From this Equation it is clear that half wave line repeats its terminating impedence.
In other words half wave line may considered as one to one transformer.

Impedence matching

The Source or i/p impedence is a fixed one, By chosing the value of load impedence to be equal to the input impedence then that is impedence matching.

Stub matching

Impedence matching is the use of an open or short circuited line of suitable length called Stub at a designated distance from the load is called Stub matching.

Single stub matching :- It has one stub to match the transmission line impedence. Its neccessities both length & location of stub to be altered for matching.

Mismatching

The load impedence is also fixed, if the load impedence is not equal to complex conjugate of the input impedence, the maximum power transfer will not take place, then this is called mismatching.

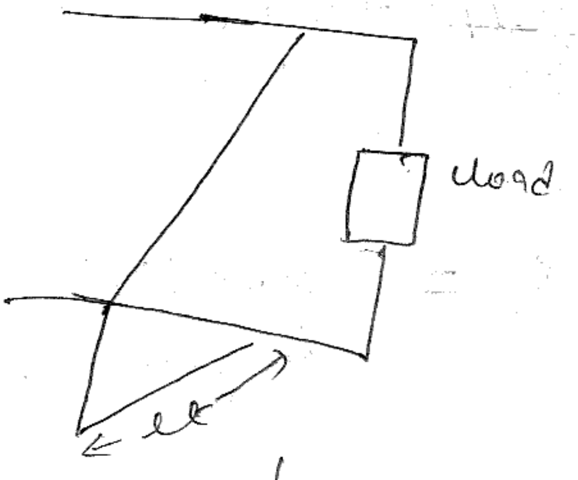
So it is necessary ~~so~~ to introduce some form of an impedence transforming section b/w source & load to acheive impedence matching.

Such a section is called impedence matching device.

Single stub matching

→ The i/p impedence at any point of a transmission line is given by

$$Z_S = Z_0 \left[\frac{Z_R + Z_0 \tan \gamma l}{Z_0 + Z_R \tan \gamma l} \right]$$



$$Z_Y = \frac{1}{Y}$$

The i/p admittance is given by

$$Y_S = Y_0 \left[\frac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l} \right]$$

For dissipation less line

$$\gamma = j\beta \quad [\because \alpha = 0]$$

$$Y_s = Y_0 \left[\frac{Y_R + jY_0 \tan\beta l}{Y_0 + jY_R \tan\beta l} \right]$$

$$\frac{Y_S}{Y_0} = \frac{Y_R + jY_0 \tan\beta l}{Y_0 + jY_R \tan\beta l}$$

$$Y_{in} = \frac{Y_S}{Y_0} = \frac{\frac{Y_R}{Y_0} + j\tan\beta l}{\cancel{\frac{Y_0}{Y_R}}\,1 + j\tan\beta l}$$

$$Y_{in} = \frac{\frac{Y_R}{Y_0} + j\tan\beta l}{\cancel{\frac{Y_0}{Y_R}}\,1 + \frac{Y_R}{Y_0} j\tan\beta l}$$

where

$$\frac{Y_S}{Y_0} = Y_{in} =$$ Normalised input ~~impedence~~ admittance.

$\frac{Y_R}{Y_0} = Y_r$ = normalised load admittance.

$$Y_{in} = \frac{Y_r + j\tan\beta l}{1 + jY_r\tan\beta l}$$

$$= \frac{Y_r + j\tan\beta l \; (1 - jY_r\tan\beta l)}{(1 + jY_r\tan\beta l)\,(1 - jY_r\tan\beta l)}$$

$$= \frac{(Y_r + j\tan\beta l)(1 - jY_r\tan\beta l)}{1 + Y_r^2\tan^2\beta l}$$

$$= \frac{Y_r\left[\left(1 + \frac{j}{Y_r}\tan\beta l\right)\left(\frac{1}{Y_r} - j\tan\beta l\right)\right]}{1 + Y_r^2\tan^2\beta l}$$

$$= \frac{Y_r\left[\frac{1}{Y_r} - j\tan\beta l + \frac{j}{Y_r^2}\tan\beta l + \frac{1}{Y_r}\tan\beta l\right]}{1 + Y_r^2\tan^2\beta l}$$

$Y_r\Big[$

$$Y_{in} = \frac{Y_r(1 + \tan^2\beta l) + j(1 - Y_r^2)\tan\beta l}{1 + Y_r^2\tan^2\beta l} \quad (1)$$

For perfect matching

$$Y_s = Y_0$$

$$\frac{Y_s}{Y_0} = y_{in} = 1$$

The stub has to be located at a point where the real part of y_{in} be 1 and

$[l = l_s]$ l_s - location of stub

$$\therefore \frac{Y_r(1+\tan^2\beta l_s)}{1+Y_r^2\tan^2\beta l_s} = 1$$

$$Y_r(1+\tan^2\beta l_s) = 1 + Y_r^2\tan^2\beta l_s$$

$$Y_r + Y_r\tan^2\beta l_s - Y_r^2\tan^2\beta l_s = 1$$

$$Y_r + \cancel{Y_r(}\tan^2\beta l_s\,(Y_r - Y_r^2) = 1$$

$$\tan^2\beta l_s\,(Y_r - Y_r^2) = 1 - Y_r$$

$$\tan^2\beta l_s = \frac{1-\cancel{Y_r}}{Y_r(1-\cancel{Y_r})} = \frac{1}{Y_r}$$

$$\tan^2\beta l_s = \frac{1}{Y_r}$$

$$\tan\beta l_s = \frac{1}{\sqrt{Y_r}}$$

$$\beta l_s = \tan^{-1}\left(\frac{1}{\sqrt{Y_r}}\right)$$

$l_s = \frac{1}{\beta} \tan^{-1}\left(\frac{1}{\sqrt{Y_r}}\right)$. $\boxed{\beta = \frac{2\pi}{\lambda}}$

$l_s = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{1}{\sqrt{Y_r}}\right)$

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Y_0}{Y_R}}}$$

~~Therefore the stub should be~~

The location of the stub

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Z_R}{Z_0}}}$$

Smith chart

1)

Gn:

characteristic imp, $Z_0 = 300\,\Omega$

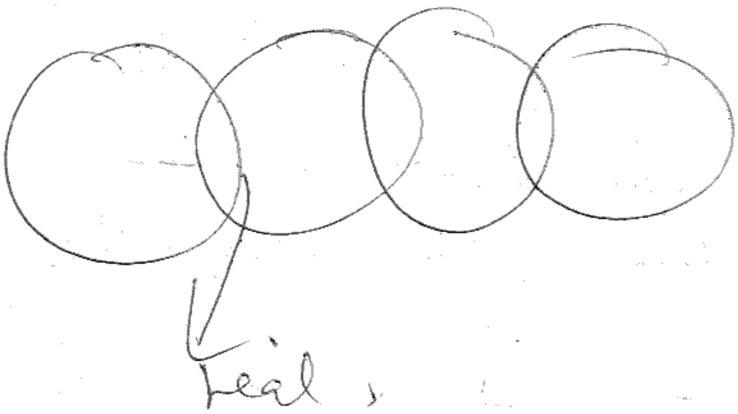
Load imp, $(Z_L \text{ or } Z_R) = 175 + j207\,\Omega$

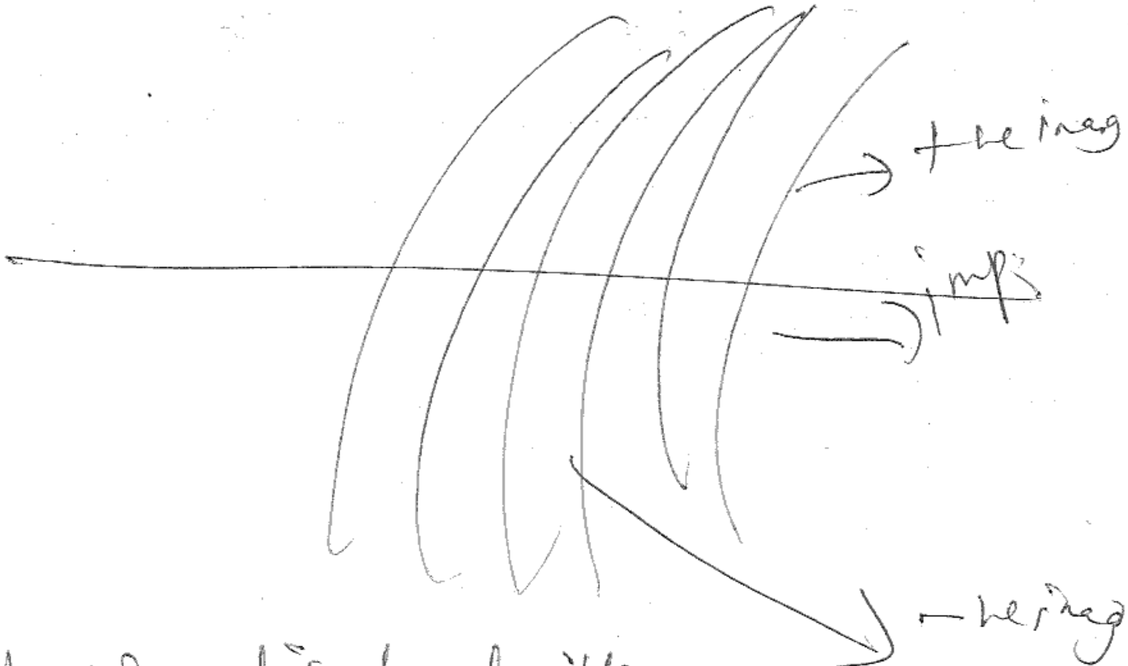
frequency, $f = 200$ MHZ,

$\lambda = \frac{c}{f} = \frac{3\times10^8}{200\times10^6} = 1.5$m.

1) Normalising imp $\Rightarrow \frac{Z_L}{Z_0} = \frac{175+j207}{300}$

$$\frac{Z_L}{Z_0} = \underbrace{0.5833}_{\text{Real}} + \underbrace{j0.69}_{\text{imaginary}}$$



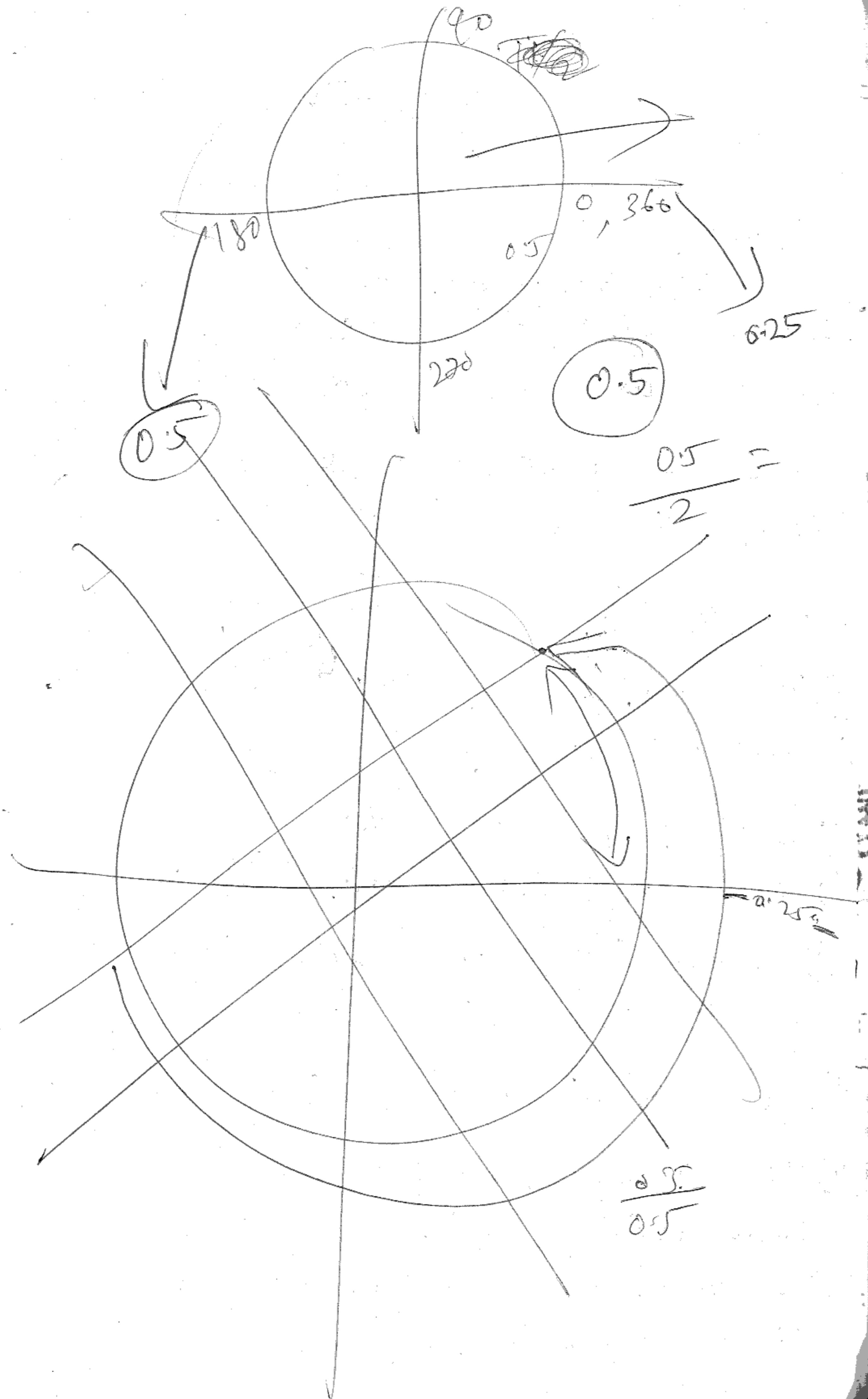


2) Normalised admittance,

$$\frac{Y}{G_0} = YZ_0 \xrightarrow{K\Omega} = (0.7 - j0.85)(300)$$

$$YZ_0 = \cancel{210 - j255}$$

$$\frac{Y}{G_0} = 0.0023 - j0.0028$$



90
0, 360
180
0.5
270
0.25
0.5
0.5
0.5 / 2 =
0.25
0.5 / 0.5

7/9/11

UNIT - IV

GUIDED WAVES

The discovery of EM waves was the outcome of Electromagnetic theory presented by professor James Clerk Maxwell.

Wave is a simple way of transporting Energy or information.

Examples of EM waves are Radio waves, TV signals, light rays and radar beams.

The three properties of EM waves are
i) EM Energy travel at high velocity
ii) while travelling they assume properties of waves.
iii) EM energy radiates outwards.

EM Energy associated with waves, Radiate over a wide area and such wave propagation is called as unguided wave propagation.

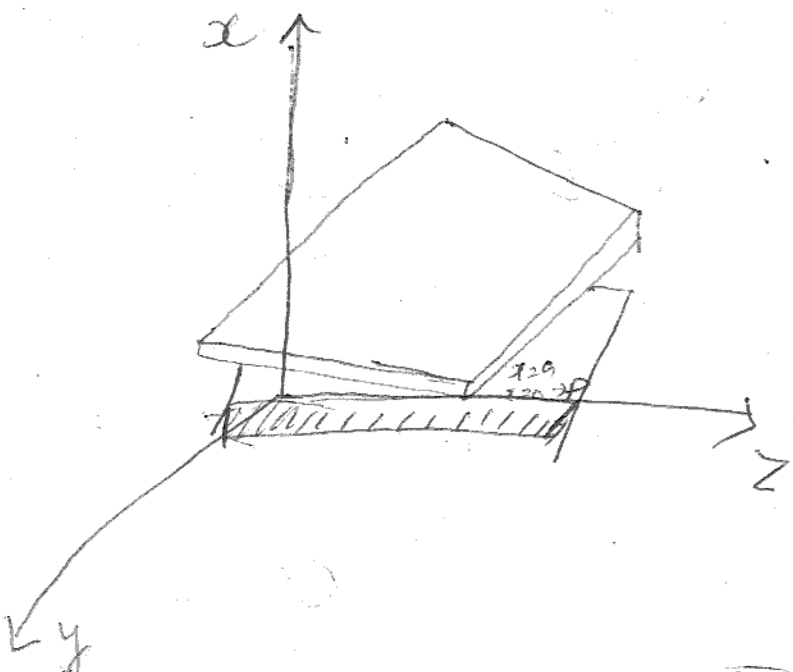
Example TV or Radio broadcasting.

- But in many applications it is necessary to confine and guide wave energy by guided structures. In such cases transmitting power is confined by the boundaries of a guided structure which is made up of a material other than that of free transmission path or media.

The waves directed or guided by guided wave structure are called guided waves.

Ex Telephone Signal Transmission.

<u>Waves b/w parallel planes of Perfect conductors</u>



Consider an EM wave propagating b/w a pair of Perfectly conducting planes.

The conducting Planes are placed at distance $x=0$ & $x=a$.

To study the behaviour of EM waves between two conducting plates, we should solve Maxwell's equation with boundary conditions.

Maxwell's curl equation in phasor form is given by

$$\nabla \times \bar{H} = (\sigma + j\omega\varepsilon)\bar{E} \quad -(1)$$

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad -(2)$$

The wave Equations in Phasor form is given by

$$\nabla^2 \bar{E} = \gamma^2 \bar{E} \quad -(3)$$

$$\nabla^2 \bar{H} = \gamma^2 \bar{H} \quad -(4)$$

In Equation (3) and (4), the propagation Constant γ is given by.

$$\bar{\gamma} = \bar{\alpha} + j\bar{\beta} = \sqrt{(\sigma + j\omega\varepsilon)(j\omega\mu)} \quad (5)$$

Simplifying Equⁿ (1) in rectangular co-ordinates.

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (8)$$

IIIrly Simplifying Equⁿ (2) in rectangular coordinates

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu \bar{H}_x \quad (9)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu \bar{H}_y \quad (10)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu \bar{H}_z \quad (11)$$

For non-conducting medium

σ becames zero and γ can be written by

$$\gamma = \sqrt{(j\omega\varepsilon)(j\omega\mu)}$$

$$= \pm j\omega\sqrt{\mu\varepsilon} \quad -(12)$$

Sub (12) in equ^n (3)

$$\nabla^2 E = \gamma^2 \bar{E}$$

$$\nabla^2 E = j^2\omega^2 \mu\varepsilon E$$

$$\nabla^2 E = -\omega^2 \mu\varepsilon \bar{E}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu\varepsilon\bar{E} \quad -(13)$$

Mly

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2\mu\varepsilon\bar{H} \quad -(14)$$

It is assumed that the ~~Propagat~~ Propagation is in the z direction and the variation of all field components in this direction may be Expressed in the form of $e^{-\gamma z}$

with

$e^{-\gamma z}$, where $\gamma = \alpha + j\beta$.

The time variation factor is combined with z variation factor and it can be given as

$$e^{j\omega t} e^{-\gamma z} = e^{j\omega t} e^{-(\alpha + j\beta) z}$$

$$= e^{j\omega t} \cdot e^{-\alpha z} \cdot e^{-j\beta z}$$

$$e^{j\omega t} e^{-\gamma z} = e^{-\alpha z} \cdot e^{j(\omega t - \beta z)}$$

If γ happens to be an imaginary then $\alpha = 0$.

Ilrly If $\beta = 0$, no wave motion, but only an exponentially decrease in amplitude.

When the variation in the z-direction of each of the field components for TE:

$$H_y = H_y^0 e^{-\gamma z}$$

where H_y is the field component in y direction.

$H_y^\circ \rightarrow$ amplitude of H_y.

diff the eqn w.r.t z

$$\frac{\partial H_y}{\partial z} = -\gamma \, \cancel{\partial} H_y^\circ e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y \quad \text{— (15)}$$

|||ly

$$\frac{\partial H_x}{\partial z} = -\gamma H_x \quad \text{— (16)}$$

Consider an Electric field

$$E_y = E_y^\circ e^{-\gamma z} \quad \text{— (17)}$$

diff w.r.t z

$$\frac{\partial E_y}{\partial z} = -\gamma E_y^\circ e^{-\gamma z}$$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y \quad \text{— (18)}$$

|||ly

$$\frac{\partial E_x}{\partial z} = -\gamma E_x \quad \text{— (19)}$$

There is no variation in the Y-direction that is derivative of y is zero.

Sub the values of (15), (16), (17) & (19) in Equation (6), (7), (8), (9), (10), (11)

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega \varepsilon E_x$$

$$\Rightarrow 0 - \frac{\partial H_y}{\partial z} = j\omega \varepsilon E_x$$

$$\boxed{\gamma H_y = j\omega \varepsilon E_x} \quad -(20) \quad ①$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega \varepsilon E_y$$

$$\boxed{-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega \varepsilon E_y} \quad -(21) \quad 2$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \varepsilon E_z$$

$$\boxed{\frac{\partial H_y}{\partial x} = j\omega \varepsilon E_z} \quad -(22) \quad 3$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega \mu H_x$$

$$0 + \gamma E_y = -j\omega \mu H_x$$

4

$$\boxed{\gamma E_y = -j\omega \mu H_x} \quad -(23)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\boxed{-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y} \quad -(24) \quad 5$$

$$\boxed{\frac{\partial E_y}{\partial x} = -j\omega\mu H_z} \quad -(25) \quad 6$$

Also put y component as zero in equn (13) & (14)

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu\epsilon\bar{E}$$

$$\left.\begin{aligned} (13) \rightarrow \frac{\partial^2 E}{\partial x^2} + \gamma^2 E &= -\omega^2\mu\epsilon\bar{E} \\ (14) \rightarrow \frac{\partial^2 H}{\partial x^2} + \gamma^2 H &= -\omega^2\mu\epsilon H \end{aligned}\right\} -(26)$$

To solve H_x component, consider equn (21) & (23)

$$(21) \rightarrow -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y.$$

$$E_y = \frac{-\gamma H_x}{j\omega\varepsilon} - \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x}$$

5 $$E_y = -\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)$$

6 (23) $\Rightarrow$ $$\gamma E_y = -j\omega\mu H_x$$

$$\boxed{H_x = -\frac{1}{j\omega\mu}\gamma E_y} \quad -(27)$$

Sub the value of E_y in equ^n (27)

$$H_x = -\frac{\gamma}{j\omega\mu} \times -\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)$$

$$= \frac{\gamma}{-\omega^2\mu\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)$$

$$H_x = -\frac{\gamma\cdot\gamma H_x}{\omega^2\mu\varepsilon} - \frac{\gamma}{\omega^2\mu\varepsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x + \frac{\gamma\,\gamma H_x}{\omega^2\mu\varepsilon} = -\frac{\gamma}{\omega^2\mu\varepsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x\left(1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = -\frac{\gamma}{\omega^2\mu\varepsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x(\omega^2\mu\epsilon + \gamma^2) = \frac{-\cancel{\omega^2\mu\epsilon}\gamma}{\cancel{\omega^2\mu\epsilon}}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x(\underbrace{\omega^2\mu\epsilon + \gamma^2}_{h^2}) = -\gamma \frac{\partial H_z}{\partial x}$$

$$\boxed{H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)}$$

To Solve. H_y

Equn 20 & 24

24 $$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

20 $$\gamma H_y = j\omega\epsilon E_x$$

$$\boxed{H_y = \frac{j\omega\epsilon E_x}{\gamma}} \quad -28$$

~~Sub 28 in equn 24~~

20 ⇓

$$\gamma E_x = j\omega\mu H_y - \gamma\frac{E_z}{\partial x}$$

$$E_x = \frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

Sub E_x in equ^n (28)

$$H_y = \frac{j\omega\varepsilon}{\gamma} \cdot \frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

$$= \frac{j\omega\varepsilon}{\gamma^2}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

$$H_y = \frac{j^2\omega^2\varepsilon\mu H_y}{\gamma^2} - \frac{j\omega\varepsilon}{\gamma^2} \cdot \frac{\partial E_z}{\partial x}$$

$$H_y\left(1 + \frac{\omega^2\varepsilon\mu}{\gamma^2}\right) = -\frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y\left(\gamma^2 + \omega^2\varepsilon\mu\right) = -j\omega\varepsilon\frac{\partial E_z}{\partial x}$$

$$\boxed{H_y = -\frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}$$

<u>To solve E_x</u>

Consider (20) & (24)

(24) $$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

Sub the value of H_y in above equ^n

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu \cdot \frac{j\omega\varepsilon}{\gamma} E_x$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = -\frac{\omega^2\mu\varepsilon}{\gamma} E_x$$

$$E_x\left(\gamma + \frac{\omega^2\mu\varepsilon}{\gamma}\right) = -\frac{\partial E_z}{\partial x}$$

$$E_x(\gamma^2+\omega^2\mu\varepsilon) = -\gamma\frac{\partial E_z}{\partial x}$$

$$\boxed{E_x = -\frac{\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}$$

<u>To solve for E_y</u>

Consider equ ⓴① & ㉓

(21) $\Rightarrow$ $-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$

(23) $\Rightarrow$ $\gamma E_y = -j\omega\mu H_x$

$$\boxed{H_x = -\frac{\gamma}{j\omega\mu}E_y}$$

Sub H_x in above equ

$$\frac{\gamma^2}{j\omega\mu}E_y - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\gamma^2}{j\omega\mu}E_y - j\omega\varepsilon E_y = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\gamma^2}{j\omega\mu} - j\omega\varepsilon\right] = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\gamma^2 + \omega^2 \varepsilon \mu}{j\omega\mu}\right] = \frac{\partial H_z}{\partial x}$$

$$E_y(\gamma^2 + \omega^2 \varepsilon \mu) = j\omega\mu \frac{\partial H_z}{\partial x}$$

$$\boxed{E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}}$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$\cancel{H_y}\ E_x = -\frac{\gamma}{h^2}\left(\frac{\partial E_x}{\partial x}\right)$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)$$

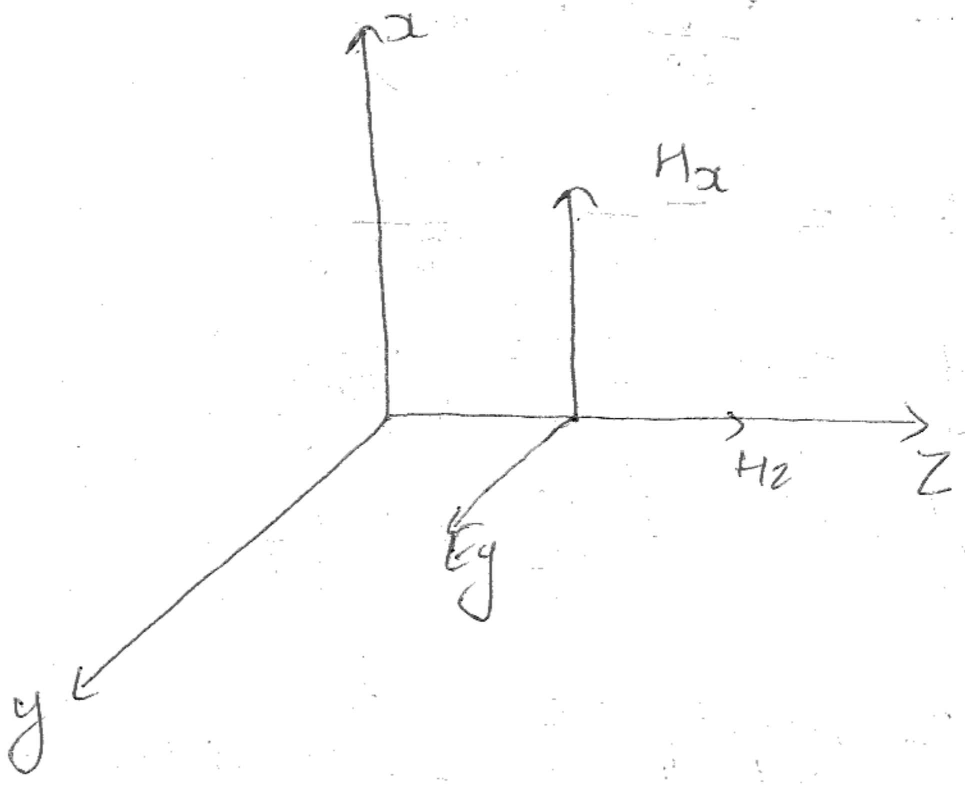
$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

Case 1 :- In first case there is a comp of E in the direction of propagation E_z but no component ~~of~~ H in z direction that type of wave called as E waves or Transverse magnetic waves (TM) ~~are~~ waves.

case 2 :- There is a component of H in H_z but no component of E in E_z, such type of waves called as H waves or Transverse Electric waves (TE).

Transverse Electric waves :-

TE waves are waves in which the Electric field strength E is Entirely transversed. It has a magnetic field strength in the direction of propagation, but no component of Electric field E_z in the same direction.



The wave Equation can be written for the component E_y

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu \epsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\omega^2 \mu \epsilon E_y - \gamma^2 E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -E_y(+\gamma^2 + \omega^2\mu\epsilon)$$

$$\frac{\partial^2 E_y}{\partial x^2} = -E_y \cdot h^2 \quad (\because \gamma^2 + \omega^2\mu\epsilon = h^2)$$

The differential equation of simple Harmonic motion and the solution of this eq^n is written in the form

$$E_y^0 = C_1 \sin hx + C_2 \cos hx \quad (C_1 \,\&\, C_2 \rightarrow \text{arbitrary Constant})$$

If E_y is expressed in ~~time~~ terms of time and direction then

$$E_y = E_y^0 e^{-\gamma z}$$ then the sol^n became

$$E_y = (C_1 \sin hx + C_2 \cos hx)\, e^{-\gamma z} \quad -(1)$$

For parallel wave guide, the boundary conditions are simple.

The tangential component of E is zero at the surface of conductors, for all values of z.

This requires that,

$$\left.\begin{array}{l} E_y = 0 \text{ at } x = 0 \\ E_y = 0 \Rightarrow x = a \end{array}\right\} \text{for all values of } z.$$

Applying the first boundary condition $x=0$ in equ^n (1)

$0 \cdot c_1 \sinh 0 + c_2 e^{-\gamma z} = 0$

$c_2 e^{-\gamma z} = 0$

$\boxed{c_2 = 0}$

Sub $c_2 = 0$ in equ^n (1)

$E_y = c_1 \sin hx \, e^{-\gamma z}$ — (2)

Apply (2) boundary condition in equ^n (2)

$0 = c_1 \sin ha \, e^{-\gamma z}$

$\sin ha = 0$

$\sin ha =$ $h = \frac{m\pi}{a}$ $\{m = 0, 1, 2 \ldots$

Sub in equ^n (2)

$E_y = c_1 \sin \frac{m\pi x}{a} e^{-\gamma z}$

diff w.r.t x

$\frac{\partial E_y}{\partial x} = c_1 \cos \frac{m\pi x}{a} \times \frac{m\pi}{a} e^{-\gamma z}$

consider eqn (23) & (25)

(23) $\Rightarrow \gamma E_y = -j\omega\mu H_x$

(25) $\Rightarrow \dfrac{\partial E_y}{\partial x} = -j\omega\mu \bar{H_z}$

from (23)

$$\bar{H_x} = \frac{\gamma}{-j\omega\mu} E_y$$

sub the value of E_y in above eqn

$$\bar{H_x} = \frac{\gamma}{-j\omega\mu} \cdot C_1 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

from eqn (25)

$$\bar{H_z} = \frac{1}{-j\omega\mu} \frac{\partial E_y}{\partial x}$$

sub the value of $\frac{\partial E_y}{\partial x}$ in above eqn

$$\bar{H_z} = -\frac{1}{j\omega\mu} \cdot C_1 \cos\left(\frac{m\pi x}{a}\right) \frac{m\pi}{a} e^{-\gamma z}$$

$$= -\frac{m\pi}{a\,j\omega\mu} \cdot C_1 \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$= j\frac{m\pi}{a\omega\mu} C_1 \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

The field strength of TE wave b/w parallel plates are

$$E_y = C_1 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$H_x = -\frac{\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

$$H_z = \frac{j m\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a} \cdot x\right) e^{-\gamma z}$$

$$\gamma = \alpha + j\beta \text{ , if } \alpha = 0$$

$$\gamma = j\beta$$

$$E_y = C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\beta z}$$

$$H_x = \frac{-\beta}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\beta z}$$

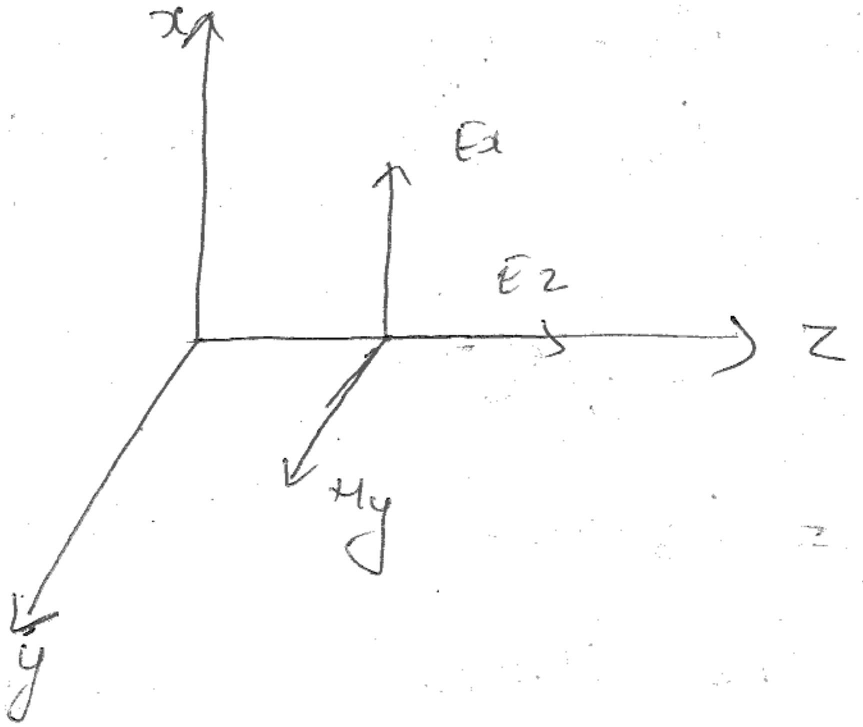
$$H_z = \frac{j m\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a} \cdot x\right) e^{-j\beta z}.$$

Each value of m specifies a particular field configuration or mode, the wave associated with the integer m is designated as ~~TM_0 wave~~ TM_0 wave or TM_0 mode.

The smallest value of $m=1$ and $n=0$ makes all field identically zero.
i.e. The lowest order mode that can exist in this case is TE_{10} mode

10/9/11

TM waves



Equation for TM waves are

$$\gamma H_y = +j\omega\varepsilon E_x$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

Wave Equation

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = -\omega^2\mu\varepsilon H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = -\omega^2 \mu \varepsilon H_y - \gamma^2 H_y$$

$$\therefore \boxed{\frac{\partial^2 H}{\partial x^2} = -h^2 H_y} \quad -① $$

Equ^n ① is second order Harmonic motion and the Solution of this Equation is - H_y^0 -

$$H_y^0 = (C_3 \sinh x + C_4 \cosh x)$$

w.k.T

$$H_y = H_y^0 e^{-\gamma z}$$

$$H_y = (C_3 \sinh x + C_4 \cosh x) e^{-\gamma z} \quad -②$$

E_z Can be obtained in terms of H_z

$$\frac{\partial H_y}{\partial x} = j\omega \varepsilon E_z$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial x}$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial (C_3 \sin hx + C_4 \cos hx) e^{-\gamma z}}{\partial x}$$

$$E_z = \frac{h e^{-\gamma z}}{j\omega\varepsilon} (C_3 \cos hx \mp C_4 \sin hx) \quad -③$$

Apply the 1st boundary condition in the above equation

$E_z = 0$ at $x = 0$

$\boxed{C_3 = 0}$

Sub $C_3 = 0$ in equ^n (3)

$$E_z = \frac{-h}{j\omega\varepsilon} C_4 \sinh hx \, e^{-\gamma z} \quad \text{—(4)}$$

Apply 2nd boundary condition in equ^n (4)

$E_z = 0, \; x = a$

$$0 = -\frac{h}{j\omega\varepsilon} C_4 \sinh ha \, e^{-\gamma z}$$

$$\sinh a = 0$$

$$h = \frac{m\pi}{a}, \quad m = 0, 1, 2 \ldots$$

Sub h in equ^n (4)

$$E_z = -\frac{m\pi}{a j\omega\varepsilon}\left[C_4 \sin\frac{m\pi x}{a}\right] e^{-\gamma z}$$

<u>To find H_y</u>

Sub $C_3 = 0$ & $h = \frac{m\pi}{a}$ in equ^n (2)

$$H_y = C_4 \cos\frac{m\pi x}{a} e^{-\gamma z}$$

W.K.T

$$\gamma H_y = j\omega\varepsilon E_x$$

$$E_x = \frac{1}{j\omega\varepsilon}\gamma H_y$$

$$E_x = \frac{\gamma}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

∴ The field strength for TM waves between parallel plates are.

$$H_y = C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$$E_x = \frac{\gamma}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

$$E_z = \frac{jm\pi}{a\omega\varepsilon} C_4 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

The TM wave associated with the integer M is designated as TM_{m0} mode if M = 0 all fields are not zero, i.e. E_x and H_y exist and only E_z becomes zero.

∴ The field for TM_{10} wave is the dominant mode.

$\gamma = \alpha + j\beta$, $\alpha = 0$, $\gamma = j\beta$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

$$E_z = \frac{j m\pi}{a\omega\varepsilon} C_4 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$$

Transverse Electro Magnetic waves (TEM)

1. For TE waves b/w the parallel plates the lowest value of m used without making all components zero, was $m = 1$. i.e TE_{10} mode.

2. For TM waves b/w the parallel plates a value of $m = 0$ does not require all the fields be zero.

Sub $m = 0$ in TM waves.

$$H_y = C_4 e^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega\varepsilon} C_4 e^{-j\beta z}$$

$$E_z = 0$$ → This type of wave is called principal wave.

For this special case of TM waves, the component of E in the direction of propagation i.e. $E_z = 0$, so that Electromagnetic waves are Completely Transversed and this type of waves are called Transverse Electromagnetic waves or principal waves.

These fields are not only Entirely transverse, but they are constant in amplitude b/w parallel planes.

Characteristics of TEM waves

1. For the lowest value $m=0$ and an ~~air~~ dielectric, the Expression for γ, β, V and λ can be reduced to

W.K.T

$$h^2 = \gamma^2 + \omega^2 \mu_0 \epsilon_0$$

$$\gamma^2 = h^2 - \omega^2 \mu_0 \epsilon_0$$

W.K.T

$$h = \frac{m\pi}{a}$$

$$\gamma = \sqrt{h^2 - \omega^2 \mu_0 \epsilon_0}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu_0 \epsilon_0}$$

$$\gamma = \sqrt{-\omega^2 \mu_0 \epsilon_0}$$

$$\gamma = j\omega \sqrt{\mu_0 \epsilon_0} \quad - (1)$$

$$\gamma = \alpha + j\beta \quad - (2)$$

compare (1) & (2)

$$\alpha = 0$$

$$\boxed{\beta = \omega \sqrt{\mu_0 \epsilon_0}}$$

$$V = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\boxed{V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\boxed{\lambda = \frac{2\pi}{\omega \sqrt{\mu_0 \epsilon_0}}}$$

$$\boxed{\lambda = \frac{c}{f}}$$ By $\left[\because C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\right]$

$$f = \frac{\omega}{2\pi}$$

Unlike TE and TM waves, the velocity of TEM waves is independent of frequency and has the value

$C = 3 \times 10^8$ m/sec.

The cutoff frequency of TEM waves is zero.

$$f_c = \frac{m}{2a\sqrt{\mu\epsilon}} = 0$$

$$\boxed{f_c = 0}$$

This above Equation shows that all frequencies down to zero can propagate along the guide. The ratio of E to h between the parallel plates for a travelling wave is

$$\frac{E_x}{H_y} = \frac{\frac{\beta}{\omega\epsilon_0} C_4 \cancel{e^{-j\beta z}}}{C_4 \cancel{e^{-j\beta z}}}$$

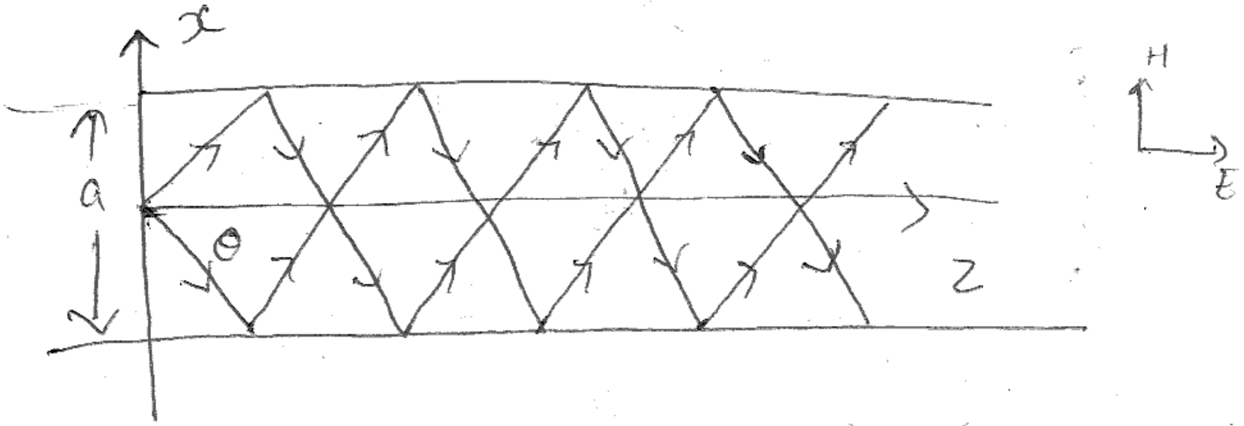
$$\left|\frac{E_x}{H_y}\right| = \frac{\beta}{\omega\varepsilon_0} = \frac{\omega\sqrt{\mu_0\varepsilon_0}}{\omega\varepsilon_0}$$

$$\left|\frac{E_x}{H_y}\right| = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

∴ ratio of E to H is ~~rotten~~ intrinsic impedence.

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Velocity of propagation



To ~~Sta~~ satisfy the bandary condition the total Electric field due to two component wave must be zero.

The cutoff frequency f_c

$$f_c = \frac{m}{2a\sqrt{\mu_0\varepsilon_0}}$$

W.K.T

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$f_c = \frac{mv}{2a}$$

$$\boxed{\therefore a = \frac{mv}{2 f_c}}$$

$$\lambda = \frac{v}{f_c}$$

$$\boxed{a = \frac{m \lambda_c}{2}}$$

$$\lambda_c = \frac{2a}{m}$$

1) when $\lambda = \lambda_c$
waves bounces back and forth b/w the walls of the guide this indicates that there is no wave motion parallel to the axis.

ii) when $\lambda < \lambda_c$

waves Travels almost parallel to the axis of the guide. This wavelength is called guided wavelength

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{m\lambda}{2a}\right)^2}}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} \quad \left[\because \lambda_c = \frac{2a}{m}\right]$$

Group velocity

The velocity with which the energy propagate along a guide is called Group velocity.

$$V_g = \frac{d\omega}{d\beta} < \text{free space velocity}$$

If the frequency spread of the group is small ~~and~~ Δf

$\frac{d\omega}{d\beta}$ may be considered to be Constant through out the group.

It is always lesser than free space velocity.

Phase velocity :- It is defined as the velocity of propagation of Equiphase Surface along a guide denoted by

$$v_p = \frac{\omega}{\beta} \Rightarrow \text{free space velocity.}$$

Phase shift is given by

$$\beta = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

Squaring on both sides.

$$\beta^2 = \omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2$$

diff above eqn

$$2\beta \, d\beta = \cancel{2 d\omega} \omega \mu \varepsilon$$

$$\frac{d\beta}{d\omega} = \frac{\cancel{2} \mu \varepsilon}{\cancel{2}\beta} = \frac{\omega \mu \varepsilon}{\beta}$$

$$\frac{d\omega}{d\beta} = \frac{\beta}{\omega \mu \varepsilon}$$

$$\frac{d\omega}{d\beta} = \frac{1}{\frac{\omega}{\beta} \mu \varepsilon}$$

$$\frac{d\omega}{d\beta} = \frac{1}{v_p \mu \varepsilon}$$

The product of the phase velocity V_p and group velocity V_g with which the Energy propagates is Equal to square of free space velocity.

$$V^2 = V_p \times V_g$$

$$V^2 = \frac{\cancel{\omega}}{\cancel{\beta}} \times \frac{\cancel{\beta}}{\cancel{\omega} \mu_0 \varepsilon_0}$$

$$V^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where $V_g \Rightarrow$ group velocity $= \frac{d\omega}{d\beta}$

$V_p \Rightarrow$ phase velocity $\Rightarrow \frac{\omega}{\beta}$

$V \Rightarrow$ free space velocity $\Rightarrow \frac{1}{\sqrt{\mu\varepsilon}}$