eq (20) & (21) represents the general solution of Transmission line.

In another way of representing eq (20) as.

$$E = \frac{E_R}{2Z_R}\left[(Z_R+Z_0)e^{\sqrt{ZY}s} + (Z_R-Z_0)e^{-\sqrt{ZY}s}\right]$$

$$I = \frac{I_R}{2Z_0}\left[(Z_R+Z_0)e^{\sqrt{ZY}s} - (Z_R-Z_0)e^{-\sqrt{ZY}s}\right]$$

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$$E = \frac{E_R}{2Z_R}\left[Z_R e^{\sqrt{ZY}s} + Z_0 e^{\sqrt{ZY}s} + Z_R e^{-\sqrt{ZY}s} - Z_0 e^{-\sqrt{ZY}s}\right]$$

$$E = \frac{E_R}{\cancel{2Z_R}}\left[\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right] + \frac{Z_0 E_R}{\cancel{2}Z_R}\left[\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right]$$

Sub. $Z_R = \frac{E_R}{I_R}$ & $\frac{E_R}{Z_R} = I_R$

$$E = E_R\left[\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right] + I_R Z_0\left[\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right] \quad (22)$$

Simillarly.

$$I = I_R\left[\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2}\right] + \frac{E_R}{Z_0}\left[\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2}\right] \quad -(23)$$

But $\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}}{2} = \cosh\sqrt{ZY}s$ &

$$\frac{e^{\sqrt{ZY}s} - e^{-\sqrt{ZY}s}}{2} = \sinh\sqrt{ZY}s$$

sub in (22) & (23)

$$E = E_R\cosh\sqrt{ZY}s + I_R Z_0 \sinh\sqrt{ZY}s \quad -(24)$$

$$I = I_R\cosh\sqrt{ZY}s + \frac{E_R}{Z_0}\sinh\sqrt{ZY}s \quad -(25)$$

The eq. (24) & (25) gives the value of E & I at any pt along the length of the line.

'x' distance of sending end. (I.X)

$$x = l - s$$

s → Receiving end. distance.

eq. (24) & (25) get transfered in terms of E_s & I_s

$$E_R = E_s \ \& \ I_R = I_s \ \& \ s = x$$

$$E = E_S \cosh(\sqrt{ZY}\,x) + I_S Z_0 \sinh\sqrt{ZY}\,x \quad -(26)$$

$$I = I_S \cosh(\sqrt{ZY}\,x) + \frac{E_S}{Z_0}\sinh\sqrt{ZY}\,x \quad -(27)$$

Physical Significance of Transmission line :-

Consider the general solution of transmission line in a recieving end is :-

$$E = E_R \cosh(\gamma s) + I_R Z_0 \sinh(\gamma s)$$

$$I = I_R \cosh(\gamma s) + \frac{E_R}{Z_0}\sinh(\gamma s)$$

sending end current can be obtained by substituting $s = l$

$$E = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l) \quad -(1)$$

$$I = I_R \cosh(\gamma l) + \frac{E_R}{Z_0}\sinh(\gamma l) \quad -(2)$$

But, $Z_R = \frac{E_R}{I_R}$

sub in (2) $Z_R = \frac{E_R}{I_R} \Rightarrow E_R = Z_R I_R$

$$I_S = I_R \cosh(\gamma l) + \frac{Z_R I_R}{Z_0} \sinh(\gamma l)$$

$$I_S = I_R \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l) \right]$$

$$\frac{I_S}{I_R} = \cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l) \quad - (3)$$

$$[\because Z_R = Z_0]$$

$$\frac{I_S}{I_R} = \cosh(\gamma l) + \sinh(\gamma l) = e^{\gamma l}$$

$$\frac{I_S}{I_R} = e^{\gamma l} \quad - (4)$$

If the line is terminated in Z_0 & using $E_R = Z_R I_R$ in eq (1) we get

$$E_S = Z_R I_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l)$$

$$E_S = I_R \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right] \quad - (5)$$

Divide eq (5) by eq (3)

$$\frac{E_S}{I_S} = \frac{\cancel{I_R}\left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}{\cancel{I_R}\left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l) \right]}$$

$$\frac{E_S}{I_S} = \frac{Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)}$$

$$Z_S = \frac{Z_0 \left[Z_R \cosh(\gamma l) + Z_0 \sinh(\gamma l) \right]}{\left[Z_0 \cosh(\gamma l) + Z_R \sinh(\gamma l) \right]} \quad \text{— (6)}$$

$$[\because Z_R = Z_0]$$

$$\boxed{Z_S = Z_0}$$

when the line is terminated in Z_0, $Z_R = Z_0$

This shows that line terminated in Z_0, its i/p impedance is characteristics impedance

Consider an infinite line with $l \to \infty$ sub in eq (6)

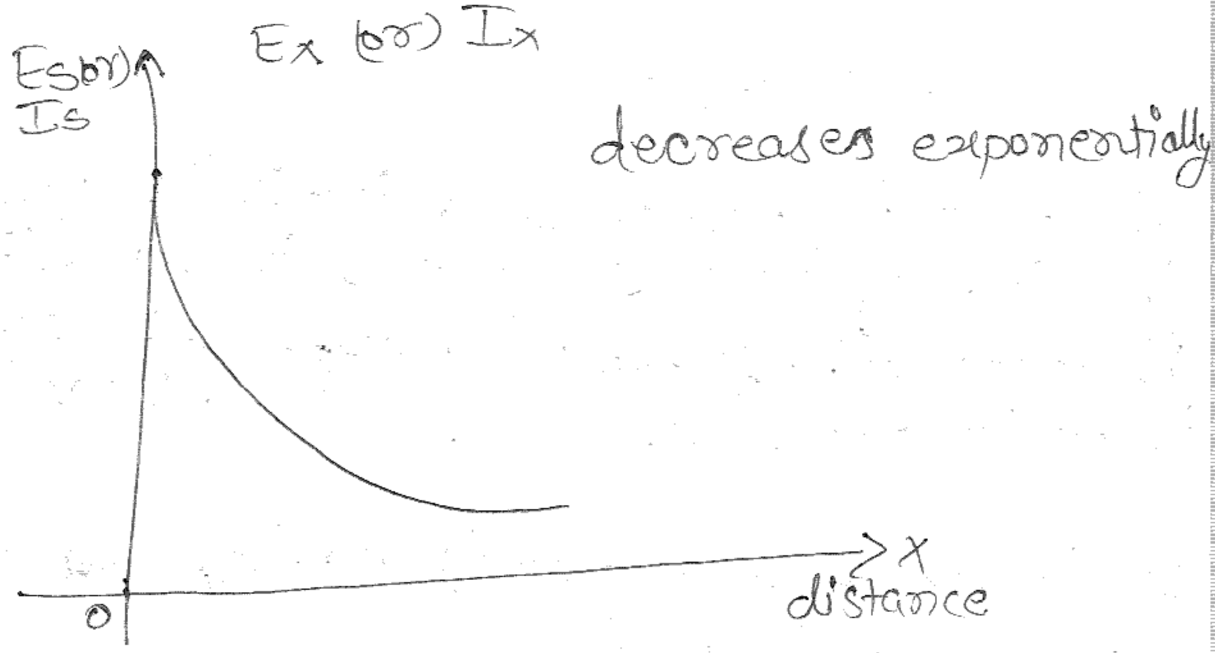
Multiplying $\frac{1}{\cosh(\gamma l)}$ in num. & denom in eq (6)

$$Z_S = \frac{Z_0 \left[Z_R \cancel{\cosh} + Z_0 \tanh(\gamma l) \right]}{\left[Z_0 + Z_R \tanh(\gamma l) \right]} \quad [\because \tanh(\infty) = 1]$$

$$Z_S = \frac{Z_0\left[\cancel{Z_R + Z_0}\right]}{\left[\cancel{Z_R + Z_0}\right]}$$

$$\boxed{Z_S = Z_0}$$

This shows that finite line terminated in its characteristics impedance behaves like an infinite line



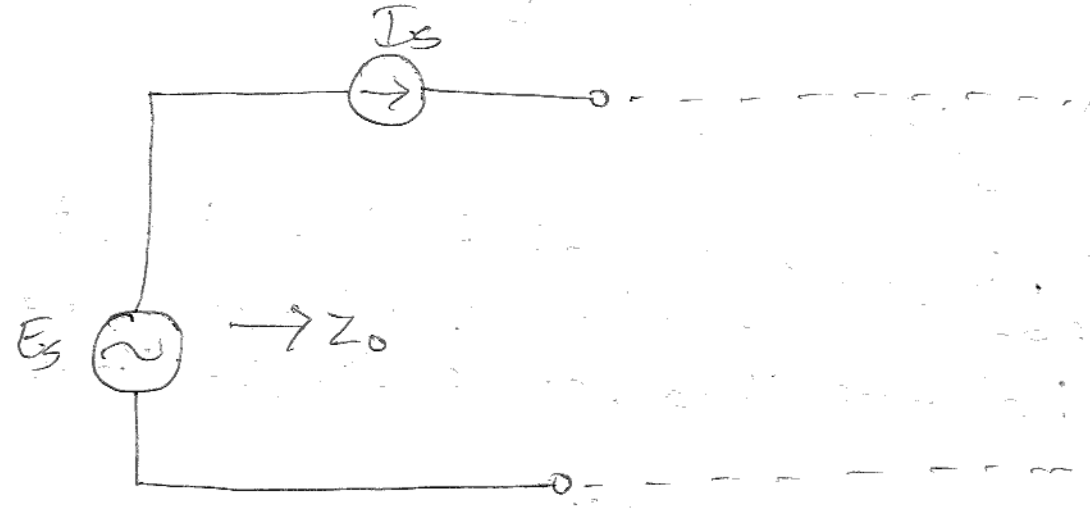
$$E_x = E_S e^{-\gamma x}$$

(or)

$$I_x = I_S e^{-\gamma x}$$

If the instruments are connected along the line, then the instruments will show the magnitude while the phase angle cannot be obtained.

Infinite Line :- (only sender not receiver)



$$Z_s = \frac{E_s}{I_s}$$

Input impedance of infinite line is Z_0 of transmission line and it is denoted by Z_0. Therefore Z_0 plays important role in Transmission line.

Z_0 is phasor quantity having magnitude $|Z_0|$ & angle is ϕ

Magnitude and angle of Z_0 is vary with freq.

Important Property of Infinite line

1. As the line as infinite length, no waves will reach the receiving end & hence there is no possibility of reflection at receiving end.

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Thus there cannot be any reflected waves & the complete power is observed by the transmission line.

2. As reflected waves are absent, Z_0 decides the flow of current when a vge is applied to the sending end. The current will not be affected by terminated impedance (Z_R) at receiving end. This condition is full filled by long line in practice.

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Distortion less line :-

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A line in which there is no phase or frequency distortion and also it is correctly terminated called as distortion less line.

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Consider $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

squaring on bothsides:-

$$\gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\gamma^2 = RG + Rj\omega C + Gj\omega L + j^2\omega^2 LC$$

$$\gamma^2 = RG + Rj\omega C + Gj\omega L - \omega^2 LC$$

$$\gamma^2 = (RG - \omega^2 LC) + j\omega(RC + GL) \quad \text{--- (1)}$$

For minimum attenuation one condition is there:-

$$L = \frac{CR}{G}$$

~~For minimum~~

$$LG = CR \quad \text{sub in (1)}$$

$$\gamma^2 = (RG - \omega^2 LC) + j\omega(RC + RC)$$

$$\gamma^2 = (RG - \omega^2 LC) + j2\omega RC$$

But $RC = LG = \sqrt{RCLG}$

$$\gamma^2 = (RG - \omega^2 LC) + j2\omega\sqrt{RCLG}$$

$$\gamma^2 = \left(\sqrt{RG} + j\omega\sqrt{LC}\right)^2$$

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC} \quad \text{--- (2)}$$

But $\gamma = \alpha + j\beta$

$$\alpha = \sqrt{RG} \quad \text{--- (3)}$$

$$\beta = \omega\sqrt{LC} \quad \text{--- (4)}$$

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It can be seen from eq (3) that α does not vary with frequency which eliminates the ~~d~~ freq. distortion.

$$v = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

Thus for condition $LG = RC$ the velocity becomes independent of frequency. This eliminates the phase distortion.

It is already proved that $RC = LG$, Z_0 become resistive and line can be correctly terminated to eliminate distortion due to Z_0 varying with frequency. Thus the overall distortion are eliminated.

$$\therefore \boxed{RC = LG} \Rightarrow \boxed{\frac{R}{G} = \frac{L}{C}}$$

Telephone Cable :-

(It is underground cable)

Since inductance & conductance is negligible it can be neglected and hence impedance & admittance of such a cable can be written by

$$Z = R$$

& $Y = j\omega C$

$$\gamma = \sqrt{ZY} = \sqrt{j\omega RC}$$

Divide by 2 in num. & denom.

$$\gamma = \sqrt{\frac{j2\omega RC}{2}}$$

$$\sqrt{j} = \sqrt{1\angle 90^\circ}$$

$$= 1\angle 45^\circ$$

$\{ j = 1 \angle \theta = 90^\circ \}$

$\{ \sqrt{\angle 90^\circ} = \angle 90^\circ/2 \}$

$$\sqrt{2j} = (1+j1)$$

$$\gamma = (1+j1)\sqrt{\frac{\omega RC}{2}}$$

$$\gamma = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}} = \alpha + j\beta$$

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$$\alpha = \sqrt{\frac{\omega RC}{2}} \quad , \quad \beta = \sqrt{\frac{\omega RC}{2}}$$

dmittance
written

$$V = \frac{\omega}{\beta} = \frac{\omega\sqrt{2}}{\sqrt{\omega RC}}$$

$$V = \sqrt{\frac{2\omega}{RC}}$$

Both α & V are the functions of frequency ω

hom.

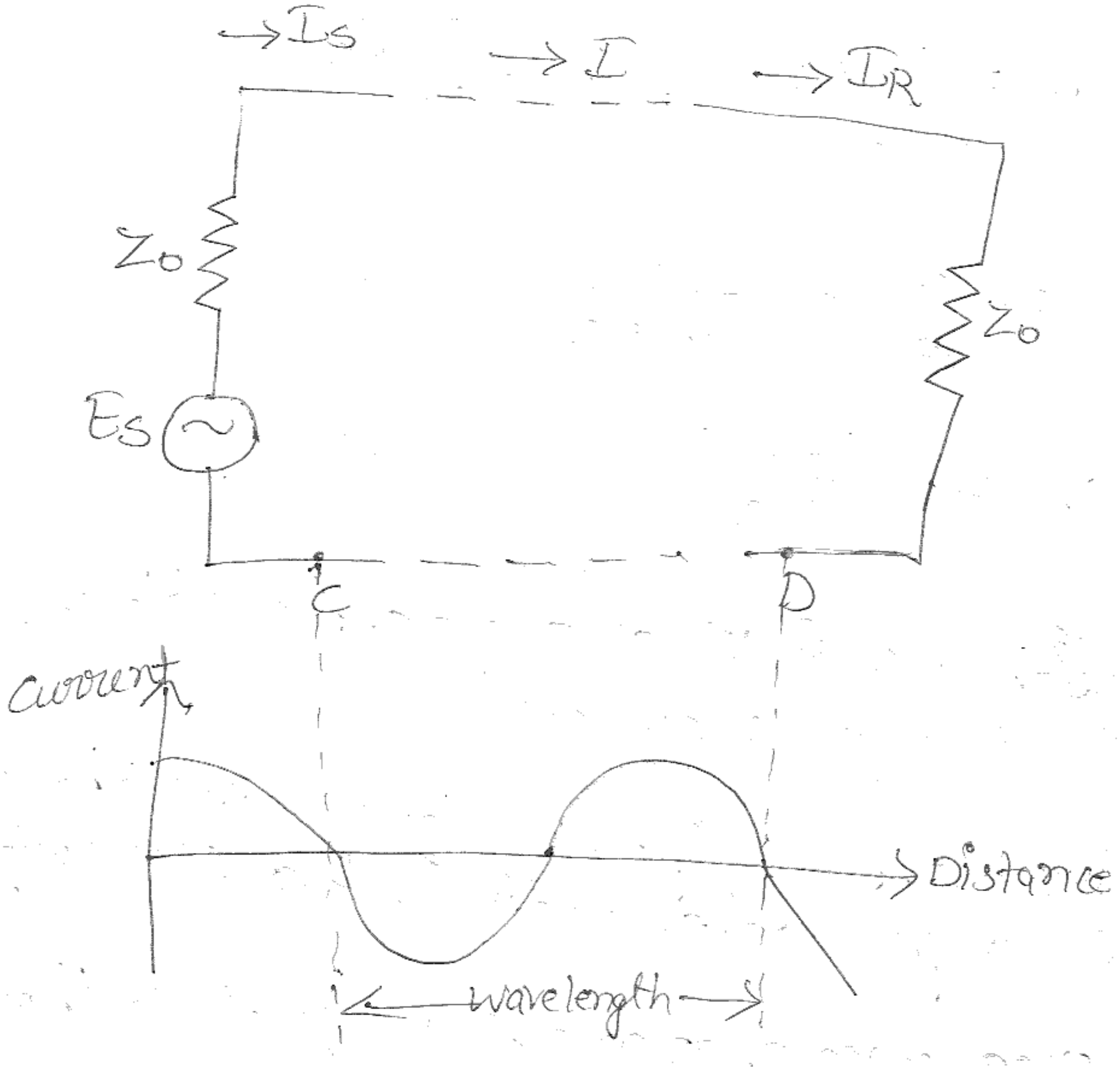
Hence for high frequencies there is large attenuation & also velocity

$\angle \theta = 90°$

Hence wave travels very fast than lower frequencies when freq. is high.

$= \angle 90°/2$

Wavelength and Velocity



The distance b/w two points along the line at which current & Vge differs in phase by 2π radian. it is called wavelength & denoted by λ.

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$$\lambda = \frac{2\pi}{\beta}$$

Phase constant(β): It is defined as radians per unit length of line.

If the frequency is f Hertz i.e. cycles/second than for one cycle the time required is called time period.

$$T = \frac{1}{f} \text{ sec/cycle.}$$

$$V = \frac{\text{distance travelled}}{\text{time taken}}$$

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$$V = \frac{\lambda}{1/f}$$

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$$\boxed{V = \lambda f}$$

$$V = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$$

Z_0

Open and Short Circuited lines

The expressions for vge and current at the sending end of a transmission line of length l are given by

$$E_S = E_R \cosh(\gamma l) + I_R Z_0 \sinh(\gamma l) \quad \text{—(1)}$$

$$I_S = I_R \cosh(\gamma l) + \frac{E_R}{Z_0} \sinh(\gamma l) \quad \text{—(2)}$$

$$V_S = V_R \cosh(\gamma l) + \frac{V_R Z_0}{Z_R} \sinh(\gamma l)$$

$$\left[\because Z_R = \frac{V_R}{I_R}\right]$$

$$V_S = V_R \left[\cosh(\gamma l) + \frac{Z_0}{Z_R} \sinh(\gamma l)\right] \quad \text{—(3)}$$

$$I_S = I_R \left[\cosh(\gamma l) + \frac{Z_R}{Z_0} \sinh(\gamma l)\right] \quad \text{—(4)}$$

The i/p impedance of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

divide ④ by ③

$$Z_S = \frac{V_R\left[\cosh(\gamma l) + \frac{Z_0}{Z_R}\sinh(\gamma l)\right]}{I_R\left[\cosh(\gamma l) + \frac{Z_R}{Z_0}\sinh(\gamma l)\right]}$$

$$= \frac{\frac{V_R}{Z_R}\left[Z_R\cosh(\gamma l) + Z_0\sinh(\gamma l)\right]}{\frac{I_R}{Z_0}\left[Z_0\cosh(\gamma l) + Z_R\sinh(\gamma l)\right]}$$

$$Z_S = \frac{Z_0}{Z_R}\frac{V_R}{I_R}\left[\frac{Z_R\cosh(\gamma l) + Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l) + Z_R\sinh(\gamma l)}\right]$$

$$\left[\frac{V_R}{I_R} = Z_R\right]$$

$$Z_S = Z_0\left[\frac{Z_R\cosh(\gamma l) + Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l) + Z_R\sinh(\gamma l)}\right]$$

If short circuited, the recieving end impedance is zero.

i.e $Z_R = 0$

$$Z_{SC} = Z_0\left[\frac{\cancel{Z_0}\sinh(\gamma l)}{\cancel{Z_0}\cosh(\gamma l)}\right]$$

$$\boxed{Z_{SC} = Z_0 \tanh(\gamma l)}$$

If open circuited the recieving end impedance is infinite.

i.e. $Z_R = \infty$

$$Z_s = Z_0 \left[\frac{\cosh(\gamma l) + \frac{Z_0}{Z_R}\sinh(\gamma l)}{\frac{Z_0}{Z_R}\cosh(\gamma l) + \sinh(\gamma l)}\right]$$

$$Z_{oc} = Z_0 \left[\frac{\cosh(\gamma l)}{\sinh(\gamma l)}\right]$$

$$Z_{oc} = Z_0 \coth(\gamma l)$$

By multiplying open circuited impedance & short circuited impedance

$$\boxed{Z_{oc} Z_{sc} = Z_0^2}$$

The characteristic impedance is given by

$$Z_0 = \sqrt{Z_{oc} Z_{sc}}$$

By dividing short circuited impedance and open circuited impedance is

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh(\gamma l)}{Z_0 \coth(\gamma l)}$$

$$\frac{Z_{sc}}{Z_{oc}} = \tanh^2(\gamma l)$$

$$\tanh(\gamma l) = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\boxed{\gamma l = \tanh^{-1}\sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

Reflection:-

When the load impedance is not equal to the characteristics impedance of transmission line, reflection takes place.

The exp. for vge & current on the transmission line are:-

$$V = \frac{V_R}{2}\left[\frac{Z_R+Z_0}{Z_R} e^{\sqrt{ZY}x} + \frac{Z_R-Z_0}{Z_R} e^{-\sqrt{ZY}x}\right]$$

$\left[\because s = x\right]$

$$\boxed{I = \frac{I_R}{2}\left[\frac{Z_R+Z_0}{Z_R} e^{\sqrt{ZY}x} - \frac{Z_R-Z_0}{Z_0} e^{-\sqrt{ZY}x}\right]}$$

$$\boxed{V = \frac{V_R}{2}\left[\frac{Z_R+Z_0}{Z_R}\right]\left\{e^{\gamma x} + \frac{Z_R-Z_0}{Z_R+Z_0} e^{-\gamma x}\right\}}$$

$\left[\because \gamma = \sqrt{ZY}\right]$

$Z_R \neq Z_0$ (mismatched conditio)

If the transmission line is not terminated with characteristic impedance, then the above exp exist & this condition is called mismatched condition.

It consists of two waves one is moving in the forward direction called incident wave other one is moving in backward direction called reflected wave.