Unit - 03

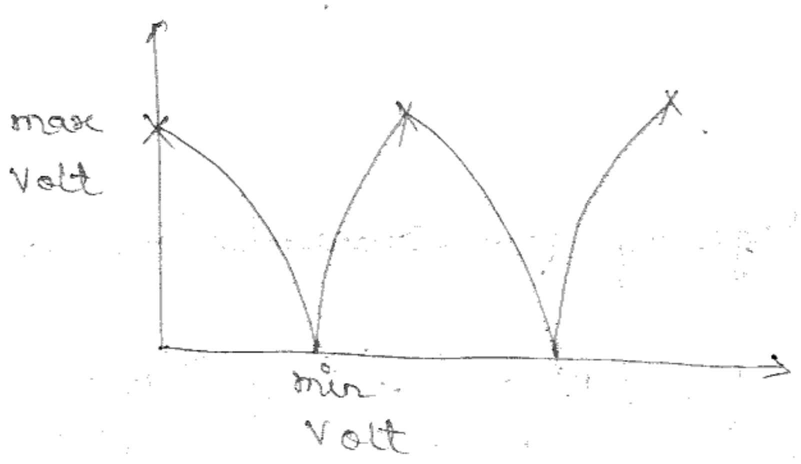
★ Standing waves and standing wave ratio

When the transmission line is not terminated by Z_0 i.e. the load impedance is not equal to characteristic impedance.

$$Z_R \neq Z_0$$

The combination of instant and reflected waves give rise to the standing wave.

If the line is not terminated by R_0 the distribution of voltage and current will be having max and mini.



2 marks The points along the line where mag. of voltage and current is zero means it is called node.

While the pts ~~to the~~ where mag. of volt and current is max it is known as anti node.

When the line is terminated by R_0 the standing waves are absent. Such line is called smooth line.

→ Standing wave ratio

It is the ratio of max to mini the mag. of voltage and current.

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| \quad \text{or} \quad \left|\frac{I_{max}}{I_{min}}\right|$$

→ Voltage Eq for transmission line

$$V = \frac{V_R}{2}\,\frac{Z_R+Z_0}{Z_R}\left[e^{\gamma x} + \frac{Z_R-Z_0}{Z_R+Z_0}\,e^{-\gamma x}\right]$$

$$V = \frac{V_R}{2}\,\frac{Z_R+Z_0}{Z_R}\left(e^{\gamma x} + K e^{-\gamma x}\right)$$

If the voltage is max means ~~it have~~ the reflec. coff is +ve and in phase.

If the voltage is mini means the reflec coff is -ve and it is out of phase.

$$V = \frac{V_R}{2} \frac{Z_R + Z_0}{Z_R} \left(e^{\gamma x} - K e^{-\gamma x}\right)$$

$$V_{max} = \frac{V_R}{2} \frac{Z_R + Z_0}{Z_R} \left(1 + |K|\right)$$

$$V_{min} = \frac{V_R}{2} \frac{Z_R + Z_0}{Z_R} \left(1 - |K|\right)$$

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| = \frac{1 + |K|}{1 - |K|}$$

or,

$$S = \frac{1 + |K|}{1 - |K|}$$

$$|K| = \frac{S - 1}{S + 1}$$

$$|K| = \frac{\left|\frac{V_{max}}{V_{min}}\right| - 1}{\left|\frac{V_{max}}{V_{min}}\right| + 1}$$

$$\boxed{|K| = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

16.8.11

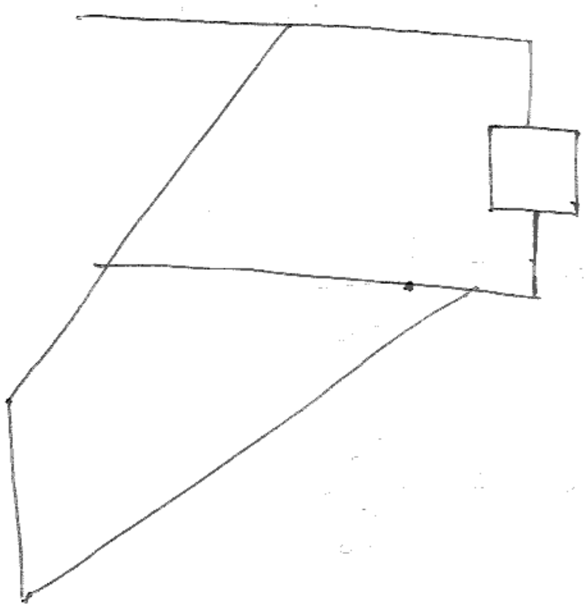
★ Stub Matching

For a maximum transfer of power the sending end impedance and receiving end impedance of a transmission line should be matched perfectly. In practical case the imp matching is not perfect. So, a stub is placed on transmission line and its length position is adjust for max. power transfer is ccalled stub matching.

→ Types

- Single stub matching
- Double stub matching

- Single stub matching



It has one stub to match the transmission line.

The input impedance

$$Z_S = Z_0 \frac{Z_R + Z_0 \tan \gamma l}{Z_0 + Z_R \tan \gamma l}$$

The input admittance of transmission line

$$Y_S = Y_0 \frac{Y_R + Y_0 \tan \gamma l}{Y_0 + Y_R \tan \gamma l}$$

If the line is zero decipation line

i.e. $\alpha = 0$

$$Y_S = Y_0 \frac{Y_R + Y_0 j \tan \beta l}{Y_0 + j Y_R \tan \beta l}$$

WKT

$$Z_{in} = \frac{Z_S}{Z_0}$$

$$Y_{in} = \frac{Y_S}{Y_0}$$

$$\therefore \frac{Y_S}{Y_0} = \frac{Y_R + Y_0 j \tan \beta l}{Y_0 + j Y_R \tan \beta l}$$

$$\therefore Y_{in} = \frac{\frac{Y_R}{Y_0} + j \tan \beta l}{1 + j \, Y_R/Y_0 \tan \beta l}$$

Take $Y_r = \frac{Y_R}{Y_0}$

$$Y_{in} = \frac{Y_r + j \tan \beta l}{1 + Y_r j \tan \beta l}$$

taking complex conjugate we get

$$Y_{in} = \frac{Y_r + j \tan \beta l}{1 + Y_r \tan \beta l j} \times \frac{1 - Y_r j \tan \beta l}{1 - Y_r j \tan \beta l}$$

$$Y_{in} = \frac{Y_r - jY_r^2 \tan\beta l + j\tan\beta l + Y_r \tan^2\beta l}{1 + Y_r^2 \tan^2 \beta l}$$

If impedance is perfectly match

where

$$Y_{in} = 1$$

The real part is giving the location of the stub and imaginary part is giving the length of the stub. When the imaginary part is equated with $Y_0 \cot\beta lt$

$$1 = \frac{Y_r(1+\tan^2\beta l_s)}{1 + Y_r^2 \tan^2 \beta l_s}$$

$$Y_r(1+\tan^2\beta l_s) = 1 + Y_r^2 \tan^2\beta l_s$$

$$Y_r \tan^2\beta l_s - Y_r^2 \tan^2\beta l_s = 1 - Y_r$$

$$\tan^2\beta l_s (Y_r - Y_r^2) = 1 - Y_r$$

$$Y_r \cancel{(1-Y_r)} \tan^2\beta l_s = \cancel{(1-Y_r)}$$

$$\tan^2\beta l_s = \frac{1}{Y_r}$$

$$\tan\beta l_s = 1/\sqrt{Y_r}$$

$$\beta l_S = \tan^{-1}\frac{1}{\sqrt{Y_r}}$$

$$\beta l_S = \tan^{-1}\sqrt{Y_0/Y_R} \qquad \left[\because Y_r = \frac{Y_r}{Y_0}\right]$$

$$l_S = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Y_0}{Y_R}}$$

Here, l_S is the location of stub

$$l_S = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_R}{Z_0}}$$

Imaginary part

$$Y_{in} = \frac{Y_S}{Y_0} = 1$$

$$Y_{in} = \frac{S}{Y_0} = \frac{(1-Y_r^2)\tan\beta l_S}{1+Y_r^2\tan^2\beta l_S}$$

$$\frac{S}{Y_0} = \frac{(1-Y_r^2)\sqrt{Y_0/Y_R}}{1+Y_r^2\,(Y_0/Y_R)}$$

$$Y_S = Y_0 \qquad Y_r = \frac{Y_R}{Y_0}$$

$$\frac{S}{Y_0} = \frac{1-\left(\frac{Y_R}{Y_0}\right)^2\sqrt{Y_0/Y_R}}{1+\left(\frac{Y_R}{Y_0}\right)^2 Y_0/Y_R}$$

$$\frac{S}{Y_o} = \frac{\cancel{\left(1+Y_R/Y_o\right)} \frac{Y_o - Y_R}{Y_o} \sqrt{Y_o/Y_R}}{\cancel{\left(1+Y_R/Y_o\right)}}$$

$= \frac{Y_r}{Y_o}$]

$$S = (Y_o - Y_R) \sqrt{\frac{Y_o}{Y_R}}$$

The S value is equating with $Y_o \cot \beta l t$

$$Y_o \cot \beta l t = (Y_o - Y_R) \sqrt{\frac{Y_o}{Y_R}}$$

$$\cot \beta l t = \frac{Y_o - Y_R}{Y_o} \sqrt{\frac{Y_o}{Y_R}}$$

$$\cot \beta l t = \frac{Y_o - Y_R}{\sqrt{Y_o Y_R}}$$

$$\cot \beta l t = \frac{1/Z_o - 1/Z_R}{\sqrt{1/Z_o \, 1/Z_R}}$$

$$= \frac{1}{Z_o} - \frac{1}{Z_R} \sqrt{Z_o Z_R}$$

$$\cot \beta l t = \frac{Z_R - Z_o}{\sqrt{Z_o Z_R}}$$

$$\tan \beta l t = \frac{\sqrt{Z_o Z_R}}{Z_R - Z_o}$$

$$\beta l_t = \tan^{-1} \frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_0 Z_R}}{Z_R - Z_0}$$

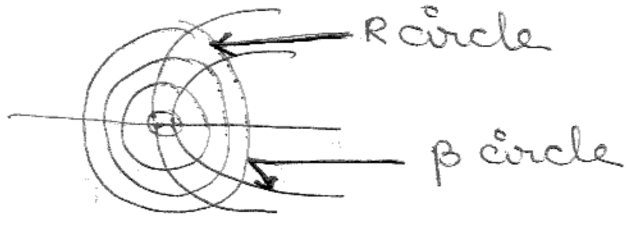
$\therefore$ It is length of stub.

★ ~~Switches~~ Smith Chart

It is a polar chart. It is used to determine SWR (standing wave ratio) and sending end impedance and load admitta[nce]. It is also used to measure ~~i/p~~ imped[ance] and admittance.

Two circles

- Const R circle
- Const X circle or β circle



Q- Determine the following standing wave ratio with load admittance

- distance b/w load and the 1st voltage mini along the transmission line for a line with char imp of 300 Ω and terminated in a load 175 + j207 Ω. An electrical signal of 200 MHz is transmitte along the line in free phase.

Sol- $Z_0 = 300\,\Omega$

Load impedance $Z_R = Z_L = 175 + j\,207\ \Omega$

Freq of elec signal = 200 MHz

wave length $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^8}{200 \times 10^6}$

$\lambda = 1.5\ m$

The normalised load imp $z_l = \dfrac{Z_R}{Z_0}$

$$z_l = \frac{175 + j207}{300} = 0.58 + j0.69$$

$$z_l = \underset{R}{0.58} + j\,\underset{\beta}{0.69}$$

After this draw a circle from $Z_L = 0.583 + j0.69$

taking it as radius of circle from center 1.0.

Now, on opp side where the circle cut the graph is ~~the~~ the value of Y

i.e. $Y = 0.72 - j0.86$

Now,

exceeding the radius to the last circle i.e. outer most one and calculated the

length of stub $= 0.5 - 0.115 = 0.385$

where

0.5 is the radius of the outer most circle.

Now,

distance b/w load imp and 1st voltage minimum $= 1.5 \times 0.385 = 0.579$ m

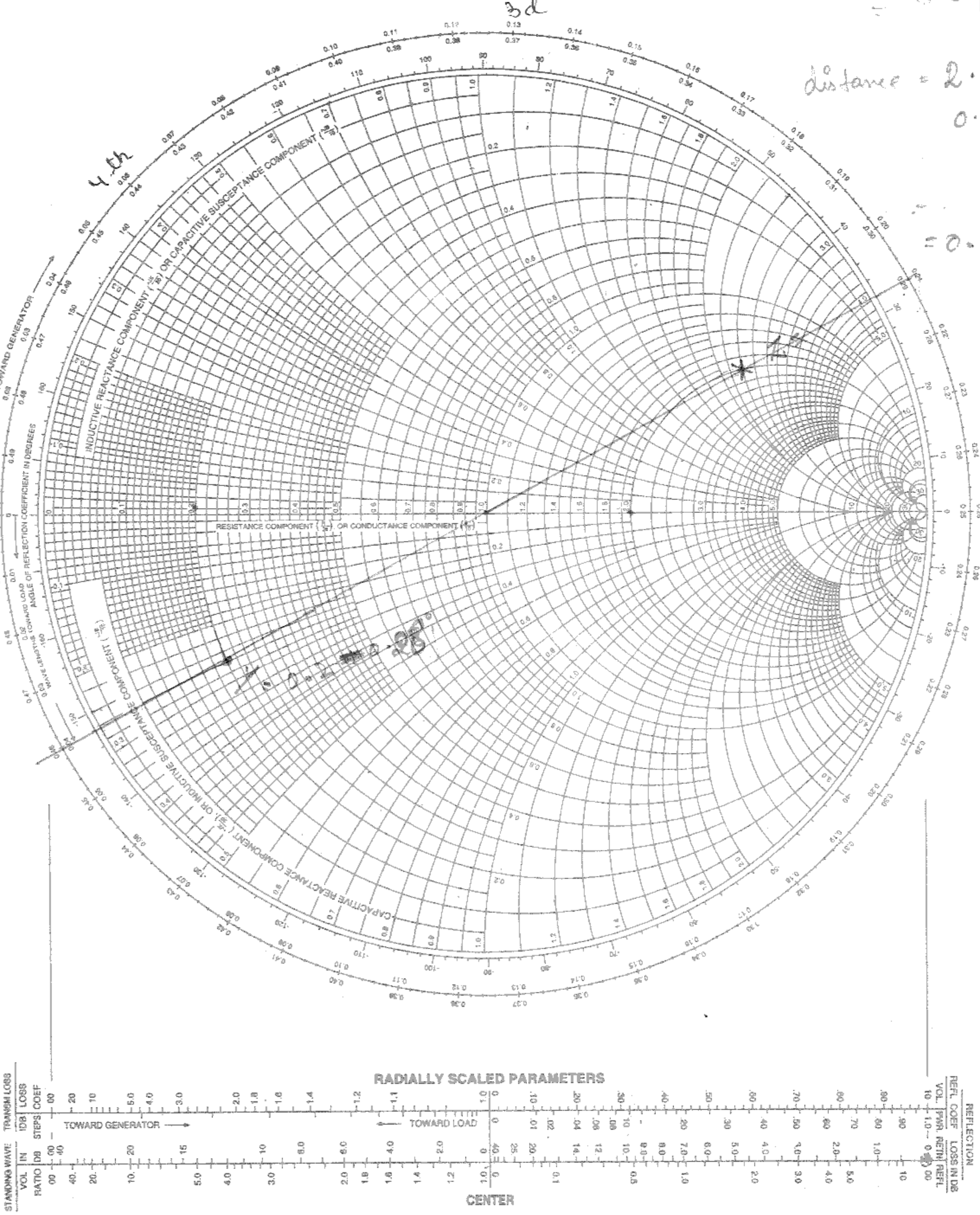
where

$\lambda = \frac{c}{f} = 1.5$

SWR = 2.8 when circle is drawn it cut the graph.

length of stub = 0.25 - 0.21
= 0.04

distance = 2.5 × 0.04
= 0.1m



SMITH CHART

M.K. STORE, Ch - 1.

following

load and 1st voltage
trans line with char imp
receiving imp 100 + 121j Ω

.5 m.

$\frac{Z_R}{Z_0} = \frac{100+121j}{50} = 2+2.42j$

2 − 0.25j

of stub = 0.25 − 0.21 = ~~0.29~~

$\lambda = 2.5$ m

distance = 2.5 × 0.04 = 0.

XH = a

Taking

1.C.

Now, o

the graf

i.e. Y

Now,
exceed

i.e. out

the

length o

outer o

Now,

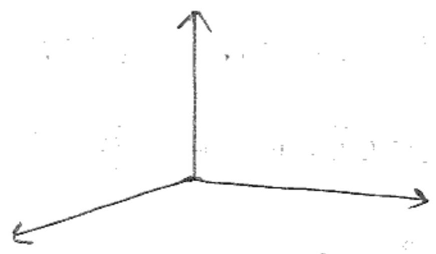
distance

minimum

SWR =

Unit - 04. Guided Waves b/w Parallel Planes

→ Application of Maxwell's equation (of wave guide) :-



The maxwell eq will be sloved to determine the electromagnetic field configuration.

$$\nabla \times H = j\omega \varepsilon E$$

$$= j\omega\varepsilon[\bar{a}_x E_x + \bar{a}_y E_y + \bar{a}_z E_z] \quad -①$$

$$\nabla \times E = -j\omega\mu H$$

$$= -j\omega\mu[\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z] \quad -②$$

The mag. field is written by matrix form

$$\nabla \times H = \begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{bmatrix}$$

$$\nabla \times H = \bar{a}_x\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \bar{a}_y\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_z}{\partial z}\right] +$$

$$\bar{a}_z\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \quad \text{—— (3)}$$

Compair eq (1) and eq (3)

$$\left.\begin{aligned}
\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\
-\frac{\partial H_z}{\partial x} + \frac{\partial H_x}{\partial z} &= j\omega\varepsilon E_y \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z
\end{aligned}\right\} \quad \text{—— (4)}$$

Now,

$$\nabla \times E = \begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{bmatrix}$$

$$\nabla \times E = \bar{a}_x\left[\frac{\partial}{\partial y}E_z - \frac{\partial E_y}{\partial z}\right] - \bar{a}_y\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_z}{\partial z}\right]$$

$$+ \bar{a}_z\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] \quad \text{—— (5)}$$

$$\left.\begin{aligned} \frac{\partial E_y}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ -\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\} \text{ — (6)}$$

The general wave equation

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

Here

$$\gamma^2 = (\sigma + j\omega\varepsilon)(j\omega\mu)$$

σ = conductivity of material

when

$\sigma = 0$ it is non-conductivity of material.

$$\therefore \gamma^2 = -\omega^2\mu\varepsilon \text{ — (7)}$$

Put (7) in general wave equation

$$\left.\begin{aligned} \nabla^2 E &= -\omega^2\mu\varepsilon E \\ \nabla^2 H &= -\omega^2\mu\varepsilon H \end{aligned}\right\} \text{ — (8)}$$

from (8a)

$$\left.\begin{aligned}\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\omega^2 \mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\omega^2 \mu\varepsilon H\end{aligned}\right\} \quad (9)$$

$$H_y = H_y^{\circ} e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -H_y^{\circ} e^{-\gamma z} \gamma$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y \quad \text{——— (10a)}$$

lly,

$$\frac{\partial E_y}{\partial z} = -\gamma E_y \quad \text{——— (10b)}$$

$$\left.\begin{aligned}\frac{\partial H_x}{\partial z} &= -\gamma H_x \\ \frac{\partial E_x}{\partial z} &= -\gamma E_x\end{aligned}\right\} \quad \text{——— (11)}$$

Sub (10) and (11) in eq (4) at $\partial/\partial y = 0$

$$\left.\begin{aligned}\gamma H_y &= j\omega\varepsilon E_x \\ -\gamma H_x - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y\end{aligned}\right\} (12)$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \quad -(12)$$

Sub (10) and (11) in (6)

$$\left.\begin{aligned} -j\omega\mu H_x &= \gamma E_y \\ -j\omega\mu H_y &= -\gamma E_x - \frac{\partial E_z}{\partial x} \\ -j\omega\mu H_z &= \frac{\partial E_y}{\partial x} \end{aligned}\right\} \quad -(13)$$

Sub $$\left.\begin{aligned} \frac{\partial^2 E}{\partial z^2} &= \gamma^2 E_z \\ \frac{\partial^2 H}{\partial z^2} &= \gamma^2 H_z \end{aligned}\right\} \quad -(a)$$

Sub (a) in eq (9)

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial \cancel{z}^2} + \gamma^2 E_z &= -\omega^2 \mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \gamma^2 H_z &= -\omega^2 \mu\varepsilon H \end{aligned}\right\} \quad -(14)$$

Consider eq (12 b) and (13 a)

(12 b) $\Rightarrow -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$

(13 a) $\Rightarrow \gamma E_y = -j\omega\mu H_x \Rightarrow H_x = \frac{-\gamma E_y}{j\omega\mu} \quad -(15)$

From (12b)

$$E_y = \mp\frac{\gamma H_x}{j\omega\varepsilon} \mp \frac{\partial H_z}{\partial x}\frac{1}{j\omega\varepsilon}$$

Sub E_y in (15)

$$H_x = \frac{\mp\gamma}{j\omega\mu}\left[\frac{\mp\gamma H_x}{j\omega\varepsilon} \mp \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{\gamma}{j\omega\mu}\left[\frac{\gamma H_x}{j\omega\varepsilon} + \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-\gamma^2 H_x}{\omega^2\mu\varepsilon} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x + \frac{\gamma^2}{\omega^2\mu\varepsilon}H_x = \frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = \frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[\omega^2\mu\varepsilon + \gamma^2\right] = -\gamma\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{\omega^2\mu\varepsilon + \gamma^2}\frac{\partial H_z}{\partial x}$$

— (15)

$$\boxed{H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}} \quad \text{— (16)}$$

From (12a)

$$\gamma H_y = j\omega\varepsilon E_x$$

$$E_x = \frac{\gamma H_y}{j\omega\varepsilon} \quad \text{—(17)}$$

From (13b)

$$-j\omega\mu H_y = -\gamma E_x - \frac{\partial E_z}{\partial x}$$

$$\Rightarrow H_y = \frac{\gamma E_x}{j\omega\mu} + \frac{\partial E_z}{j\omega\mu\,\partial x} \quad \text{—(18)}$$

Sub (17) in (18)

$$\Rightarrow H_y = \frac{\gamma^2 H_y}{j^2\omega^2\varepsilon\mu} + \frac{\partial E_z}{j\omega\mu\,\partial x}$$

$$H_y = \frac{-\gamma^2 H_y}{\omega^2\mu\varepsilon} + \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$H_y\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{1/j\omega\mu \, \frac{\partial E_z}{\partial x}}{1 + \gamma^2/\omega^2\mu\varepsilon}$$

$$\boxed{H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} \quad \text{--- (19)}$$

where

$$h^2 = \omega^2\mu\varepsilon + \gamma^2$$

cosider (12a)

$$\gamma H_y = j\omega\varepsilon$$

$$H_y = \frac{j\omega\varepsilon E_x}{\gamma}$$

from (13b)

$$j\omega\mu H_y = \gamma E_x + \frac{\partial E_x}{\partial x}$$

$$\boxed{E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}} \quad \text{--- (20)}$$

consider (12b)

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$H_x = \frac{-j\omega\varepsilon E_y}{\gamma} - \frac{\partial H_z}{\partial x}\frac{1}{\gamma}$$

From eq (13a)

$$-j\omega\mu H_x = \gamma E_y$$

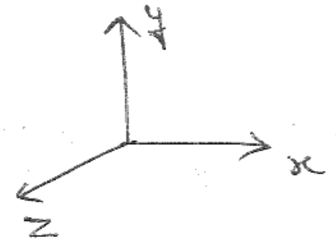
$$\frac{j^2\omega^2\mu\varepsilon E_y}{\gamma} - \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} = \gamma E_y$$

$$E_y\left[-\gamma + \frac{j^2\omega^2\mu\varepsilon}{\gamma}\right] = \frac{\partial H_z}{\partial x}\frac{j\omega\mu}{\gamma}$$

$$\boxed{E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}} \quad \text{— (21)}$$

★ Transverse Electric Waves

The TE waves are waves in which electric field strength E is entirely transvers. It has no magnetic field strength H. And no component of electric field Ez is zero.



Sub $E_z = 0$

$$E_x = -\gamma/h^2 \, \frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial^2}$$

E_x and $H_y = 0$

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu\varepsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\omega^2 \mu\varepsilon E_y - \gamma^2 E_y$$

$$= -(\omega^2 \mu\varepsilon + \gamma^2) E_y$$

$$= -h^2 E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0 \quad \text{—— (1)}$$

This is the differential eq. The solution of this equation is given by

$$E_y' = C_1 \sin hx + C_2 \cos hx$$

where

c_1, c_2 = arbitrary constant

$$E_y = E_y' e^{-\gamma z}$$

$$E_y = (C_1 \sin hx + C_2 \cos hx) e^{-\gamma z} \quad \text{--- (2)}$$

The arbitrary const C_1 and C_2 are determined from boundary condition.

$E_y = 0$, at $x = 0$

$E_y = 0$, at $x = a$

Applying $E_y = 0$ in eq (2) we get

$$E_y = 0$$

$$C_1 \times 0 + C_2 e^{-\gamma z} = 0$$

$$\therefore C_2 e^{-\gamma z} = 0$$

$$\therefore C_2 = 0$$

Sub $C_2 = 0$ in eq (2) we get

$$E_y = C_1 \sin hx \, e^{-\gamma z} \quad \text{--- (3)}$$

Applying $E_y = 0$ at $x = a$ in eq (3)

$$C_1 \sin ha \, e^{-\gamma z} = 0$$

Here, $\sin ha = 0$

$ha = 2n\pi$

$$h = \frac{m\pi}{a}$$

where

$m = 0, 1, 2, \ldots$

Sub $h = m\pi/a$ in eq (3)

$$E_y = C_1 \sin \frac{m\pi}{a} x \, e^{-\gamma z} \quad \text{--- (4)}$$

diff eq (4) with res to x

$$\frac{\partial E_y}{\partial x} = \frac{m\pi}{a} \times a \cos\left(\frac{m\pi}{a}\right) x \, e^{-\gamma z} \quad \text{--- (5)}$$

From (13a).

$$\gamma \cdot E_y = -j\omega \mu H_x$$

$$H_x = \frac{-\gamma E_y}{j\omega\mu}$$

Subt (4) in H_x we get

$$H_x = \frac{-\gamma}{j\mu\omega} (C_1 \sin\left(\frac{m\pi}{a}\right) x \, e^{-\gamma z} \quad \text{--- (6)}$$

From (13c)

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$H_z = \frac{-1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$

$$\therefore H_z = \frac{-1}{j\omega\mu}\frac{m\pi}{a} C_1 \cos\left(\frac{m\pi}{a}\right)x\, e^{-\gamma z} \quad (7)$$

The wave guide without attenuation

$$\alpha = 0 \,, \gamma = j\beta$$

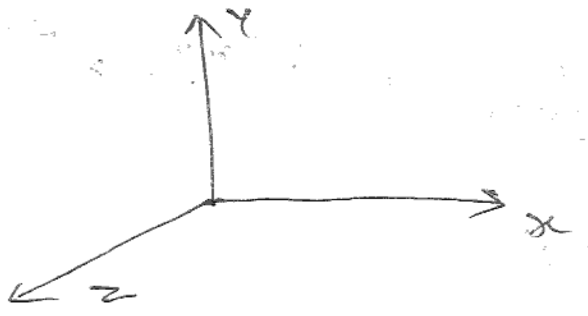
H_x ; H_z , E_y will be

$$H_z = \frac{-1}{j\omega\mu}\frac{m\pi}{a} C_1 \cos\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$$

$$H_x = \frac{-\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$$

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$$

* Transverse magnetic waves



The TM waves are waves in which the magnetic field strength H is entirely transversed. It has an electric field strength E_z in the direction of propagation and no component of magnetic field. H_z in same direction. Here, $H_z = 0$

So, $H_x = 0$ and $E_y = 0$.

H_y

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = -\omega^2 \mu \varepsilon H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_y = 0$$

$$\frac{\partial^2 H_y}{\partial x^2} + H_y h^2 = 0 \quad \text{—— (1)}$$

The solution of this eq is

$$H_y' = C_3 \sin hx + C_4 \cos hx$$

we know that

$$H_y^{\circ} = H_y' e^{-\gamma z}$$

$$H_y = (C_3 \sin hx + C_4 \cos hx) e^{-\gamma z} \quad — ②$$

diff ② w r t x

$$\frac{\partial H_y}{\partial x} = (C_3 \cos hx - C_4 \sin hx) h e^{-\gamma z} \quad — ③$$

W.K.T from 12 C

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial x}$$

$$E_z = \frac{1}{j\omega\varepsilon} (C_3 \cos hx - C_4 \sin hx) h e^{-\gamma z} \quad — ④$$

Applying boundary condition in ④

$E_z = 0$ at $x = 0$

$$0 = \frac{h}{j\omega\varepsilon} C_3 e^{-\gamma z}$$

$$C_3 = 0$$

sub $C_3 = 0$ in eq ④

$\sin hx = h \cos hx$

$$E_z = -h/j\omega\varepsilon \; C_4 \sin hx \, e^{-\gamma z} \quad — ⑤$$

$E_z = 0$ at $x = a$

$$0 = \frac{-h}{j\omega\varepsilon} c_4 \sinh a \, e^{-\gamma z}$$

Here

$\sinh a = 0$

$ha = m\pi$

$$h = \frac{m\pi}{a}$$

Sub h in eq (5)

$$E_z = \frac{-m\pi/a}{j\omega\varepsilon} c_4 \sin\left(\frac{m\pi}{a}\right)x \, e^{-\gamma z}$$

consider $C_3 = 0$ and $h = \frac{m\pi}{a}$

In (2)

$$H_y = c_4 \cos\left(\frac{m\pi}{a}\right)x \, e^{-\gamma z} \quad \text{--- (7)}$$

From (12a)

$$\gamma H_y = j\omega\varepsilon E_x$$

$$E_x = \frac{\gamma H_y}{j\omega\varepsilon}$$

$$E_x = \frac{\gamma}{j\omega\varepsilon}\left[c_4 \cos\left(\frac{m\pi}{a}\right)x \, e^{-\gamma z}\right] \quad \text{--- (8)}$$

wave guide is without attenuation $\alpha = 0$

$$\gamma = j\beta$$

$$E_x = \frac{\beta}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$$

$$E_z = \frac{-m\pi}{aj\omega\varepsilon} C_4 \sin\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$$

* Transverse electromagnetic waves

The TEM waves are waves in which both electric and magnetic fields are transverse entirely but have no component of E and H in same direction. ~~The field strength for In TEM wave the electric field E along~~ In transverse magnetic wave in which electric field E is along the direction of propagation is zero. The field strength for TM wave is

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$$

$$E_{\cancel{z}} = \frac{-m\pi}{aj\omega\varepsilon} C_4 \sin\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$$

$x=0$

$$E_x = \frac{\beta}{j\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}\right)x \, e^{-j\beta z}$$

And for TE wave

$E_z = 0$ and $m = 0$

$\therefore$ In TEM wave

$H_y = C_4 \, e^{-j\beta z}$

$E_x = \beta/\omega\varepsilon \; C_4 \, e^{-j\beta z}$

$E_z = 0$

The propagation const for TEM for

$$\gamma^2 = (\sigma + j\omega\varepsilon) j\mu\omega$$

$$\gamma^2 = j\omega\mu\sigma - \omega^2\varepsilon\mu$$

Here $\sigma = 0$

$$\therefore \gamma^2 = -\omega^2\varepsilon\mu$$

$$\gamma = \sqrt{-\omega^2\varepsilon\mu}$$

$$\gamma = j\omega\sqrt{\varepsilon\mu}$$

WKT

$$\gamma = \alpha + j\beta$$

paral
e
the
se
field
gation
M

$$\therefore \beta = \omega\sqrt{\varepsilon\mu}$$

velocity of wave guide

$$V = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\therefore V = \frac{1}{\sqrt{\mu\varepsilon}}$$

- Velocity propagation

- Characteristic impedance
  It is the ratio of the component of electric field to the component of magnetic field.

$$Z_{xy} = \frac{E_x}{H_y}$$

lly $Z_{y\cancel{x}} = \frac{E_Y}{H_{\cancel{x}}}$

If the direction is positive the ratio of electric and magnetic field is

$$Z_{zx} = \frac{E_z}{H_{\cancel{x}}}$$

$$Z_{xy} = \frac{E_x}{H_y}$$

$$Z_{yz} = \frac{E_y}{H_z}$$

In -ve direction

$$Z_{yx} = \frac{-E_y}{H_x}$$

$$Z_{zy} = \frac{-E_z}{H_y}$$

of

$$Z_{xz} = \frac{-E_x}{H_z}$$

Calculate the characteristic eq for TM waves

$$Z_0(TM) = \frac{E_x}{H_y}$$

$$Z_0(TM) = \frac{\beta}{\omega\varepsilon}$$

$$h = \frac{m\pi}{a}$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\left(\frac{m\pi}{a}\right)^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon \Rightarrow \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon}$$

when

$f < f_c$

$\gamma = \alpha$ and $\beta = 0$

$f > f_c$

$\gamma = j\beta$ and $\alpha = 0$

$f = f_c$

$\gamma = 0$

consider for above

$$f = f_c$$

$$\left(\frac{m\pi}{a}\right)^2 = \omega_c^2 \mu\varepsilon$$

$$\omega_c \sqrt{\mu\varepsilon} = \frac{m\pi}{a}$$

$$2\pi f_c \sqrt{\mu\varepsilon} = \frac{m\pi}{a}$$

$$f_c = \frac{m\pi}{2a\sqrt{\mu\varepsilon}}$$

$$\gamma = \sqrt{\omega_c^2 \mu\varepsilon - \omega^2 \mu\varepsilon}$$

$$\gamma = \sqrt{\mu\varepsilon(\omega_c^2 - \omega^2)}$$

$$\gamma = \sqrt{-\mu\varepsilon(\omega^2 - \omega_c^2)}$$

$$\gamma = j2\pi\sqrt{\mu\varepsilon}\sqrt{f^2 - f_c^2}$$

$$\beta = 2\pi\sqrt{f^2 - f_c^2}\sqrt{\mu\varepsilon}$$

WKT

$$Z_0(T_m) = \frac{\beta}{\omega\varepsilon}$$

$$\therefore \frac{\beta}{\omega\varepsilon} = \frac{\cancel{2\pi}\sqrt{\mu\varepsilon}\sqrt{f^2 - f_c^2}}{\cancel{2\pi} f \varepsilon}$$

$$Z_0(TM) = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \frac{f_c^2}{f^2}}$$

Let

$$\sqrt{\frac{\mu}{\varepsilon}} = \eta$$

$$Z_0 = \eta\sqrt{1 - f_c^2/f^2}$$

For TE waves

char imp

$$Z_0(TE) = \frac{-E_y}{H_x} = \frac{-C_1 \sin\left(\frac{m\pi}{a}\right) e^{-j\beta z}}{-\frac{\beta}{\mu\omega} C_1 \sin\left(\frac{m\pi}{a}\right) e^{-j\beta z}}$$

$$= \frac{\mu\omega}{\beta} = \frac{\cancel{2\pi} f \mu}{\cancel{2\pi}\sqrt{\mu\varepsilon}\sqrt{f^2 - f_c^2}}$$

$$Z_0(TE) = \sqrt{\frac{\mu}{\varepsilon}}\frac{f}{\sqrt{f^2 - f_c^2}} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}$$

char imp of TEM wave

$$Z_0(TEM) = \frac{E_x}{H_y} = \frac{\frac{\beta C_4}{\omega\varepsilon} e^{-j\beta z}}{C_4 e^{-j\beta z}}$$

$$= \frac{\beta}{\omega\varepsilon} = \frac{\cancel{\omega}\sqrt{\mu\varepsilon}}{\cancel{\omega}\varepsilon}$$

$$= \sqrt{\mu/\varepsilon}$$

$$\boxed{Z_0(TEM) = \eta}$$

★ Attenuation :-

$$\alpha = \frac{\text{Power lost/unit length}}{2 \times \text{power transmitter.}}$$

i.e.

It is the ratio of power lost per unit length to the twice of power transmitter.

Unit - 5. Wave Guides

Rectangular Waveguide

The wave guide shape is rectangular.

Wave eq. of rec. wave guide is

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{—} (1)$$

mag. field eq.

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{—} (2)$$

Field equation of rect. waveguide

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y} \quad \text{—} (3)$$

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \quad \text{—} (4)$$

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial y} \quad \text{—} (5)$$

$$H_y = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x} \quad \text{—} (6)$$

Transverse Magnetic Waves In Rect Wave Guide

The wave equation for rect. wave guide

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

Here, x and y is the individual variable with respect to z.

$$E_z = x.y$$

$$\frac{\partial^2 xy}{\partial x^2} + \frac{\partial^2 \cancel{x}y}{\partial y^2} + \gamma^2 xy = -\omega^2 \mu \varepsilon xy$$

$$\text{or, } y\frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 \cancel{x}y}{\partial y^2} + \gamma^2 xy = -\omega^2 \mu \varepsilon xy$$

$$Y\frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 \cancel{x}y}{\partial y^2} + xy[\gamma^2 + \omega^2 \mu \varepsilon] = 0$$

$$Y\frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 \cancel{x}y}{\partial y^2} + h^2 xy = 0$$

$$Y\frac{\partial^2 x}{\partial x^2} + x\frac{\partial^2 \cancel{y}}{\partial y^2} + h^2 xy = 0$$

Dividing by xy

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + \frac{1}{y}\frac{\partial^2 y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + h^2 = -{}^1/_y \frac{\partial^2 y}{\partial y^2}$$

Here, x and y are independent variable So it is possible to equate with other constant.

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + h^2 = A^2$$

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + h^2 - A^2 = 0$$

Let $h^2 - A^2 = B^2$

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + B^2 = 0$$

The solution for this differential eq.

$$X = C_1 \cos Bx + C_2 \sin Bx$$

Ily,

$$-\frac{1}{y}\frac{\partial^2 y}{\partial y^2} = A^2$$

$$\frac{1}{y}\frac{\partial^2 y}{\partial y^2} + A^2 = 0$$

The solution

$$y = C_3 \cos Ay + C_4 \sin Ay$$

WKT

$$E_z = XY$$

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)\, X$$
$$(C_3 \cos Ay + C_4 \sin Ay)$$

$$E_z = C_1C_3 \cos Bx \cos Ay + C_1C_4 \cos Bx \sin Ay + C_2C_3 \sin Bx \cos Ay + C_2C_4 \sin Bx \sin Ay \quad \text{—— (1)}$$

The constants C_1, C_2, C_3, C_4, A and B are determined by boundary condition.

when $E_z = 0$, at $x = 0$ (i), $x = a$ — (iii)

$E_z = 0$ at $y = 0$, (ii) $y = b$ (iv)

Apply boundary cond (i) in eq (1)

$$0 = C_1C_3 \cos Ay + C_1C_4 \sin Ay$$

$c_1 = 0$

Put $c_1 = 0$ in eq ①

$$E_z = c_2 c_3 \sin Bx \cos Ay + c_2 c_4 \sin Bx \sin Ay \quad -②$$

Apply 2nd boundary cond in eq ②

$$0 = c_2 c_3 \sin Bx$$

Here, This is possible only if either c_2 or $c_3 = 0$

If $c_2 = 0$ eq ② $E_z = 0$ not possible

If $c_3 = 0$ the eq ② becomes

$$E_z = c_4 c_2 \sin Bx \sin Ay \quad \cancel{-③}$$

Assum $c_2 c_4 = c$

$$\therefore E_z = c \sin Bx \sin Ay \quad -③$$

Apply 3rd boundary cond in eq ③

$$0 = c \sin Ba \sin Ay$$

Let, $\sin Ba = 0$

$$Ba = m\pi$$

$$B = \frac{m\pi}{a} \qquad [m = 0, 1, 2 \ldots]$$

$$\therefore E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin Ay \quad \text{—— (4)}$$

Apply 4th boundary condition to eq (4)

$$0 = C \sin\left(\frac{m\pi}{a}\right)x \sin Ab$$

Let

$$\sin Ab = 0$$

$$Ab = n\pi$$

$$A = \frac{n\pi}{b}$$

$$\therefore E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

From 3rd eq

$$E_z = C \sin Bx \sin Ay$$

$$\frac{\partial E_z}{\partial x} = BC \cos Bx \sin Ay$$

Subti the value $\frac{\partial E_z}{\partial x}$ and $\frac{\partial H_z}{\partial x} = 0$ in field equation

$$E_x = \frac{-\gamma}{h^2} BC \cos Bx \sin Ay$$

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y}$$

Here

$$\frac{\partial E_z}{\partial y} = CA \cos Ay \sin Bx$$

$$E_y = \frac{-\gamma}{h^2} CA \cos Ay \sin Bx$$

$$H_x = \frac{j\omega\varepsilon}{h^2} \cos Ay \sin Bx (CA)$$

$$H_y = \frac{-j\omega\varepsilon}{h^2} CB \cos Bx \sin Ay$$

$$B^2 = h^2 - A^2$$

$$A^2 + B^2 = h^2$$

$$A^2 + B^2 = \gamma^2 + \omega^2 \mu\varepsilon$$

$$\gamma^2 = A^2 + B^2 - \omega^2 \mu\varepsilon$$

$$\gamma = \sqrt{A^2 + B^2 - \omega^2 \mu\varepsilon}$$

$$\boxed{\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu\varepsilon}}$$

Cut off frequency

$$\gamma = \alpha + j\beta$$

$$A^2 + B^2 = h^2$$

$$\Rightarrow \omega_c^2 \mu\varepsilon = A^2 + B^2$$

$$\omega_c^2 \mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

wave length

$$\lambda_c = \frac{V}{f_c} = \frac{V}{\frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Here, $V = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$

⇒ Transverse Electric Waves for Rec Wave Guide

Wave eq. for TE waves

$$\frac{\partial^2 H_Z}{\partial x} + \frac{\partial^2 H_Z}{\partial y} + \gamma^2 H_Z = -\omega^2 \mu \varepsilon H_Z$$

Here, x and y are the individual variable

$$H_Z = X \cdot Y$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY = -\omega^2 \mu \varepsilon XY$$

$$Y\frac{\partial^2 x}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + XY\left(\gamma^2 + \omega^2 \mu \varepsilon\right) = 0$$

$$Y\frac{\partial^2 x}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + XYh^2 = 0$$

dividing both side by XY we get

$$\frac{1}{X}\frac{\partial^2 x}{\partial x^2} + h^2 = A^2$$

$$\cancel{\frac{1}{y}\frac{\partial^2 y}{\partial y^2} + B^2 = 0}$$

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + h^2 - A^2 = 0$$

$$h^2 - A^2 = B^2 \quad \text{(assume)}$$

$$\frac{1}{x}\frac{\partial^2 x}{\partial x^2} + B^2 = 0$$

Sol of eq :- $X = C_1 \cos Bx + C_2 \sin Bx$

lly

$$-\frac{1}{y}\frac{\partial^2 y}{\partial y^2} = A^2$$

$$\frac{1}{y}\frac{\partial^2 y}{\partial y^2} + A^2 = 0$$

$$Y = C_3 \cos AY + C_4 \sin AY$$

we know that

$$H_z = xy$$

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos AY + C_4 \sin AY)$$

$$H_z = C_1C_3\cos Bx\cos Ay + C_1C_4\cos Bx\sin Ay + C_2C_3\sin Bx\cos Ay + C_2C_4\sin Bx\sin Ay \quad \text{—(1)}$$

The const C_1, C_2, C_3, C_4, A and B are determined by the boundary condition

when,

$H_z = 0$ if $x = 0$ —(i)

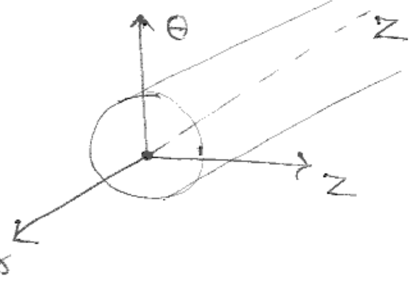
$H_z = 0$ if $x = a$ —(ii)

$H_z = 0$ if $y = 0$ —(iii)

$H_z = 0$ if $y = a$ —(iv)

Same as previous one

★ Cylindrical Waveguides (Circular W/G)



It is having both electric and magnetic field.

$\nabla \times H = j\omega\epsilon E$

$= j\omega\epsilon\,[E_r\bar{a}_r + E_\theta\bar{a}_\theta + E_z\bar{a}_z]$ —(1)

Magnetic field is written as matrix form

$$\nabla \times H = \begin{bmatrix} \frac{\bar{a}_r}{r} & \bar{a}_\theta & \frac{\bar{a}_z}{r} \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial z \\ H_r & r.H_\theta & H_z \end{bmatrix}$$

$$\nabla \times H = \frac{\bar{a}_r}{r}\left[\frac{\partial H_z}{\partial \theta} - \frac{r\partial H_\theta}{\partial z}\right] - \bar{a}_\theta\left[\frac{\partial H_z}{\partial r} - \frac{\partial H_r}{\partial z}\right]$$

$$+ \frac{\bar{a}_z}{r}\left[\frac{r\partial H_\theta}{\partial r} - \frac{\partial H_r}{\partial \theta}\right] \quad -(2)$$

comparing eq (1) and (2)

$$\frac{1}{r}\frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z} = j\omega\varepsilon E_r \quad -(3)$$

$$\frac{\partial H_z}{\partial r} - \frac{\partial H_r}{\partial z} = -j\omega\varepsilon E_\theta \quad -(4)$$

$$\frac{1}{r}\left[r\frac{\partial H_\theta}{\partial r} - \frac{\partial H_r}{\partial \theta}\right] = j\omega\varepsilon E_z \quad -(5)$$

$$\frac{\partial H_\theta}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial \theta} = j\omega\varepsilon E_z \quad -(5)$$

we know that

$$H_\theta = H_\theta^\circ e^{-\gamma z}$$

$$\frac{\partial H_\theta}{\partial z} = -\gamma H_\theta^\circ e^{-\gamma z} = -\gamma H_\theta$$

$$\frac{\partial H_r}{\partial z} = -\gamma H_r$$

subt the value (above) in eq (3) and (4)

$$\frac{1}{r}\frac{\partial H_z}{\partial \theta} + \gamma H_\theta = j\omega \varepsilon E_r \quad \text{--- (6)}$$

$$\frac{\partial H_z}{\partial r} + \gamma H_r = -j\omega \varepsilon E_\theta \quad \text{--- (7)}$$

The Maxwell equation for electric field

$$\nabla \times E = -j\omega \mu H$$

$$= -j\mu\omega\left[\bar{a}_r H_r + \bar{a}_\theta H_\theta + \bar{a}_z H_z\right] \quad \text{--- (8)}$$

The electric field in matrix form

$$\nabla \times E = \begin{bmatrix} \frac{a_r}{r} & a_\theta & \frac{a_z}{r} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial z \\ E_r & rE_\theta & E_z \end{bmatrix}$$

$$\nabla \times E = \frac{a_r}{r}\left[\frac{\partial E_z}{\partial \theta} - r\frac{\partial E_\theta}{\partial z}\right] - a_\theta\left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right]$$

$$+ \frac{a_z}{r}\left[r\frac{\partial E_\theta}{\partial r} - \frac{\partial E_r}{\partial \theta}\right] \quad \text{— (9)}$$

compairing (8) and (9) we get

$$\frac{\partial E_z}{r\partial \theta} - \frac{\partial E_\theta}{\partial z} = -j\mu\omega H_r \quad \text{— (10)}$$

$$\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} = +j\omega\mu H_\theta \quad \text{— (11)}$$

$$\frac{\partial E_\theta}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \theta} = -j\mu\omega H_z \quad \text{— (12)}$$

we know that

$$E_\theta = E_\theta^{\circ}\, e^{-\gamma z}$$

$$\frac{\partial E_\theta}{\partial z} = -\gamma E_\theta$$

$$\frac{\partial E_r}{\partial z} = -\gamma E_r$$

sub above value in (10) and (11)

$$\frac{\partial E_z}{\partial r} + \gamma E_r = j\omega\mu H_\theta \quad \text{— (14)}$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} + \gamma E_\theta = -j\omega\mu H_r \quad \text{--- (13)}$$

From eq (7)

$$E_\theta = -\frac{1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial r} + \gamma H_r\right]$$

Sub value of E_θ in eq (13) we get

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial r} + \gamma H_r\right] = -j\omega\mu H_r$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial r} + \gamma H_r\right] + j\omega\mu H_r = 0$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial r} - \frac{\gamma^2 H_r}{j\omega\varepsilon} + j\omega\mu H_r = 0$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial r} - \left[\frac{\gamma^2 + \omega^2\mu\varepsilon}{j\omega\varepsilon}\right]H_r = 0$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial r} = \frac{h^2}{j\omega\varepsilon} H_r$$

$$H_r = \frac{j\omega\varepsilon}{h^2}\left[\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial r}\right]$$

$$\boxed{H_r = \frac{j\omega\varepsilon}{r.h^2}\frac{\partial E_z}{\partial \theta} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial r}}$$

from eq (6)

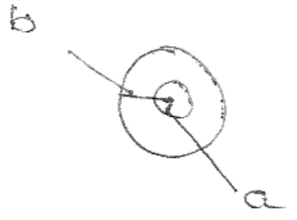
$$E_r = \frac{1}{j\omega\varepsilon r}\frac{\partial H_z}{\partial \theta} + \frac{\gamma H_\theta}{j\omega\varepsilon}$$

sub in eq (14)

From eq (6)

* Transverse Electromagnetic Waves (TEM) in coaxial Cable.

$E_z = 0$ and $H_z = 0$



$$\nu = \frac{c}{\sqrt{\epsilon \mu}}$$

where,

ϵ = gap permitivity

μ = relative permeability

A/c to Maxwell equation

$$f(r, \phi)\, e^{i(kz - \omega t)} \quad \text{—— (1)}$$

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$B = \nabla \times A \quad \text{—— (2)}$$

where

A → Potential vector

A/c to Lorentz Gauge eqn.

$$\nabla \cdot A = -\frac{1}{v^2}\frac{\partial V}{\partial t} \quad \text{—— (3)}$$

A/c to wave equa

$$\nabla^2 V = \frac{1}{v^2}\frac{\partial^2 V}{\partial t^2} \quad \text{—— (4)}$$

Taking the scalar potential

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial f}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} - k^2 f = -\frac{w^2 f}{v^2} \quad \text{—— (5)}$$

$$\because \quad k = \frac{w}{v} \quad \text{—— (6)}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) = 0 \quad ; \quad \frac{\partial f}{\partial r} = \frac{k}{r}$$

$$f = k \ln \frac{r}{r_0} \quad \cancel{(r > b)}$$

$$f = \begin{cases} 0 & (r > b) \\ V_0 \dfrac{\ln(r/b)}{\ln(a/b)} & (d < r < b) \\ V_0 & (r < a) \end{cases}$$

Scalar potential

$$V = \begin{cases} 0 \quad ; & r > b \\ -V_0 \ln(r/b)\, e^{i(kz-\omega t)} & (a<r<b) \\ V_0\, e^{i(kz-\omega t)} & (r<a) \end{cases}$$

The electric and magnetic field follows from (2)

$$E_r = -\frac{\partial V}{\partial r} = \begin{cases} 0 & (r>b) \\ \dfrac{V_0\, e^{i(kz-\omega t)}}{r \ln(b/a)} & (a<r<b) \\ 0 & (r<a) \end{cases}$$

★ Stub Matching

i) Single stub

ii) Double stub

If the load impedance is not equal to the i/p impedance, the max power transfer will not takes place. This is known as missmatching

So, it is necessary to introduce some form of an impedance transforming section b/w source and load to achieve impedance matching. Such section is known as impedance matching device. i.e. quarter wave line or transformer.

Another method of impedance matching is the use of an open or short ckted line of suitable length called <u>stub</u> at a designated distance from the load. This called stub matching.