UNIT I
PART B

1. Explain the characteristics impedance of symmetrical network.

4-2. Characteristic impedance of symmetrical networks

When $Z_1 = Z_2$ or the two series arms of a T network are equal, or $Z_a = Z_c$ and the shunt arms of a π network are equal, the networks are said to be *symmetrical*.

Filter networks are ordinarily set up as symmetrical sections, basically of the T or π type, such as shown at (b) and (d), Fig. 4-2.

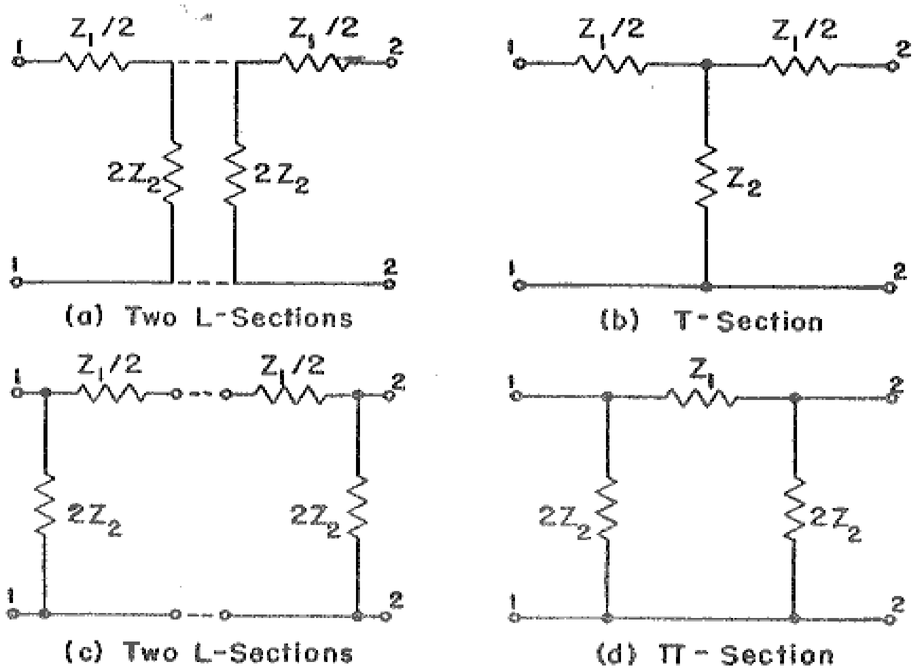


Fig. 4-2. The T and π sections as derived from unsymmetrical L sections, showing notation used in symmetrical network analysis.

Attention is called to the peculiarities of notation employed on the various arms. This peculiarity is largely dictated by custom, arising from the fact that both T and π networks can be considered as built of unsymmetrical L half sections, connected together in one fashion for the T network, and oppositely for the π network as at (a) and (c), Fig. 4-2. A series connection of several T or π networks leads to so-called "ladder networks," which are indistinguishable one from the other except for the end or terminating L half sections, as can be seen in Fig. 4-3.

For a symmetrical network the image impedances Z_{1i} and Z_{2i}, of Eqs. 3-15 and 3-16, are equal to each other, and the image impedance is then called the *characteristic impedance* or the *iterative impedance*, Z_0. That is, if a symmetrical T network is terminated in Z_0, its input impedance will also be Z_0, or its impedance transformation ratio is unity. The term iterative impedance is apparent if the terminating impedance Z_0 is considered as the input impedance

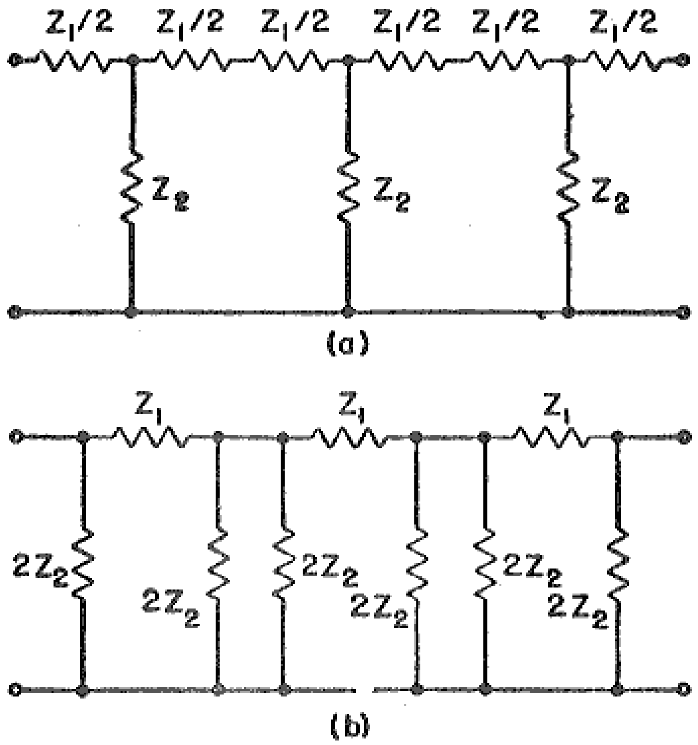


Fig. 4-3. (a) Ladder network made from T sections; (b) ladder network built from π sections. The parallel shunt arms will be combined.

of a chain of similar networks, in which case Z_0 is iterated at the input to each network.

The value of Z_0 for a symmetrical network can be easily determined. For the T network of Fig. 4-4(a), terminated in an impedance Z_0, the input impedance is

$$Z_{1\,\text{in}} = \frac{Z_1}{2} + \frac{Z_2(Z_1/2 + Z_0)}{Z_1/2 + Z_2 + Z_0} \tag{4-9}$$

It can be assumed that if Z_0 is properly chosen in terms of the network arms, it should be possible to make $Z_{1\,\text{in}}$ equal to Z_0. Requiring this equality gives

$$Z_0 = \frac{Z_1^2/4 + Z_1Z_2 + Z_2Z_0 + Z_1Z_0/2}{Z_1/2 + Z_2 + Z_0}$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

For the symmetrical T section, then,

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2} = \sqrt{Z_1Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} \qquad (4\text{-}10)$$

becomes the characteristic impedance. This result could also have been immediately obtained from Eqs. 3-15 and 3-16 for the image

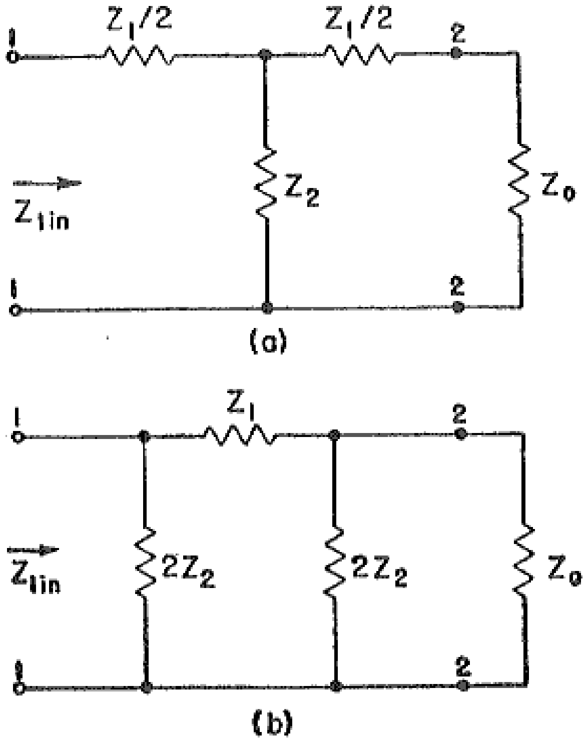


Fig. 4-4. Determination of Z_0: (a) for a T section; (b) for a π section.

impedance of a T section, by using the values of the arms of Fig. 4-4.

Similarly, for the π section of Fig. 4-4 (b) the input impedance is

$$Z_{1\,\text{in}} = \frac{\left[Z_1 + \left(\dfrac{2Z_2Z_0}{2Z_2 + Z_0}\right)\right]2Z_2}{Z_1 + \dfrac{2Z_2Z_0}{2Z_2 + Z_0} + 2Z_2}$$

Requiring that $Z_{1\,\text{in}} = Z_0$ leads to

$$Z_{0\pi} = \sqrt{\frac{Z_1Z_2}{1 + Z_1/4Z_2}} \qquad (4\text{-}11)$$

which is the characteristic impedance of the symmetrical π section.

In Chapter 1, certain information concerning networks was developed from measurements of Z_{oc} and Z_{sc}. If these measurements are made on the T section of (a), Fig. 4-4, exclusive of the load Z_0, then

$$Z_{1oc} = Z_{oc} = \frac{Z_1}{2} + Z_2$$

$$Z_{1sc} = Z_{sc} = \frac{Z_1}{2} + \frac{Z_1 Z_2/2}{Z_1/2 + Z_2}$$

$$Z_{oc}Z_{sc} = \frac{Z_1^2}{4} + Z_1 Z_2 = Z_{0T}^2 \qquad (4\text{-}12)$$

Similar work for the π section leads to

$$Z_{oc}Z_{sc} = \frac{4Z_2^2 Z_1}{Z_1 + 4Z_2} = Z_{0\pi}^2$$

Therefore, for a symmetrical network,

$$Z_0 = \sqrt{Z_{oc}Z_{sc}} \qquad (4\text{-}13)$$

This result could have been directly obtained from the image impedance relations of Section 3-3. It is a valuable relationship, since it supplies an easy experimental means of determining the Z_0 of any symmetrical network.

2. Develop the differential equations governing the voltage and current at any point on a uniform transmission line, and then solve these to obtain the voltage and current in terms of the load current and voltage.
3. Design a low pass filter (both T & Π) having cut off frequency 1 KHZ to
   i) Operate with a terminated load resistance of 200Ω
   ii) Find the frequency at which this filter offers attenuation of 19.1 db.
4. Explain about filter fundamentals.

4-6. Filter fundamentals; pass and stop bands

Ideally it is desired that a filter network transmit or *pass* a desired frequency band without loss, whereas it should *stop* or completely *attenuate* all undesired frequencies. The propagation constant $\gamma = \alpha + j\beta$, being a function of frequency by Eq. 4-38, can supply information on the ability of the filter to perform as desired. If $\alpha = 0$ or $I_1 = I_2$, then there is no attenuation, only a phase shift, in transmitting a signal through the filter, and operation is in a *pass band* of frequencies. When α has a positive value, then I_2 is smaller in magnitude than I_1, attenuation has occurred and operation is in an attenuation or *stop band* of frequencies.

The propagation constant γ may be conveniently studied by use of Eq. 4-36:

$$\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} \tag{4-43}$$

It will first be assumed that the network contains only pure reactances, and thus $Z_1/4Z_2$ will be real, and either positive or negative, depending on the type of reactance used for Z_1 and Z_2. Expanding gives

$$\sinh \frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2} + \frac{j\beta}{2}\right)$$

$$= \sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} \tag{4-44}$$

as an equation containing much information.

If Z_1 and Z_2 *are the same type of reactance* then $|Z_1/4Z_2| > 0$, or the ratio is positive and real. This requires that $\sinh \gamma/2$ be real, which mea... that the imaginary term in Eq. 4-44 must equal zero and that

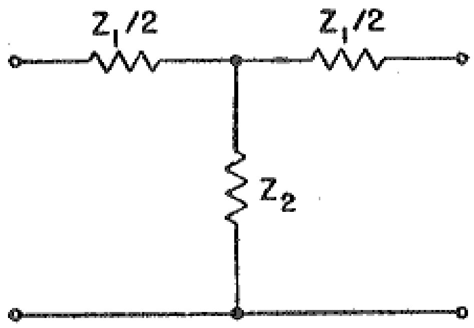


Fig. 4-7. Symmetrical T network.

(a) $\cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = 0$

(b) $\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

are simultaneously satisfied.

From (a),

$$\sin \frac{\beta}{2} = 0; \qquad \beta = n\pi \qquad \text{where } n = 0, 2, 4, \ldots$$

From (b), since $\cos \beta/2 = 1$, then

$$\sinh \frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

and the attenuation will be given by

$$\alpha = 2 \sinh^{-1} \sqrt{\frac{Z_1}{4Z_2}} \tag{4-45}$$

Thus the *condition that* $|Z_1/4Z_2| > 0$ *implies a stop or attenuation band of frequencies.*

If Z_1 *and* Z_2 *are opposite types of reactance* then $Z_1/4Z_2$ is negative, $|Z_1/4Z_2| < 0$, and the radical of Eq. 4-43 is imaginary. The real term in Eq. 4-44 must then be zero, so that

(c) $$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0$$

(d) $$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

must be satisfied.

Two conditions are possible from the above:

I. $\sinh\frac{\alpha}{2} = 0$; therefore

$$\alpha = 0; \qquad \beta \neq 0; \qquad \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

II. $\cos\frac{\beta}{2} = 0$; therefore $\sin\frac{\beta}{2} = \pm 1$ and

$$\alpha \neq 0; \qquad \beta = (2n - 1)\pi; \qquad \cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Condition I leads to a pass band, or region of zero attenuation, which is limited by the upper limit on the sine, or by $\sin\frac{\beta}{2} = 1$, or it is required that

$$-1 < \frac{Z_1}{4Z_2} < 0$$

The phase angle in this pass band will be given by

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (4\text{-}46)$$

Condition II leads to a stop or attenuation band since $\alpha \neq 0$. The phase angle is π, and the attenuation is given by

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (4\text{-}47)$$

Because the hyperbolic cosine has no value below unity, it appears that the region in which condition II applies is a stop band where

$$\frac{Z_1}{4Z_2} < -1$$

Values of $Z_1/4Z_2$ can then be classified into three regions, with corresponding values of α and β, these regions being bounded by $Z_1/4Z_2$ values of $+\infty$, 0, -1 and $-\infty$, as given below:

$Z_1/4Z_2 =$	$+\infty$ to 0	0 to -1	-1 to $-\infty$
Reactance type:	Same	Opposite	Opposite
Band:	Stop	Pass	Stop
α:	$2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	0	$2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$
β:	π	$2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	π

The frequencies at which the network changes from a pass network to a stop network, or vice versa, are called *cutoff frequencies.* These frequencies occur when

$$\left.\begin{aligned} \frac{Z_1}{4Z_2} &= 0 \quad \text{or} \quad Z_1 = 0 \\ \frac{Z_1}{4Z_2} &= -1 \quad \text{or} \quad Z_1 = -4Z_2 \end{aligned}\right\} \tag{4-48}$$

where Z_1 and Z_2 are opposite types of reactance.

Since Z_1 and Z_2 may have a number of configurations, as L and C elements, or as parallel and series combinations, a variety of types of performance are possible.

The elements considered above were assumed pure reactances, and design is ordinarily carried out on this basis. Measurements of actual performance are then made and adjustments introduced into the design to compensate for deviation of the results from the ideal. In addition to minimizing the losses of physical elements it is also necessary to reduce stray electric and magnetic couplings between elements to obtain more nearly the predicted performance.

5. Explain about behavior of the characteristics impedance.

4-7. Behavior of the characteristic impedance

It has been shown that for a symmetrical T network

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

In a network made up entirely of pure reactances this expression for the characteristic impedance becomes

$$Z_{0T} = \sqrt{-X_1 X_2 \left(1 + \frac{X_1}{4X_2}\right)} \qquad (4\text{-}49)$$

where the X terms will carry their own signs, the minus sign under the radical being due to j^2.

In Section 4-6 it was shown that a stop band exists where X_1 and X_2 are the same type of reactance. The ratio $X_1/4X_2$ will be real and positive, and the characteristic impedance will be a pure reactance in this attenuation region.

A pass band was shown to exist where X_1 and X_2 were of opposite reactance types and $-1 < X_1/4X_2 < 0$. Placing these conditions in Eq. 4-49 results in the product X_1X_2 being negative, with the bracketed term positive. The over-all sign under the radical will be positive and Z_0 will be real, and thus able to absorb power from a source.

A stop band exists with X_1 and X_2 of opposite types, but with $X_1/4X_2 < -1$. This implies that the product X_1X_2 is a negative term, and that the bracketed term is negative. When combined with the negative sign present in Eq. 4-49, the over-all sign under the radical will be negative and Z_0 will be a pure reactance in this stop region.

It has been shown that in a pass band Z_0 is real and positive. If the reactive network is terminated with a resistive $Z_0 = R_0$, then the input impedance is R_0, and the network can accept power and will transmit it to the resistive load without loss or attenuation. If the network is supplied by a source having R_0 as its internal impedance, the system will be matched at each set of terminals, and maximum power will be delivered from generator to load.

In a stop band Z_0 has been shown to be reactive. If the network is terminated in its reactive Z_0, it will appear as a totally reactive circuit and as such cannot accept or transmit power, since there is no resistive element in which the power may be dissipated. The network may transmit voltage or current, but with a 90° phase angle between the two and with considerable attenuation.

Similar reasoning may be applied to the Z_0 for a π network if it is noted that

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} \qquad (4\text{-}50)$$

and Z_1Z_2 is always real for Z_1 and Z_2 as pure reactances. Thus it is seen that the conditions developed for pass and stop bands for T sections likewise apply for π sections.

6. What is constant k filter? & derive the expressions for LPF.

4-8. The constant-*k* low-pass filter

If Z_1 and Z_2 of a reactance network are unlike reactance arms, then

$$Z_1Z_2 = k^2$$

where k is a constant independent of frequency. Networks or filter sections for which this relation holds are called *constant-k* filters.

As a special case, let $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$, then the product

$$Z_1Z_2 = \frac{L}{C} = R_k^2 \tag{4-51}$$

The term R_k is used since k must be real if Z_1 and Z_2 are of opposite type. A T section so designed would appear as at (a), Fig. 4-8.

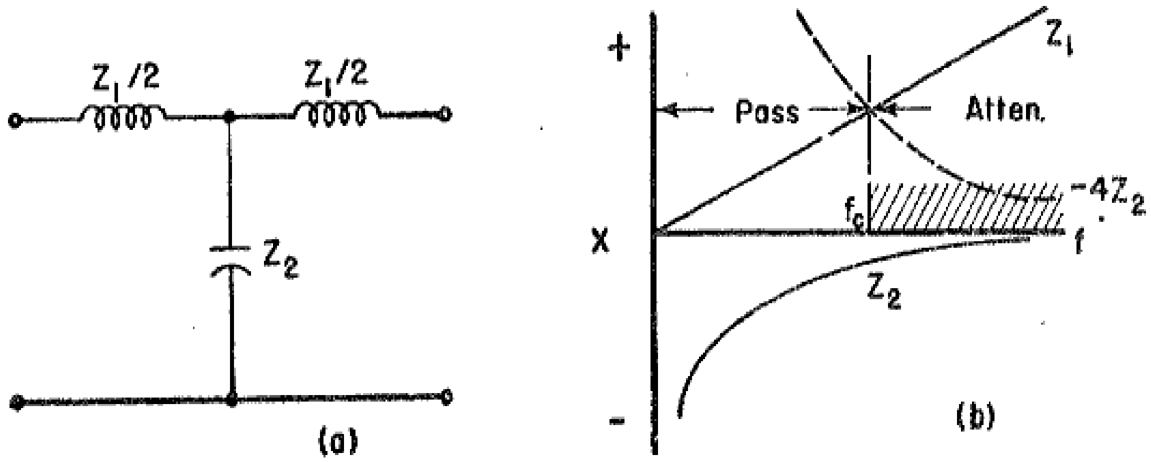


Fig. 4-8. (a) Low-pass filter section; (b) reactance curves demonstrating that (a) is a low-pass section or has a pass band between $Z_1 = 0$ and $Z_1 = -4Z_2$.

The reactances of Z_1 and $4Z_2$ will vary with frequency as sketched at (b), Fig. 4-7. The curve representing $-4Z_2$ may be drawn and compared with the curve for Z_1. It has been shown by Eq. 4-48 that a pass band starts at the frequency at which $Z_1 = 0$ and runs to the frequency at which $Z_1 = -4Z_2$. Thus the reactance curves show that a pass band starts at $f = 0$ and continues to some higher frequency f_c. All frequencies above f_c lie in a stop, or attenuation, band. Thus the network is called a *low-pass* filter.

The cutoff frequency f_c may be readily determined, since at that point

$$Z_1 = -4Z_2, \qquad j\omega_c L = \frac{4j}{\omega_c C}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} \tag{4-52}$$

This expression may be used to develop certain relations applicable to the low-pass network. Then $\sinh \gamma/2$ may be evaluated as

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{j\omega\sqrt{LC}}{2}$$

and in view of Eq. 4-52 this is

$$\sinh\frac{\gamma}{2} = j\frac{f}{f_c} \tag{4-53}$$

Then if the frequency f is in the pass band or $f/f_c < 1$, so that $-1 < Z_1/4Z_2 < 0$, then

$$\frac{f}{f_c} < 1, \qquad \alpha = 0, \qquad \beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)$$

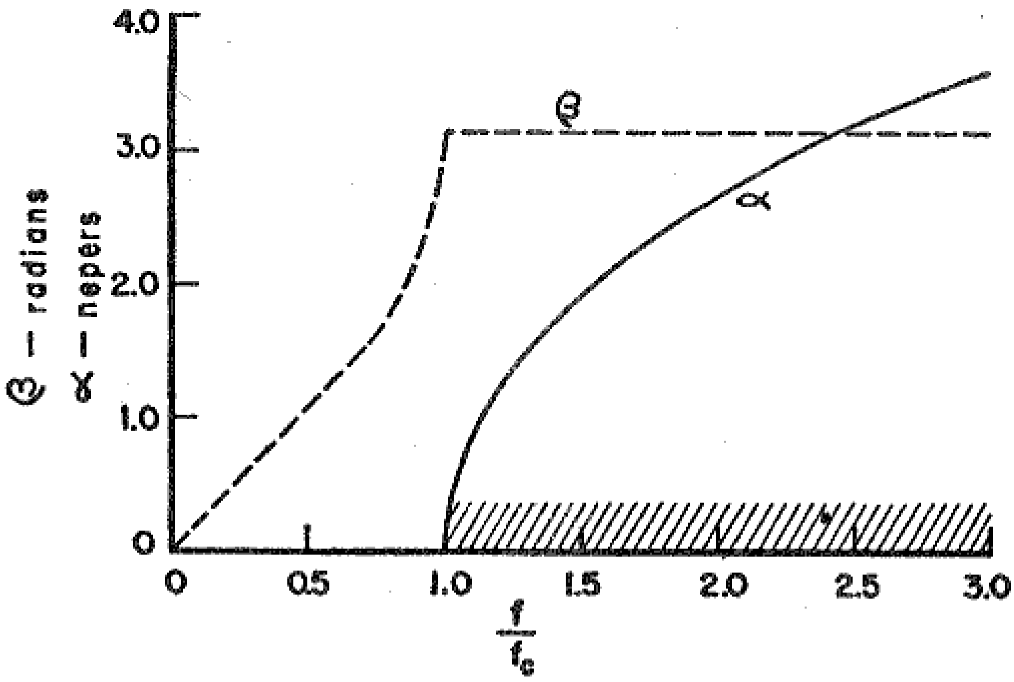


Fig. 4-9. Variation of α and β with frequency for the low-pass section.

whereas if frequency f is in the attenuation band or $f/f_c > 1$, so that $Z_1/4Z_2 < -1$, then

$$\frac{f}{f_c} > 1, \qquad \alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right), \qquad \beta = \pi$$

thereby allowing determination of α and β. The variation of α and β is plotted in Fig. 4-9 as a function of f/f_c. This method shows that the attenuation α is zero throughout the pass band but rises gradually from the cutoff frequency at $f/f_c = 1.0$ to a value of ∞ at infinite frequency. The phase shift β is zero at zero frequency and increases gradually through the pass band, reaching π at f_c and remaining at π for all higher frequencies.

The characteristic impedance of a T section was obtained as

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

which becomes
$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)} \qquad (4\text{-}54)$$

for the low-pass constant-k section under discussion. By use of Eq. 4-52 the characteristic impedance of a low-pass filter may be stated as

$$Z_{0T} = \sqrt{\frac{L}{C}\left[1 - \left(\frac{f}{f_c}\right)^2\right]} \qquad (4\text{-}55)$$

$$= R_k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \qquad (4\text{-}56)$$

in accordance with the definition of R_k in Eq. 4-51. Values of Z_{0T}/R_k are plotted against f/f_c in Fig. 4-10. It may be seen that Z_{0T} varies throughout the pass band, reaching a value of zero at cutoff, then becomes imaginary in the attenuation band, rising to infinite reactance at infinite frequency.

A low-pass filter may be designed from a knowledge of the cutoff frequency desired and the load resistance to be supplied. It is desirable that the Z_0 in the pass band match the load; but because of the nature of the Z_0 curve in Fig. 4-10, this result can occur at only one frequency. This match may be arranged to occur at any frequency which it is desired to favor by an impedance match.

For reasons which will appear in Section 4-13, the load is chosen as $R = R_k = \sqrt{L/C}$, which will favor zero frequency for a low-pass filter.

The design of a low-pass filter may be readily carried out. From

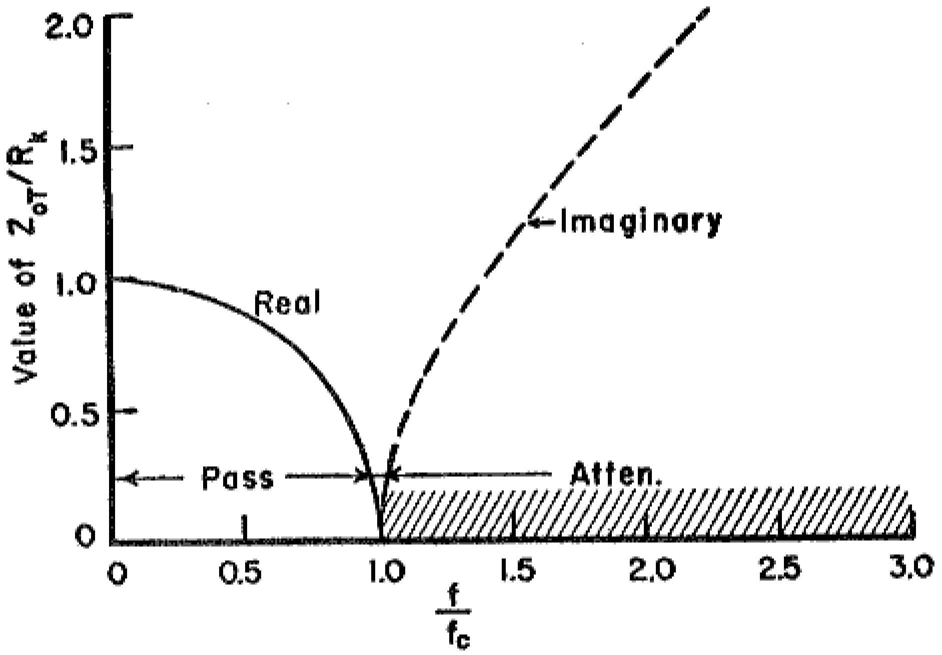


Fig. 4-10. Variation of Z_{0T}/R_k with frequency for the low-pass section.

the relation that at cutoff

$$Z_1 = -4Z_2$$

it is seen that

$$\omega_c L = \frac{4}{\omega_c C}$$

Using the cutoff frequency equation changes this to

$$\pi^2 f_c^2 LC = 1$$

and use of the relation $R = \sqrt{L/C}$ gives for the value of the shunt capacitance arm

$$C = \frac{1}{\pi f_c R} \qquad (4\text{-}57)$$

By similar methods the inductance for Z_1 is obtained as

$$L = \frac{R}{\pi f_c} \qquad (4\text{-}58)$$

Since the design is based on an impedance match at zero frequency only, power transfer to a matched load will drop at higher pass-band frequencies. This condition may be undesirable in certain applications, and a remedy will be discussed in Section 4-13.

A network such as is described here is called a *prototype section*. It may be employed when a sharp cutoff is not required, although cutoff may be sharpened by using a number of such networks in cascade. This is not usually an economic use of circuit elements, and introduces excessive losses over other available methods of raising the attenuation near the cutoff frequency.

7. i) Design a constant k BPF with fc of 3khz & 7.5 khz & nominal characteristics impedance or Ro=900Ω
   ii) design a constant BSF with fc of 3khz & 7.5 khz & nominal characteristics impedance or Ro=900Ω

8. Explain about m derived filter of T & Π sections.

4-10. The *m*-derived T section

The constant-*k* prototype filter section, though simple, has two major disadvantages. The attenuation does not rise very rapidly at cutoff, so that frequencies just outside the pass band are not appreciably attenuated with respect to frequencies just inside the pass band. Also, the characteristic impedance varies widely over the pass band, so that a satisfactory impedance match is not possible. In cases where an impedance match is not important, the attenuation may be built up near cutoff by cascading or connecting a number of constant-k sections in series.

It is more economical to attempt to raise the attenuation near cutoff by other means. Consider first the circuit of (a), Fig. 4-12. The reactance curves sketched at (b) show that this circuit is a low-pass filter. However, it can be seen that the shunt arm is a series

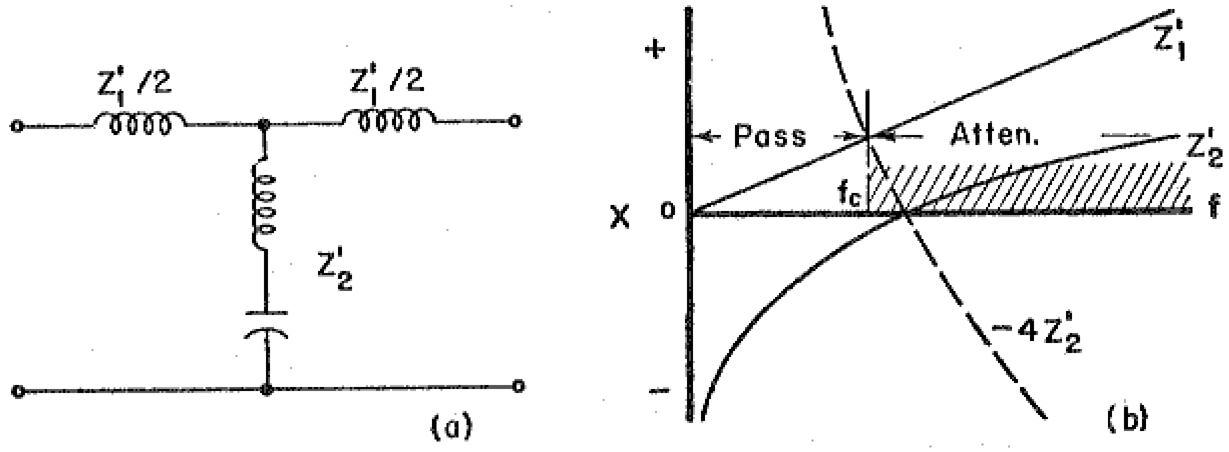


Fig. 4-12. (a) Derivation of a low-pass section having a sharp cutoff action; (b) reactance curves for (a).

circuit resonant at a frequency above f_c. At this resonant frequency the shunt arm appears as a short circuit on the network, or the attenuation becomes infinite. This frequency of infinite or high attenuation is called f_∞; and by reason of the requirement that below f_c the shunt circuit appear as a capacitance, the frequency of resonance, f_∞, will always be higher in value than f_c. If, then, f_∞ can be chosen arbitrarily close to f_c, the attenuation near cutoff may be made high.

The attenuation above f_∞ will fall to low values, so that if high attenuation is desired over the whole attenuation band, it is necessary to use a section such as in Fig. 4-12 for high attenuation near cutoff, in series with a prototype section to provide high attenuation at frequencies well removed from cutoff. For satisfactory matching of several such types of filters in series, it is necessary that the Z_0 of all be identical at all points in the pass band. They will consequently also all have the same pass band.

The network of Fig. 4-12 may be derived by assuming that

$$Z_1' = mZ_1 \qquad (4\text{-}65)$$

the primes indicating the *derived section*. It is then necessary to find the value for Z_2' such that $Z_0' = Z_0$. Setting the characteristic impedances equal,

$$Z_0' = Z_0$$

$$\frac{(mZ_1)^2}{4} + mZ_1Z_2' = \frac{Z_1^2}{4} + Z_1Z_2$$

$$Z_2' = \frac{Z_2}{m} + \frac{1 - m^2}{4m} Z_1 \qquad (4\text{-}66)$$

It then appears that the shunt arm Z_2' consists of two impedances in series, as shown in Fig. 4-13. As required, the characteristic impedance and f_c remain equal to those of the T section prototype containing Z_1 and Z_2 values.

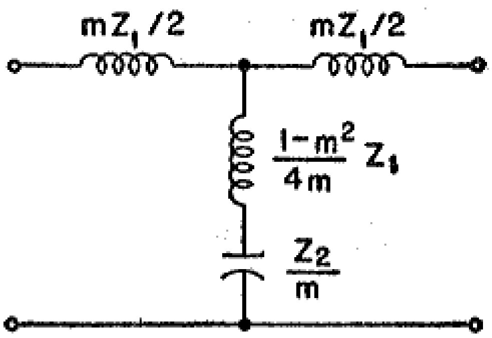


Fig. 4-13. The *m*-derived low-pass filter.

Since m is arbitrary, it is possible to design an infinite variety of filter networks meeting the required conditions on Z_0 and f_c. However, Z_2 will be opposite in sign to Z_1, and it is desired that this relation continue in the two series impedances given by Eq. 4-66 for the Z_2' arm. Equation 4-66 then indicates that $(1 - m^2)/4m$ must be positive, forcing the terms $1 - m^2$ and m always to be positive. Thus m must always be chosen so that

$$0 < m < 1$$

Filter sections obtained in this manner are called *m-derived sections.*

The shunt arm is to be chosen so that it is resonant at some frequency f_∞ above f_c. This means that at the resonant frequency

$$\left|\frac{Z_2}{m}\right| = \left|\frac{1 - m^2}{4m} Z_1\right| \qquad (4\text{-}67)$$

and for the *low-pass filter*

$$\frac{1}{2\pi f_\infty mC} = \frac{1-m^2}{4m} 2\pi f_\infty L$$

$$f_\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}}$$

Since the cutoff frequency for the low-pass filter is

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

the frequency of infinite attenuation will be

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \tag{4-68}$$

from which

$$m = \sqrt{1-(f_c/f_\infty)^2} \tag{4-69}$$

This equation determines the m to be used for a particular f_∞.

Similar relations for the *high-pass filter* can be derived as

$$f_\infty = f_c\sqrt{1-m^2} \tag{4-70}$$

and

$$m = \sqrt{1-(f_\infty/f_c)^2} \tag{4-71}$$

The m-derived section is designed following the design of the prototype T section. The use of a prototype and one or more m-derived sections in series results in a *composite filter*. If a sharp cutoff is desired, an m-derived section may be used with f_∞ near f_c, followed by as many m-derived sections as desired to place frequencies of high attenuation where needed to suppress various signal components or to produce a high attenuation over the entire attenuation band.

The variation of attenuation over the attenuation band for a *low-pass m-derived section* in the stop band is dependent on the sign of the reactances or

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} \quad \text{or} \quad \alpha = 2\sinh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

$$f_c < f < f_\infty \qquad\qquad f_\infty < f$$

For $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the prototype, then

$$\left|\frac{Z_1}{4Z_2}\right| = \frac{m\omega L}{4[1/m\omega C - \omega L(1 - m^2)/4m]}$$

so that for $f_c < f < f_\infty$

$$\alpha = 2 \cosh^{-1} \frac{mf/f_c}{\sqrt{1 - f^2/f_\infty^2}} \qquad (4\text{-}72)$$

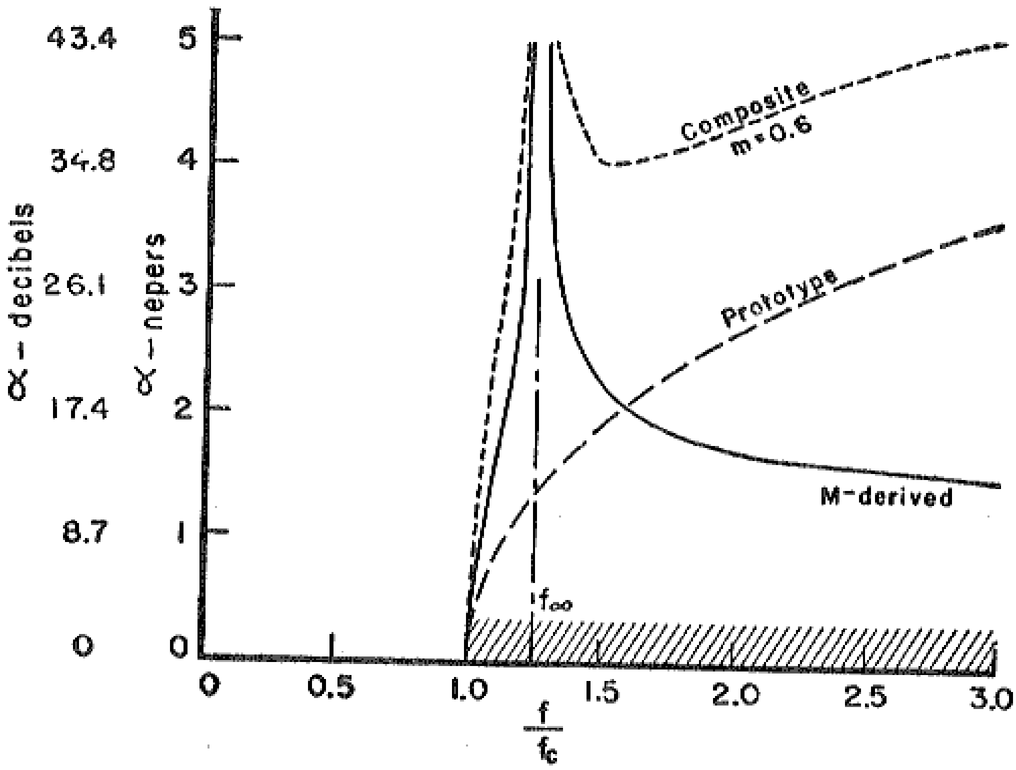


Fig. 4-14. Variation of attenuation for the prototype and m-derived sections, and the composite result of the two in series.

and for $f_\infty < f$

$$\alpha = 2 \sinh^{-1} \frac{mf/f_c}{\sqrt{f^2/f_\infty^2 - 1}} \qquad (4\text{-}73)$$

The value of α may be determined from this expression. Figure 4–14 is a plot of α against f/f_c for $m = 0.6$, which gives a value of f_∞ equal to 1.25 times the cutoff frequency f_c. The great increase in sharpness of cutoff for the m-derived section over the prototype is apparent. The higher attenuation over the whole attenuation band obtained by use of a prototype section and an m-derived section in series as a composite filter is also readily seen.

Again following the procedure of Section 4–8, the phase shift constant β may be determined; in the pass band, from

$$\beta = 2 \sin^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

$$= 2 \sin^{-1} \frac{mf/f_c}{\sqrt{1 - (f^2/f_c^2)(1 - m^2)}} \qquad (4\text{-}74)$$

In the attenuation band, up to f_∞, β has the value π. Above f_∞ the value of β drops to zero, because the shunt arm becomes inductive above resonance. The phase shift of the m-derived section is plotted as a function of f/f_c in Fig. 4-15.

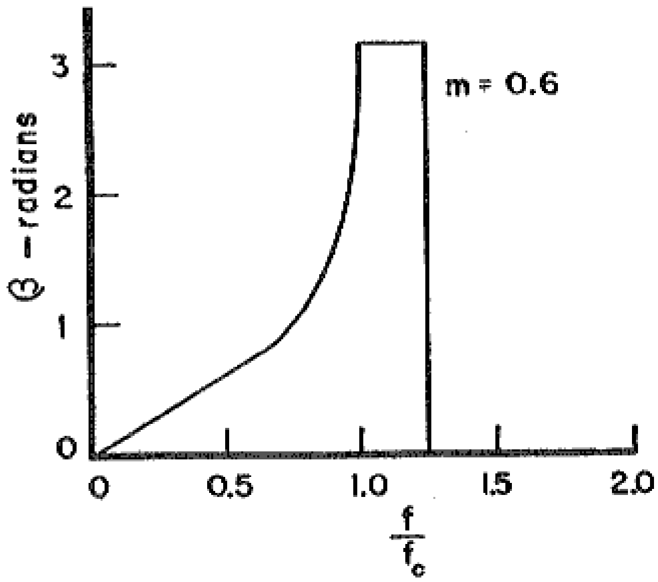


Fig. 4-15. Variation of phase shift β, for the m-derived filter.

This material demonstrates the ability of the m-derived section to overcome the lack of a sharp cutoff in the simple prototype filter. Although it may be noted that the sharpness of cutoff increases for small values of m, the attenuation beyond the point of peak attenuation becomes smaller for small m. This emphasizes the necessity of supplementing the m-derived section with a prototype section in series to raise the attenuation for frequencies well removed from cutoff.

4-11. The m-derived π section

An m-derived π section may also be obtained. The characteristic impedance of the π section is

$$Z_{0\pi} = \frac{Z_1Z_2}{\sqrt{Z_1Z_2(1 + Z_1/4Z_2)}}$$

The characteristic impedances of the prototype and m-derived sections are to be equal so that they may be joined without mismatch. By use of the transformation for the shunt arm,

$$Z_2' = \frac{Z_2}{m} \tag{4-75}$$

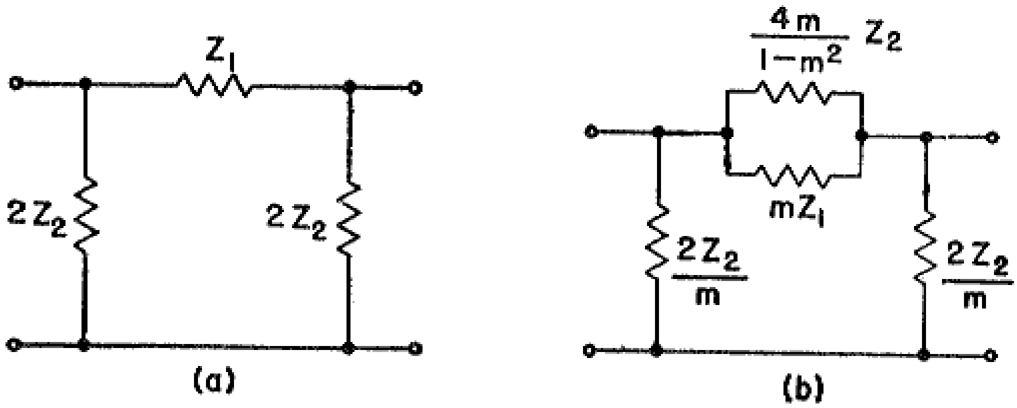


Fig. 4-16. (a) Usual symmetrical π section; (b) the m-derived π filter.

it is possible to equate the characteristic impedances as

$$\frac{Z_1'Z_2/m}{\sqrt{Z_1(Z_2/m)(1 + Z_1'm/4Z_2)}} = \frac{Z_1Z_2}{\sqrt{Z_1Z_2(1 + Z_1/4Z_2)}}$$

from which

$$Z_1' = \frac{1}{\dfrac{1}{mZ_1} + \dfrac{1}{\dfrac{4m}{1-m^2}Z_2}} \tag{4-76}$$

It is apparent that the series arm Z_1' is represented by two impedances in parallel, one being mZ_1, the other being $4m/(1 - m^2)Z_2$ in value.

Equations 4-75 and 4-76 thus give the values to be used in designing the m-derived π section. The circuit is drawn in Fig. 4-16.

4-12. Variation of characteristic impedance over the pass band

It has been shown in Section 4-8 that for a low-pass T section

$$Z_{0T} = R_k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \tag{4-77}$$

The characteristic impedance for a π section is

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 (1 + Z_1/4Z_2)}}$$

Since $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the *low-pass filter*, use of the cutoff frequency expression permits the characteristic impedance of

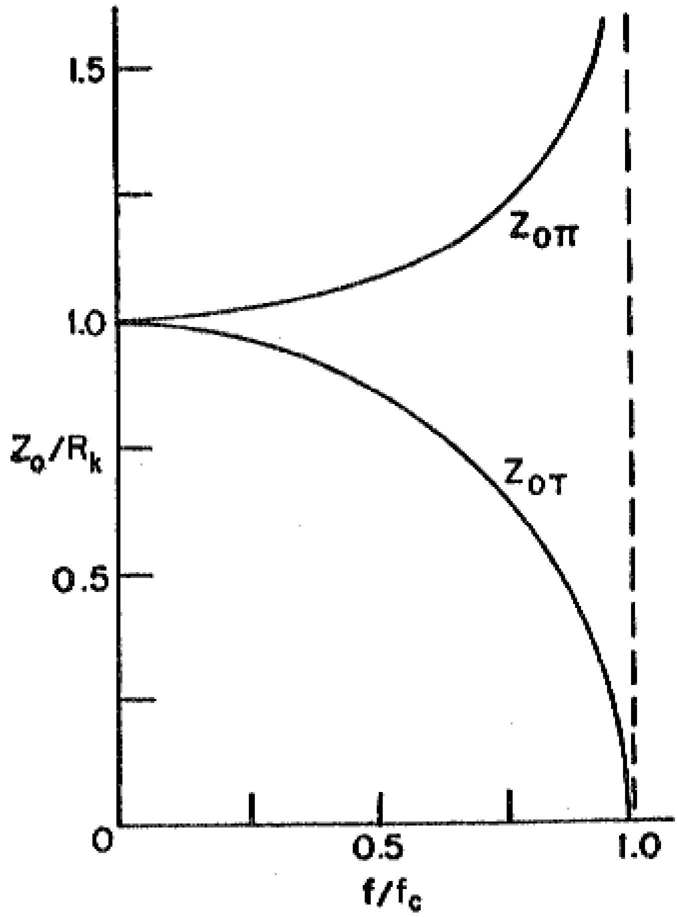


Fig. 4-17. Manner of variation of Z_0 over the pass band for the T and π networks.

the low-pass π section to be expressed as

$$Z_{0\pi} = \frac{L/C}{\sqrt{L/C[1 - (f/f_c)^2]}}$$

$$= \frac{R_k}{\sqrt{1 - (f/f_c)^2}} \qquad (4\text{-}78)$$

The π-section characteristic impedance is plotted over the pass band in Fig. 4-17 as a function of f/f_c and is compared with the curve for the T section, reproduced from Fig. 4-10.

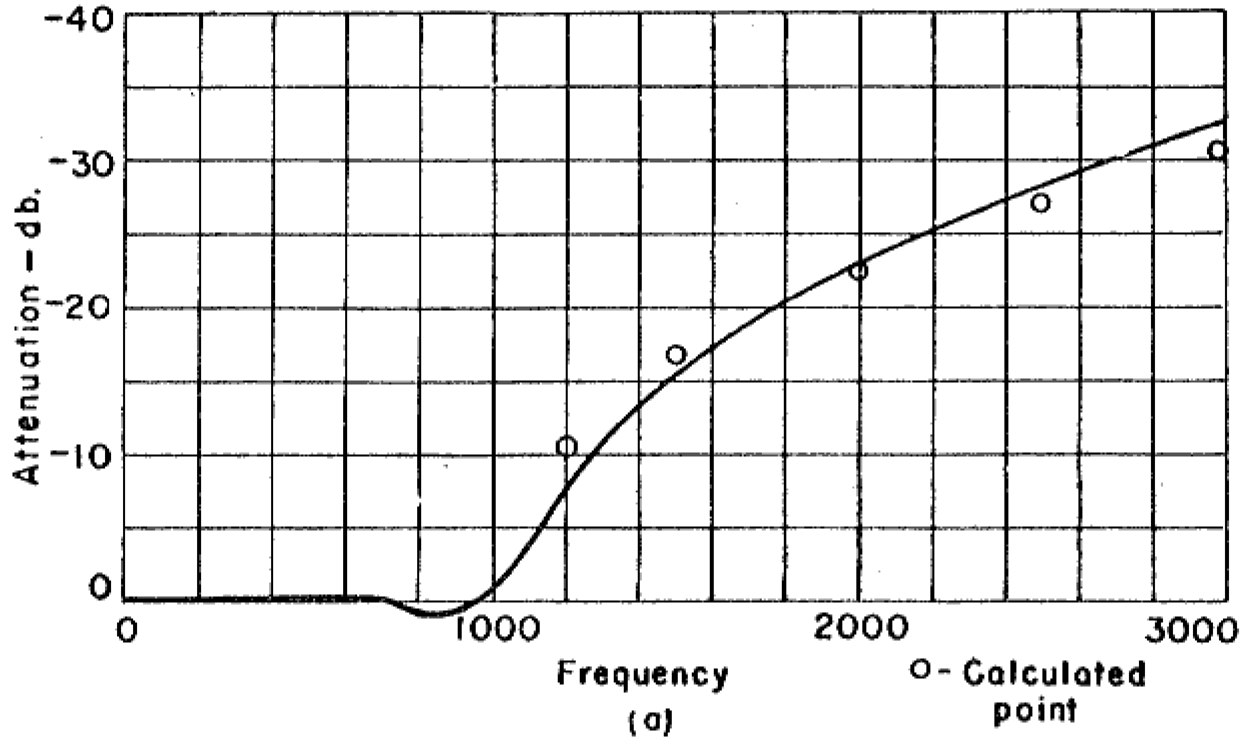
The curves show that the characteristic impedance of neither section is sufficiently constant over the pass band that a load equal to R_k will give a satisfactory impedance match.

9. Explain about filter performance

4-17. Filter performance

To illustrate the sort of approach to the theoretical ideal which is possible in filter design, laboratory filters were assembled in accordance with the designs of Fig. 4-27. The inductors used were toroids on compressed molybdenum-permalloy dust cores, and had Q values of approximately 40. These inductors would not be considered as having very high Q by commercial standards, as values of 100 to 300 are available. The filters were designed for 500 ohms resistance termination, and were so used for each measurement.

Attenuation measurements were made over the pass and stop bands on each of the filter sections. The attenuation of the prototype section, terminated in 500 ohms, is shown in (a), Fig. 4-28. The presence of resistance and the insertion loss of the section causes a rounding of the attenuation curve near cutoff, but otherwise the shape of the curve reasonably fits the theoretical curve of Fig. 4-9. Calculation of attenuation, based on pure reactances and the



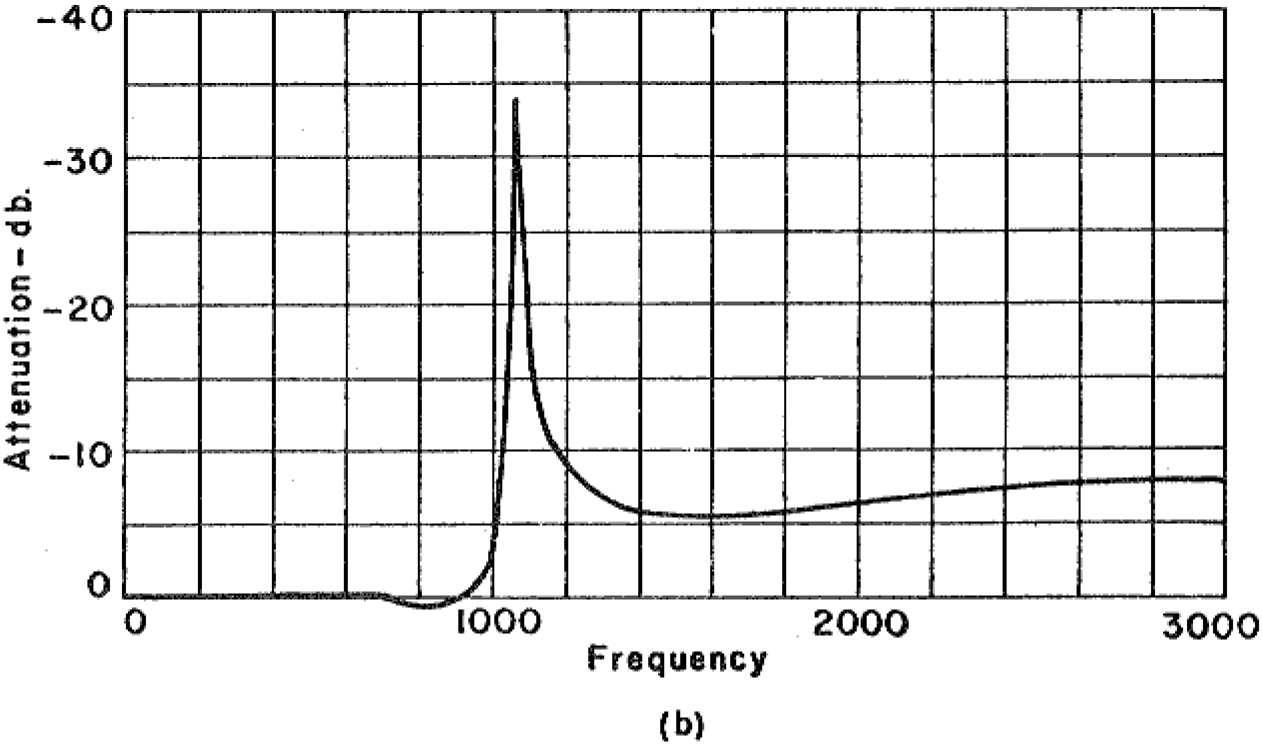
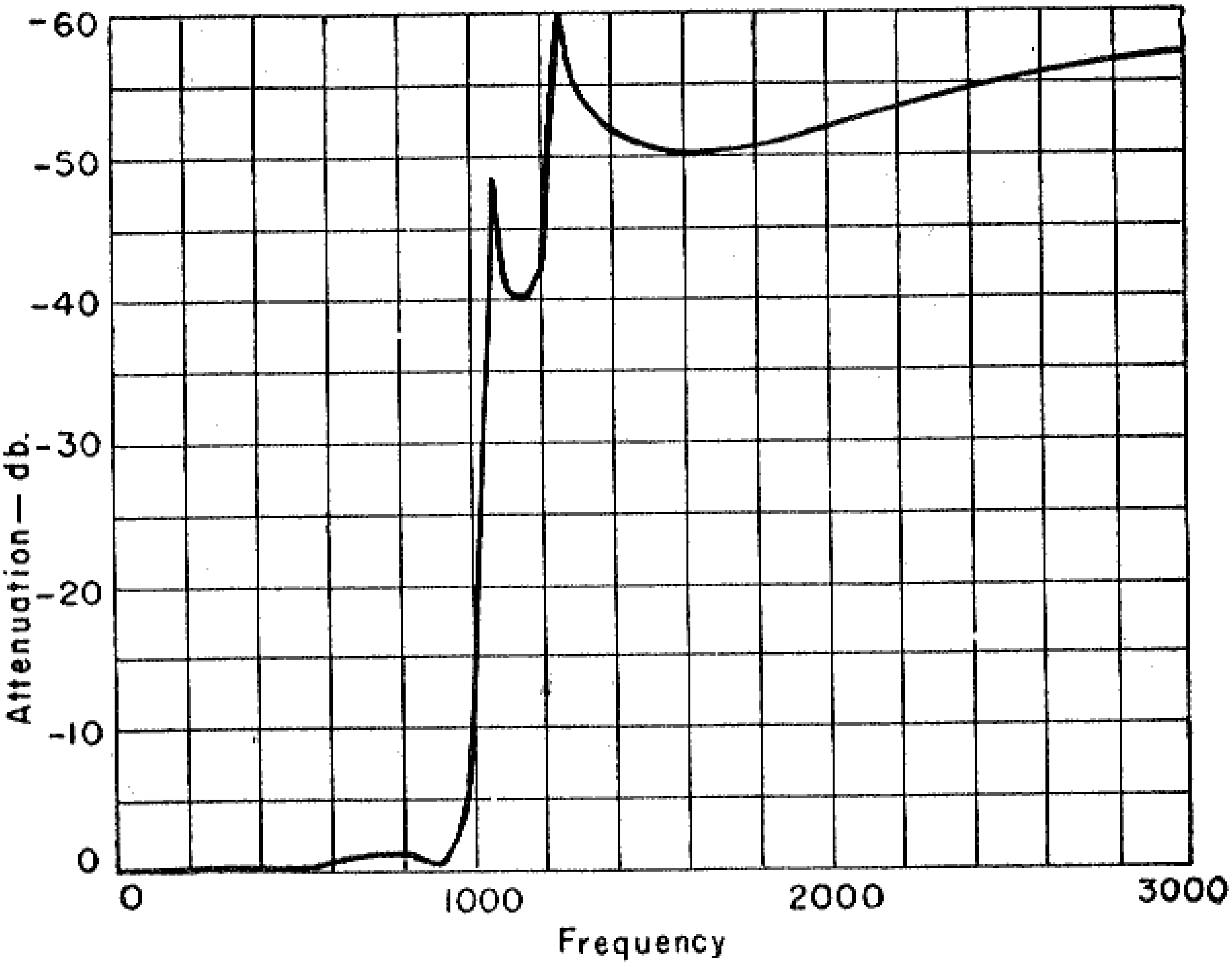


Fig. 4-28. (a) Attenuation of the prototype: $f_c = 1000$ cycles, $R_k = 500$ ohms. (b) Attenuation of the m-derived section: $m = 0.346$, $f_c = 1000$, $f_\infty = 1065$, $R_k = 500$ ohms. (c) Action of composite filter of Fig. 4-27(b).

theoretical equations of Section 4-8, gives 16.7 db attenuation at 1500 cycles or $f/f_c = 1.5$. The measured curve shows 15.5 db, which is a reasonable check. Calculated values of α for a pure



(c)

Fig. 4-28. *(cont.)*.

reactance network are shown as marked points for a few frequencies in Fig. 4-28(a), for comparison with the measurements.

Figure 4-28(b) illustrates the measured attenuation for the m-derived section alone, with $m = 0.343$. This section was designed to have $f_\infty = 1065$ cycles and a cutoff of 1000 cycles, and a peak of attenuation of -34 db is obtained at the frequency specified. The usually undesirable low α values above f_∞ are also found, the slight rise at 3000 cycles probably being due to increased losses in the coils used.

Figure 4-28(c) shows the over-all attenuation of the complete filter of Fig. 4-27, when the prototype and m-derived sectons of (a) and (b) are combined with terminal half sections having $m = 0.6$ and terminated in 500 ohms. The terminal half sections give an additional value of high attenuation at $f_\infty = 1250$ cycles.

Due to probable slight mismatches and losses, there is a small irregularity in the pass band, but reasonably sharp cutoff characteristics are obtained.

10. What is crystal filter & explain it.

4-18. Crystal filters

The lattice structure can also be shown to have filter properties. Considering the network of Fig. 4-29,

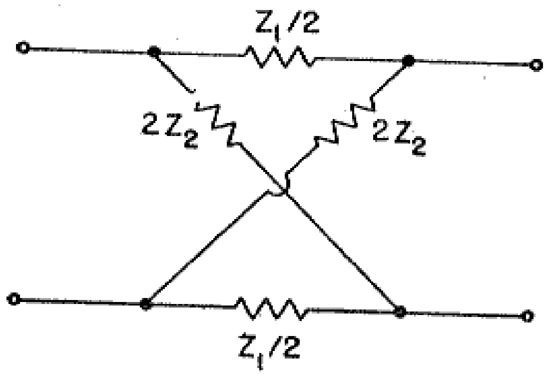


Fig. 4-29. Lattice filter section.

$$Z_{oc} = \frac{(Z_1/2 + 2Z_2)^2}{2(Z_1/2 + 2Z_2)}$$

$$= \frac{Z_1}{4} + Z_2 \qquad (4\text{-}113)$$

$$Z_{sc} = \frac{Z_1 Z_2}{Z_1/2 + 2Z_2} + \frac{Z_1 Z_2}{Z_1/2 + 2Z_2}$$

$$= \frac{Z_1 Z_2}{Z_1/4 + Z_2} \qquad (4\text{-}114)$$

The characteristic impedance of the lattice section then is

$$Z_{0L} = \sqrt{Z_{oc} Z_{sc}} = \sqrt{Z_1 Z_2} \qquad (4\text{-}115)$$

Thus if the section elements are reactive, Z_{0L} is real, or a pass band exists for frequencies for which Z_1 and Z_2 are of opposite sign. Over ranges where Z_1 and Z_2 have the same sign, an attenuation band exists.

Propagation can be investigated further by noting that

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{\frac{Z_1}{Z_2}} \left[\frac{1}{1 + Z_1/4Z_2}\right] \qquad (4\text{-}116)$$

It may be noted that Z_{0L} depends on the product of Z_1 and Z_2, whereas γ depends on the ratio of Z_1 to Z_2. This feature permits somewhat greater versatility in design of the lattice section over the T or π section, especially for filters in which certain of the elements are constructed of piezoelectric crystals. These crystals have a resonant frequency of mechanical vibration dependent on certain of their dimensions; and because of the very high equivalent Q of the crystals, it is possible to make very narrow band filters and filters in which the attenuation rises very rapidly at cutoff.

The equivalent electric circuit of a quartz mechanical-filter crystal is shown in Fig. 4-30(a), which shows a possibility of both resonance and antiresonance occurring. The inductance L_x is very large, being in henrys for crystals resonating near 500 kc, so that while R_x may approximate a few hundred or few thousand ohms, the effective Q may be in the range of 10,000 to 30,000. Considering the properties of resonant circuits, such as Q would provide a band width of 20 to 50 cycles at 500 kc.

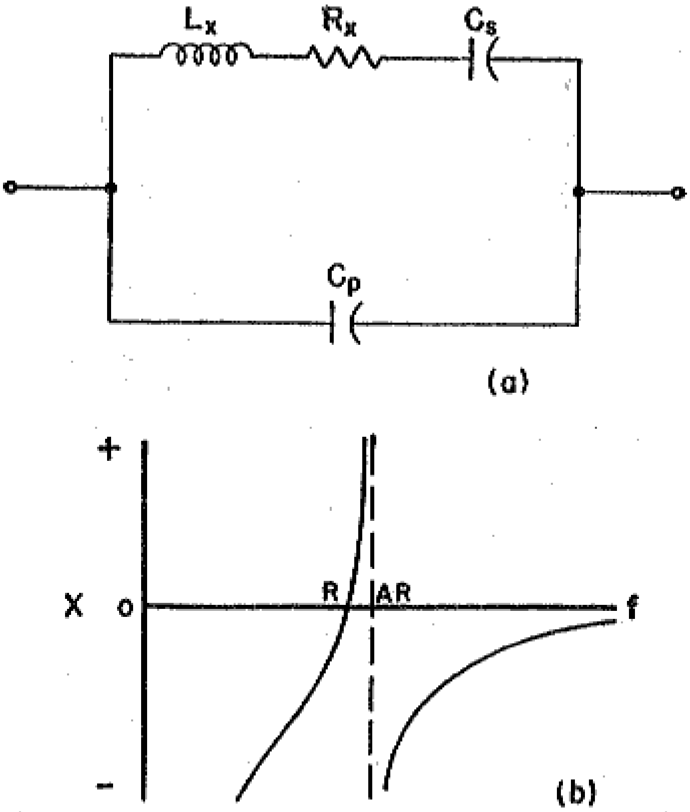


Fig. 4-30. (a) Equivalent electrical circuit for a piezoelectric crystal; (b) reactance curves for the circuit of (a).

The resistance of the crystal is due largely to mechanical damping introduced by the electrodes and by the surrounding atmosphere. By placing a crystal in an evacuated container, the value of Q can be notably increased. The electrodes are normally electroplated onto the crystal faces and need not introduce much damping.

Capacitance C_s is the equivalent series capacitance of the crystal forming a resonant circuit with L_x. Capacitance C_p is the parallel capacitance introduced by the crystal electrodes. The values of C_s and C_p are such that $C_p \gg C_s$, so that the resonant and antiresonant frequencies of the circuit lie very close together, differing by a fraction of 1 per cent of the resonant frequency. The reactance-curve sketch of Fig. 4-30(b) shows the resonant frequency below the antiresonant one. By placing adjustable capacitors in parallel with the crystal, C_p can be increased, resulting in the antiresonant frequency being moved closer to the resonant point.

Since the crystal represents either a resonant or antiresonant circuit, it may be used to replace the normal elements of the band-pass or band-elimination filter. As previously shown for band-pass action, the resonant frequency of one arm must equal the anti-resonant frequency of the other arm. The pass band with crystal

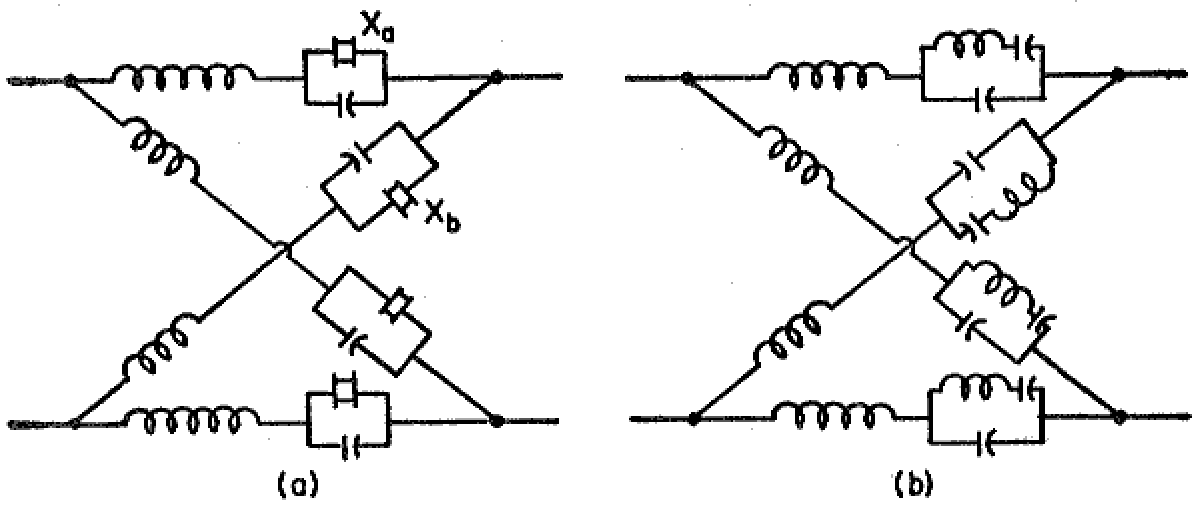


Fig. 4-31. (a) Circuit of a lattice crystal filter with series inductors and parallel capacitors; (b) the electrical equivalent of (a).

elements will then be found to extend from the lowest crystal resonant frequency to the highest crystal antiresonant frequency, or a width of pass band equal to twice the separation of the resonant and antiresonant frequencies of one crystal. This range will result in a pass band a fraction of 1 per cent wide. The band width can be reduced by putting adjustable capacitors in parallel with the crystal, furnishing a means of adjustment of the width of the pass band.

By the addition of coils in series with the crystals the pass bands may be widened. Since the added coils have Q values very much below those of the crystals, there will be some loss in sharpness at cutoff. A circuit including series coils is shown in Fig. 4-31(a), with its equivalent drawn at (b). The reactance curves for the A and B portions of this circuit are drawn in Fig. 4-32(a), which shows how the resonances and antiresonances are arranged. The presence of the series coil adds an additional resonance, and the pass band exists from the lowest resonance of one crystal to the highest resonance of the other. If f_1 and f_2 are the frequencies of resonance of one of the circuits and f_R is that of the antiresonance, then

$$f_{1,2} = f_R \sqrt{1 \mp \frac{C_s}{C_p}}$$

The separation of f_1 and f_2 represents two-thirds of the pass band and is seen to depend on the $\sqrt{C_s/C_p}$ ratio. Since C_s/C_p may be of the order of 0.01, it can be seen that the separation of f_1 and f_2 may be of

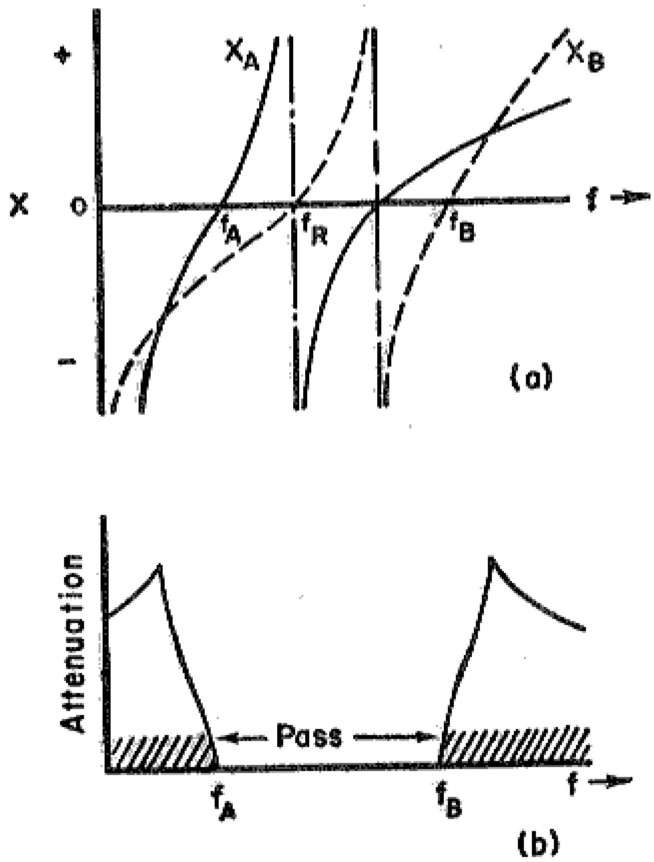


Fig. 4-32. (a) Reactance curves for the circuit of Fig. 4-31(a); (b) attenuation curves for that circuit.

the order of 0.10 f_R, or 10 per cent of the resonant frequency. By placing coils in series with the crystals, it has been possible to widen the pass band considerably. By adjustment of C_p it is then possible to narrow the band to any desired amount.

Thus the use of coils permits the bands to be widened to pass speech frequencies, and crystal filters are quite generally used to separate the various channels in carrier telephone circuits, in the range above 50 kilocycles.