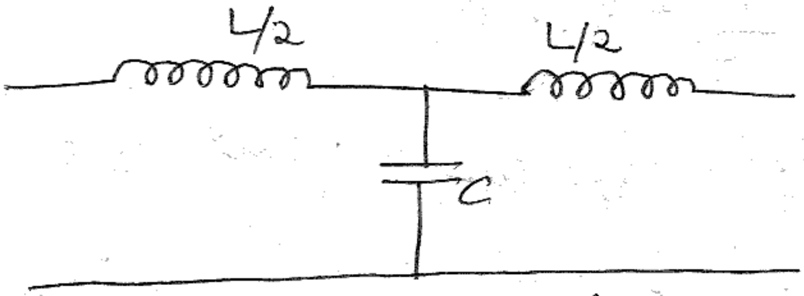
Constant T or π sectn filter is also called as Prototype section filters.

~~The Constant~~

PROTO Types Filters

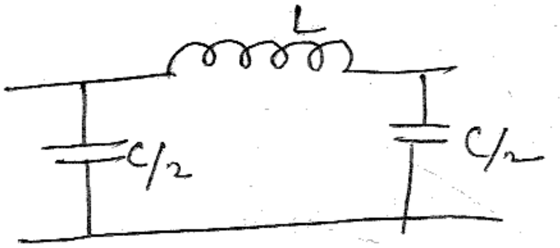
LOW PASS FILTER :-

LOW PASS FILTER



T constant T sect^n

Constant π-sect^n



Design section :-

$Z_1 = j\omega L$ $\quad$ Z_1 = Total series amp resistance

$Z_2 = \frac{-j}{\omega C}$ $\quad$ Z_2 = Total shunt amp resi.

$R_o^2 = Z_1 Z_2$

$R_o^2 = \frac{\omega L}{\omega C}$

$\Rightarrow R_o = \sqrt{\frac{\omega L}{\omega C}} = \sqrt{\frac{L}{C}}$

$$\boxed{R_o = \sqrt{\frac{L}{C}}}$$

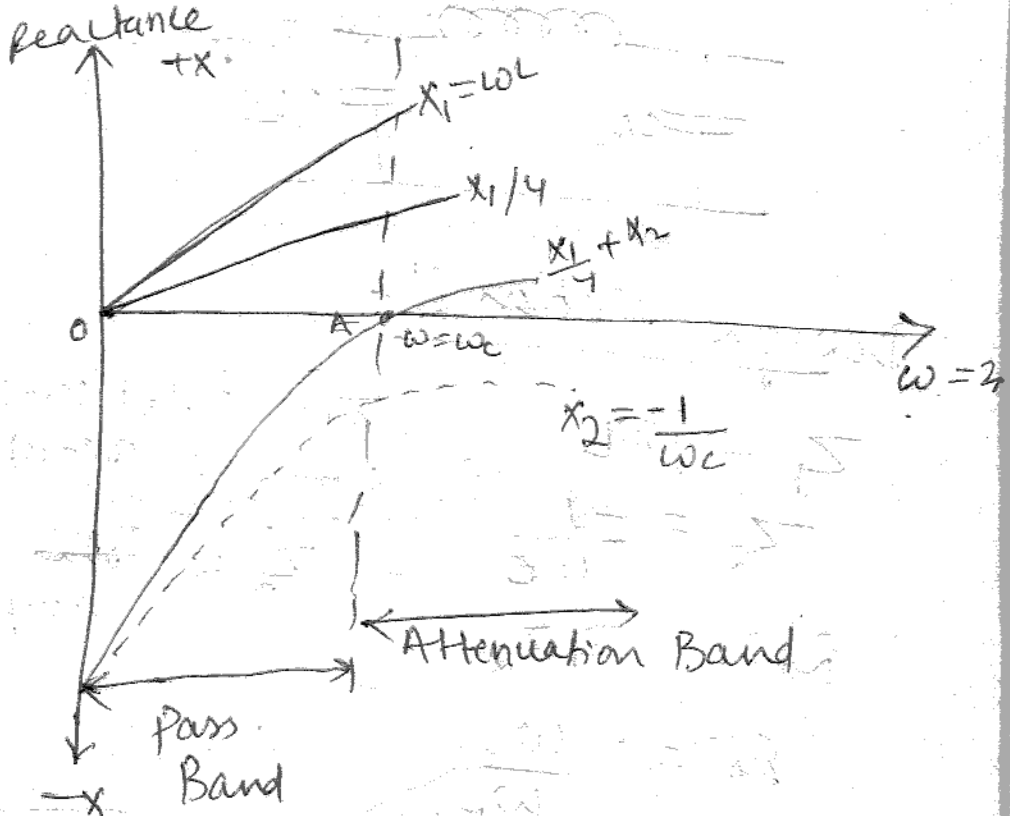
Reactance Curves & Cut off frequency Expr

$\because \frac{X_1}{4} + X_2 = 0$

also $X_1 = \omega L$ and $X_2 = \frac{-1}{\omega C}$

$\Rightarrow \frac{X_1}{4} + X_2 = \frac{\omega L}{4} - \frac{1}{\omega C} = 0$ —①

For LPF, reactance curve is given by



Hence all the frequencies upto point A gives Pass Band, above point A gives stop band

Thus the point A is the cut off frequency given by $\omega = \omega_c$.

quency Exp | from ①,

$-\frac{1}{\omega C}$

0 —①

ven by

$\omega = 2$

d

A gives

stop band

requency

at point, $\omega = \omega_c$

eqn ① becomes,

$$\frac{\omega_c L}{4} - \frac{1}{\omega_c C} = 0$$

$$\Rightarrow \frac{\omega_c L}{4} = \frac{1}{\omega_c C}$$

$$\Rightarrow \omega_c^2 LC = 4$$

$$\Rightarrow \omega_c^2 = \frac{4}{LC}$$

$$\Rightarrow \boxed{\omega_c = \frac{2}{\sqrt{LC}}}$$

$$\Rightarrow 2\pi f_c = \frac{2}{\sqrt{LC}}$$

$$\Rightarrow \boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

By Algebrically

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$\boxed{Z_{OT} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}}$$

From the above Expression Z_{OT} is real when $\frac{\omega^2 LC}{4} < 1$. and Z_{OT} is Img when

$$\frac{\omega^2 LC}{4} > 1.$$

So at cut off frequency.

$$\frac{\omega^2 LC}{4} - 1 = 0$$

$$\Rightarrow \frac{\omega^2 L_C}{4} = 1$$

$$\Rightarrow \omega_c^2 = \frac{4}{LC}$$

$$\omega = \sqrt{\frac{4}{LC}} = \frac{2}{\sqrt{LC}}$$

$$\boxed{\omega_c = \frac{2}{\sqrt{LC}}}$$

Thus above prototype section passes all frequencies of $\frac{2}{\sqrt{LC}}$, while attenuat all frequency above cut off frequency

<u>Variation of Z_{OT} and $Z_{O\pi}$ with frequency</u> :-

$$Z_{OT} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}}$$

is real

s img whe

asses all

attenuati

equency

th

$$Z_{0T} = R_0 \sqrt{1 - \frac{f^2}{f_c^2}}$$

$$\because \quad Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$\boxed{Z_{0\pi} = \frac{R_0^2}{R_0 \sqrt{1 - \left(\frac{f}{f_c}\right)^2}}}$$

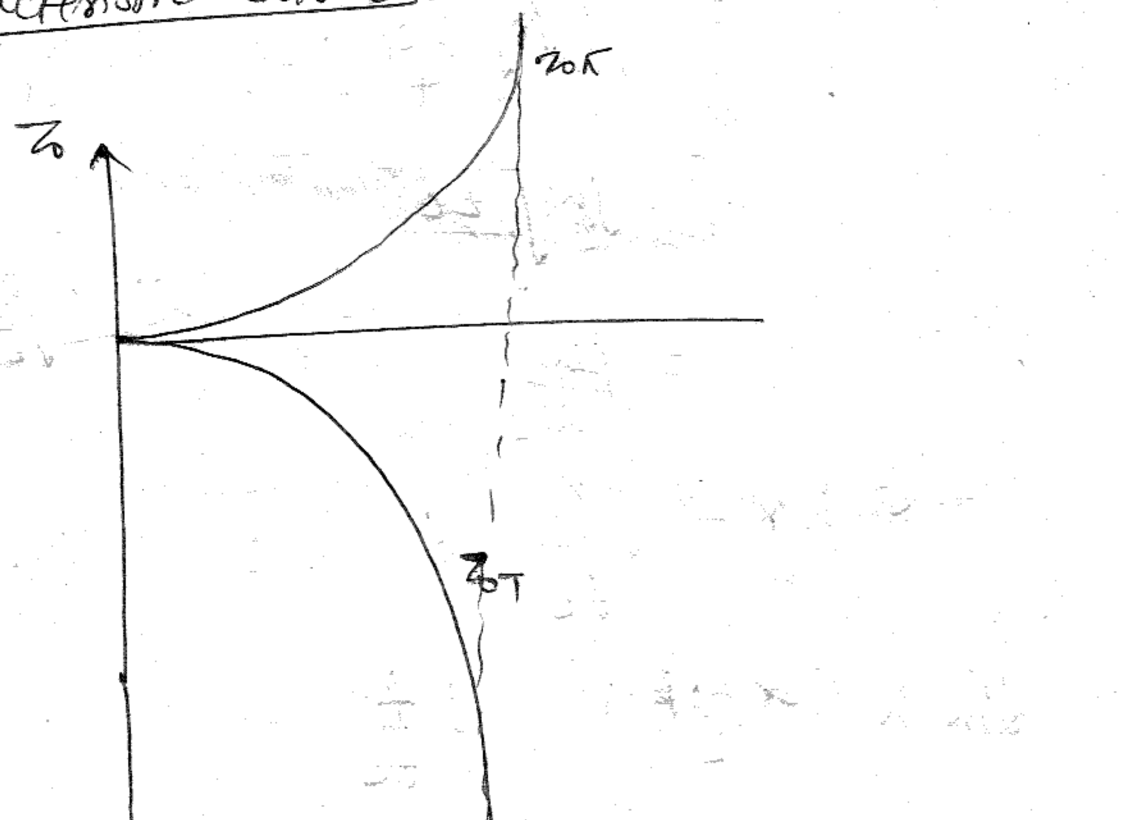
the frequency increases from zero to f_c.

t decreases R_0 to zero in pass band.

n passband as frequency ↑ from zero to f_c,

$Z_{0\pi}$ ↑ from R_1 to ∞.

<u>haracteristic curve</u>:-



<u>Variation of attenuation constant α with frequency</u>:-

we have $\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

$$Z_1 = j\omega L \quad , \quad Z_2 = \frac{-j}{\omega C}$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$= \sqrt{\frac{j\omega L}{4\left(\frac{-j}{\omega C}\right)}}$$

$$= \sqrt{j\omega L \times \frac{\omega C}{-4j}}$$

$$= j\omega\sqrt{\frac{LC}{4}}$$

$$= j\omega\sqrt{\frac{LC}{4}}$$

$$= \frac{j\omega}{\omega_c} \qquad \because \omega_c = \frac{2}{\sqrt{LC}}$$

$$\sinh\frac{\gamma}{2} = j\frac{f}{f_c}$$

$$\sinh\left(\frac{\alpha + j\beta}{2}\right) = j\frac{f}{f_c}$$

$$\sinh\frac{\alpha}{2}\cosh\frac{j\beta}{2} + \cosh\frac{\alpha}{2}\sinh\frac{j\beta}{2} = j\left(\frac{f}{f_c}\right)$$

$$\Rightarrow \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = j\left(\frac{f}{f_c}\right) \quad \text{--- (1)}$$

For ideal filter. $\overset{P.B}{\alpha = 0}, \; \overset{S.B.}{\beta = \pi}$

$$\cosh\frac{\pi}{2} = 0, \quad \sinh\frac{\pi}{2} = 1.$$

So eqⁿ becomes,

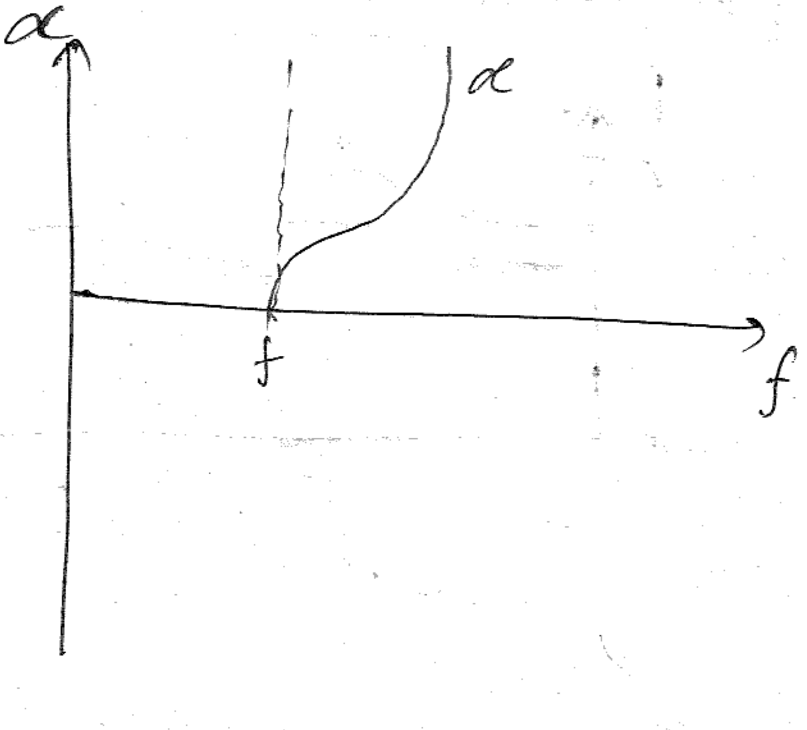
$$\cancel{\cosh\frac{\alpha}{2}} \; \cancel{\sinh} \; \cancel{1 = j\left(\frac{f}{f_c}\right)}$$

$$j\cosh\frac{\alpha}{2} = \cancel{j}\left(\frac{f}{f_c}\right)$$

$$\Rightarrow \boxed{\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)}$$

In stop band, as the frequency f increases above f_c, the attenuation α also ↑.

Plot betⁿ attenuation α & frequency f_c,



Variation of Phase Constant β with frequency :-

Put $\alpha = 0$, $\beta = \gamma$ in ①

we get $\boxed{\sinh \frac{\gamma}{2} = j \frac{f}{f_c}}$

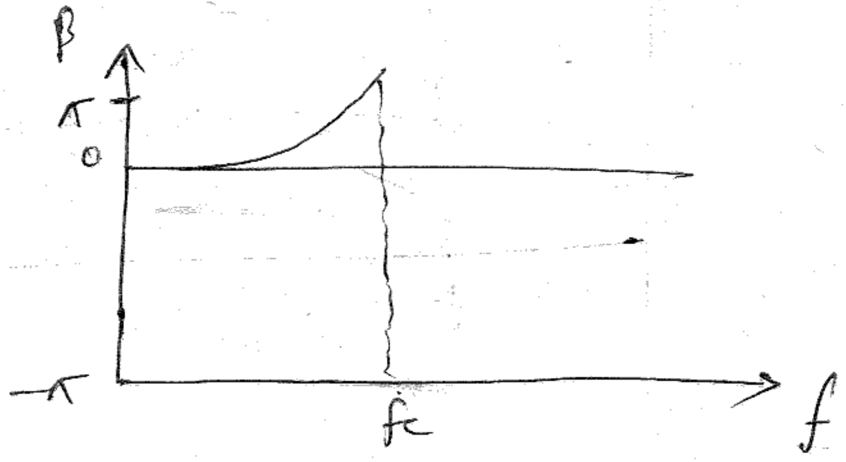
Now, in stop band, phase constant $\beta = \pi$

and $\alpha =$

we get $j \sinh \frac{\beta}{2} = j \frac{f}{f_c}$

$\Rightarrow \boxed{\beta = 2 \sinh^{-1} \frac{f}{f_c}}$

As frequency increases from zero to f_c, β also ↑ from 0 to π values.



Design Eqⁿ for Prototype low pass filter :-

$$\because \quad R_0 = \sqrt{\frac{L}{C}},$$
$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$\frac{R_0}{f_c} = \sqrt{\frac{L}{C}} \div \frac{1}{\pi\sqrt{LC}}$$
$$= \sqrt{\frac{L}{C}} \times \pi\sqrt{LC}$$
$$= \frac{\sqrt{L}}{\sqrt{C}} \times \pi\sqrt{L}\sqrt{C}$$
$$\frac{R_0}{f_c} = L\pi$$

$$\boxed{L = \frac{R_0}{\pi f_c}}$$

Q. Design a prototype LPF if design imp $R_0 = 500\,\Omega$, & $f_c = 2000$ Hz.

$L = 79.57$

Solⁿ $\quad L = \dfrac{R_0}{\pi f_c} = \dfrac{500\,\Omega}{3.14 \times 2000} = 79.57$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$
$$\Rightarrow 2000 = \frac{1}{\pi\sqrt{79.57C}}$$
$$\Rightarrow (2000)^2 = \frac{1}{\pi^2(79.57C)}$$

$\Rightarrow \quad C = \cancel{\dfrac{1}{\pi^2 (79.57) \times (20000)^2}}$

$=$

$R_0 f_c = \sqrt{\dfrac{L}{C}} \; \dfrac{1}{\pi\sqrt{LC}}$

$= \dfrac{1}{\pi C}$

$\Rightarrow \quad C = \dfrac{1}{\pi R_0 f_c}$

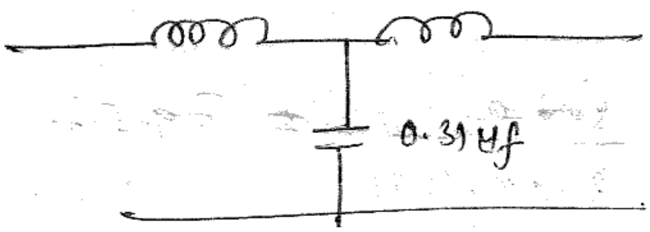
$= \dfrac{1}{3.14 \times 500 \times 2000}$

$= 0.3184 \times 10^{-6}$

~~$= 3.1 \mu f$~~

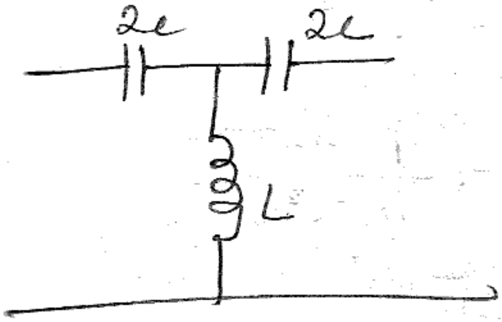
$= 0.31 \mu f$

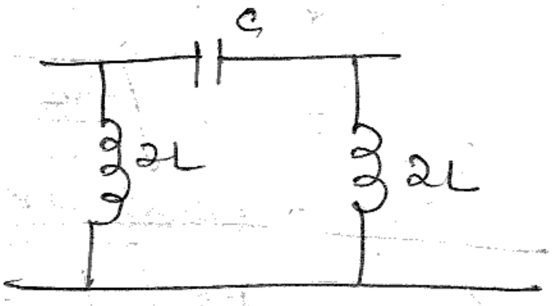
$C = 0.31 \mu f$



Prototype π - sectⁿ

T - sectⁿ





find R_0 :

Solⁿ :- Total series arm resistance

$\because Z_1 = \frac{-j}{\omega C}$ $Z_2 = j\omega L$ Total shunt arm resistance.

$Z_1 Z_2 = R_0^2$

$\Rightarrow \frac{L}{C} = R_0^2$

$\Rightarrow R_0 = \sqrt{\frac{L}{C}}$

for cut off freq :- $X_1 = \frac{-1}{\omega C}$

$X_2 = \omega L$

$\frac{X_1}{4} + X_2 = 0$

$\Rightarrow -\frac{1}{\omega C} + \omega L = 0$

$\Rightarrow \omega L = \frac{1}{\omega C}$

$\omega^2 = \frac{1}{LC}$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$X_1 + 4X_2 = 0$

$\Rightarrow$

$-\frac{1}{\omega c} + 4\omega L = 0$

$4\omega L = \frac{1}{\omega C}$

$\Rightarrow \omega^2 \frac{1}{4LC}$

$\Rightarrow \omega = \frac{1}{2\sqrt{LC}}$

$\Rightarrow \omega_c L - \frac{1}{4\omega_c C} = 0$

$\Rightarrow \omega_c L = \frac{1}{4\omega_c C}$

$\Rightarrow \omega_c^2 = \frac{1}{4LC}$

$\Rightarrow \boxed{\omega_c = \frac{1}{2\sqrt{LC}}}$

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad , R_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$\boxed{Z_{OT} = R_0 \sqrt{1 - \frac{1}{4\omega^2 LC}}}$$

$\Rightarrow$ Z_{OT} is real $\frac{1}{4\omega^2 LC} < 1$

$\Rightarrow$ Z_{OT} is Img when $\frac{1}{4\omega^2 LC} > 1$.

<u>at critical freqn</u>

$$1 - \frac{1}{4\omega^2 LC} = 0$$

$$\Rightarrow 1 = \frac{1}{4\omega^2 LC}$$

$$\Rightarrow \omega^2 = \frac{1}{4LC}$$

$$\Rightarrow \boxed{\omega = \frac{1}{2\sqrt{LC}}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

2/7/11

(Major drawbacks) :-

Disadvantage of Prototype Section :-

(i) The attenuation should ideally, attenuat" should change sharply in the attenuation band, but in all the prototype filter section changes gradually. ~~in the~~ in the attenuation band.

Hence the frequency near cut off frequency are passed through the filter.

(ii) Ideally in the pass band, the o/p of filter ~~is~~ should remain, constant. i.e Z_0 is constant.

This indicates that the characteristic impedance should remain constant. but it is observed that characteristic impedances varies with the frequency from value R_0, through the Passband.

~~M-ARRY~~

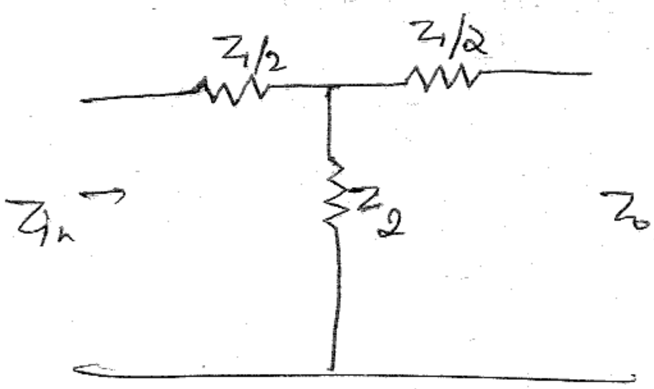
M- Derived filter Section :-

If two sections of same type are cascaded the attenuation in the attenuation band gets double and giving much sharper cut-off characteristics.

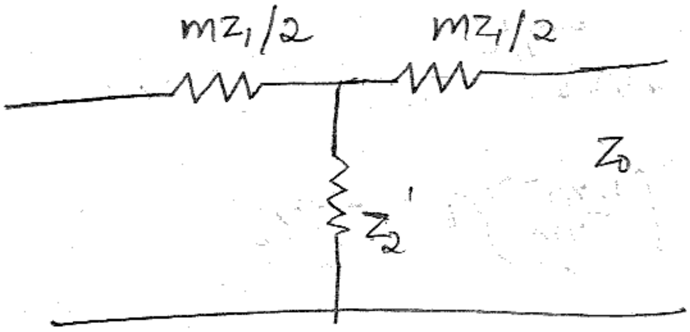
Derivation of M-derived section :-

It is necessary to design a new section filter having same cut off frequency, but different attenuation characteristic. So both the section (constant k & M-derive) must have same characteristic impedance

General T- Section



For M-derive section :-



$$\frac{Z_1}{2} = \frac{mZ_1}{2} \quad \& \cdot \cancel{\frac{Z_1}{2} = m} \quad Z_2 = Z_2'.$$

For prototype sectⁿ

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad —① $$

$$Z_{0T} = \sqrt{\frac{mZ_1^2}{4} + \frac{mZ_1Z_2'}{\cancel{2}}} \quad —②$$

① = ②

$$\Rightarrow \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{mZ_1^2}{4} + mZ_1Z_2'}$$

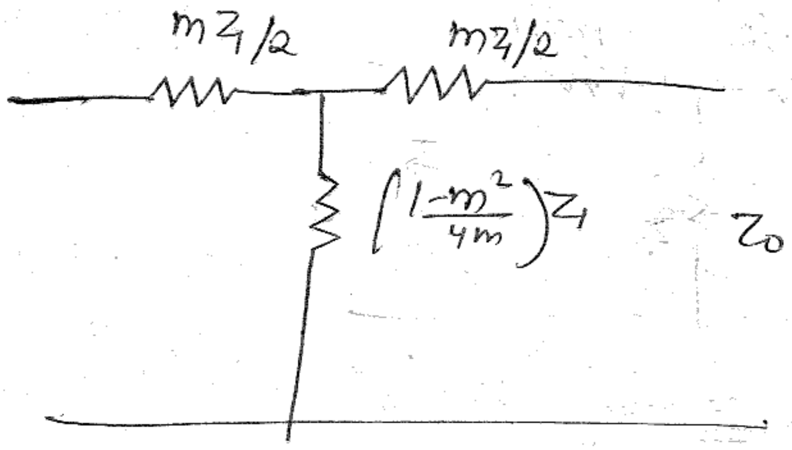
$$\Rightarrow \frac{Z_1^2}{4} + Z_1 Z_2 = \frac{mZ_1^2}{4} + mZ_1Z_2'$$

$$\Rightarrow \frac{Z_1^2}{4} - \frac{mZ_1^2}{4} = mZ_1Z_2' - Z_1Z_2$$

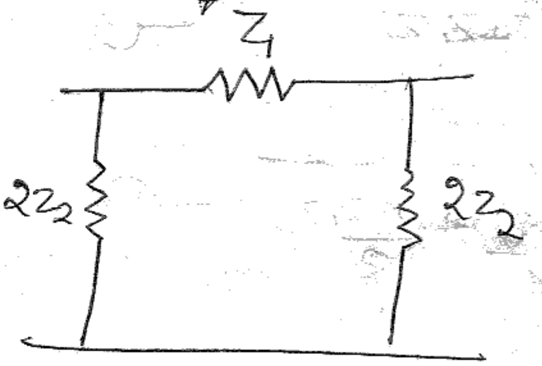
$$\Rightarrow \cancel{\frac{Z_1^2}{4}(1-m) = mZ_1}$$

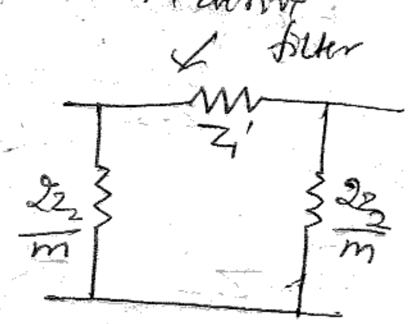
$$\Rightarrow \frac{Z_1^2}{4} + Z_1 Z_2 - \frac{mZ_1^2}{4} = mZ_1Z_2'$$

$$\Rightarrow Z_2' = \frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{Z_1}{4} = \left(\frac{1-m^2}{4m}\right)Z_1 + \frac{Z_2}{m}$$



For Constant k- section :-





$$Z_{OT} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} \quad -(1)$$

$$Z_{OT} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$$

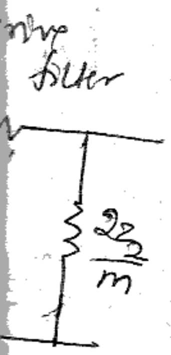
$$= \sqrt{\frac{Z_1'\left(\frac{Z_2}{m}\right)}{1 + \frac{Z}{4\left(\frac{Z_2}{m}\right)}}} = \sqrt{\frac{\frac{Z_1' Z_2}{m}}{1 + \frac{m Z_1'}{4Z_2}}}$$

$-(2)$

$\therefore$ ① = eqⁿ ②,

$$\frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4}+Z_1 Z_2}} = \sqrt{\frac{\frac{Z_1' Z_2}{m}}{1+\frac{m Z_1'}{4 Z_2}}}$$

$$\Rightarrow \frac{(Z_1 Z_2)^2}{\frac{Z_1^2}{4}+Z_1 Z_2} = \frac{\frac{Z_1' Z_2}{m}}{1+\frac{m Z_1'^2}{4 Z_2}}$$



$$\Rightarrow \cancel{(Z_1 Z_2)^2}$$

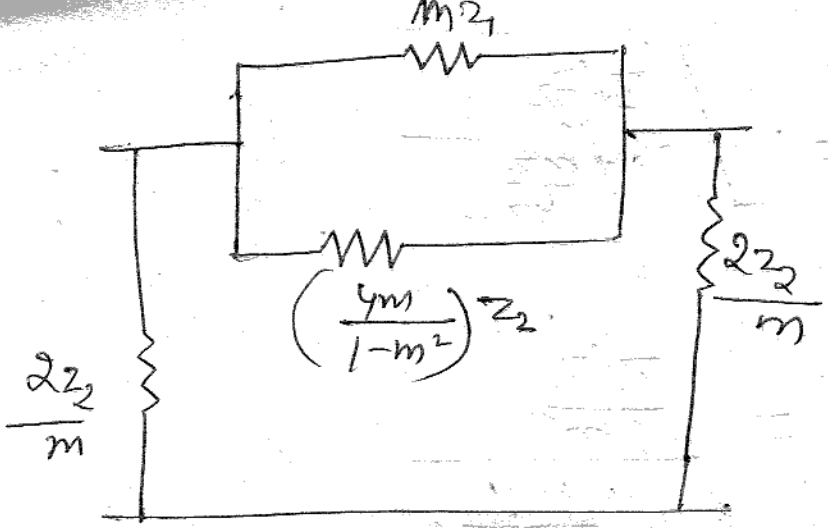
$$\Rightarrow \frac{Z_1 Z_2}{\frac{Z_1^2}{4}+Z_1 Z_2} = \frac{\frac{Z_1' Z_2}{m}}{1+\frac{m Z_1'}{4 Z_2}}$$

$$\Rightarrow Z_1 Z_2\left(1+\frac{m Z_1'}{4 Z_2}\right) = Z_1' Z_2$$

$$Z_1'\left[\frac{Z_2}{m}+\frac{Z_1}{4m}(1-m^2)\right] = Z_1 Z_2$$

$$Z_1' = \frac{Z_1 Z_2}{\frac{Z_2}{m}+\left(\frac{1-m^2}{4m}\right)Z'}$$

$$Z_1' = \frac{(m Z_1)\left(\frac{4m}{1-m^2}\right)Z_2}{\left(\frac{4m}{1-m^2}\right)Z_2 + m Z_1}$$



13/7/4:-

M- Derived LOW PASS FILTER SECTION

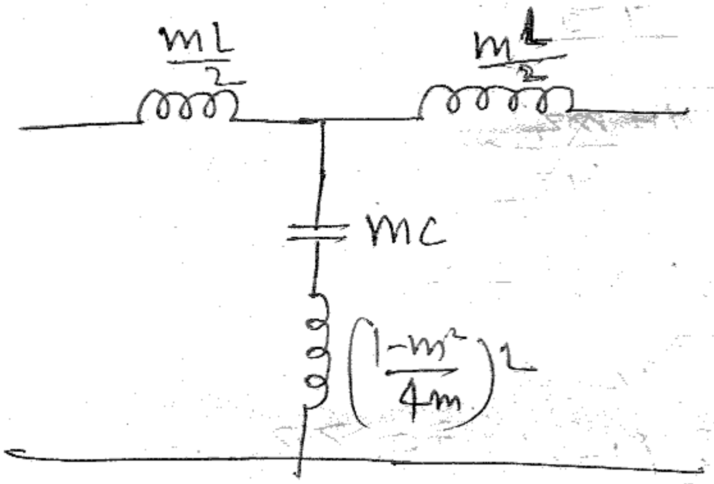


fig (a)

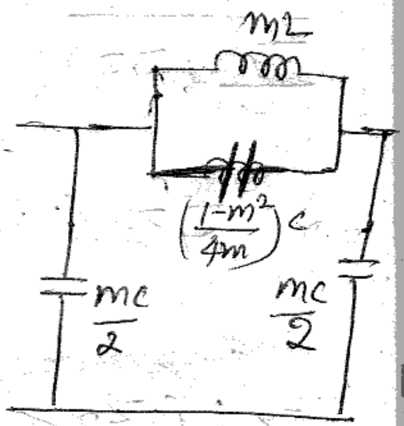


fig (b)

$$f_\infty = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{\left(\frac{1-m^2}{4m}\right)L\, mC}}$$

$$= \frac{1}{2\pi\sqrt{\frac{(L-m^2L)C}{4}}}$$

$$= \frac{1}{2\pi\sqrt{\frac{LC - m^2 LC}{4}}}$$

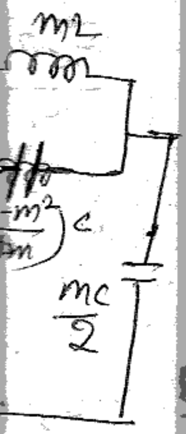
$$= \frac{1}{2\pi\sqrt{\frac{LC(1-m^2)}{4}}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$= \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

TION



$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} = \frac{\frac{1}{\pi\sqrt{LC}}}{\frac{f_c}{\sqrt{1-m^2}}} = \frac{1}{\pi\sqrt{LC}}\,\frac{\sqrt{1-m^2}}{f_c} = f_\infty$$

$$\Rightarrow \quad 1 - m^2 = \left(\frac{f_c}{f_\infty}\right)^2$$

$$\boxed{m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}}$$

Q. Design M-derived T-type LPF to work into the load of 500 Ω, & cut off frequency of 4000 Hz & peak attenuation is 4.5 kHz.

$$L = \frac{R_0}{\pi f_c} \text{ for prototype}$$

Sol^n. $f_c = 4\text{kHz}$, $R_0 = 500\,\Omega$, $f_\infty = 4.5$

$$C = \frac{1}{\pi f_c R_0}$$

$$L = \frac{\cancel{500}}{3.14 \times 4 \times 10^{\cancel{3}}} = \frac{5}{12.56 \times 10} = \cancel{0.039}\ 39.8\text{ mH}.$$

$$C = \frac{1}{\pi f_c R_o} = \frac{1}{3.14 \times 4\times10^{3} \times 500}$$

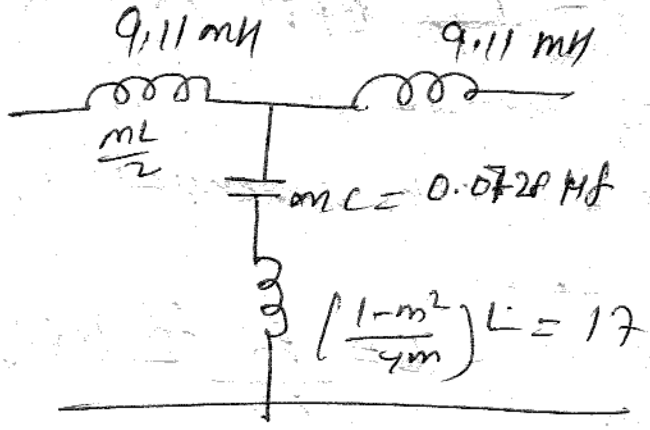
$$= 0.159\ \mu f.$$

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

$$= \cancel{\sqrt{1-(4\times10^{3})^2}}$$

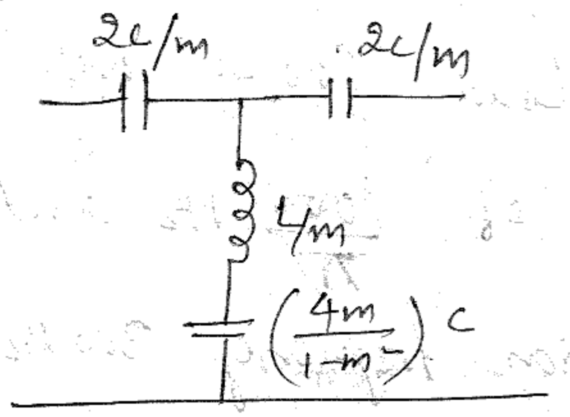
$$= \sqrt{1-\left(\frac{4}{4.5}\right)^2}$$

$$= \sqrt{1-\frac{16}{(4.5)^2}} = 0.458$$



← Constant π section

Constant T-section :-



$$f_\infty = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{\frac{L}{m} \times \frac{4mC}{1-m^2}}}$$

$$= \frac{1}{2\pi\sqrt{\frac{4LC}{1-m^2}}}$$

$$= \frac{1}{4\pi\sqrt{\frac{LC}{1-m^2}}}$$

$$f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

$$f_\infty' = f_c\sqrt{1-m^2} \Rightarrow \frac{f_\infty}{f_c} = \sqrt{1-m^2}$$

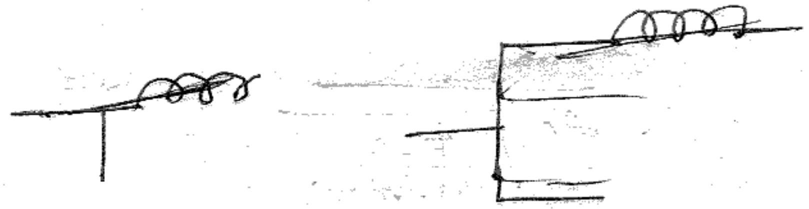
$$\Rightarrow \left(\frac{f_\infty}{f_c}\right)^2 = 1-m^2$$

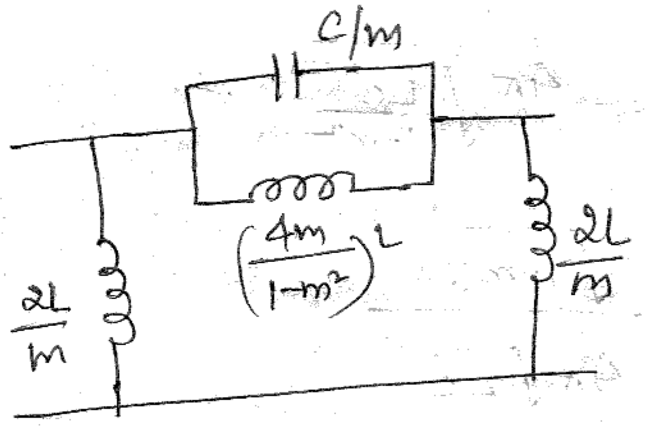
$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

Q2 Design M-derive HPF π section to work into the load 600 Ω, with Cut-off frequency of $\frac{1000}{\pi}$ Hz and Peak attenuation frequency 300 Hz.

Sol^n

$$R_0 = 600\ \Omega,$$
$$f_c = \frac{1000}{\pi}$$
$$f_\infty = 300\ \text{kHz}$$





Constant π Section.

$$L = \frac{R_0}{4\pi f_c} = \frac{600}{4\cancel{\pi} \times \frac{1000}{\cancel{\pi}}}$$
$$= \frac{600}{4000} = \frac{6}{40} = \frac{3}{20}$$
$$= 0.15$$

do

th

nd

Hz.

ction.

$\frac{3}{20}$

0.15

$$C = \frac{1}{\pi f_c R_o}$$

$$= \frac{1}{4\pi \times \frac{1000}{\pi} \times 600} = \frac{1 \times 10^{-5}}{4 \times 6} = 4.16 \times 10^{-7}.$$

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2}$$

$$= \sqrt{1 - \left(\frac{300}{\frac{1000}{\pi}}\right)^2} = 0.335$$

$$m = 0.335$$

$$\frac{C}{m} = \frac{4.16 \times 10^{-7}}{0.335} = 1.24$$

$$\frac{2L}{m} = \cancel{\frac{2 \times 0.15}{0.335}} = \cancel{\frac{0.30}{0.335}} = \frac{2 \times 0.15}{0.335}$$

$$= 0.8939$$

$$= 893.9\ mH$$

$$= \cancel{\frac{2 \times 150}{0.335}} =$$

$$\left(\frac{4m}{1-m^2}\right) L = \frac{4 \times 0.335 \times \cancel{0.}150}{1-(0.335)^2}$$

$$= 225.6\ mH.$$

19/7/11

UNIT 2

Relationship between Primary & Secondary Constants:-

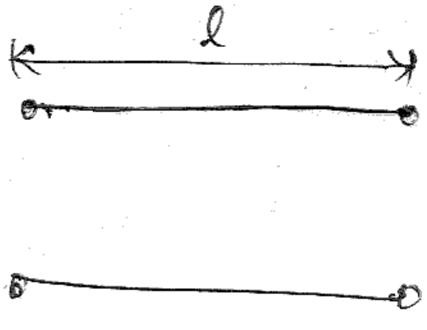
Primary Constant	Unit
R - Resistance	(Ω)
G - Conductance	(mho) or $\mho$
L - Inductance	(Henry).
C - Capacitance.	(Farad)

$\Rightarrow$ Primary Constants r independent of frequency.

secondary Constant
Z_0 - o/p impedance
γ -
α - attenuation Constant
β -

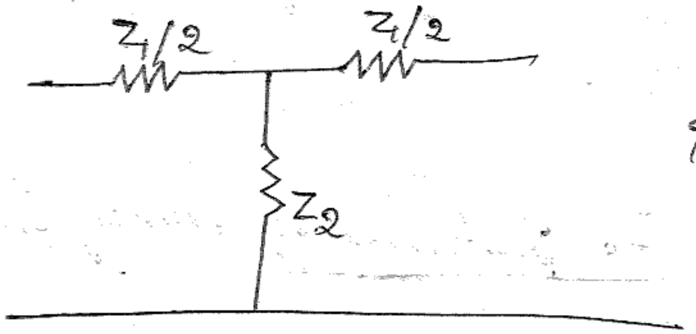
$\Rightarrow$ secondary Constant are dependent on frequency.

Consider a short length of line l (mts), this section will have resistance R, conductance G, inductance L, & Capacitance C



Symmetric
T- NWK.

Also it has Characteristic impedance Z_0.



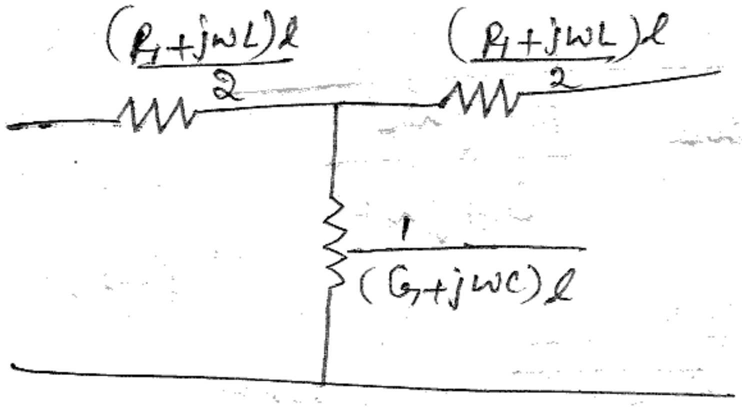
T-NWK'

If the length of the line is very small, than is represented as serious amp resistance

$$Z_1 = (R_1 + jwL)\,l$$

$$Z_2 = \frac{1}{(G_1 + jwC)\,l}$$

<u>T-section can be represented as:-</u>



~~Determi~~ The total series amp impedance per unit length is denoted as ~~Z0~~ 'Z'.

$\boxed{Z = R + jwL}$

||y The total parallel impedance per unit length is denoted as $\boxed{Y = Z_{shunt} = \frac{1}{Z_0} = G + jwC}$

This assumption is valied only when the length of line is very very small.

ie $l \to 0$

Determination of Z_0 in terms of Primary Constan

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{[(R+jwL)l]^2}{4} + \frac{[(R+jwL)l]}{(G+jwC)l}}$$

$$= \sqrt{\frac{(R+jwL)^2 l^2}{4} + \frac{R+jwL}{G+jwC}}$$

$$= \sqrt{\frac{R+jwL}{G+jwC}}$$

$\because l \to 0$

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}}}$$

when

$Z = R + jwL$

$Y = \frac{1}{G+jwC}$

Determination of γ in terms of Primary Constant :-

$$e^{\gamma l} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{2Z_2}$$

$$= 1 + \frac{(R+j\omega L)l}{\frac{2}{(G+j\omega C)l}} + \frac{Z_0}{\frac{1}{(G+j\omega C)l}}$$

$$= 1 + (R+j\omega L)l \times \frac{(G+j\omega C)l}{2} + Z_0(G+j\omega C)l$$

$$= 1 + \frac{(R+j\omega L)(G+j\omega C)l^2}{2} + $$