$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

Assume

$R_1 = R_2, \quad C_1 = C_2$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\omega = \frac{1}{\sqrt{R^2 C^2}} = \frac{1}{RC}$$

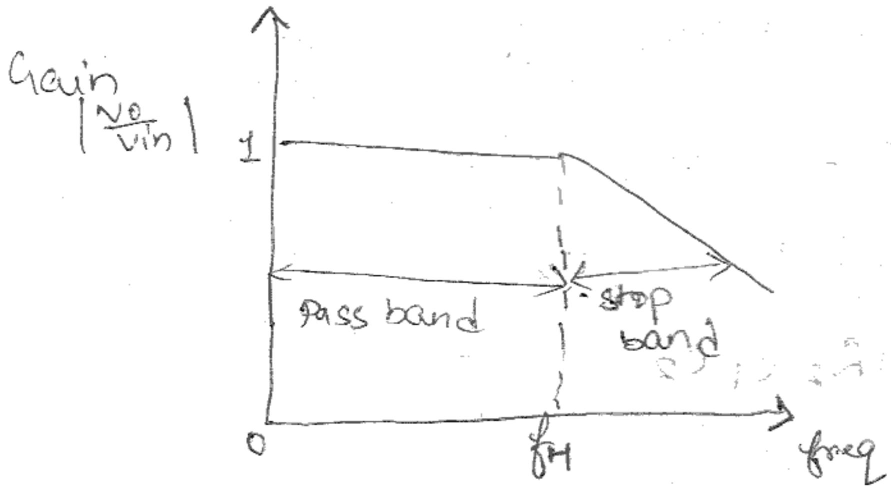
$$2\pi f = \frac{1}{RC}$$

$$\boxed{f = \frac{1}{2\pi RC}}$$

J.S.
01/07/11

FILTERS

Low pass filters :-

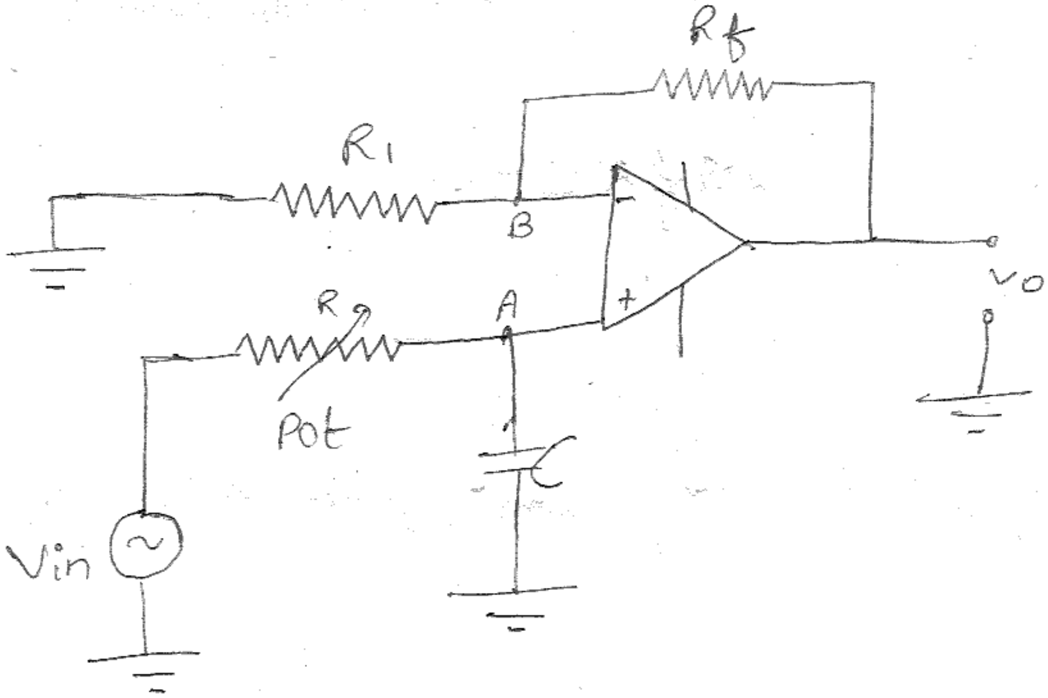


→ First order Butterworth LPF.

→ Second " " " .

First order LP Butterworth filter :-

It is realized by an RC circuit used with an op-amp in an non-inverting configuration. This is also called one pole LP Butterworth filter. The resistance R_f and R_i decides the the gain of the filter in the Pass band.



Analysis

The impedence of the capacitor 'C' is $-jX_c$.

where $X_c = \frac{1}{2\pi fc}$

By using potential divider Rule, the voltage at the non-inverting input terminal A, across the capacitor C is given by

$$V_A = \frac{-jX_C \cdot V_{in}}{R - jX_C}$$

$$= \frac{\frac{-j}{2\pi f C} \cdot V_{in}}{R - \frac{j}{2\pi f C}}$$

$$= \frac{-j}{2\pi f RC - j} \; V_{in}$$

$$= \frac{V_{in}}{1 - \frac{2\pi f RC}{j}}$$

But, $-j = \frac{1}{j}$, $\frac{-1}{j} = j$

$$V_A = \frac{V_{in}}{1 + j 2\pi f RC}$$

Since the op-amp is in the non-inverting configuration,

$$V_o = \left(1+\frac{R_f}{R_1}\right) V_{in}$$

$$= \left(1+\frac{R_f}{R_1}\right)\frac{V_{in}}{1+2\pi f RC}$$

$$\frac{V_o}{V_{in}} = \frac{\left(1+\frac{R_f}{R_1}\right)}{1+2\pi f RC}$$

$$\frac{V_o}{V_{in}} = \frac{A_F}{1+2\pi f RC}$$

where A_F is the gain of the Passband frequency and $f_H = \frac{1}{2\pi RC}$ = high cutoff frequency of the filter and $f \to$ operating frequency. $\frac{V_o}{V_{in}}$ is the transfer function of the filter and can be expressed in the polar form as $\cancel{\frac{V_o}{V_{in}}}$

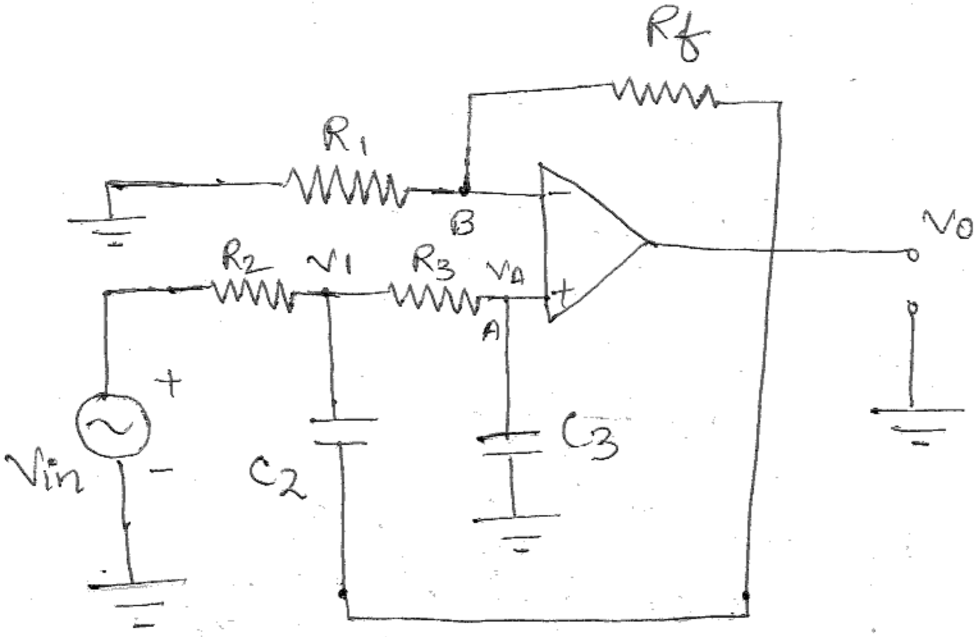
$$\frac{V_o}{V_{in}} = \left|\frac{V_o}{V_{in}}\right| < \phi$$

where,

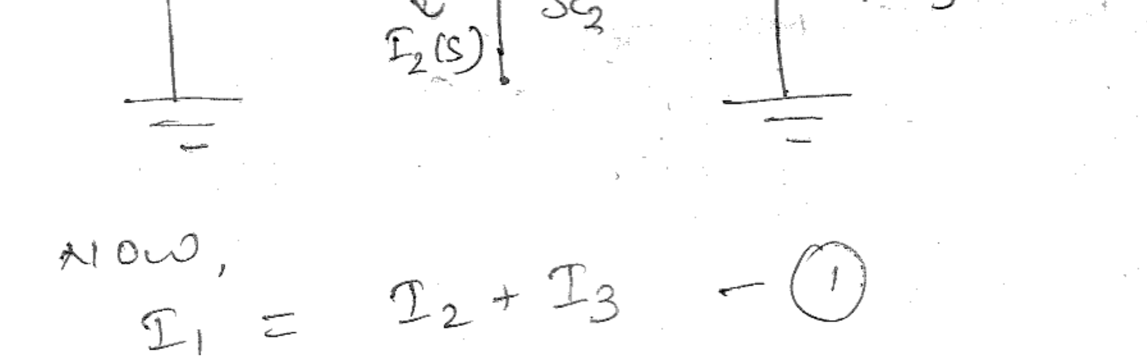
$$\frac{V_o}{V_{in}} = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}$$

and $\varphi = -\tan^{-1}\left(\frac{f}{f_H}\right)$

Second order Butterworth LPF :-



Analysis :-



Now,

$$I_1 = I_2 + I_3 \quad -(1)$$

$$\frac{V_{in} - V_1}{R_2} = \frac{V_1 - V_0}{1/SC_2} + \frac{V_1 - V_A}{R_3} \quad -(2)$$

using potential divider rule,

$$V_A = V_1 \left[\frac{1/sC_3}{R_3 + \frac{1}{sC_3}} \right]$$

$$V_A = V_1 \left[\frac{1/sC_3}{\frac{1+sR_3C_3}{sC_3}} \right]$$

$$V_1 = V_A(1+sR_3C_3) \quad — ③$$

Sub V_1 in ②

$$\frac{V_{in} - V_A(1+sR_3C_3)}{R_2} = \frac{V_A(1+sR_3C_3) - V_O}{1/sC_2} + \frac{V_A(1+sR_3C_3) - V_A}{R_3}$$

$$\Rightarrow \frac{V_{in}}{R_2} - \frac{V_A(1+sR_3C_3)}{R_2} = \frac{V_A(1+sR_3C_3)}{1/sC_2} - \frac{V_O}{1/sC_2} + \frac{V_A(1+sR_3C_3)}{R_3} - \frac{V_A}{R_3}$$

$$\Rightarrow \frac{V_{in}}{R_2} + \frac{V_O}{1/sC_2} = \frac{V_A(1+sR_3C_3)}{1/sC_2} + \frac{V_A(1+sR_3C_3)}{R_3} + \frac{V_A(1+sR_3C_3)}{R_2} - \frac{V_A}{R_3}$$

② $$\Rightarrow \frac{V_{in}}{R_2} + V_O(sC_2) = V_A\left[\frac{1+sR_3C_3}{1/sC_2} + \frac{1+sR_3C_3}{R_3} + \frac{1+sR_3C_3}{R_2} - \frac{1}{R_3}\right]$$

$$= V_A \frac{\left[R_3 (1+sR_3C_3) + sC_2R_2R_3 (1+sR_3C_3) + R_2(1+sR_3C_3) - R_2 \right]}{R_2R_3}$$

$$V_{in}R_3 + V_o(sC_2)R_2R_3 = V_A \left[R_3(1+sR_3C_3) + sC_2R_2R_3(1+sR_3C_3) + R_2(1+sR_3C_3) - R_2 \right]$$

$$V_A = \frac{V_{in}R_3 + V_o(sC_2)R_2R_3}{\left[R_3(1+sR_3C_3) + sC_2R_2R_3(1+sR_3C_3) + R_2(1+sR_3C_3) - R_2 \right]}$$

$$V_A = \frac{V_{in}R_3 + V_o(sC_2)R_2R_3}{(1+sR_3C_3)\left[(R_3 + sC_2R_2R_3 + R_2) \right] - R_2}$$

For op-amp in the non-inverting configuration

$$A_F = 1 + \frac{R_f}{R_1}$$

$$V_o = \left(1 + \frac{R_f}{R_1}\right) V_A$$

$$V_o = A_F \left[\frac{V_{in}R_3 + V_o sC_2R_2R_3}{(1+sR_3C_3)\left[R_3 + sC_2R_2R_3 + R_2\right] - R_2} \right]$$

$$\frac{A_F R_3 V_{in}}{(1+SR_3C_3)[R_3+SC_2R_2R_3+R_2]-R_2} = V_0\left[1 - \frac{A_F S R_2 C_2 R_3}{((1+SR_3C_3)[R_3+SC_2R_2R_3+R_2]-R_2)}\right]$$

$$A_F R_3 V_{in} = V_0\left[(1+SR_3C_3)(R_3+SC_2R_2R_3+R_2) - R_2 - A_F S R_2 C_2 R_3\right]$$

$$\frac{V_0}{V_{in}} = \frac{A_F R_3}{[(1+SR_3C_3)(R_3+SC_2R_2R_3+R_2)-R_2-A_F S R_2 C_2 R_3]}$$

$$= \frac{\cancel{A_F} A_F R_3}{R_3+SC_2R_2R_3+R_2+SR_3^2C_3+S^2R_3^2C_2R_2C_3 + SR_3C_3R_2 - R_2 - A_F S R_2 C_2 R_3}$$

$$= \frac{\cancel{A_F} A_F}{1 + SC_2R_2 + \cancel{\frac{R_2}{R_3}} + SR_3C_3 + S^2R_3C_2R_2C_3 + SC_3R_2 - \cancel{\frac{R_2}{R_3}} - A_F S R_2 C_2}$$

$$= \frac{A_F}{1 + R_2C_2S + R_3C_3S + S^2R_2R_3C_2C_3 + SC_3R_2 - A_F S R_2 C_2}$$

$$\frac{V_0}{V_{in}} = \frac{A_F}{S^2R_2R_3C_2C_3 + S(R_2C_2+R_3C_3+C_3R_2-A_FR_2C_2) + 1}$$

$$\frac{V_o}{V_{in}} = \frac{A_F}{S^2 + S\frac{(R_2C_2 + R_3C_3 + R_2C_3 - A_F R_2 C_2)}{R_2C_2R_3C_3} + \frac{1}{R_2C_2R_3C_3}} \quad (4)$$

$\because$ the gain of the filter has an order of 2. It is said to be second order filter.

Then, the transfer function of the II order system is

$$\frac{V_o(S)}{V_{in}(S)} = \frac{A}{S^2 + 2\xi\omega_n S + \omega_n^2} \quad (5)$$

Comparing equ^n (4) & (5)

$$\omega_n^2 = \frac{1}{R_2R_3C_2C_3}$$

In the case of filters, this equation is based upon the cutoff frequency.

$$\omega_n = \frac{1}{\sqrt{R_2R_3C_2C_3}}$$

Since we are finding the higher cutoff frequency

$$\omega_H = \frac{1}{\sqrt{R_2R_3C_2C_3}}$$

$$2\pi f_H = \frac{1}{\sqrt{R_2 R_3 C_2 C_3}} \quad -④$$

$$\frac{1}{R_2 C_2 R_3 C_3}$$

$$\boxed{f_H = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}}$$

is an
second

order

-⑤

equation

ency.

ver

In the polar form

$$\left|\frac{V_o}{V_{in}}\right| < \phi$$

where,

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_H}\right)^4}}$$

Where A_F is gain of the filter in Pass band.

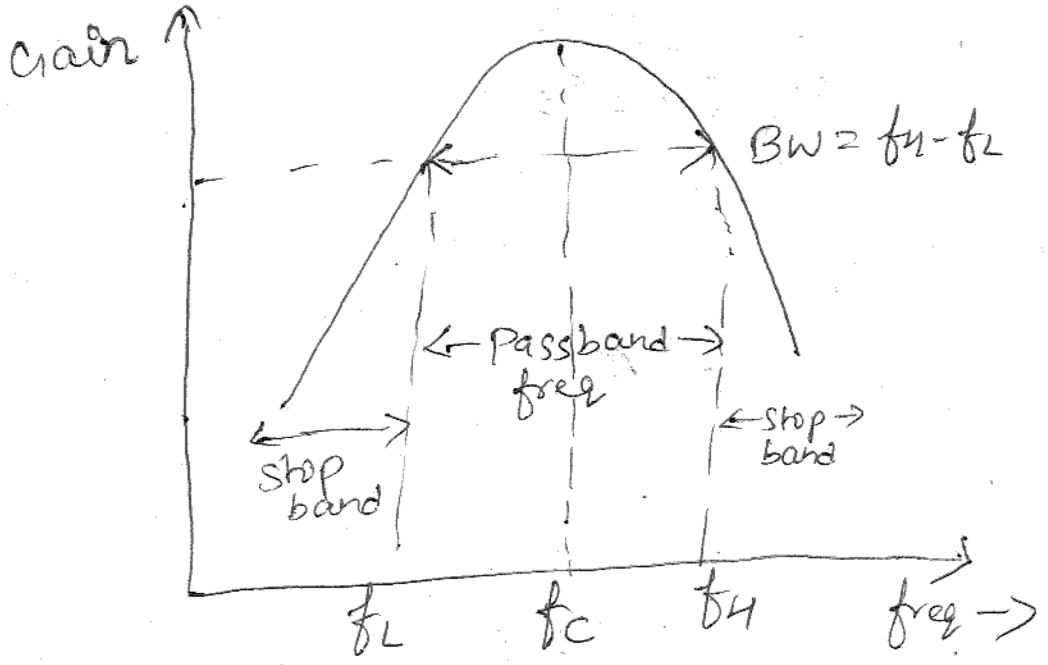
f is input frequency and f_H is high cutoff frequency.

Band pass filters :-

A Combination of high pass & low pass filter is called Band pass filter.

A Band pass filter is basically a frequency selector, it allows a particular band of frequency to pass by thus the passband is between the two frequencies f_H and f_L.

where $f_H > f_L$



There are two types of band pass filters classified based on the figure of merits (or) quality factor (Q) $= \frac{f_0}{BW}$

→ Wide BPF ⇒ $Q < 10$

→ Narrow BPF ⇒ $Q > 10$

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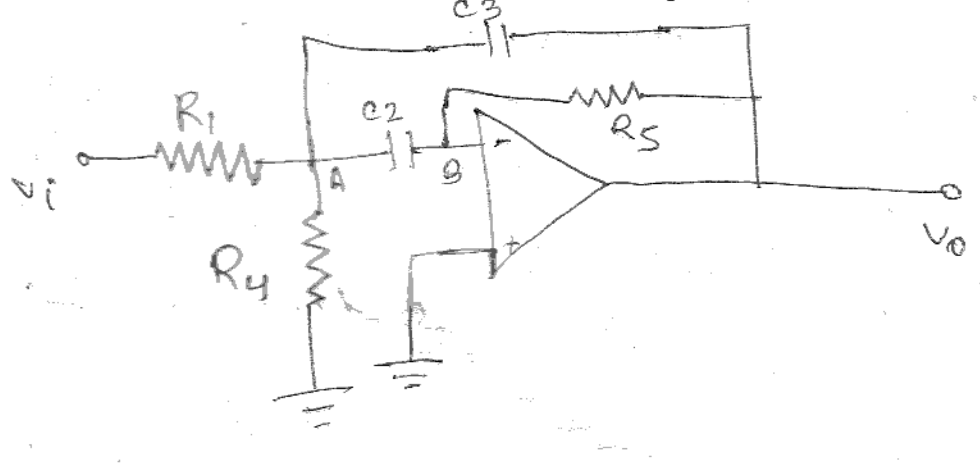
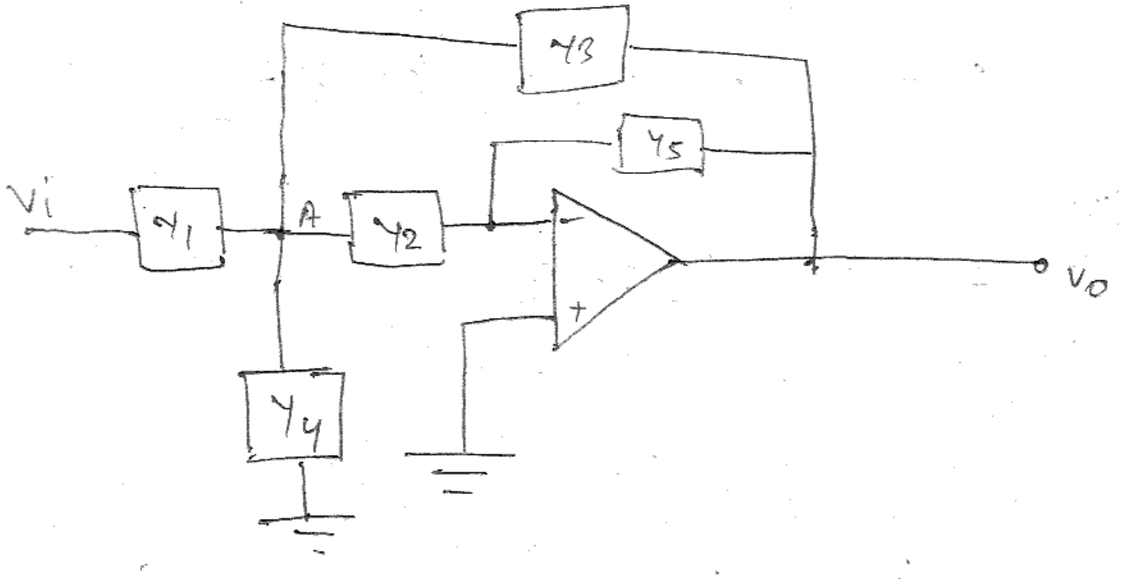
Narrow Band pass filter :-





At node 'A'

$$V_i Y_1 + V_o Y_3 = V_A(Y_1 + Y_2 + Y_3 + Y_4) \quad \text{--- (1)}$$

filters
merits.

At node 'B'

Assume, $V_B = 0$

$$V_A Y_2 + V_o Y_5 = 0$$

$$V_A = \frac{-V_o Y_5}{Y_2} \quad \text{--- (2)}$$

Sub equ^n (2) in (1)

$$V_i Y_1 + V_o Y_3 = \frac{-V_o Y_5}{Y_2}(Y_1 + Y_2 + Y_3 + Y_4)$$

$$V_i Y_1 = \frac{-V_o Y_5 (Y_1 + Y_2 + Y_3 + Y_4)}{Y_2} - V_o Y_3$$

$$V_i Y_1 = -V_o \left[\frac{Y_5 Y_1 + Y_5 Y_2 + Y_5 Y_3 + Y_5 Y_4 + Y_2 Y_3}{Y_2}\right]$$

$$\frac{V_o}{V_i} = \frac{-Y_1 Y_2}{[Y_5 Y_1 + Y_5 Y_2 + Y_5 Y_3 + Y_5 Y_4 + Y_2 Y_3]} \quad -(3)$$

Assume

$Y_1 = G_1$, $Y_2 = SC_2$, $Y_3 = SC_3$,

$Y_4 = G_4$, $Y_5 = G_5$.

Sub the values in eqn (3)

Hence the transfer function as given by

$$\frac{V_o(s)}{V_i(s)} = -\left[\frac{G_1 S C_2}{G_5 G_1 + G_5 SC_2 + G_5 SC_3 + G_5 G_4 + SC_2 \cdot SC_3}\right]$$

$$= -\left[\frac{S G_1 C_2}{G_5 G_1 + G_5 SC_2 + G_5 SC_3 + G_5 G_4 + S^2 C_2 C_3}\right]$$

$$= \frac{-S G_1 C_2}{S^2 C_2 C_3 + S(G_5 C_2 + G_5 C_3) + G_5(G_1 + G_4)}$$

$$= \frac{-S G_1 C_2}{S^2 C_2 C_3 + S(C_2 + C_3) G_5 + G_5(G_1 + G_4)} \quad -(4)$$

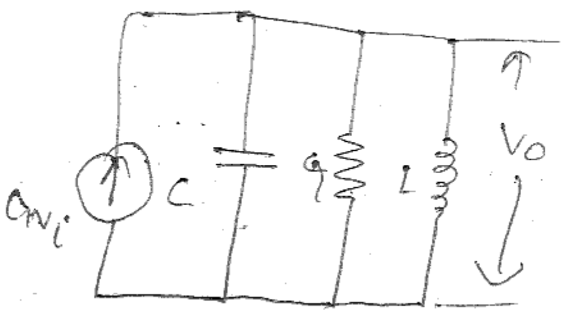
Dividing num & deno by SC_2

$$\frac{V_o(S)}{V_i(S)} = \frac{-G_1}{SC_3 + \frac{(C_2 + C_3) G_5}{C_2} + \frac{G_5(G_1 + G_4)}{SC_2}} \quad -(5)$$

∴ The transfer function is equivalent to the gain of the given RLC circuit with the band pass filter.

Gain exp of this ckt is,

$$\frac{V_o(s)}{V_i(s)} = -\frac{G'}{Y}$$



$$= \frac{-G'}{sC + G + \frac{1}{sL}} \quad \text{— (6)}$$

By comparing (5) & (6)

$$G' = G_1, \quad C = C_3, \quad G = \frac{G_5(C_2+C_3)}{C_2},$$

$$L = \frac{C_2}{G_5(G_1+G_4)}$$

At the resonant condition the circuit has an unity power factor

i.e. the imaginary part equal to zero.

Thus the resonant frequency ω_0 is given by

$$\omega_0^2 = \frac{1}{LC}$$

$$= \frac{1}{\frac{C_2 \quad C_3}{G_5(G_1+G_4)}}$$

$$\omega_0^2 = \frac{G_5(G_1+G_4)}{C_2 C_3}$$

The gain at the resonance is

$$\frac{V_0}{V_i} = -\frac{G'}{G} = \frac{-G_1}{\frac{G_5(C_2+C_3)}{C_2}} \quad \text{— (7)}$$

$$= \frac{-G_1 C_2}{G_5(C_2+C_3)}$$

$$= -\cancel{\frac{R_5}{R_5}}$$

$$= -\frac{R_5 \, C_2}{R_1 (C_2+C_3)} \quad \text{— (8)}$$

Quality factor :-

$$Q = \frac{\omega_0 L}{R} = \omega_0 RC = \frac{\omega_0 C}{G}$$

$$Q = \frac{\omega_0 C_3 C_2}{G_5(C_2+C_3)} \quad \text{— (9)}$$

$$BW = \frac{f}{Q} = \frac{\omega_0}{2\pi Q_0}$$

$$= \frac{\dfrac{G_5 (G_1 + \cancel{G_4})}{C_2 C_3}}{\dfrac{\omega_0 C_3 C_2}{G_5 (C_2 + C_3)}}$$

$$= \frac{\cancel{\omega_0} G_5 (C_2 + C_3)}{2\pi \cancel{\omega_0} C_3 C_2}$$

$$\boxed{BW = \frac{G_5 (C_2 + C_3)}{2\pi C_3 C_2}}$$

Assume $C_2 = C_3 = C$

$$BW = \frac{G_5 \times \cancel{2} C}{\cancel{2} \pi \times C^2} = \frac{G_5}{\pi C}$$

$$= \frac{\frac{1}{R_5}}{\pi C} = \frac{1}{\pi R_5 C}$$

$$\boxed{BW = \frac{1}{\pi R_5 C}}$$

C