UNIT-2

TRANSMISSION LINE PARAMETERS

Transmission line:

It acts as a medium to transfer energy from one point to another.

Transmission line can be classified as

(i) open wire or parallel wire or balanced wire.

(ii) Co-axial Cable.

① Open wire:

- It is the basic form of transmission line.
- It is employed where at least two connectors are used.
- They are twisted in pairs, Coming together and they are covered by a protective load or sheet.

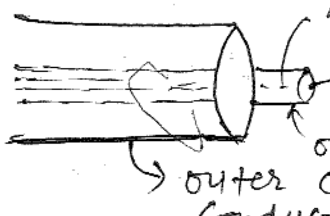
② Co-axial Cable

Disadvantages

- High requirement of telephone cost and towers are needed.
- It is high initial cost.
- Maintain is difficult
- Possibility of short circuiting.

(2) Co-axial Cable

- It has got inner and outer conductors respectively.



- It is used for higher voltage level transmission
- It is used as Ultra high frequency (UHF).
  Micro wave freq. (MHF)
- It is employed where one

Wave guides

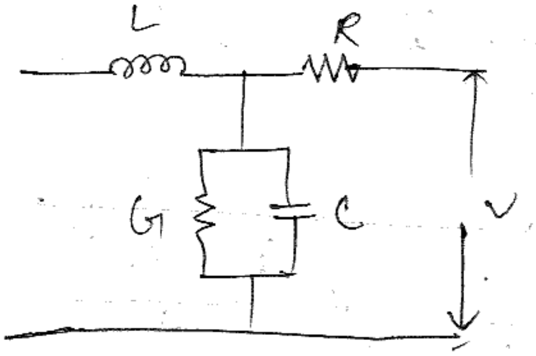
- It is employed ~~when~~ where one conductor is used.
- " " " at microwave frequencies - (high range of frequencies).

(M) ~~UHF~~ microwave freq → (1GHz - 100GHz
$10^9 Hz - 10^{11} Hz$)

Eg. Rectangular wave guides.
Circular " ".
Parallel Plates.

primary parameters of transmission line :

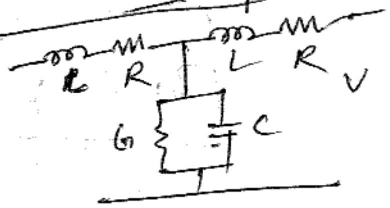
R → Resistance → loop resistance (ohms/km)
L → Inductance → loop Inductance per unit length (Henry/km)
G – conductance → shunt conductance per unit length (mho/km)
C – capacitance → shunt capacitance (farad/km).



[Z → series impedance
Y → shunt ~~impedance~~ admittance]

V. Imp.

$$\boxed{\begin{aligned} Z &= R + j\omega L \\ Y &= G + j\omega C \end{aligned}}$$



secondary parameters

* Z_0 → characteristic impedance

* P or ~~δ~~ γ (Gamma) → propagation constant.

$$Z_0 = \sqrt{Z/Y}$$

$$\gamma = \sqrt{ZY}$$

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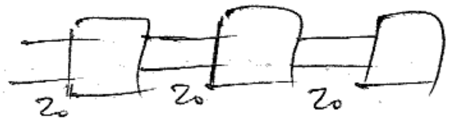
$$\gamma = \alpha + j\beta$$

α → Attenuation constant
β → phase constant.

$$\gamma = \ln\left(\frac{I_S}{I_R}\right) \text{ (or) } \ln\left(\frac{V_S}{V_R}\right)$$

$$Z_0 = \sqrt{Z/Y}$$



$$Z_{oT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} \quad - (1)$$

T-Network

Total series imp → Z_1 → $(R+j\omega L)$ length.

Total shunt impedence of T-N/w Z_2 → $\left(\frac{1}{G+j\omega C}\right)$ length. } (2)

from (1) & (2)

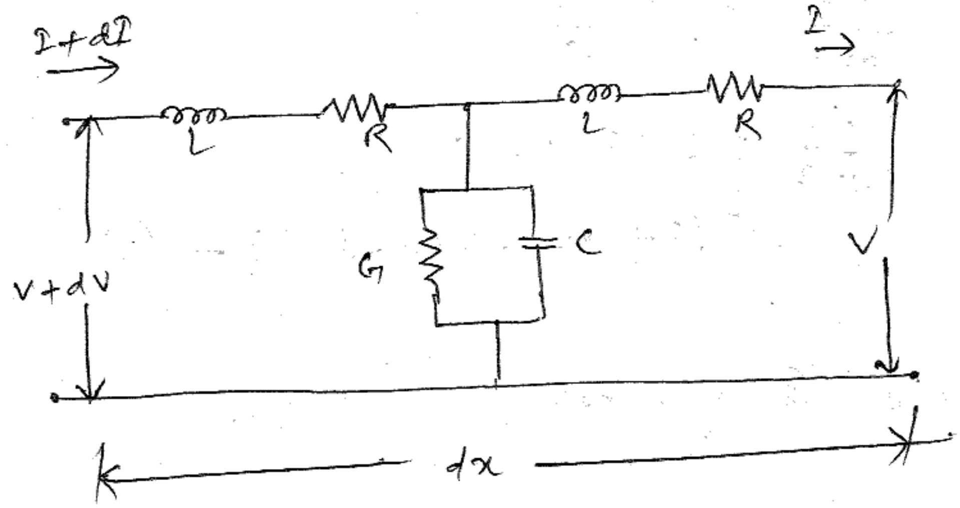
$$Z_{oT} = \sqrt{(R+j\omega L)\frac{1}{(G+j\omega C)} + \frac{(R+j\omega L)^2}{4}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\boxed{Z_0 = \sqrt{Z/Y}}$$

V. Imp.

General Solution of transmission



series impedance $(Z) = R + j\omega L$

admittance $Y = G + j\omega C$

Series impedance of small section which can be expressed as $\rightarrow (R + j\omega L)dx$

Shunt admittance of small section can be expressed as $\rightarrow (G + j\omega C)\,dx$

The potential difference between two ~~et~~ ends of the T-network is given by

$$(\cancel{V} + dV) - \cancel{V} = I\,(R + j\omega L)\,dx$$

$$\frac{dV}{dx} = I(R + j\omega L) = IZ \qquad \text{— (1)}$$

$$(\cancel{I} + dI) - \cancel{I} = V(G + j\omega C)\,dx$$

$$\frac{dI}{dx} = V(G + j\omega C) = V\,\cancel{Z}Y \qquad \text{— (2)}$$

Diffn ① w.r. to x

$$\frac{d^2V}{dx^2} = Z\frac{dI}{dx}$$

$$\frac{d^2V}{dx^2} = Z.V(G+j\omega C)$$

$$=(R+j\omega L).V(G+j\omega C) \quad —③$$

Diffn ② w.r. to x

$$\frac{d^2I}{dx^2} = \frac{dV}{dx}.Y.$$

$$\frac{d^2I}{dx^2} = I(R+j\omega L)(G+j\omega C) \quad —④$$

But we know,

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\frac{d^2V}{dx^2} = \gamma^2 V \quad —⑤$$

$$\frac{d^2I}{dx^2} = \gamma^2 I \quad —⑥$$

The soln of eqns ⑤ & ⑥ in terms of linear diffn. eqn.

$$V = Ae^{\gamma x} + Be^{-\gamma x} \quad —⑦$$

$$I = Ce^{\gamma x} + De^{-\gamma x} \quad —⑧$$

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Diffn (8) w.r.to x.

$$\frac{dV}{dx} = Ae^{\gamma x}\gamma - Be^{-\gamma x}\gamma \quad -(9)$$

lly,
$$\frac{dI}{dx} = Ce^{\gamma x}\gamma - De^{-\gamma x}.\gamma \quad -(10)$$

we know,

$$\frac{dV}{dx} = IZ \quad -(11) \; [\because \text{from } (1)]$$

comparing (9) & (11)

$$IZ = Ae^{\gamma x}\gamma - B\gamma e^{-\gamma x}$$

$$I = \frac{Ae^{\sqrt{ZY}x}\sqrt{ZY} - Be^{-\sqrt{ZY}x}\sqrt{ZY}}{Z} \quad [\because \gamma = \sqrt{ZY}]$$

$$\therefore \boxed{I = Ae^{\sqrt{ZY}x}\sqrt{\frac{Y}{Z}} - Be^{-\sqrt{ZY}.x}\sqrt{Y/Z}} \quad -(11)$$

again,

$$\frac{dI}{dx} = VY$$

$$Ce^{\gamma x}.\gamma - De^{-\gamma x}\gamma = VY$$

$$\frac{Ce^{\sqrt{ZY}x}\sqrt{ZY} - De^{-\sqrt{ZY}.x}\sqrt{ZY}}{Y} = V.$$

$$Ce^{\sqrt{ZY}x}\sqrt{Z/Y} - De^{-\sqrt{ZY}x}\sqrt{Z/Y} = V$$

$$\therefore \boxed{V = C\sqrt{Z/Y}\,e^{\sqrt{ZY}x} - D\sqrt{Z/Y}\,e^{-\sqrt{ZY}x}} \quad -(12)$$

~~V = Ae~~ (short ckt)

Being small transmission line, $x = 0$
and voltage at receiving $(V_R) = V$
current " " $(I_R) = I$

Sub. above values in (11) & (12).,

Then,

$V_R = A + B \rightarrow$ (13) [from eqⁿ (7) & 8]

$I_R = C + D \longrightarrow$ (14)

$\therefore$ ~~$V_R = A$~~ again from (11) & (12)

$I_R = A\sqrt{Y/Z} - B\sqrt{Y/Z}$ — (15)

$V_R = C\sqrt{Z/Y} - D\sqrt{Z/Y}$ — (16)

$x = \sqrt{Z/Y}$

$1/x = \sqrt{Y/Z}$

(from above)

$I_R = \frac{A}{x} - \frac{B}{x}$ — (17)

$V_R = Cx - Dx$ — (18)

We know

$I_R = C + D$ [$\because$ from (14)

By comparing,

$C + D = \frac{1}{x}(A - B)$

$Cx + Dx = A - B$ — (19)

$V_R = Cx - Dx$ [from 18]

$A + B = Cx - Dx$. [from (13) & (18)] — (20)

solving eqns (13) & (20)

~~$A - B + A + B$~~

$A + \cancel{B} + A - \cancel{B} = Cx - \cancel{Dx} + Cx + \cancel{Dx}$

$\boxed{A = Cx}$ — (21)

sub $A = Cx$ in (20), then

$\cancel{Cx} + B = \cancel{Cx} - Dx$

$\boxed{B = -Dx}$ — (22)

$\therefore \boxed{V_R = Cx - Dx}$ — (23)

But $I_R = C + D$, multiply by 'x' on both sides,

$$\left.\begin{aligned} I_R x &= Cx + Dx \\ V_R &= Cx - Dx \end{aligned}\right\} \text{— (24)}$$

Adding (23) & (24)

$V_R + I_R x = Cx - \cancel{Dx} + Cx + \cancel{Dx}$

$V_R + I_R x = 2Cx$

$\therefore C = \dfrac{V_R}{2x} + \dfrac{I_R}{2}$ — (25)

sub: 24 from 23

$$V_R - I_R x = \cancel{Cx} + Dx - \cancel{Cx} - Dx$$

$$\frac{V_R}{2x} - \frac{I_R}{2} = -D$$

$$D = \frac{I_R}{2} - \frac{V_R}{2x} \quad \text{—— } (26)$$

$$C = \frac{V_R}{2x} + \frac{I_R}{2} \qquad \left[\text{from } 25 \text{ \& } x = \sqrt{Z/Y}\right]$$

$$= \frac{I_R}{2} + \frac{V_R}{2\sqrt{Z/Y}}$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{Y/Z} \quad \text{—— } (27)$$

$$D = \frac{I_R}{2} - \frac{V_R}{2x} \qquad \left[\because \text{from } 26 \text{ \& } x = \sqrt{Z/Y}\right]$$

$$= \frac{I_R}{2} - \frac{V_R}{2}\sqrt{Y/Z} \quad \text{—— } (28)$$

we know,

$$A = Cx$$

$$\therefore \boxed{A = \frac{I_R}{2}x + \frac{V_R}{2}} \quad \text{—— } (29)$$

$$B = -Dx$$

$$\boxed{B = -\frac{I_R}{2}x + \frac{V_R}{2}} \quad \text{—— } (30)$$

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$$A = Cx$$

$$B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{Z/Y} \qquad [B = -Dx]$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{Y/Z}$$

$$D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{Y/Z}$$

$$V_R = A\sqrt{Z/Y}\, e^{\sqrt{ZY}x} - B\sqrt{Z/Y}\, e^{-\sqrt{ZY}x}$$

$$I_R = C\sqrt{Y/Z}\, e^{\sqrt{ZY}x} - D\sqrt{Y/Z}\, e^{-\sqrt{ZY}x}$$

$$A = Cx$$

$$= \frac{I_R}{2}\sqrt{Z/Y} + \frac{V_R}{2} \qquad [\because x = \sqrt{Z/Y}]$$

$$v = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right] e^{\sqrt{ZY}x} + \frac{V_R}{2}\left[1 - \frac{Z_0}{Z_R}\right] e^{-\sqrt{ZY}x}$$

$[\because V = IR$

$V_R = I_R Z_R]$

$V_R/Z_R = I_R$

$$A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{Z/Y}$$

$$\boxed{A = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]} \quad - (31) \qquad \left(Z_0 = \sqrt{Z/Y}\right)$$

$$\boxed{B = \frac{V_R}{2}\left[1 - Z_0/Z_R\right]} \quad - (32)$$

$$C = \frac{V_R}{2Z_R} - \frac{V_R}{2}\frac{1}{Z_0}$$

$$\boxed{C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]} \quad \text{— (33)}$$

$$\boxed{D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]} \quad \text{— (34)}$$

$$V = \frac{V_R}{2}\left[1 + \frac{Z_0}{Z_R}\right] e^{\sqrt{ZY}x} + \frac{V_R}{2}\left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY}x} \quad \text{— (35)}$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right) e^{\sqrt{ZY}x} - \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right) e^{-\sqrt{ZY}x} \quad \text{— (36)}$$

$$V = \frac{V_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY}x} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY}x}\right] \quad \text{— (37)}$$

$$I = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right) e^{\sqrt{ZY}x} - \left(1 - \frac{Z_R}{Z_0}\right) e^{-\sqrt{ZY}x}\right] \quad \text{— (38)}$$

$$V = V_R\left[\frac{e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}}{2}\right] + I_R Z_0\left[\frac{e^{\sqrt{ZY}x} - e^{-\sqrt{ZY}x}}{2}\right]$$

$$\boxed{\begin{aligned} V &= V_R \cosh\sqrt{ZY}x + I_R Z_0 \sinh\sqrt{ZY}x \quad \text{— (39)} \\ I &= I_R \cosh\sqrt{ZY}x + \frac{V_R}{Z_0}\sinh\sqrt{ZY}x \quad \text{— (40)} \end{aligned}}$$

~~Phy~~ ~~Physical sign solution of~~ :

Physical significance of general solution of ~~line~~ transmission line

From general solution,

$\gamma = \sqrt{ZY}$

$x = l$

$V = V_s = V_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$ — ①

$I = I_s = I_R \cosh \gamma l + \frac{V_R}{Z_0} \sinh \gamma l$ — ②

$V = IZ$

$V_s \rightarrow$ sending voltage

$I_s \rightarrow$ " current.

we know that, $V_R = I_R Z_R$ sub in ① & ②

$V_s = I_R \left[Z_R \cosh \gamma l + Z_0 \sinh \gamma l \right]$ — ③

$I_s = I_R \left[\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l \right]$ — ④

$$Z_s = \frac{V_s}{I_s} = \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l} \right]$$

$$Z_s = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right] \quad \text{— ⑤}$$

when receiving end impedence is terminated by its characteristic impedance Z_0 then the eqn ⑤ can be written as

i.e. $Z_0 \leftrightarrow Z_R$

$$Z_S = Z_0\left(\frac{\cancel{Z_0}\,(\cosh\gamma l + \sinh\gamma l)}{\cancel{Z_0}\,(\cosh\gamma l + \sinh\gamma l)}\right)$$

$\therefore Z_0 = Z_L$ terminated.

$$\therefore Z_S = Z_0\left(\frac{\cancel{\cosh\gamma l + \sinh\gamma l}}{\cancel{\cosh\gamma l + \sinh\gamma l}}\right)$$

$Z_0 = Z_L$
No power loss.

$$\therefore \boxed{Z_S = Z_0} \quad \text{— (6)}$$

Now, consider an infinite length or infinite transmission line with length $l \to \infty$, so eqn (5)

$$Z_S = Z_0\left(\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right)$$

$$= Z_0\,\frac{\cancel{\cosh\gamma l}}{\cancel{\cosh\gamma l}}\left(\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right)$$

$$= Z_0\left(\frac{Z_R + Z_0\tanh\gamma\infty}{Z_0 + Z_R\tanh\gamma\infty}\right) \quad \left(\because l \to \infty,\ \tanh\gamma\infty = 1\right)$$

$$= Z_0\left(\frac{\cancel{Z_R + Z_0}}{\cancel{Z_0 + Z_R}}\right)$$

$$= Z_0$$

$$\therefore \boxed{Z_S = Z_0}$$ //

→ It is the ratio of reflected voltage at the load to the incident voltage at the load.

It can be denoted as 'K'.

i.e. $K = \frac{V_R}{V_S}$, $-K = \frac{I_R}{I_S}$

Also,
$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

Incident voltage

reflected voltage

$$V = \frac{V_R}{2}\left[\frac{Z_R+Z_0}{Z_R} \cdot e^{\gamma x} + \frac{Z_R-Z_0}{Z_0} e^{-\gamma x}\right]$$

$$= \frac{V_R(Z_R+Z_0)}{2Z_R} e^{\gamma x} + \frac{V_R(Z_R-Z_0)}{2Z_0} e^{-\gamma x}$$