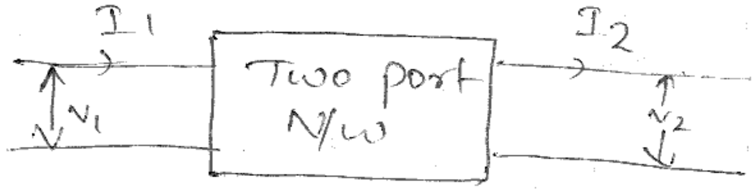
# UNIT- I

## Nepers and decibels

Nepers :- It is defined as the natural logarithm of the ratio of i/p voltage or current to that of output voltage or current, provided the n/w is properly terminated with Zo.



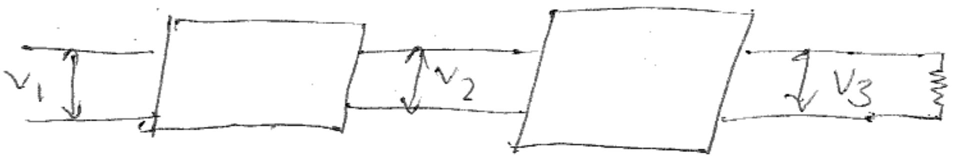

$$\therefore\ N = \log_e \left| \frac{V_1}{V_2} \right| = \log_e \left| \frac{I_1}{I_2} \right| \quad - (1)$$

$$\Rightarrow\ e^N = \left| \frac{V_1}{V_2} \right| = \left| \frac{I_1}{I_2} \right|$$

Hence two voltage or currents differs by 1 neper when one of them is e times as larger as the other.

Neper can be Expressed in terms of i/p, o/p power as

$$N = \frac{1}{2} \log_e \left| \frac{P_1}{P_2} \right| \quad - (2)$$




$$N_1 = \log_e \left| \frac{V_1}{V_2} \right|$$

$$N_2 = \log_e \left| \frac{V_2}{V_3} \right|$$

The total attenuation 'N' through the Combination is obtained by adding the attenuation of separate n/w.

$$N = N_1 + N_2$$

$$= \log_e \left| \frac{V_1}{V_2} \right| + \log_e \left| \frac{V_2}{V_3} \right|$$

$$= \log_e \left| \frac{V_1}{\cancel{V_2}} \times \frac{\cancel{V_2}}{V_3} \right|$$

$$\boxed{N = \log_e \left| \frac{V_1}{V_3} \right|}$$

Thus the addition of logarithm is Equivalent to multiplying the voltage ratio. Similarly the Subtraction of the log is Equivalent to dividing the voltage ratio.

Decibels :-

It is the 10 times common algorithm of the ratio of the input power to the O/P power.

$$\therefore \quad D = 10\log_{10}\frac{P_1}{P_2} \quad - (1)$$

If the powers $P_1$ & $P_2$ are associated with equal impedences, the power ratio can be expressed as the square of either the voltage or the current ratio under this condition equ (1) can be written as

$$D = 20\log_{10}\left|\frac{V_1}{V_2}\right| = 20\log_{10}\left|\frac{I_1}{I_2}\right| - (2)$$

~~Neighe~~ Neither neper nor decibel is an absolute unit each gives the ratio b/w i/p & o/p power.

WKT

$$\log_e x = \log_{10} x \times \log_e 10 \quad - (1)$$

Sub

$$x = \frac{P_1}{P_2} \quad \& \quad \log_e 10 = 2.3026 \text{ in } (1)$$

$$\log_e \frac{P_1}{P_2} = \log_{10}\frac{P_1}{P_2} \times \cancel{\log_e 10} = 2.3026 \quad - (2)$$

Sub

$$N = \frac{1}{2}\log_e \frac{P_1}{P_2} \quad \&$$

$$D = 10 \log_{10} \frac{P_1}{P_2} \quad \text{in } (2)$$

$$2N = \frac{D}{10} \times 2.3026$$

$$\boxed{1 \text{ neper} = 8.686 \text{ dB}}$$

$$\boxed{1 \text{ dB} = 0.115 \text{ neper}}$$

## Networks

An Electrical n/w is any inter-connection of electric elements or branches.
N/w Elements are impedances and Sources. Each branch may include R, L, C or other types of Elements.

## Passive N/w

A n/w containing ckt Elements without Energy Sources.

## Active n/w

A n/w containing generator or Energy Sources as well as other Elements.

## Nodes

A terminal of any branch of a n/w or the terminal common to n branches.

## Resonance :-

The property of cancellation of reactance when inductor & capacitor reactance are in series (or) cancellation of susceptance is when inductor & capacitor are in Parallel.

## Filters :-

A n/w which freely passes desired band of freq while almost supresses other band of frequencies called filters.

## Active filters :-

The filters which contain active elements is called active filters such as transistor, op-amp.

The disadvantage is it req. additional Power Supply.

Voltage, current, power gain is possible.

## Passive filters :-

The filter consists of passive Element like R, L, C is called passive filters.

It does not req. additional power supply.

None of gain is possible.

## Pass Band :

The range in which the attenuation is zero.

Stop Band :- (attenuation band)

The range of freq. over which the attenuation is infinite.

cutoff frequencies :-

The freq. which separates the pass band from the stop band is called cutoff freq ($f_c$).

We know that

$$I = \frac{I_R (Z_R + Z_0)}{2Z_0}\left[e^{\gamma s} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma s}\right]$$

sub $s = l$

$$I_S = \frac{I_R (Z_R + Z_0)}{2Z_0}\left[e^{\gamma l} - \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l}\right]$$

$$I_S = \frac{I_R (Z_R + Z_0)}{2Z_0}\left[e^{\gamma l} - K e^{-\gamma l}\right]$$

$\Rightarrow Z_S = \frac{E_S}{I_S}$ & $E_S = 1\angle 0^\circ \, V$

$$I_S = \frac{E_S}{Z_S} = \frac{1\angle 0^\circ}{624.733 \angle 1.7301^\circ}$$

$$I_S = 1.600 \times 10^{-3} \angle -1.7301^\circ$$

$$Z_R + Z_0 = 200\angle 0^\circ + 696.204\angle -10.825$$
$$= 893.23 \angle -8.414^\circ$$

$$Ke^{-\gamma l} = (0.5608\angle 173.289)(0.4676 \angle -208.38^\circ)$$

$$Ke^{-\gamma l} = 0.2622\angle -35.091^\circ$$

$I_S \Rightarrow$

$$1.600\times 10^{-3}\angle -1.7301^\circ = \frac{I_R (893.23\angle -8.414^\circ)}{2\times 696.204\angle -10.825}$$
$$\left[2.138\angle 208.38^\circ - 0.2622\angle -35.091^\circ\right]$$

$$1.600\times10^{-3}\angle-1.7301^\circ = I_R\left[\frac{893.23\angle-8.414^\circ}{1392.408\angle-10.825}\right]\times$$

$$\left\{\cancel{1.8758}\left[2.138\angle208.38^\circ - 0.262\angle-35.091^\circ\right]\right.$$

$$1.600\times10^{-3}\angle1.7301^\circ = I_R\left[0.6415\angle2.411^\circ\right]\left[2.138\angle208.38^\circ - 0.262\angle-35.091^\circ\right]$$

$$I_R = \left(\frac{1.600\times10^{-3}\angle1.7301^\circ}{\left[0.6415\angle2.411^\circ\right]\left[2.138\angle208.38^\circ - 0.262\angle-35.091^\circ\right]}\right)$$

$$= \frac{1.600\times10^{-3}\angle1.7301^\circ}{(0.6415\angle2.411^\circ)(1.876\angle173.289^\circ)}$$

$$= \frac{1.600\times10^{-3}\angle1.7301^\circ}{1.2034\angle175.7^\circ}$$

$$I_R = 1.329\times10^{-3}\angle-177.429^\circ$$

$$I_R = 0.001329\angle-177.4296^\circ$$

$$E_R = I_R Z_R$$

$$= 1.329\times10^{-3}\angle-177.429^\circ\times200\angle0^\circ$$

$$= 265.8\times10^{-3}\angle-177.429^\circ$$

$$\boxed{E_R = \cancel{0.266}\ 0.2658\angle-177.42^\circ}$$

$P_S = E_S I_S \cos(E_S \wedge I_S)$

$= 1\angle 0^\circ \times 1.599\times10^{-3}\angle -1.7301^\circ$

$\cos(1\angle 0^\circ \wedge 1.599\times10^{-3}\angle -1.7301^\circ)$

$= 1.599\angle -1.7301^\circ \cos(1\angle 0^\circ \wedge$

$P_R = I_R^2 R$

$= 2.38$

~~FILTER FUNDAMENTAL~~

# FILTERS FUNDAMENTALS

If $\alpha = 0$, or $I_1 = I_2$ then there is no attenuation, only phase shift occur.

1) $\alpha = 0 \rightarrow \boxed{I_1 = I_2} \rightarrow \beta$ exist

These conditions occurs only in pass band

2) $\alpha = +ve \rightarrow \boxed{I_1 < I_2}$, $\alpha$ exist.

It occurs in stop Band.

3) The propagation constant $\gamma$ for symmetrical T section is given by

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2} + j\frac{\beta}{2}\right)$$

$$\sinh\frac{\gamma}{2} = \sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} + \cosh\frac{\alpha}{2} \cdot \sin\frac{\beta}{2} \quad -(1)$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2} \cdot \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad -(2)$$

From the above two eqn $\alpha$ & $\beta$ depends on $Z_1$ & $Z_2$

Case 1: If $Z_1$ and $Z_2$ are the same type of reactance then $\frac{Z_1}{4Z_2}$ should be greater than zero or the ratio is positive and real

$$Z_1 = Z_2, \quad \frac{Z_1}{4Z_2} > 0 \rightarrow \text{real}$$

$$\sin \beta/2 \cdot \cosh \frac{\alpha}{2} = 0 \rightarrow (3) \quad \{\text{imaginary part zero}\}$$

$$\sinh \frac{\alpha}{2} \cdot \cos \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{--- } (4)$$

from eqn (3)

$$\sin \frac{\beta}{2} = 0, \quad \beta = n\pi, \; n = 0, 1, 2 \ldots$$

from eqn (4) $\quad [\because \cos 0 = 1]$

$$\sinh \frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}},$$

$$\alpha = 2 \sinh^{-1} \sqrt{\frac{Z_1}{4Z_2}}$$

Thus the condition that $\frac{Z_1}{4Z_2} > 0$ implies a stop band or attenuation band of frequencies.

Case 2: If $z_1$ & $z_2$ are opposite types of reactance than $\frac{z_1}{4z_2} = -ve$

Real part becomes zero.

$$\cosh\frac{\alpha}{2} \cdot \sin\frac{\beta}{2} = \sqrt{\frac{z_1}{4z_2}} \quad -⑥$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} = 0 \quad -⑤$$

from equ$^n$ ⑤

$$\sinh\frac{\alpha}{2} = 0$$

$$\boxed{\alpha = 0}, \quad \boxed{\beta \neq 0}$$

$$\sin\frac{\beta}{2} = \sqrt{\frac{z_1}{4z_2}} \quad \left\{\cosh\frac{\alpha}{2} = 1\right\}$$

from equ$^n$ ⑥

$$\cos\frac{\beta}{2} = 0, \quad \sin\beta/2 = \pm 1 \ \& \ \alpha \neq 0$$

$$\cosh\frac{\alpha}{2} = \sqrt{\frac{z_1}{4z_2}}$$

from $\sin\frac{\beta}{2} = \sqrt{\frac{z_1}{4z_2}}$

$$\beta = 2\sin^{-1}\left(\sqrt{\frac{z_1}{4z_2}}\right) \quad -⑦$$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

Condition (1) gives a passband as infinity i.e. zero attenuation. Then the phase angle in the pass band is given by

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} \quad - (7)$$

Condition (2) gives a stopband. Then the phase angle is $\pi$ and attenuation is given by

$$\boxed{\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \quad - (8)$$

From equn (7) & (8), we can easily calculate $\alpha$ in the stop band in which $\beta$ is $\pi$ and the phase shift $\beta$ in PB in which $\alpha = 0$.

The frequencies at which the network changes from a pass network to the stop network called cutoff frequency ($f_c$). These frequencies occurs when

$$\frac{Z_1}{4Z_2} = 0 \quad (\text{or}) \quad Z_1 = 0 \rightarrow (9)$$

~~Hence~~ or

$$\frac{Z_1}{4Z_2} = -1 \quad \text{or} \quad Z_1 = -4Z_2 \rightarrow (10)$$

Where $Z_1$ & $Z_2$ are opposite type of reactance.

HW

Q. An open wire telephone line has $R = 10\,\Omega/km$, $L = 0.004\ H/km$, $C = 0.008 \times 10^{-6}\ F/km$ and $G = 0.4 \times 10^{-6}$ mho/km. Find $Z_0, \alpha, \beta$ at 1 KHz

Soln

$$Z = R + j\omega L$$

$$= 10 + j \times 2\pi \times 10^3 \times 0.004$$

$$= 10 + 25.132j$$

$$= 27.0484 \angle 68.30^\circ$$

$$Y = G + j\omega C$$

$$= 0.4 \times 10^{-6} + j \times 2\pi \times 10^3 \times 0.008 \times 10^{-6}$$

$$= 5.0267 \times 10^{-5} \angle 89.54$$

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{27.0484 \angle 68.30^\circ}{5.0267 \times 10^{-5} \angle 89.54}}$$

$$= \sqrt{5.3810 \times 10^{5} \angle -21.24^\circ}$$

$$= 733.552 \angle -10.62^\circ$$

$$\gamma = \sqrt{ZY}$$

$$= \sqrt{5.0267 \times 10^{-5} \times 27.043 \angle 157.84^\circ}$$

$$= \sqrt{135.967 \times 10^{-5} \angle 157.84^\circ}$$

$$= 36.8456 \times 10^{-3} \angle 78.92^\circ$$

$$\gamma = 7.0809 \times 10^{-3} + 0.0361\, j$$

$\alpha = 7.0809 \times 10^{-3}$, $\beta = 0.0361$ Ans.

Q3. A telephone line has $R = 20\,\Omega$, $L = 10\,mH$, $C = 0.1\,\mu f$ & insulation resistance of 100 kΩ/km. Find the i/p impedence at angular frequency 5000 rad/sec if the line is very long

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{20 + j \times 5000 \times 10^{-2} \times 10 \times 10^{-3}}{0.01 \times 10^{-3} + j \times 5000 \times 10^{-2} \times 0.1 \times 10^{-6}}}$$

$$= \sqrt{\frac{20 + j \times 5000 \times 10^{-4}}{0.01 + j \times 500\cancel{0} \times \cancel{0.1} \times 10^{-8}}}$$

$$= \sqrt{\frac{20.0062 \angle 1.432}{0.0100 \angle 0.0286}}$$

$$= \sqrt{2000.62 \angle 1.4034}$$

$$Z_0 = 44.72 \angle 0.701^\circ$$

## Behaviour of characteristics impedence

In the symmetrical T network, the chara. impedence is given by

$$Z_0 = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

For pure reactances this expression for the characteristics impedence becomes

$$Z_0 = \sqrt{-x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)}$$

## Constant K Filters

1. A T or π Section in which series & shunt impedences $Z_1$ & $Z_2$ satisfy the relationship

$$\boxed{Z_1 Z_2 = R_0^2}$$

where $R_0$ is a Real constant, it is denoted as K or $R_K$, where $R_K$ is constant independent of frequency.

$$\boxed{Z_1 Z_2 = K^2} \quad \text{or} \quad \boxed{Z_1 Z_R = R_K^2}$$

Networks are filters sections for which this reatti relation holds for constant K filters.

## Constant K LPF

In low pass filters the series impedance $Z_1 = j\omega L$ & shunt impedence is $Z_2 = -\frac{j}{\omega C}$

Then

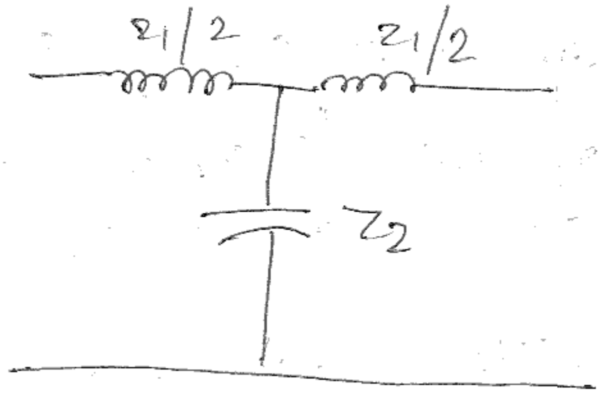
$$Z_1 Z_2 = -j\omega L \cdot \frac{j}{\omega C}$$

$$\boxed{Z_1 Z_2 = \frac{L}{C}} = R_k^2$$

The term $R_k$ is used since k is real and constant if $Z_1$ & $Z_2$ are opposite type.

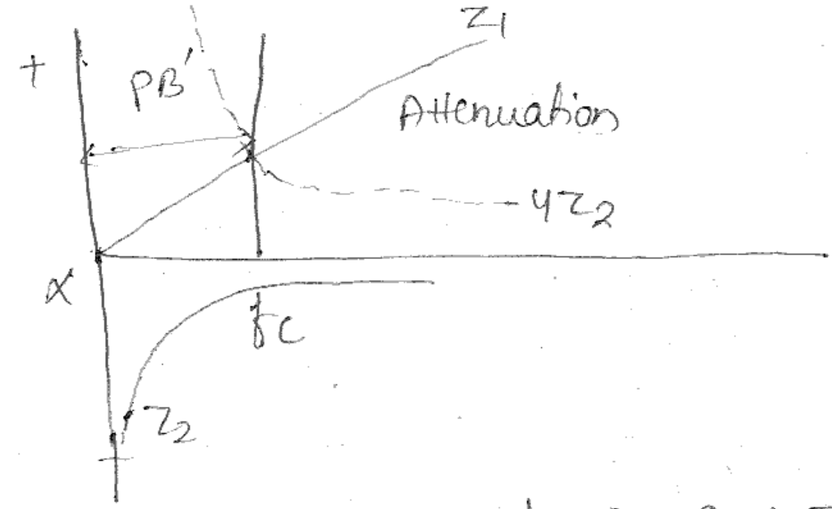
$$R_0^2 = \frac{L}{C} \qquad \boxed{R_0 = \sqrt{\frac{L}{C}}}$$

(a)




LPF section.

(b)




The reactances of $Z_1$ & $4Z_2$ will vary with frequency

WKT

$\boxed{Z_1 = -4Z_2}$ Since it is varying with freq, Thus the reactance shows that the PB starts at $f=0$ & continues to fc. All frequencies above fc lies is the Attenuation or Stop band. Thus the network is called LPF.

The cutoff frequency fc may be obtained by using $Z_1 = -4Z_2$ — (1)

Sub $Z_1 = j\omega L$ & $Z_2 = -\frac{j}{\omega C}$ in equ (1)

$$j\omega_c L = +4\frac{j}{\omega_c C}$$

$$\omega_c^2 = \frac{4}{LC}$$

$$(2\pi f_c)^2 = \frac{4}{LC}$$

$$4\pi^2 f_c^2 = \frac{4}{LC}$$

$$f_c^2 = \frac{4}{4\pi^2 LC}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \quad - (3)$$

constant β with frequency

For passband α is zero and β is π then

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$= \sqrt{\frac{j\omega L}{-4\frac{j}{\omega C}}} = \sqrt{\frac{\cancel{j}\omega L \times \omega C}{-4\cancel{j}}}$$

$$\sinh\frac{\gamma}{2} = \sqrt{-\frac{\omega^2 LC}{4}}$$

$$\sinh\frac{\gamma}{2} = j\frac{\omega}{2}\sqrt{LC}$$

WKT

$f_c$ is $\frac{1}{\pi\sqrt{LC}}$ & $\omega = 2\pi f_c$

$$\cancel{\sinh\frac{\gamma}{2} = \frac{j2\pi \times 1}{\pi\sqrt{LC}}}$$

$$\sinh\frac{\gamma}{2} = \frac{j2\pi f}{2}\cdot\frac{1}{\pi f_c}$$

$$\boxed{\sinh\frac{\gamma}{2} = \frac{jf}{f_c}} \quad \left\{\because \text{For PB}, \alpha = 0, \ \sin\frac{\beta}{2} = \frac{f}{f_c}\right\}$$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)}$$

The characteristics impedence of T section can be calculated by

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{j^2\omega^2L^2}{4} + j\omega L\cdot\frac{-j}{\omega C}}$$

$$= \sqrt{\frac{-\omega^2L^2}{4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}}\cdot\sqrt{1 - \frac{\omega^2 LC}{4}}$$

WKT

$$R_0 = \sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Z_0 = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}}$$

From the above Expression it is clear that $Z_0$ is Real if $\frac{\omega^2 LC}{4}$ is lesser than 1 and imaginary if $\frac{\omega^2 LC}{4} > 1$. Hence condition $\frac{\omega^2 LC}{4} - 1 = 0$ which gives

$$\frac{\omega^2 LC}{4} = 1$$

$$\omega^2 LC = 4$$

$$\omega^2 = \frac{4}{LC}$$

$$\boxed{\omega = \frac{2}{\sqrt{LC}}}$$

Thus the above Expression shows it Passes all frequencies below $\omega = \frac{2}{\sqrt{LC}}$ which attenuates all frequencies above $\frac{2}{\sqrt{LC}}$.

$\therefore$ The cutoff frequency of LPF is given by

$$\omega_c = \frac{2}{\sqrt{LC}}$$

$$2\pi f_c = \frac{2}{\sqrt{LC}}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

is

Variation of $Z_0$ with frequency

Consider $Z_0 = R_0\sqrt{1 - \frac{\omega^2 LC}{4}}$ — (1)

$= 0$

$$\omega_c = \frac{2}{\sqrt{LC}} \quad \text{or} \quad \omega_c^2 = \frac{4}{LC}$$

$$Z_0 = R_0\sqrt{1 - \frac{4 \times \cancel{LC}\,\omega^2}{\cancel{LC} \times 4\,\omega_c^2}}$$

$$Z_0 = R_0\sqrt{1 - \left(\frac{2\pi f}{2\pi f_c}\right)^2}$$

$$\boxed{Z_{0T} = R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

bove

$M|^{ly}$ WKT

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

is

$$Z_{0\pi} = \frac{Z_1 Z_2}{R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

WKT

$$Z_1 Z_2 = R_0^2$$

$$\therefore Z_{0\pi} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

Variation of attenuation constant $\alpha$ with frequency

$$\alpha \neq 0, \quad \beta = \pi$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$= \sqrt{\frac{j\omega L \times \omega C}{4(-j)}}$$

(for LPF $Z_1 = j\omega L$, $Z_2 = \frac{-j}{\omega C}$)

$$\sinh\frac{\gamma}{2} = \cancel{\sqrt{\phantom{x}}}\ \frac{jf}{f_c}$$

$$\sinh\frac{(\alpha+j\beta)}{2} = \frac{jf}{f_c}$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{j\beta}{2} + \cosh\frac{\alpha}{2} \cdot \sin\frac{j\beta}{2} = \frac{jf}{f_c}$$

w.k.T

$$\sinh j\theta = j\sin\theta$$

$$\cosh j\theta = \cos\theta$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} + \cosh\frac{\alpha}{2} \cdot j\sin\frac{\beta}{2} = j\left(\frac{f}{f_c}\right)$$

w.k.T

$$\alpha = 0 \quad \& \quad \beta = \pi$$

$$\sinh(0) = 0, \quad \cos\frac{\pi}{2} = 0; \quad \sin\frac{\beta}{2} = \sin\frac{\pi}{2} = 1$$

$$j\cosh\frac{\alpha}{2} = j\left(\frac{f}{f_c}\right)$$

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

<u>Finding Inductance & capacitance by using $R_0$ and $f_c$</u>

$$R_0 = \sqrt{\frac{L}{C}} \quad \text{--- (1)}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

divide (1) & (2)

$$\frac{R_0}{f_c} = \sqrt{\frac{L}{C}} \times \pi\sqrt{LC} = \frac{L\pi\cancel{\sqrt{C}}}{\cancel{\sqrt{C}}}$$

$$\frac{R_o}{f_c} = \pi L$$

$$\therefore \boxed{L = \frac{R_o}{\pi f_c}}$$

Multiply (1) & (2)

$$R_o f_c = \sqrt{\frac{L}{C}} \times \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi C}$$

$$\boxed{C = \frac{1}{\pi R_o f_c}}$$

## CONSTANT K HIGH PASS FILTER

In high pass filter the series impedence $Z_1 = -\frac{j}{\omega C}$ & shunt impedence is

$$Z_2 = j\omega L$$

$$Z_1 Z_2 = -\frac{j}{\omega C} \times j\omega L = \frac{L}{C}$$

$$\boxed{Z_1 Z_2 = \frac{L}{C}} \quad \text{--- (1)}$$

WKT

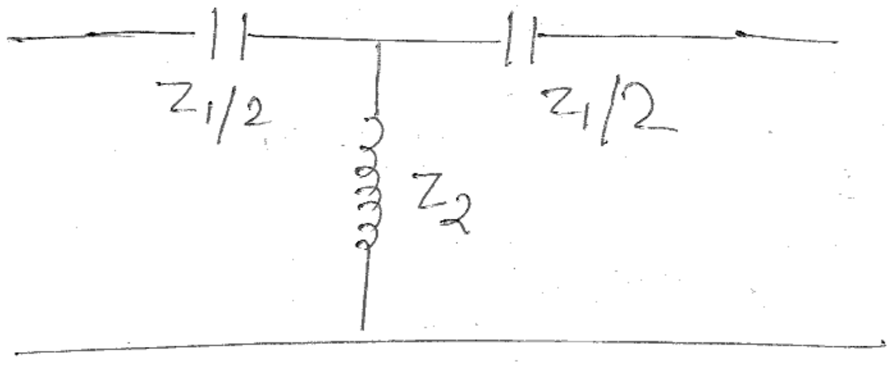
Constant K Section

$$\boxed{Z_1 Z_2 = R_0^2}$$

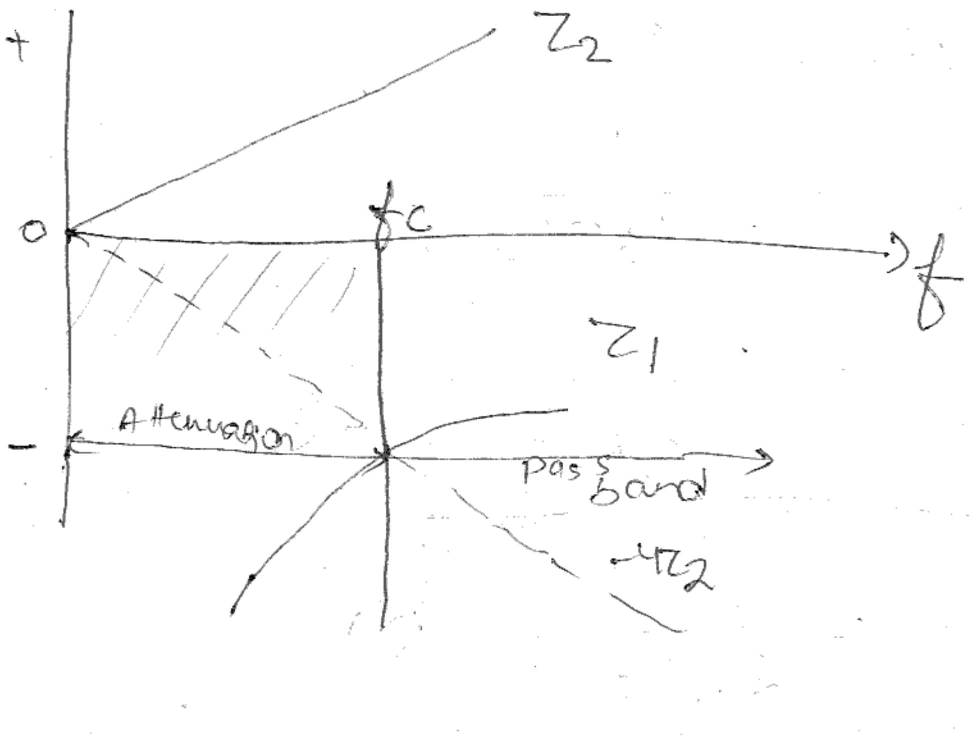
Sub in equn (1)

$$R_0^2 = \frac{L}{C}$$

$$\therefore \boxed{R_0 = \sqrt{\frac{L}{C}}}$$




HPF Section



$Z_1$ is compared with $-4Z_2$ showing a cutoff frequency at the point at which $\boxed{Z_1 = -4Z_2}$ This network is high pass filter all frequencies below fc lie in a attenuation or stopband

The cutoff frequency is determined as the frequency at which $Z_1 = -4Z_2$ — (2)

Sub $Z_1 = \frac{-j}{\omega C}$ & $Z_2 = j\omega L$

$$\cancel{-}\frac{j}{\omega C} = \cancel{-}j\omega L \times 4$$

$$\frac{1}{2\pi f_c C} = 4 \times 2\pi f_c L$$

$$\frac{1}{8\pi L \cdot 2C} = f_c^2$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}} \quad \text{— (4)}$$

$$\boxed{\omega_c = \frac{1}{2\sqrt{LC}}} \quad \text{— (3)}$$

i) Variation of attenuation with frequency

$\alpha$ with f

The propagation constant

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{— (5)}$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{\frac{-j}{\omega C}}{4 \times j\omega L}}$$

$$= \sqrt{\frac{-\cancel{j}}{4\,\omega C \cdot \cancel{j}\omega L}} \cdot \sqrt{\frac{-1}{4\omega^2 LC}}$$

$$= \frac{j}{2\omega} \frac{1}{\sqrt{LC}} = \sqrt{-1} \cdot \frac{1}{}$$

$$\sinh\frac{\gamma}{2} = \frac{j}{2\omega} \cdot \frac{1}{\sqrt{LC}} \quad \text{— (6)}$$

sub

$$\omega_c = \frac{1}{2\sqrt{LC}}$$

$$\boxed{\sinh\frac{\gamma}{2} = j\left(\frac{\omega_c}{\omega}\right)}$$

$$\sinh\frac{\gamma}{2} = j\left(\frac{\cancel{2\pi} f_c}{\cancel{2\pi} f}\right)$$

$$\sinh\frac{\gamma}{2} = j\left(\frac{f_c}{f}\right) \quad \text{— (7)}$$

Now $\gamma = \alpha + j\beta$

$$\sinh\frac{(\alpha + j\beta)}{2} = j\left(\frac{f_c}{f}\right)$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{j\beta}{2} + \cosh\frac{\alpha}{2} \cdot \sin\frac{j\beta}{2} = j\left(\frac{f_c}{f}\right)$$

W.K.T

$$\cosh j\theta = \cos\theta$$

$$\sinh j\theta = j\sin\theta$$

$$\sinh\frac{\alpha}{2} \cdot \cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2} \cdot \sin\frac{\beta}{2} = j\left(\frac{f_c}{f}\right) \quad - (8)$$

$\alpha = 0, \quad \beta = \pi$

$$0 + j\cosh\frac{\alpha}{2} \cdot \sin\frac{\pi}{2} = j\frac{f_c}{f}$$

$$\cancel{j}\cosh\frac{\alpha}{2} = \cancel{j}\frac{f_c}{f}$$

$$\boxed{\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)}$$

ii) Variation of $\beta$ with frequency

from eqn (8)

$\sinh\frac{\alpha}{2}\cdot\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\cdot\sin\frac{\beta}{2} = j\left(\frac{f_c}{f}\right)$ ... (8)

Put $\alpha = 0$ in equ= (8)

$\sinh(0) = 0$, $\cosh(0) = 1$

$\cancel{j}\sin\frac{\beta}{2} = \cancel{j}\frac{f_c}{f}$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)}$$

(iii) Variation of $Z_0$ with frequency:

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_0 = \sqrt{\frac{\left(\frac{-j}{\omega C}\right)^2}{4} + \frac{-j}{\omega C}\times j\omega L}$$

$$Z_0 = \sqrt{\frac{j^2}{4\omega^2 C^2} + \frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$Z_0 = R_0\sqrt{1 - \frac{1}{4\omega^2 LC}} \quad \left[\omega_c^2 = \frac{1}{4LC}\right]$$

$$Z_{oT} = R_0 \sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$Z_{oT} = R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

IIIry

$$Z_{o\pi} = \frac{Z_1 Z_2}{Z_{oT}}$$

$$Z_{o\pi} = \frac{R_0^{\cancel{2}}}{\cancel{R_0} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{Z_{o\pi} = \frac{R_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

iv) Finding Inductance & Capacitance values by using $R_0$ & $f_c$

$$R_0 = \sqrt{\frac{L}{C}} \quad \text{--- (1)}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad \text{--- (2)}$$

divide (1) & (2)

$$\frac{R_o}{f_c} = \frac{\sqrt{\frac{L}{C}}}{\frac{1}{4\pi\sqrt{LC}}}$$

$$= \sqrt{\frac{L}{C}} \times 4\pi\sqrt{LC}$$

$$\frac{R_o}{f_c} = 4\pi L$$

$$\boxed{L = \frac{R_o}{4\pi f_c}}$$

Multiply (1) & (2)

$$R_o f_c = \sqrt{\frac{L}{C}} \cdot \frac{1}{4\pi\sqrt{LC}}$$

$$= \frac{1}{4\pi C}$$

$$\boxed{C = \frac{1}{4\pi R_o f_c}}$$

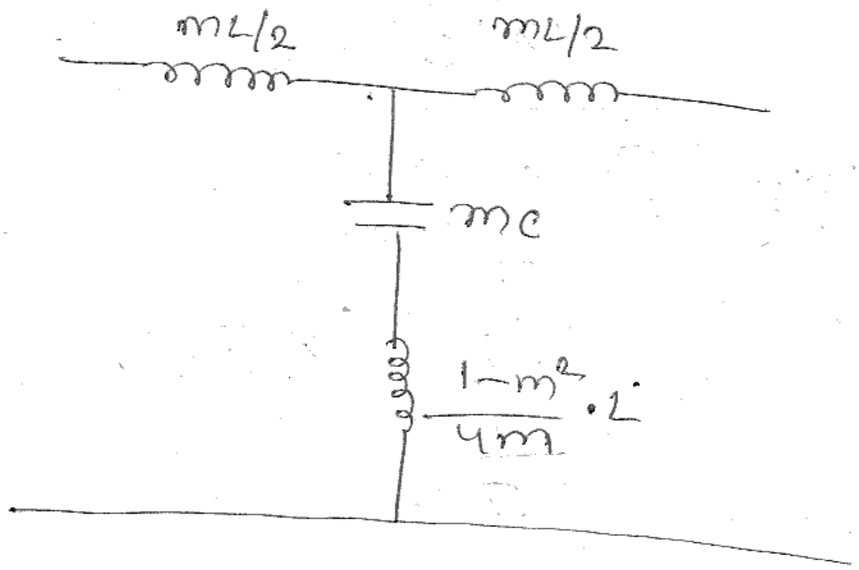
Disadvantages of Constant k filter

i) Attenuation doesnot increase Rapidly beyond cutoff frequencies.

ii) Characteristics impedences varies widely in the transmission or passband from the desired value.

Q. Design m-derived LPF (T or π) having design impedence
1) $R_0 = 500\Omega$, $f_c = 1500 Hz$ & infinite attenuation freq $f_\alpha = 2000 HZ$

m-derived LPF T-section




1) $L = \frac{R_0}{\pi f_c} = \frac{500}{\pi 1500} = 0.106 mH$

2) $C = \frac{1}{\pi R_0 f_c} = \frac{1}{\pi 500 \times 1500} = 4.24 \times 10^{-7}$
$= 0.424 \mu F$

3) $m = \sqrt{1 - \left(\frac{f_c}{f_\alpha}\right)^2} = \sqrt{1 - \left(\frac{1500}{2000}\right)^2} = \sqrt{1 - \frac{25}{16}}$

$= \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{16}$

$= 0.661$

ving

$$\frac{mL}{2} = \frac{0.661 \times 0.106}{2} = 0.035$$

$$= 35.03\,mH$$

$$mC = 0.661 \times 0.424 \times 10\ \mu F$$

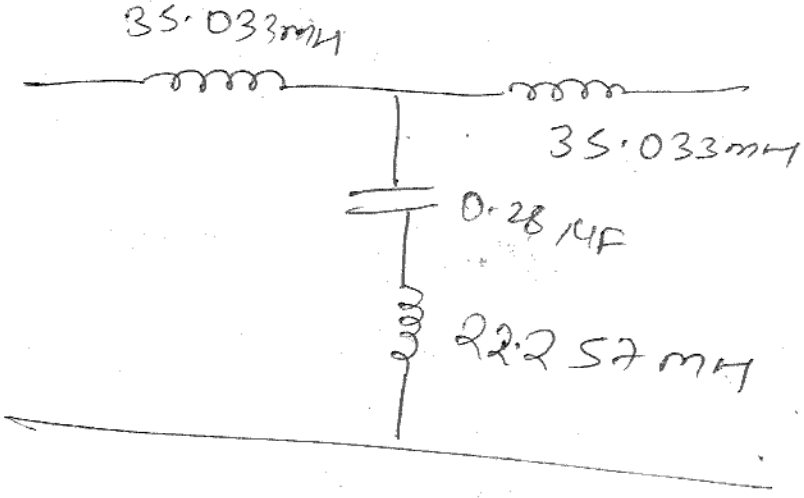
$$= 0.28\ \mu F$$

$$\frac{1-m^2}{4m}L = \frac{1-(0.661)^2}{4 \times 0.661} \times 0.106$$

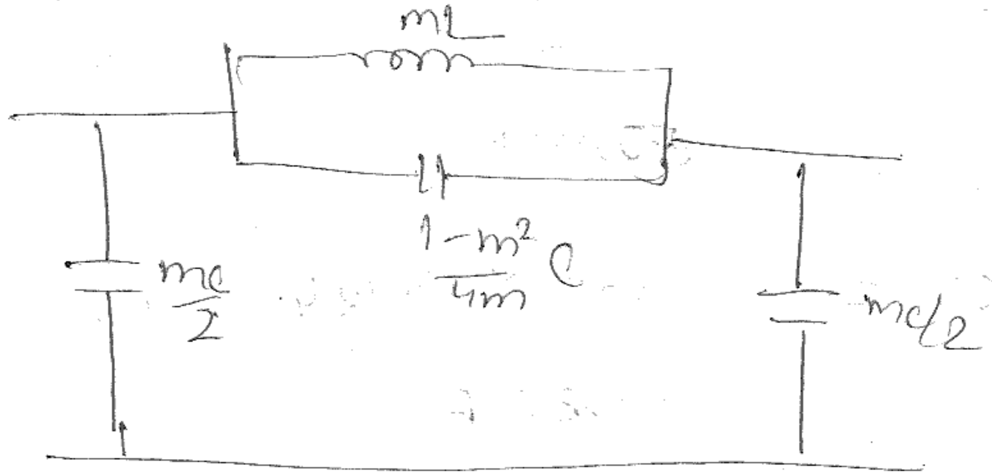
$$= \frac{0.5638}{4 \times 0.661} \times 0.106$$

$$= 0.022$$

$$= 22.257\ mH$$



## $\pi$ Section



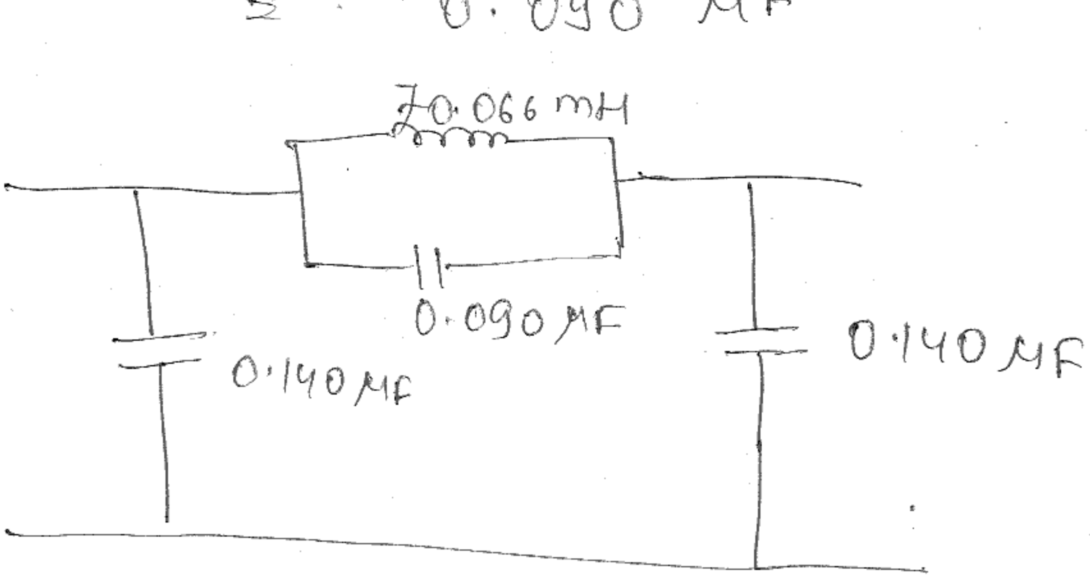

$$mL = 0.661 \times 0.106 = 0.070$$

$$= 70.066 \text{ mH}$$

$$\frac{mc}{2} = \frac{0.661 \times 0.424}{2} = 0.140 \text{ MF}$$

$$\frac{1-m^2}{4m} c = \frac{1-(0.661)^2}{4 \times 0.661} \times 0.424$$

$$= 0.090 \text{ MF}$$



Q2. Design a prototype Band pass filter section having cutoff frequencies of 3000 Hz & 6000 Hz. And nominal characteristics impedence of 600 Ω. Also find the resonant frequency of shunt arm.

Sol$^n$. Band pass

$R_0 = 600\,\Omega$

Cutoff frequency $f_1 = 3000\text{ Hz} = 3\text{ kHz}$

$f_2 = 6000\text{ Hz} = 6\text{ kHz}$

i) $L_1 = \dfrac{R_0}{\pi(f_2 - f_1)} = \dfrac{\cancel{600}^{\,2}}{\pi \times \cancel{3000}_{\,10}} = 63.66\text{ mH}$

ii) $C_1 = \dfrac{f_2 - f_1}{R_0\, 4\pi f_1 f_2} = \dfrac{3\times10^3}{600\times4\times\pi\times18\times10^6} = 0.0221\,\mu F$

iii) $L_2 = \dfrac{R_0(f_2 - f_1)}{4\pi f_1 f_2} = \dfrac{600\times3\times10^3}{4\pi\times18\times10^6} = 7.957\text{ mH}$

iv) $C_2 = \dfrac{1}{\pi R_0 (f_2 - f_1)} = \dfrac{1}{\pi\times600\times3\times10^3} =$

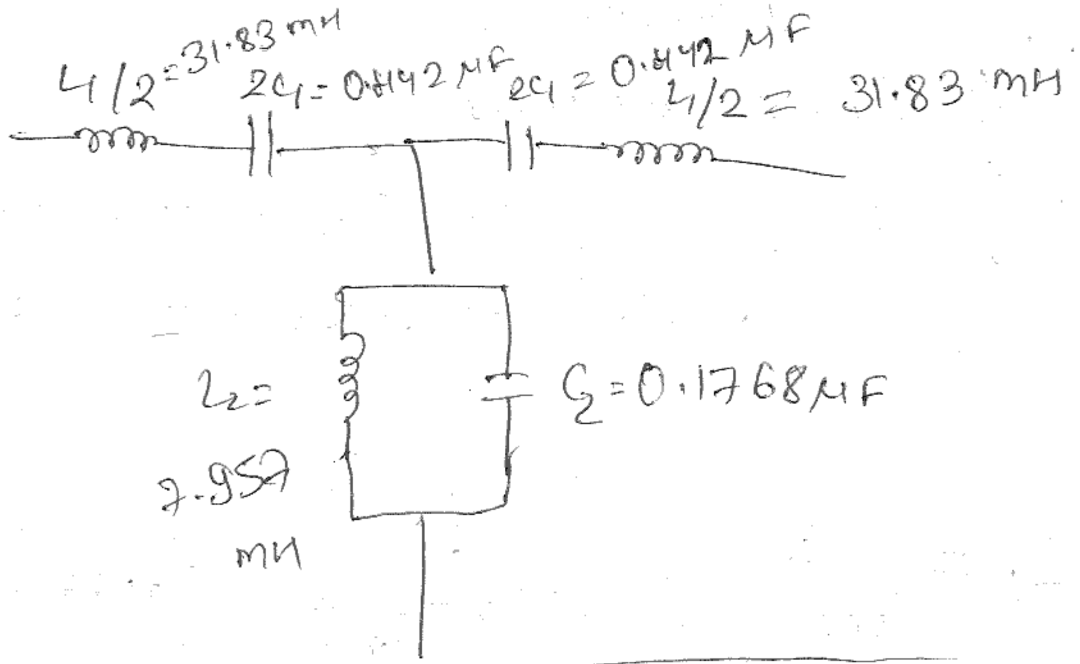
$0.1768\,\mu F$

Resonant frequency ($f_0$) = $\sqrt{f_1 \cdot f_2}$

$= \sqrt{18\times10^6}$

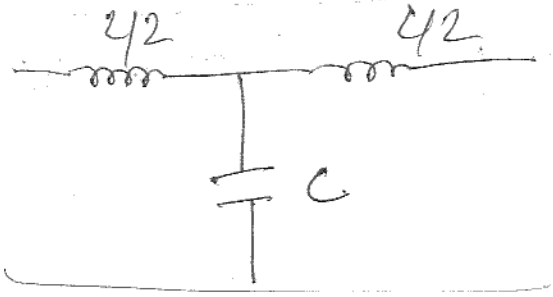
$= 4.24\times10^3$

= 4.24 KHz = 4242.64 HZ



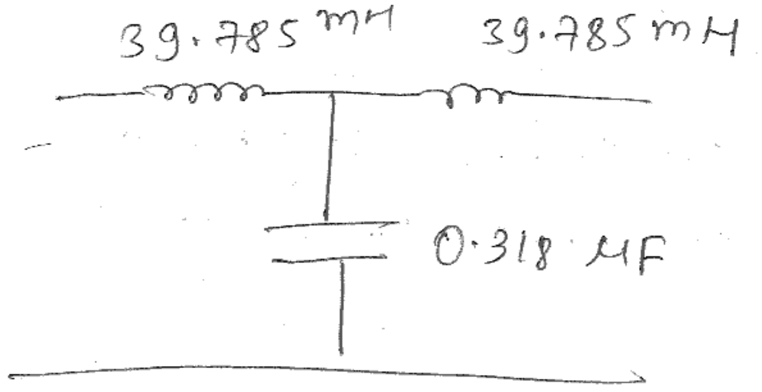

(constant K)

Q: Design a prototype LPF (constant K) section, if design impedence $R_0 = 500\,\Omega$ and cutoff frequency $f_c = 2000$ HZ



1) $L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 2000} = 79.57\ mH$

2) $C = \frac{1}{\pi R_0 f_c} = 0.318\ \mu F$.




LPF (constant K)

II nd unit

Q. A transmission line of 2 miles long operates at 1 KHZ & has parameters $R = 10.4\ \Omega/\text{mile}$, $L = 0.00367\ H/\text{mile}$, $C = 0.00835\ \mu F/\text{mile}$ $G = 0.8\ \mu mho/\text{mile}$

Find the characteristics impedence $Z_0$, $\gamma$, $\alpha$, $\beta$, $\lambda$, $V_p$ (phase velocity).

→ $\omega = 2\pi f$

→ Series impedence $(Z) = R + j\omega L$

→ Shunt admittance $(Y) = G + j\omega C$

$Z_0 = \sqrt{\frac{Z}{Y}}$ → angle, $\gamma = \sqrt{ZY}$ complex $\alpha$ $\lambda = \frac{2\pi}{\beta}$

$V_p = \frac{\omega}{\beta}$ (formula)

Q2. The transmission line having resistance
$R = 6\Omega$, $L = 2.2\,mH$, $C = 0.005\,\mu F$,
& $G = 0.05\,\mu\mho$

Calculate

At freq. 1 KHZ

(i) The terminating impedence for which no reflection will be set up in the line.

ii) $\gamma$, $V_p$, $\alpha$, $\beta$ & $\lambda$

→ when

→ $\omega = 2\pi f$

→ $Z = R + j\omega L$

& $Y = G + j\omega C$

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta$$

→ $V_p = \dfrac{\omega}{\beta}$, $\lambda = \dfrac{2\pi}{\beta}$.

Q1. $R = 10.4\,\Omega/\text{mile}$ , $L = 0.00367\ H/\text{mile}$

$C = 0.00835\ \mu F/\text{mile}$ , $G = 0.8\ \mu\mho/\text{mile}$

$Z = R + j\omega L$

$\omega = 2\pi f$

$\omega = 2\pi \times 10^3$

$Z = 10.4 + j(2\pi \times 10^3 \times 0.00367)$

$= 10.4 + j(23.059)$

$Z = 25.29 \angle 65.72°$

$Y = G + j\omega C$

$= 0.8\times10^{-6} + j(2\pi\times10^3 \times 0.00835\times10^{-6})$

$= 0.8\times10^{-6} + j(5.246\times10^{-5})$

$= 5.246\times10^{-5} \angle 89.126°$

$Z_0 = \sqrt{\dfrac{Z}{Y}}$

$= \sqrt{\dfrac{25.29 \angle 65.72}{5.246\times10^{-5} \angle 89.126°}}$

$= \sqrt{4.82\times10^{5} \angle -23.406°}$

$\boxed{Z_0 = 692.53 \angle -11.703°}$

$\gamma = \sqrt{ZY}$

$= \sqrt{2529 \angle 65.72^\circ \times 5.246 \times 10^{-5} \angle 89.126^\circ}$

$= \sqrt{0.00132 \angle 154.846^\circ}$

$= 0.0364 \angle 77.423^\circ$

$$\boxed{\gamma = 0.00792 + 0.03552j}$$

$\gamma = \alpha + j\beta$

$\alpha = 0.00792, \quad \beta = 0.03552$

$\lambda = \frac{2\pi}{\beta} = \frac{2 \times 3.14}{0.03552}$

$$\boxed{\lambda = 176.88 \text{ Km}}$$

$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 1000}{0.03552}$

$= \frac{6283.185}{0.03552}$ Km/sec

$$\boxed{V_p = 1.76 \times 10^5 \text{ Km/sec}}$$

Q2. $R = 6\,\Omega$, $L = 2.2\,mH$, $C = 0.005\,\mu F$

$G = 0.05\,\mu \mho$.

$\omega = 2\pi \times 1000$

$Z = R + j\omega L$

$= 6 + j(2\pi \times 1000 \times 2.2 \times 10^{-3})$

$= 6 + j(13.823)$

$Z = 15.06 \angle 66.53^\circ$

$Y = G + j\omega C$

$= 0.05 \times 10^{-6} + j(2\pi \times 10^{3} \times 0.005 \times 10^{-6})$

$= 0.05 \times 10^{-6} + j(3.142 \times 10^{-5})$

$= 3.141 \times 10^{-5} \angle 89.90^\circ$

$Z_0 = \sqrt{\dfrac{Z}{Y}} = \sqrt{\dfrac{15.06 \angle 66.53^\circ}{3.141 \times 10^{-5} \angle 89.90^\circ}}$

$= \sqrt{4.79 \times 10^{5} \angle -23.32^\circ}$

$$\boxed{Z_0 = 692.098 \angle -11.685^\circ}$$

$$\gamma = \sqrt{ZY}$$

$$= \sqrt{15.06 \angle 66.53^\circ \times 3141 \times 10^{-5} \angle 89.90^\circ}$$

$$= \sqrt{47.30 \times 10^{-5} \angle 156.43^\circ}$$

$$= 0.0217 \angle 78.215^\circ$$

$$\boxed{\gamma = 0.0044 + 0.0212j}$$

$\alpha = 0.0044$ nepers/km

$\beta = 0.0212$ radians/km.

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0212} = 296.37 \text{ km}$$

$$\boxed{\lambda = 296.37 \text{ km}}$$

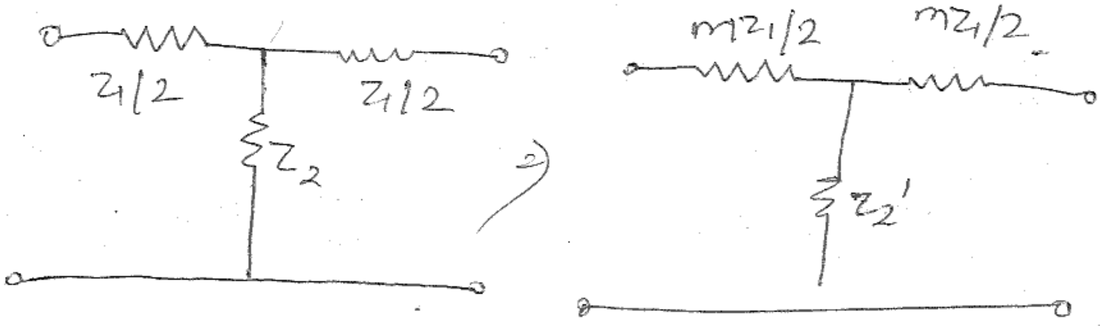
$$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 1000}{0.0212}$$

$$\boxed{V_p = 2.96 \times 10^5 \text{ km/sec}}$$ miles

10/8/11

# M - derived T. Section

The attenuation characteristics of the filter and very high value of attenuation is achieved at a frequency near the cutoff frequency called frequency of infinite or high attenuation ($f_\infty$) using m - derived filters.




Prototype m - derived.

Consider T n/w as the series arm modified by some constant m and shunt impedence $Z_2$ modified by $Z_2'$

W.K.T

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad \text{— (1)}$$

For modified n/w.

$Z_1 = mZ_1$ and $Z_2 = Z_2'$, sub in (1)

$$Z_{0T} = \sqrt{\frac{m^2 Z_1^2}{4} + mZ_1 \cdot Z_2'} \quad \text{— (2)}$$

Sub in $Z_{0T}$ and Equate.

$$\sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

Squaring both sides

$$\frac{m^2 Z_1^2}{4} + m Z_1 Z_2' = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$\frac{Z_1^2}{4}(m^2-1) + m Z_1 Z_2' = Z_1 Z_2 .$$

$$m Z_1 Z_2' = \frac{4 Z_1 Z_2 (m^2-1)}{Z_1^2}$$

$$Z_2' = \frac{4 Z_1 Z_2 (m^2-1)}{m Z_1^2}$$

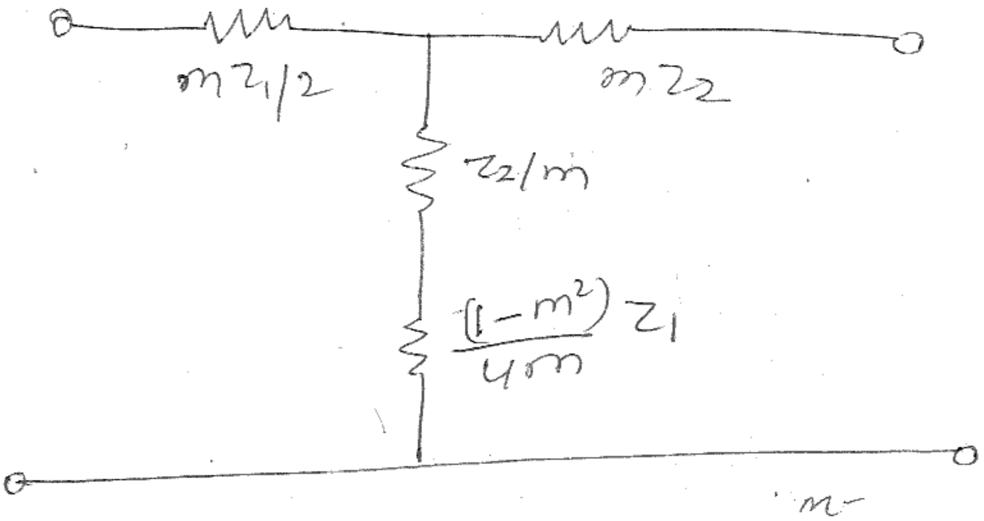
$$Z_2' = \frac{4 Z_2 (m^2-1)}{m Z_1^2}$$

$$m Z_1 Z_2' = Z_1 Z_2 - \frac{Z_1^2}{4}(m^2-1)$$

$$Z_2' = \frac{Z_2}{m} + \frac{(1-m^2) Z_1^2}{4m}$$

The above equⁿ gives than $Z_2'$ must impedence $\frac{Z_2}{m}$ in series with an impedence $\frac{(1-m^2) Z_1}{4m}$.

and these can be physically realisable if $0 < m < 1$



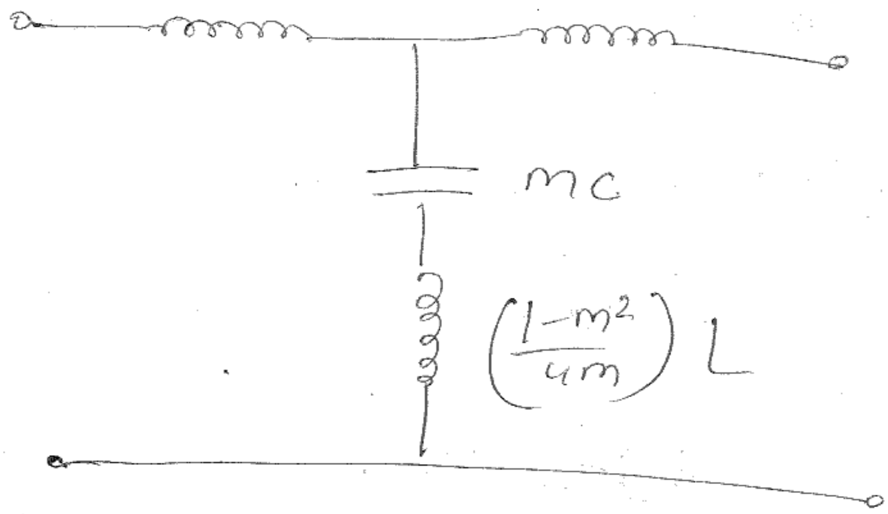

This is the complete m-derived T section.

## M-derived LPF

Consider the shunt arm of T-section Resonates at the frequency of infinite impedence ie. $f_\infty$ which is selected just above cutoff frequency $f_c$. Hence the frequency of resonance is given by

$$\left|\frac{Z_2}{m}\right| = \left|\frac{(1-m^2)\cdot Z_1}{4m}\right| \quad — (1)$$

for LPF, $Z_1 = j\omega L$, $Z_2 = -j/\omega C$




Sub $Z_1 = j2\pi f_\infty L$, $Z_2 = \frac{-j}{2\pi f_\infty C}$ in (1)

$$\left|\frac{Z_2}{m}\right| = \left(\frac{-j}{2\pi f_\infty C m}\right) = \left|\frac{(1-m^2)}{4m} j 2\pi f_\infty L\right|$$

$$\frac{1}{m\, 2\pi f_\infty C} = \frac{(1-m^2)}{4m} \cdot 2\pi f_\infty L$$

$$f_\infty^2 = \frac{\cancel{4m}}{\cancel{4}\pi^2 \cancel{m}(1-m^2) LC}$$

$$f_\infty^2 = \frac{1}{\pi^2 (1-m^2) LC}$$

$$\boxed{f_\infty = \frac{1}{\pi\sqrt{(1-m^2) LC}}} \quad \text{— (2)}$$

∴ cutoff frequency of LPF is

$$f_c = \frac{1}{\pi\sqrt{LC}} \rightarrow (3)$$

$$f_\infty = \frac{f_c}{\sqrt{(1-m^2)}}$$

$$f_\infty^2 = \frac{f_c^2}{1-m^2}$$

$$1-m^2 = \frac{f_c^2}{f_\infty^2}$$

$$1 - \frac{f_c^2}{f_\infty^2} = m^2$$

$$m^2 = 1 - \left(\frac{f_c}{f_\infty}\right)^2$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

Variation of attenuation constant ($\alpha$) and phase constant ($\beta$) with frequency.

The variation of attenuation over the attenuation band for a LPF m-derived section in the stop band is dependent on the sign of the reactance.

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} \quad \text{for } f_c < f < f_\infty$$

$$\alpha = 2\sinh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} \quad \text{for } f_\infty < f$$

For $Z_1 = j\omega L$ and $Z_2 = \frac{-j}{\omega C}$, then

$$\left|\frac{Z_1}{4Z_2}\right| = \left|\frac{j\omega L \cdot \omega C}{-\cancel{j}\cdot 4}\right|$$

$$= \frac{\omega^2 LC}{4}$$

In terms of m derived

$$\left|\frac{Z_1}{4Z_2}\right| = \frac{Z_1 m}{4 Z_2'}$$

$$\left|\frac{j\omega L\omega C}{4(-j)}\right| = \frac{m^2 \cancel{j} \cancel{Z_1}}{4\left[\frac{Z_2}{m} + \frac{(1-m^2)Z_1^2}{4m}\right]}$$

$$\frac{\omega^2 LC}{4} = \frac{m^2 Z_1}{\frac{4Z_2}{m} + \frac{(1-m^2)Z_1^2}{m}}$$

$$\frac{\omega^2 LC}{4} = \frac{m^2 Z_1 \cancel{\times m}}{4Z_2 + (1-m^2)Z_1^2}$$

$$\frac{\omega^2 LC}{4} = \frac{m^2 Z_1}{4Z_2 + (1-m^2)Z_1^2}$$

$$\frac{\omega^2 LC}{4} = \frac{m^2 (j\omega L)}{-\frac{4j}{\omega C} + (1-m^2)\, j\omega L}$$

$$\frac{\omega^2 LC}{4} = \frac{j\, \omega L \cdot m^2\, \omega C}{-4j + (1-m^2)\, j\omega^2 LC}$$

$$\frac{\omega^2 LC}{4} = \frac{m^2 \cancel{j} \cdot \omega^2 LC}{\cancel{j}\,(-4 + (1-m^2)\,\omega^2 LC)}$$

$$\frac{\cancel{\omega^2 LC}}{4} = \frac{m^2 \cancel{\omega^2 LC}}{(1-m^2)\,\omega^2 LC - 4}$$

$$\frac{1}{4} = \frac{m^2}{(1-m^2)\,\omega^2 LC - 4}$$

$$4m^2 = (1-m^2)\,\omega^2 LC - 4$$

$$4(m^2+1) = (1-m^2)\,\omega^2 LC$$

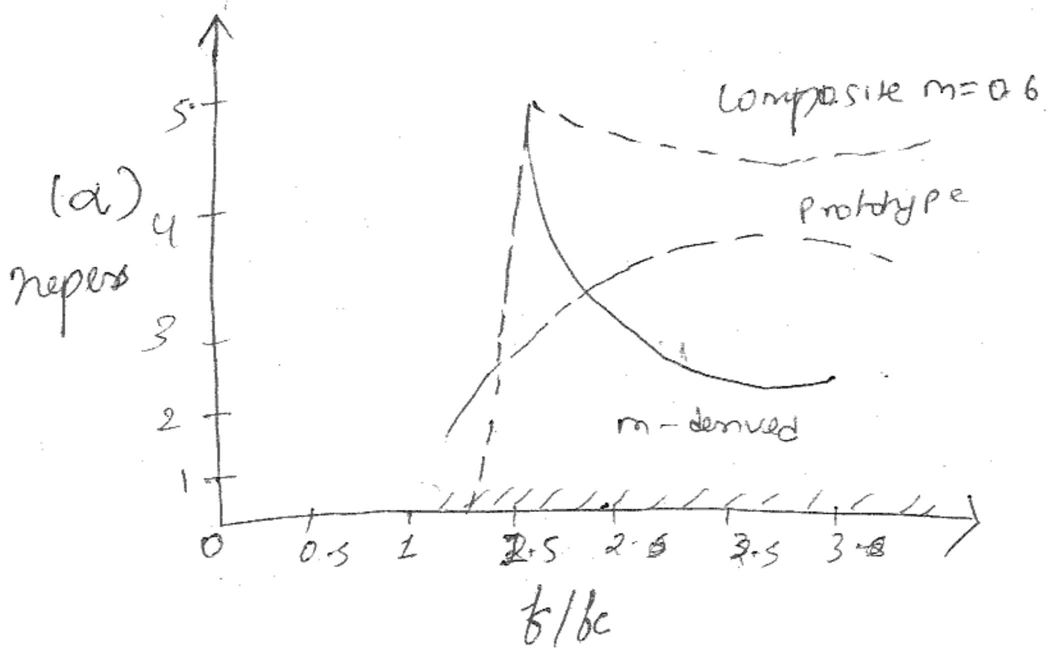
$$\omega^2 = \frac{4(m^2+1)}{(1-m^2)\,LC}$$

$$(2\pi f_\infty)^2 = \frac{4(m^2+1)}{(1-m^2)\,LC}$$

$$4\pi^2 f_\infty^2 = \frac{4(m^2+1)}{(1-m^2)\,LC}$$

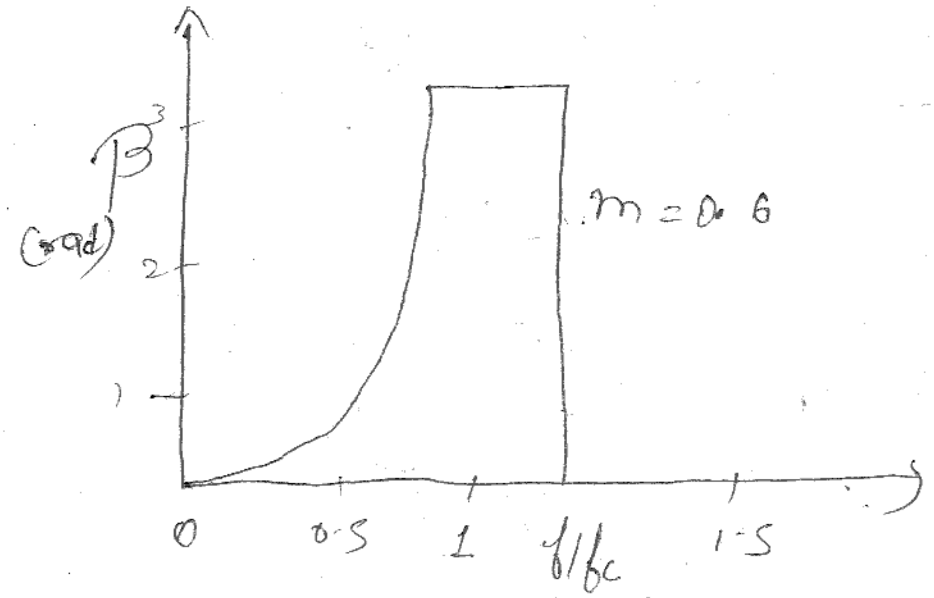
$$f_\infty^2 = \frac{1}{\pi^2}\,\frac{(m^2+1)}{(1-m^2)\,LC}$$

$$f_\infty = \frac{1}{\pi}\sqrt{\frac{m^2+1}{(1-m^2)\,LC}}$$

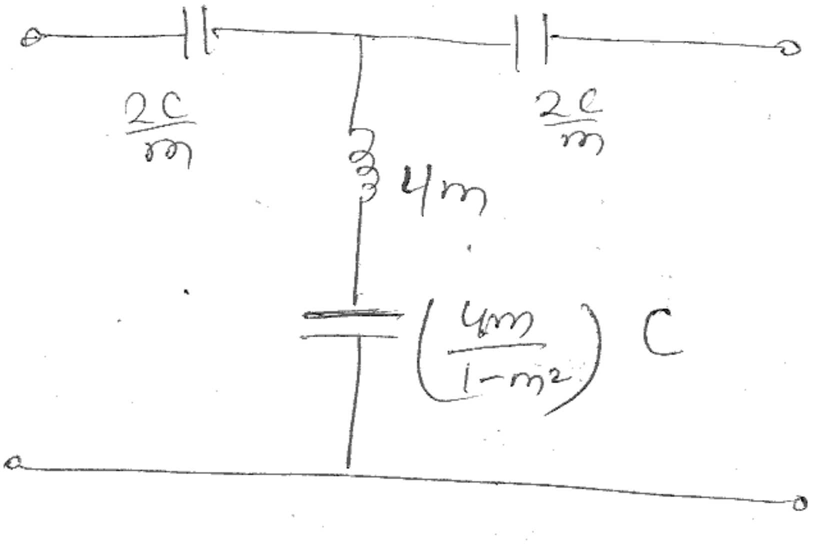



IIIr'ly for phase constant determine in Passband given by $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$ and

$$\beta = 2\sin^{-1}\frac{m f/f_c}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1-m^2)}}$$



# M - derived High pass filter.




Consider the Shunt arm of T-section resonates at frequency of infinite attenuation which is selected just below cutoff frequency. This frequency of resonance is given by

sub

$$Z_1 = \frac{-j}{\omega C} \quad \text{and} \quad Z_2 = j\omega L$$

$$\left|\frac{Z_2}{m}\right| = \left|\frac{(1-m^2)Z_1}{4m}\right|$$

$$\left|\frac{j\omega L}{m}\right| = \left|\frac{(1-m^2)(-j)}{\omega C \cdot 4m}\right|$$

$$\frac{2\pi f_\infty L}{m} = \frac{(1-m^2)}{4m\,2\pi f_\infty C}$$

$$f\infty^2 = \frac{(1-m^2) \times m}{4m\, 2\pi C \times 2\pi L}$$

$$f\infty^2 = \frac{(1-m^2)}{16\pi^2 LC}$$

$$f\infty = \frac{1}{4\pi}\sqrt{\frac{1-m^2}{LC}}$$

$$fc = \frac{1}{4\pi\sqrt{LC}}$$

$$\left|\frac{Z_2}{m}\right| = \sqrt{1-m^2}$$

$$f\infty = fc\sqrt{1-m^2}$$

Squaring both side

$$f\infty^2 = fc^2(1-m^2)$$

$$f\infty^2 = fc^2 - m^2 fc^2$$

$$m^2 fc^2 = fc^2 - f\infty^2$$

$$m^2 = 1 - \left(\frac{f\infty}{fc}\right)^2$$

$$\boxed{m = \sqrt{1-\left(\frac{f\infty}{fc}\right)^2}}$$

Variation of attenuation constant $\alpha$ and phase constant $\beta$ with frequency.

$$\alpha = 2\cosh^{-1}\left[\frac{m\,(f_c/f)}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}\right] \quad f_\infty < f < f_c$$

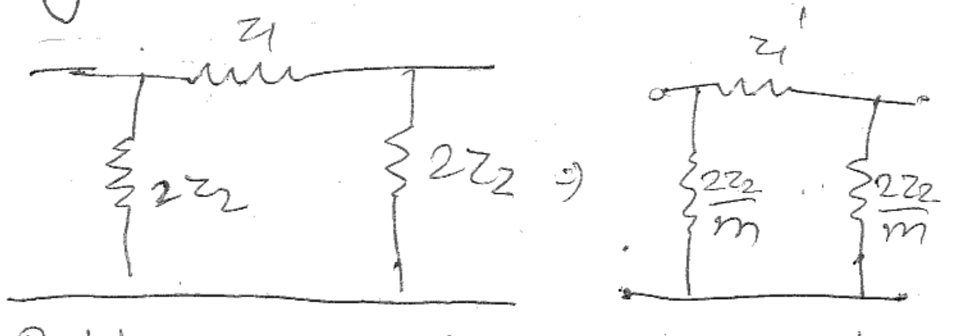
$$\alpha = 2\sinh^{-1}\left[\frac{m\,(f_c/f)}{\sqrt{\left(\frac{f_\infty}{f}\right)^2 - 1}}\right] \quad f < f_\infty$$

For phase constant '$\beta$'

$$\beta = 2\sinh^{-1}\left[\frac{m\,(f_c/f)}{\sqrt{1-(1-m^2)\left(\frac{f_c}{f}\right)^2}}\right]$$

M derived $\pi$ Section

Consider $\pi$ network of m-derived Section, $Z_1'$ be series impedence & $2Z_2'$ being the Shunt impedence.




Prototype - $\pi$ section m-derived.

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} \quad - (1)$$

For modified n/w

$$Z_1 = Z_1' , \quad Z_2 = \frac{Z_2}{m} \quad - \text{in } (1)$$

$$Z_{0\pi} = \sqrt{\frac{Z_1' \cdot \frac{Z_2}{m}}{1+\frac{Z_1' m}{4Z_2}}} \quad - (2)$$

$$= \sqrt{\frac{\frac{Z_1' Z_2}{m} \times 4Z_2}{4Z_2 + Z_1' m}}$$

$$Z_{0\pi} = \sqrt{\frac{4Z_1' Z_2^2}{4Z_2 m + Z_1'}} \quad - (2)$$

Equate (1) & (2)

$$\frac{Z_1 \cancel{Z_2}}{1+\frac{Z_1}{4Z_2}} = \cancel{\frac{4Z_1' Z_2^2}{4Z_2 m + Z_1'}} \quad \frac{\frac{Z_1' \cancel{Z_2}}{m}}{1+\frac{Z_1' m}{4Z_2}}$$

$$\frac{Z_1}{1+\frac{Z_1}{4Z_2}} = \frac{Z_1'}{m+\frac{Z_1'}{4Z_2}}$$

$$mZ_1 + \frac{Z_1' Z_1}{4Z_2} = Z_1' + \frac{Z_1' Z_1}{4Z_2}$$

$$z_1' = m z_1 + \cancel{\frac{z_1' z_1}{4z_2}} - \cancel{\frac{z_1' z_1}{4z_2}}$$

$$z_1' = m z_1 + \cancel{z_1' z_1}$$

$$z_1' = m z_1$$

$$z_1 \cancel{z_2} \left(1 + \frac{m z_1'}{4z_2}\right) = \frac{z_1' \cancel{z_2}}{m} \left(1 + \frac{z_1}{4z_2}\right)$$

$$z_1' = \frac{z_1 m \left(1 + \frac{m z_1'}{4z_2}\right)}{\left(1 + \frac{z_1}{4z_2}\right)}$$

$$z_1' = \frac{z_1 m (4z_2 + m z_1')}{(4z_2 + z_1)}$$

$$z_1' = \frac{4 z_1 z_2 m + m^2 z_1' z_1}{4z_2 + z_1}$$

$$z_1' = \left[z_1 m + \frac{z_1 z_1' m^2}{4z_2}\right]\left[\frac{1}{1 + \frac{z_1}{4z_2}}\right]$$

$$z_1' - \frac{z_1 z_1' m^2}{4z_2} \cdot \frac{1}{1 + \frac{z_1}{4z_2}} = \frac{z_1 m}{1 + \frac{z_1}{4z_2}}$$

$$z_1'\left(1 - \frac{z_1 m^2}{\cancel{4z_2}} \times \frac{\cancel{4z_2}}{4z_2+z_1}\right) = \frac{z_1 m}{1+\frac{z_1}{4z_2}}$$

$$z_1'\left(1 - \frac{z_1 m^2}{4z_2+z_1}\right) = \frac{4z_1 z_2 m}{4z_2+z_1}$$

$$z_1'\left[\frac{4z_2+z_1-z_1 m^2}{\cancel{4z_2+z_1}}\right] = \frac{4z_1 z_2 m}{\cancel{4z_2+z_1}}$$

$$\cancel{z_1' = 4z_1 z_2 m}$$

Divide by $4m$.

$$z_1' = \frac{z_1 z_2}{\frac{z_1(1-m^2)}{4m} + \frac{\cancel{4}z_2}{\cancel{4}m}}$$

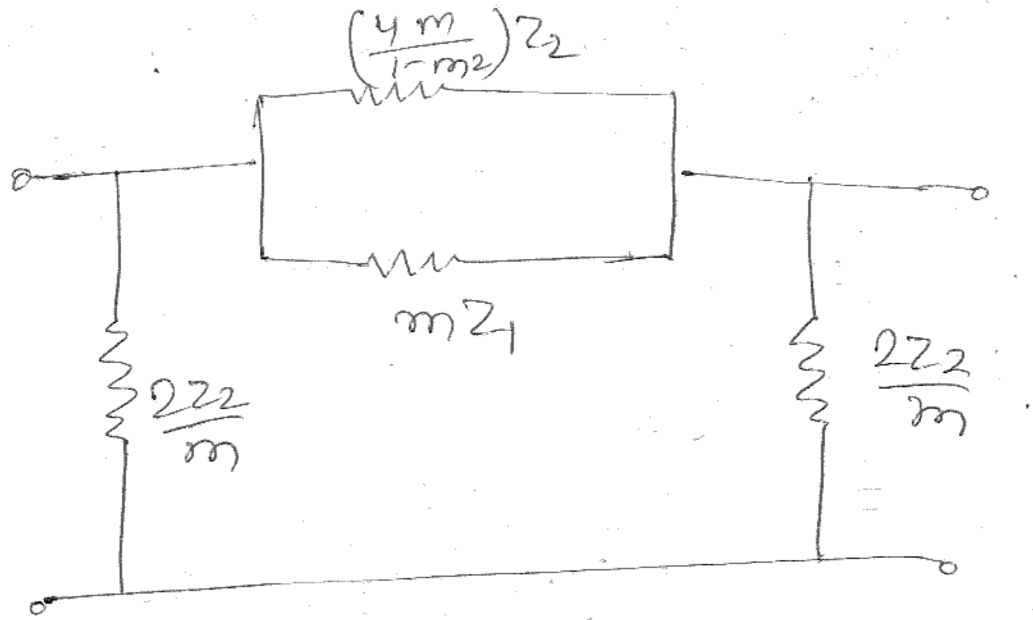
$$z_1' = \frac{z_1 z_2}{\frac{z_2}{m} + \frac{z_1}{4m}(1-m^2)}$$

multiply num & deno by $\frac{4m^2}{1-m^2}$

$$z_1' = \frac{z_1 z_2\left(\frac{4m^2}{1-m^2}\right)}{\frac{z_2}{\cancel{m}} \times \frac{4m^2}{1-m^2} + \frac{z_1}{\cancel{4m}}\cancel{(1-m^2)} \times \frac{\cancel{4}m^2}{\cancel{1-m^2}}}$$

$$z_1' = \frac{z_1 z_2\left(\frac{4m^2}{1-m^2}\right)}{z_1 m + \frac{4z_2 m}{(1-m^2)}}$$

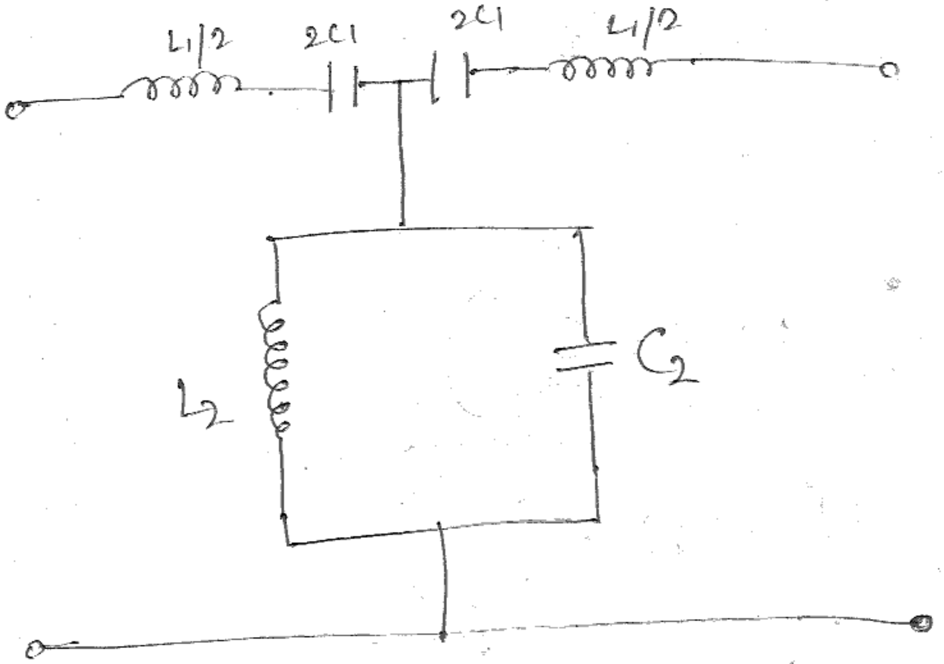
$$\boxed{Z_1' = \frac{mZ_1 \cdot \frac{4m}{(1-m^2)} Z_2}{Z_1 m + \frac{4m}{(1-m^2)} \cdot Z_2}}$$




The above equation $Z_1'$ it is a combination of two parallel impedences namely $Zm_1$ & $\frac{4m}{1-m^2} Z_2$.

17/8/11

# BPF




It consists of two parallel L & C connected. Hence at Resonance for each of the Series and shunt arm

$$\omega_0^2 L_1 C_1 = 1 \quad - (1)$$

$$\omega_0^2 L_2 C_2 = 1 \quad - (2)$$

Series impedance is $Z_1$ & shunt impedance is $Z_2$.

Equate (1) & (2)

$$\omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2 = 1$$

$$L_1 C_1 = L_2 C_2$$

$$\boxed{\frac{L_1}{C_2} = \frac{L_2}{C_1}}$$

The impedence of the arms are

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_1 = j\omega L_1 - \frac{j}{\omega C_1}$$

$$\boxed{Z_1 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)}$$

$$Z_2 = \frac{j\omega L_2\left(\frac{-j}{\omega C_2}\right)}{j\omega L_2 - \frac{j}{\omega C_2}}$$

$$= \frac{\frac{L_2}{C_2}}{j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)}$$

$$Z_2 = \frac{\frac{L_2}{C_2} \times \omega C_2}{j(\omega^2 L_2 C_2 - 1)} = \frac{\omega L_2}{j(\omega^2 L_2 C_2 - 1)}$$

$$\boxed{Z_2 = \frac{+j\omega L_2}{1 - \omega^2 L_2 C_2}}$$

W.K.T for constant K - filter

$$Z_1 Z_2 = R_0^2$$

$$Z_1 Z_2 = \left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(\frac{j\omega L}{1-\omega^2 L_2 C_2}\right)$$

$$= j\left(\frac{\omega^2 L_1 C_1 - 1}{\cancel{j}\omega C_1}\right)\left(\frac{j\omega L_2}{1-\omega^2 L_2 C_2}\right)$$

$$= \frac{(1-\omega^2 L_1 C_1)\,\omega L_2}{\omega C_1 (1-\omega^2 L_2 C_2)}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} \cdot \frac{(1-\omega^2 L_1 C_1)}{(1-\omega^2 L_2 C_2)}$$

$$\{\because L_1 C_1 = L_2 C_2\}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} \cdot \frac{\cancel{(1-\omega^2 L_2 C_2)}}{\cancel{(1-\omega^2 L_2 C_2)}}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} = R_0^2 \quad \text{— (3)}$$

(or)

$$\frac{L_1}{C_2} = R_0^2 \quad \text{— (3a)}$$

$$\boxed{R_0 = \sqrt{\frac{L_2}{C_1}}} \quad \text{— 3(b)}$$

At the cutoff frequencies:

$$Z_1 = -4Z_2 \quad — (4)$$

multiply $Z_1$ in eqn (4)

$$Z_1^2 = -4Z_1Z_2$$

$$Z_1^2 = -4R_0^2$$

$$Z_1 = \pm 2jR_0 \rightarrow (5)$$

So that $Z_1$ at lower cutoff frequency $f_1$ = $-Z_1$ at upper cutoff frequency $f_2$.

$$\left(j\omega_1 L_1 \mp \frac{j}{\omega_1 C_1}\right) = -\left(+j\omega_2 L_1 + \frac{(-j)}{\omega_2 C_1}\right)$$

$$\cancel{j}\left(\omega_1 L_1 \mp \frac{1}{\omega_1 C_1}\right) = \cancel{-j}\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\frac{1-\omega_1^2 L_1 C_1}{\omega_1 \cancel{C_1}} = \frac{\omega_2^2 C_1 L_1 - 1}{\omega_2 \cancel{C_1}}$$

$$(1-\omega_1^2 L_1 C_1) = \frac{\omega_1}{\omega_2}(\omega_2^2 C_1 L_1 - 1) \quad — (6)$$

from (1)

$$\omega_0^2 L_1 C_1 = 1$$

$$L_1 C_1 = \frac{1}{\omega_0^2} \quad \text{— (7)}$$

sub (7) in (6),

$$\left(1 - \omega_1^2 \cdot \frac{1}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$\left(1 - \frac{\omega_1^2}{(\omega_0)^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$\left[1 - \left(\frac{f_1}{f_0}\right)^2\right] = \frac{f_1}{f_2}\left[\left(\frac{f_2}{f_0}\right)^2 - 1\right]$$

$$\frac{f_0^2 - f_1^2}{\cancel{f_0^2}} = \frac{f_1}{f_2}\left(\frac{f_2^2 - f_0^2}{\cancel{f_0^2}}\right)$$

$$f_0^2 - f_1^2 = \frac{f_1}{f_2}\left(f_2^2 - f_0^2\right)$$

$$f_0^2 - f_1^2 = f_1$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = \frac{f_1 f_2}{f_0^2} - \frac{f_1}{f_2}$$

$$1 + \frac{f_1}{f_2} = \frac{f_1 f_2}{f_0^2} + \frac{f_1^2}{f_0^2}$$

$$1+\frac{f_1}{f_2} = \frac{f_1 f_2 + f_1^2}{f_0^2}$$

$$f_0^2 = \frac{\frac{f_1 f_2 + f_1^2}{\cancel{f_0^2}}}{1+\frac{f_1}{f_2}}$$

$$f_0^2 = \frac{(f_1 f_2 + f_1^2) f_2}{f_2 + f_1}$$

$$= \frac{f_1 \cancel{(f_1 + f_2)}}{\cancel{(f_2 + f_1)}} f_2$$

$$f_0^2 = f_1 f_2$$

$$\boxed{f_0 = \sqrt{f_1 \cdot f_2}} \quad -(8)$$

## Design of BPF

At lower cutoff frequency

* $Z_1 = -j2R$

$$Z_1 = j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -j2R.$$

$$\cancel{-j}\left(-\omega_1 L_1 + \frac{1}{\omega_1 C_1}\right) = \cancel{-j}\, 2R$$

$$\frac{1 - \omega_1^2 L_1 C_1}{\omega_1 C_1} = 2R$$

from eqn (7) $L_1 C_1 = \frac{1}{\omega_0^2}$

$$1 - \omega_1^2 L_1 C_1 = 2R\omega_1 C_1$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2R\omega_1 C_1$$

$\Rightarrow$ $\cancel{\omega_0^2 - \omega_1^2}$

$$1 - \frac{f_1^2}{f_0^2} = 2R \times 2\pi f_1 C_1$$

$$\Rightarrow \quad 1 - \frac{f_1^2}{f_0^2} = 4\pi R f_1 C_1$$

$$C_1 = \frac{f_0^2 - f_1^2}{4\pi R f_1 f_0^2}$$

$$C_1 = \frac{f_1 f_2 - f_1^2}{4\pi R f_1 \cdot f_1 f_2} \qquad \{f_0 = \sqrt{f_1 f_2}\}$$

$$C_1 = \frac{f_1 (f_2 - f_1)}{4\pi R f_1^2 f_2}$$

$$\boxed{C_1 = \frac{f_2 - f_1}{4\pi R f_1 f_2}} \quad - \quad (9)$$

from equ$^n$ 7 & 8

$$L_1 C_1 = \frac{1}{\omega_0^2} \quad , \quad f_0 = \sqrt{f_1 f_2}$$

$$f_0^2 = f_1 f_2$$

$$L_1 = \frac{1}{\omega_0^2 C_1}$$

$$L_1 = \frac{1}{(2\pi f_0)^2 \left( \frac{f_2 - f_1}{4\pi R f_1 f_2} \right)}$$

$$= \frac{4\pi R f_1 f_2}{4\pi^2 f_0^2 (f_2 - f_1)}$$

$$= \frac{\cancel{4}\cancel{\pi} R \cancel{f_1 f_2}}{\cancel{4}\pi^{\cancel{2}} \cancel{f_1 f_2} (f_2 - f_1)}$$

$$\boxed{L_1 = \frac{R}{\pi (f_2 - f_1)}} \quad \text{— (10)}$$

from equ$^n$ (3)

$$\frac{L_2}{C_1} = R_0^2 \quad \text{— (11)}$$

sub $C_1$ in (11)

$$\frac{L_2 \, 4\pi R f_1 f_2}{(f_2 - f_1)} = R^2$$

$$\boxed{\therefore L_2 = \frac{R(f_2 - f_1)}{4\pi f_1 f_2}} \quad \text{— (12)}$$

from 3(9)

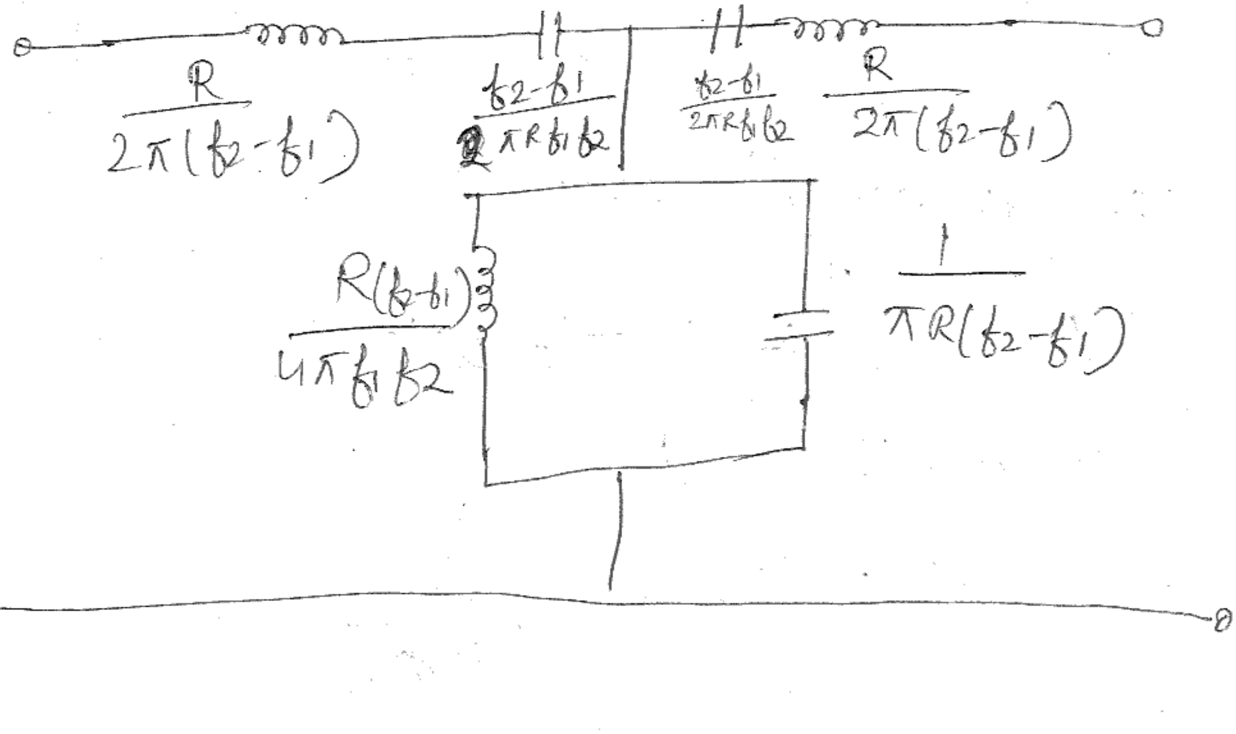
$$\frac{L_1}{C_2} = R^2 \quad \text{— (13)}$$

$$C_2 = \frac{L_1}{R^2}$$

$$C_2 = \frac{R}{\pi(f_2 - f_1) \cdot R^2}$$

$$\boxed{C_2 = \frac{1}{\pi R(f_2 - f_1)}} \quad \text{— (13)}$$

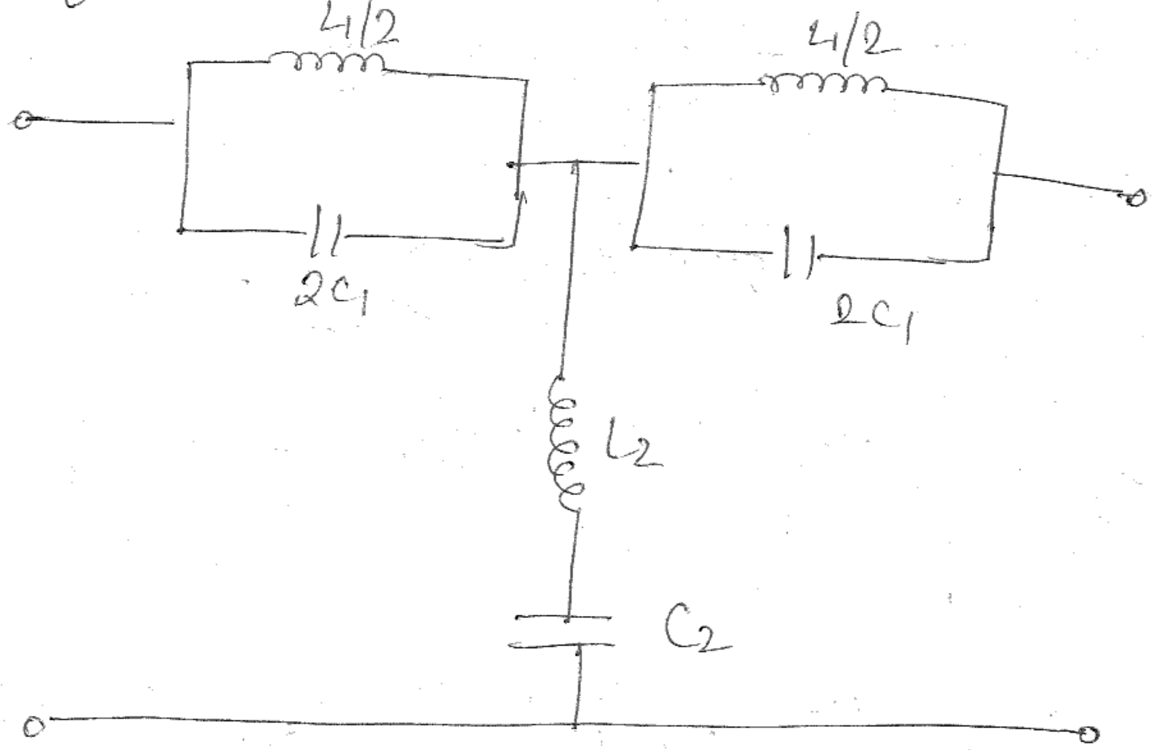
This completess the design of prototype BP filter.



# Band Elimination Filter

If the series and parallel tuned circuits or parallel tuned arms of the Band Pass filter are interchanged, then the result is Band Elimination filter.

Here the cutoff frequency of HPF is higher than the cutoff frequency of LPF




At frequency response

$$\omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2 = 1 \quad (1)$$

from eqn (1)

$$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0^2} \quad (2)$$

$$\frac{L_1}{C_2} = \frac{L_2}{C_1} = \frac{R_0^2}{\cancel{C_0}} \quad -(3)$$

$$f_0 = \sqrt{f_1 f_2} \quad -(4)$$

At $f_c$

$$Z_1 = -4Z_2 \quad -(5)$$

multiply $Z_2$ in equⁿ (5)

$$Z_1 Z_2 = -4Z_2^2$$

$$\cancel{Z_1 Z_2 =} \quad R_0^2 = -4Z_2^2$$

$\cancel{Z_2 =}$

$$-\frac{R_0^2}{4} = Z_2^2$$

$$\boxed{Z_2 = \pm\frac{j}{2} R_0} \quad -(6)$$

If the ~~term~~ filter is terminated in a load $R = R_0$, then lower cut-off frequency

$$Z_2 = \frac{j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right)}{1 - \omega_1^2 L_2 C_2} = \frac{jR_0}{2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{2\pi f_1 C_2 R}{2} \quad \left[\because L_2 C_2 = \frac{1}{\omega_0^2}\right]$$

$$1 - \frac{f_1^2}{f_0^2} = \pi f_1 C_2 R.$$

$$C_2 = \frac{f_1 f_2 - f_1^2}{f_1 f_2 \cdot \pi f_1 R} = \frac{f_1(f_2 - f_1)}{f_1^2 f_2 \pi R}.$$

$$\boxed{C_2 = \frac{f_2 - f_1}{\pi f_1 f_2 R}} \quad \text{— (7)}$$

from eqn (2) & (4)

$$L_2 C_2 = \frac{1}{\omega_0^2} \text{ — (8)}, \qquad f_0 = \sqrt{f_1 f_2}$$

w.k.T $\omega_0^2 = (2\pi f_0)^2$.

$$L_2 = \frac{1}{4\pi^2 f_0^2 C_2}.$$

$$= \frac{1 \cdot \pi \cancel{f_1 f_2} R}{4\pi^2 \cancel{f_1 f_2} (f_2 - f_1)}$$

$$\boxed{L_2 = \frac{R}{4\pi (f_2 - f_1)}} \quad \text{— (9)}$$

$\overset{|||}{}$ly the values for the series arm obtained as

$$\frac{L_1}{C_2} = R_0^2.$$

$$\frac{L_1}{\frac{f_2-f_1}{\pi f_1 f_2 R}} = R^2$$

$$L_1 = \cancel{R^2}.\ \frac{f_2-f_1}{\pi f_1 f_2 \cancel{R}}$$

$$\boxed{L_1 = \frac{R(f_2-f_1)}{\pi f_1 f_2}} \quad \text{— (10)}$$

$$\frac{L_2}{C_1} = R_0^2$$

$$C_1 = \frac{L_2}{R_0^2}$$

$$= \frac{\cancel{R}}{4\pi(f_2-f_1)R^{\cancel{2}}}$$

$$\boxed{C_1 = \frac{1}{4\pi R(f_2-f_1)}} \quad \text{— (11)}.$$

equn (7), (9), (10) (11), computes the design the prototype Band Elimination filter.

The Resistance of a Copper coaxial line is given by $R_{ac} = 4.16 \times 10^{-8} \sqrt{f}\left(\frac{1}{b}+\frac{1}{a}\right)$ $\Omega/m$.

where a and b are Outer radius of inner conductor and inner radius of outer conductor respectively.

The Shunt Susceptance and power factor are

$$Y = G + j\omega C$$

$$P.F = \frac{G}{\sqrt{G^2+\omega^2C^2}}$$

If $g << \omega C$ then

$$P.F = \frac{g}{\omega C}$$

$$\boxed{G = \omega C \times PF}$$

Dissipation factor:- The ratio of Energy dissipated and to the Energy Stored in the Dielectric per cycle and it is Proportional to the tangent of angle $\phi$. If $g >> \omega C$, the dissipation factor and power factor are equal in magnitude.

## Constants for the line of zero-dissipation

For transmission of energy at high frequencies where the power efficiency is high, it is possible to assume negligible losses are zero.

The line parameters of the line of zero dissipation are

$$Z = j\omega L, \quad Y = j\omega C$$

Characteristics impedance $Z_0 = \sqrt{\frac{Z}{Y}}$

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}\ \Omega}$$

Propagation constant, $\gamma = \sqrt{ZY}$

$$\gamma = \sqrt{j\omega L \cdot j\omega C} = \sqrt{-1\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}} \quad -(1)$$

W.K.T

$$\gamma = \alpha + j\beta \quad \text{—} \ (2)$$

$$\alpha = 0, \quad \beta = \omega\sqrt{LC}$$

Velocity of propagation $V_p = \frac{\omega}{\beta}$

$$= \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\boxed{V_p = \frac{1}{\sqrt{LC}} \ m/sec}$$ $[\because V = 3 \times 10^8 \ m/s]$

wavelength $\lambda = \frac{2\pi}{\beta}$

$$\boxed{\lambda = \frac{2\pi}{\omega\sqrt{LC}}} = \frac{2\pi}{2\pi f\sqrt{LC}}$$

$$\lambda = \frac{1}{f\sqrt{LC}}$$

## Voltages and currents on the dissipation line

The voltage at any point distant S units from the receiving end of a transmission line is

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\gamma s} + K e^{-\gamma s}\right) \quad \text{—(1)}$$

For zero dissipation line, $\alpha = 0$, and $Z_0 = R_0$

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{j\beta s} + K e^{-j\beta s}\right) \quad \text{—(2)}$$

where $e^{j\beta s}$ is nothing but incident wave moves from source to load.

$e^{-j\beta s}$ is reflected wave moves from load to source (L–S).

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R} e^{j\beta s} + K \cdot \frac{E_R(Z_R+Z_0)}{2Z_R} e^{-j\beta s}.$$

$$= \underbrace{\frac{E_R Z_R}{2Z_R}\left(e^{j\beta s} + K e^{-j\beta s}\right)}_{K=1} + \frac{E_R Z_0}{2Z_R}\underbrace{\left(e^{j\beta s} + K e^{-j\beta s}\right)}_{K=-1}$$

$$= \frac{E_R Z_R}{Z_R}\left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + j\frac{E_R Z_0}{Z_R}\left\{\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right\}$$

$$= \frac{E_R Z_R}{Z_R}\left(\cosh \beta s\right) + j\frac{E_R Z_0}{Z_R}\sinh \beta s.$$

$$E = E_R(\cos \beta s) + j I_R Z_0 \sin \beta s \quad -(3)$$

Ill^ly

$$I = I_R \cos \beta s + j \frac{E_R}{Z_0} \sin \beta s \quad -(4)$$

W.K.T

$$\beta = 2\pi/\lambda \quad \text{in equ}^n \ (3) \ \& \ (4)$$

$$E = E_R\left(\cos \frac{2\pi}{\lambda} s\right) + j I_R Z_0 \sin \frac{2\pi}{\lambda} s$$

$$I = I_R\left(\cos \frac{2\pi}{\lambda} s\right) + j \frac{E_R}{Z_0} \sin \frac{2\pi}{\lambda} s$$

Therefore, the voltage and current distribution is the sum of cosine & sine distribution.

If the line is open circuited, $I_R = 0$ then.

$$E_{oc} = E_R \cos \frac{2\pi}{\lambda} s$$

$$I_{oc} = j \frac{E_R}{Z_0} \sin \frac{2\pi}{\lambda} s$$

If the line is short circuited $E_R = 0$, then

$$E_{sc} = j I_R Z_0 \sin \frac{2\pi}{\lambda} s.$$

$$I_{sc} = I_R\left(\cos \frac{2\pi}{\lambda} s\right)$$

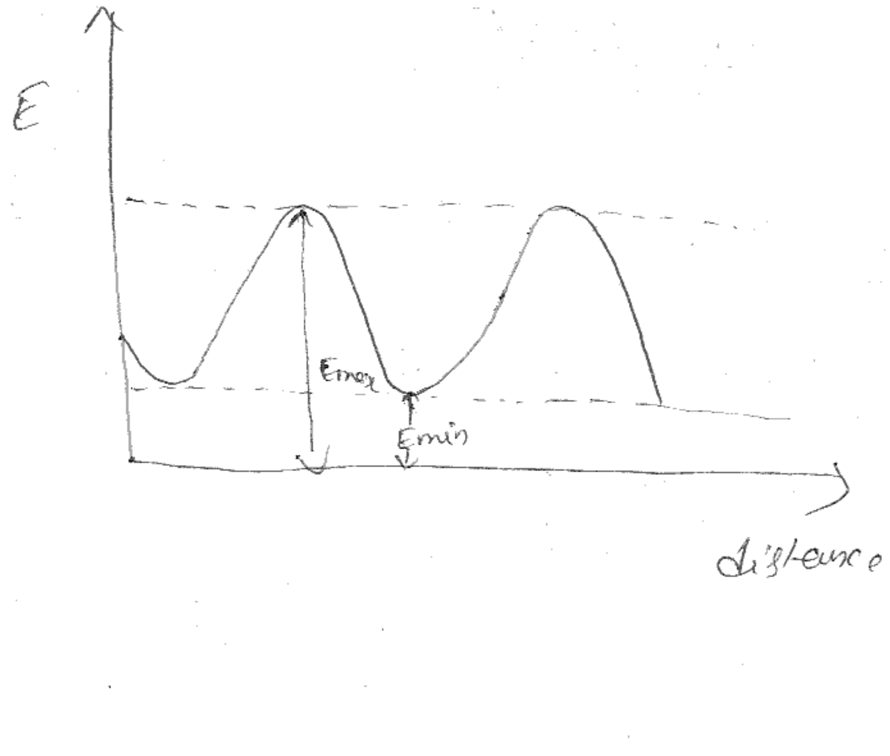
From equⁿ (2) the incident and Reflected voltage waves, if the line is terminated in $Z_R = Z_0$, the reflection coefficient & reflected waves becames zero.

$$E = E_R \cdot e^{j\beta s}$$

$$I = I_R \cdot e^{j\beta s}$$

2mark

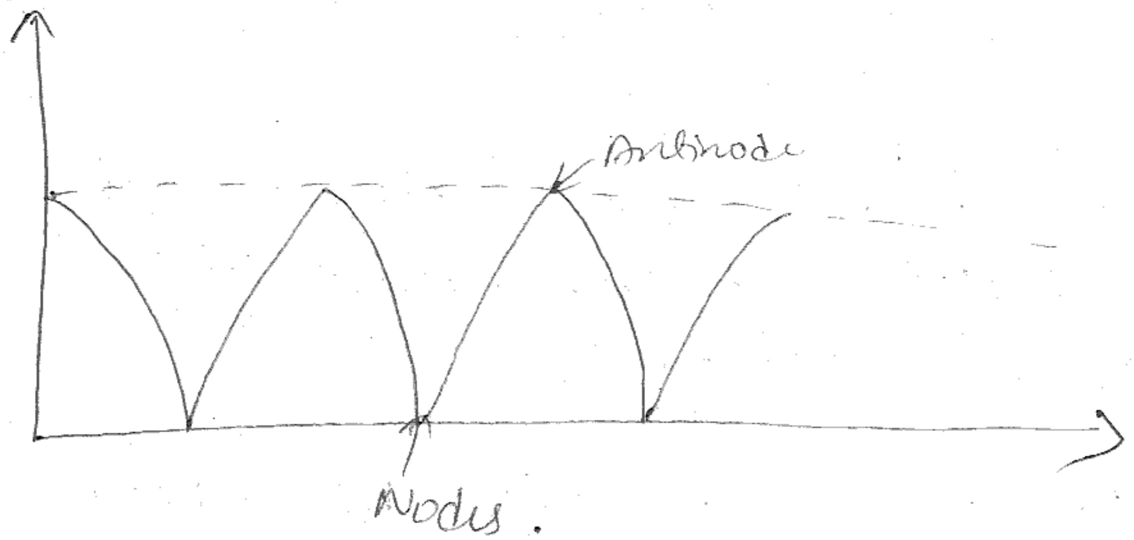
Standing waves :- The total voltage or current wave appears to stand on the line, oscillating in magnitude with time but having fixed position of maxima and minima, such a wave is known as standing waves.



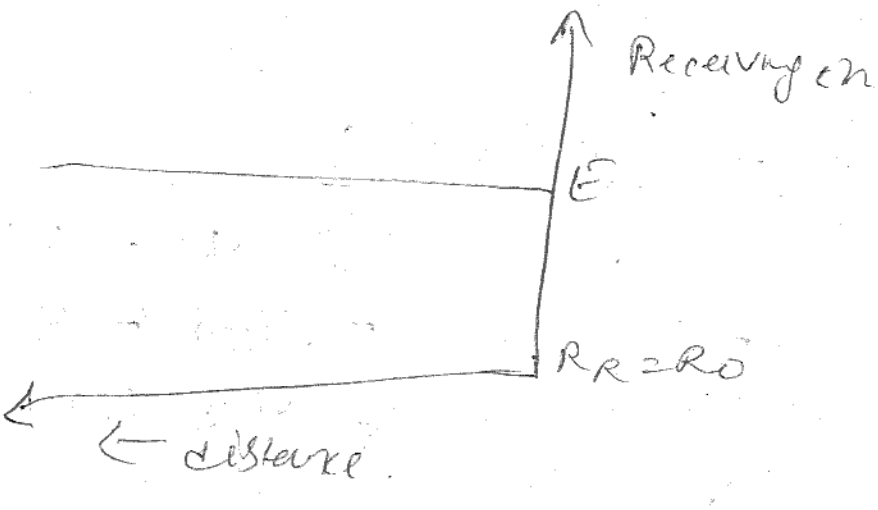
Nodes and Antinodes ;-

The point along the line where the magnitude of voltage or current is zero is called Nodes.

The point along the line where magnitude of voltage or current is maximum are called Antinodes




Smooth line ;- When a line is terminated in $Z_0$, the standing waves are absent. such a line is smooth line



Standing wave Ratio (SWR) :-

The ratio of the maximum to minimum magnitudes of current or voltage on a line having standing waves is called as Standing wave Ratio. It is denoted by S.

$$S = \frac{|E_{max}|}{|E_{min}|} \quad \text{or} \quad S = \frac{|I_{max}|}{|I_{min}|}$$

The standing wave Ratio S is measured by RF voltmeter across the line at a point.

The ratio of Emax to Emin is reffered as VSWR.

llly the ratio of Imax to Imin is reffered as ISWR.

24/8/11

<u>One Eight wave line</u>

For the transmission line, the voltage and current at any point distant x from the receiving end of the transmission line is given by

$$V = \frac{V_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma x} + K e^{-\gamma x}\right]$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\gamma x} - K e^{-\gamma x}\right]$$

The term with $\gamma x$ is defined as the incident wave progressing forward from the source to the load whereas the term involving $-\gamma x$ is the reflected wave ~~transm~~ travelling from the load back ward towards the source.

For the line of zero dissipation, the attenuation constant $\alpha = 0$

i.e. $\gamma = j\beta$, $Z_0 = R_0$ (zero dissipation line)

$$V = \frac{V_R(Z_R+R_0)}{2Z_R}\left(e^{j\beta x} + K e^{-j\beta x}\right)$$

III'ly

$$I = \frac{I_R(Z_R+R_0)}{2R_0}\left(e^{j\beta x} - K e^{-j\beta x}\right)$$

Here $K = \dfrac{Z_R - R_0}{Z_R + R_0}$

$$V_\phi = \frac{V_R\,(Z_R+R_0)}{2Z_R}\left(e^{j\beta x} + \frac{Z_R-R_0}{Z_R+R_0}e^{-j\beta x}\right)$$

$$= \frac{V_R\,(Z_R+R_0)}{2Z_R}e^{j\beta x} + \frac{V_R(Z_R+R_0)}{2Z_R}\left(\frac{Z_R-R_0}{Z_R+R_0}\right)e^{-j\beta x}$$

$$V_\phi = \frac{V_R(Z_R+R_0)}{2Z_R}e^{j\beta x} + \frac{V_R}{2Z_R}(Z_R-R_0)e^{-j\beta x}$$

$$= \frac{V_R}{2Z_R}\left[(Z_R+R_0)e^{j\beta x} + (Z_R-R_0)e^{-j\beta x}\right]$$

$$= \frac{V_R}{2Z_R}\left[Z_R(e^{j\beta x}+e^{-j\beta x}) + R_0(e^{j\beta x}-e^{-j\beta x})\right]$$

$$= \frac{V_R}{\cancel{2}Z_R}\left[Z_R\left(\frac{e^{j\beta x}+e^{-j\beta x}}{2}\right) + R_0 j\left(\frac{e^{j\beta x}-e^{-j\beta x}}{2j}\right)\right]$$

$$= \frac{V_R}{Z_R}\left[Z_R\cos\beta x + jR_0\sin\beta x\right]$$

$$V_\phi = V_R\cos\beta x + jR_0 I_R\sin\beta x.$$

IIIrly

$$I_\phi = I_R\cos\beta x + j\frac{V_R}{R_0}\sin\beta x.$$

The i/p impedence of a dissipation line is

$$Z_S = \frac{V}{I}$$

$$Z_S = \frac{V_R \cos\beta x + j R_0 I_R \sin\beta x}{I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x}$$

$\cancel{Z_S =}$

$$V_R = I_R Z_R$$

$$Z_S = \frac{I_R Z_R \cos\beta x + j R_0 I_R \sin\beta x}{I_R \cos\beta x + j \frac{I_R Z_R}{R_0} \sin\beta x}$$

$$Z_S = \frac{Z_R \cos\beta x + j R_0 \sin\beta x}{\cos\beta x + j \frac{Z_R}{R_0} \sin\beta x}$$

$$= R_0 \left[ \frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x} \right]$$

Divide $\cos\beta x$ in num & deno

$$Z_S = R_0 \left[ \frac{Z_R + j R_0 \tan\beta x}{R_0 + j Z_R \tan\beta x} \right] \quad (1)$$

For an Eight ~~wavelength~~ waveline,

$$x = \frac{\lambda}{8}, \quad \beta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

Put $\beta x = \frac{\pi}{4}$ in equ$^n$ (1)

$$Z_S = R_0 \left[ \frac{Z_R + j R_0 \tan\frac{\pi}{4}}{R_0 + j Z_R \tan\frac{\pi}{4}} \right]$$

$$\boxed{Z_S = R_0 \left[ \frac{Z_R + j R_0 \cdot 1}{R_0 + j Z_R} \right]}$$

If such a line is terminated with pure resistance i.e. $Z_R = R_R$.

$$Z_S = R_0 \left[ \frac{R_R + j R_0}{R_0 + j R_R} \right]$$

Thus the magnitude of input impedence

$$|Z_{in}| = R_0 \left[ \frac{\sqrt{R_R^2 + R_0^2}}{\sqrt{R_0^2 + R_R^2}} \right]$$

$$\boxed{|Z_{in}| = |Z_S| = R_0}$$

## Quarter wave line and impedence matching

The i/p impedence of a dissipation transmission line is.

$$Z_S = R_O \left[ \frac{Z_R + jR_O \tan\beta x}{R_O + jZ_R \tan\beta x} \right]$$

divide $\tan\beta x$ in num & deno

$$Z_S = R_O \left[ \frac{\frac{Z_R}{\tan\beta x} + jR_O}{\frac{R_O}{\tan\beta x} + jZ_R} \right]$$

$$x = \frac{\lambda}{4}, \quad \beta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}.$$

Put $\beta x = \frac{\pi}{2}$ in above eqn.

$$Z_S = R_O \left[ \frac{\frac{Z_R}{\tan\frac{\pi}{2}} + jR_O}{\frac{R_O}{\tan\frac{\pi}{2}} + jZ_R} \right]$$

$$Z_S = R_O \left[ \frac{jR_O}{jZ_R} \right]$$

$$\boxed{Z_S = \frac{R_O^2}{Z_R}}$$

A quarter wave section of a line may be considered as transformer to match a load of $Z_R$ to a source is $Z_s$. Such a match can be obtained if the characteristics impedence $R_0'$ of the matching quarter wave section of the line is properly choosen.

$$R_0' = \sqrt{Z_s \cdot Z_R}$$

The $R_0'$ of the matching section should be equal to geometric mean of source & load impedence.

A quarter wave transformer may also be used, if the load is not a pure resistance. It should then be connected b/w points corresponding to $I_{max}$ or $V_{min}$, at which places the transformation line as resistive impedence given by $\frac{R_0}{S}$ or $R_0 \cdot S$.

→ Thus for step up down in impedence from value of $R_0$, the characteristics impedence of the matching section $R_0'$ can be given by

$$R_0' = \sqrt{R_0\left(\frac{R_0}{S}\right)}$$

$$\boxed{R_0' = \frac{R_0}{\sqrt{S}}}$$

The step up in impedence from value of Ro the characteristics impedence of matching

$$R_0' = \sqrt{R_0(R_0 \cdot S)}$$

$$\boxed{R_0' = R_0\sqrt{S}}$$

Half wave line

The i/p impedence of dissipation less line is given by

$$Z_S = R_0\left[\frac{Z_R + jR_0 \tan\beta x}{R_0 + jZ_R \tan\beta x}\right]$$

$$x = \frac{\lambda}{2}, \quad \beta x = \frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{2} = \pi$$

$$Z_S = R_0\left[\frac{Z_R + j\,\cancel{R_0 \tan}}{R_0}\right] \quad \tan\pi = 0$$

$$\boxed{Z_S = Z_R}$$