1] Soln:

Given

$$G = \left| \begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right| \quad - (1)$$

$(6,3) = (n,k)$

$n = 6, \; k = 3$

$q = 6 - 3 = 3$

The code vectors can be obtained through the following steps:

Step-1: To determine the P submatrix from generator matrix.

we know,

$$G = [I_k : P_{k \times q}] \quad — (2)$$

Now, Comparing (1) & (2)

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

and

$$P_{k \times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Step-2: To obtain the eqns for check bits.

$k = 3, \; q = 3, \; n = 6.$

i.e. the block size of the message vector is 3 bits. Hence, there will be total 8 possible message vectors as shown following.

Sr. No.	Bits of msg. vector in one block. m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

& we have, $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

For the check bit vector, there will be the three bits. They can be obtained using.

$$[c_1\ c_2\ c_3] = [m_1\ m_2\ m_3]_{1\times 3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3} \quad — ①$$

where. m_1, m_2 & m_3 are ~~code~~ msg vectors

& c_1, c_2 & c_3 are code vectors.

$$\therefore [c_1\ c_2\ c_3] = [m_2 \oplus m_3 \quad m_1 \oplus m_3 \quad m_1 \oplus m_2] \quad — ②$$

~~Equating~~ Equating the eqn ②

$\therefore$ $c_1 = m_2 \oplus m_3$

$c_2 = m_1 \oplus m_3$

$c_3 = m_1 \oplus m_2$

Step-3: To determine the check bits & code vectors for every message vector:

~~Code~~

The following table lists all the message bits, their check bits and code vectors.

Sr. No.	Bits of msg. vector in one block			Check bits			Complete code vector					
	m_1	m_2	m_3	$c_1 = m_2 \oplus m_3$	$c_2 = m_1 \oplus m_3$	$c_3 = m_1 \oplus m_2$	m_1	m_2	m_3	c_1	c_2	c_3
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	1	0	0	0	1	1	1	0
3	0	1	0	1	0	1	0	1	0	1	0	1
4	0	1	1	0	1	1	0	1	1	0	1	1
5	1	0	0	0	1	1	1	0	0	0	1	1
6	1	0	1	1	0	1	1	0	1	1	0	1
7	1	1	0	1	1	0	1	1	0	1	1	0
8	1	1	1	0	0	0	1	1	1	0	0	0

Code vectors of (6,3) block code.

37 soln:

Given, $(6,3) = (n, k)$

$n = 6, \; k = 3, \; q = 3.$

&

$$\left.\begin{aligned} c_4 &= d_1 \oplus \cancel{d_2} \oplus d_3 \\ c_5 &= d_1 \oplus d_2 \\ c_6 &= d_1 \oplus d_3 \end{aligned}\right\} \quad (1)$$

i) To obtain generator matrix:

We know that, the check bits, message bits and parity matrix are related as follows,

$$[c_4 \; c_5 \; c_6] = [d_1 \; d_2 \; d_3] \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$c_4 = d_1 P_{11} \oplus d_2 P_{21} \oplus d_3 P_{31}$$
$$c_5 = d_1 P_{12} \oplus d_2 P_{22} \oplus d_3 P_{32}$$
$$c_6 = d_1 P_{13} \oplus d_2 P_{23} \oplus d_3 P_{33} \quad \text{--- (1)}$$

A/c to qsn.

$$c_4 = d_1 \oplus d_3$$
$$c_5 = d_1 \oplus d_2 \oplus d_3$$
$$c_6 = d_1 \oplus d_2 \quad \text{--- (2)}$$

$P_{11} = 1 \quad P_{21} = 0 \quad P_{31} = 1$

$P_{12} = 1, \; P_{22} = 1, \; P_{32} = 1$

$P_{13} = 1, \; P_{23} = 1, \; P_{33} = 0$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Comparing (1) & (2), we get,

$$\cancel{P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times3}}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times3} \quad \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{4} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

we know,

Generator Matrix, $G = \cancel{P : I_q} \, [I_k \; P_{k \times q}]$

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

ii) To obtain all the code vectors.

Sr. No.	msg. bits.			check bits.			code vectors. (x)						weight of code vector W(x).
1	d_1	d_2	d_3	c_4	c_5	c_6	d_1	d_2	d_3	c_4	c_5	c_6	
2	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	1	1	1	0	3
4	0	1	0	0	1	1	0	1	0	0	1	1	3
5	0	1	1	1	0	1	0	1	1	1	0	1	4
6	1	0	0	1	1	1	1	0	0	1	1	1	4
7	1	0	1	0	0	1	1	0	1	0	0	1	3
8	1	1	0	1	0	0	1	1	0	1	0	0	3
9	1	1	1	0	1	0	1	1	1	0	1	0	4

iii)

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5) Soln:

Given,

$$G = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{vmatrix}$$

$(6,3) = (n,k)$

$n = 6,\ k = 3,\ q = 3.$

$$G = \left[I_q : P_{k \times q} \right]_{k \times n}$$

where, $I_q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\& \ P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

1

0 0 1

0 0 0 1

ector [101]

attern

]

mitted code.

the 1st

we know $C = MP$.

$$[C_1 \ C_2 \ C_3] = [m_1 \ m_2 \ m_3] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Eqns for check bits are

$$\left.\begin{aligned} C_1 &= m_1 \oplus m_2 \\ C_2 &= m_1 \oplus m_2 \oplus m_3 \\ C_3 &= m_1 \oplus m_3 \end{aligned}\right\} - (1)$$

Now,

Sr. no.	msg bits			check bits			Code vectors (X)						wt. of the code W(X)
	m_1	m_2	m_3	c_1	c_2	c_3	m_1	m_2	m_3	c_1	c_2	c_3	
1.	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	1	1	0	0	1	0	1	1	3
3	0	1	0	1	1	0	0	1	0	1	1	0	3
4	0	1	1	1	0	1	0	1	1	1	0	1	4
5	1	0	0	1	1	1	1	0	0	1	1	1	4
6	1	0	1	1	0	0	1	0	1	1	0	0	3
7	1	1	0	0	0	1	1	1	0	0	0	1	3
8	1	1	1	0	1	0	1	1	1	0	1	0	3

$d_{min} = 3$

$d_{min} = [w(x)] = 3$

· Error detection & correction

detection

$d_{min} \geq s+1$

$3 \geq s+1$

$s \leq 2$

Hence two errors can be detected.

Correction,

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$t \leq 1$

∴ only one error can be corrected.

To determine message bit sequence.

$S = YH^T$ where, $[Y = 101101]$

$H = [P^T : I_q]_{n \times q}$

$$H^T = \begin{bmatrix} P \\ \cdots \\ I_q \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = YH^T = [1\ 0\ 1\ 1\ 0\ 1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s = [1 \oplus 1 \quad 1 \oplus 1 \quad 1 \oplus 1 \oplus 1]$$

$$s = [0 \quad 0 \quad 1]$$

Here, the syndrome is non-zero.

Hence, there is an error in the received codevector.

To locate the error,

On comparing syndrome $s = 001$ with the rows of H^T, we find that 6th row matches with syndrome.

Hence, 6th bit is in error.

∴ The error vector can be written as,

$$E = [0\,0\,0\,0\,0\,1]$$

$$\therefore \quad X = Y \oplus E$$

$$= [1\,0\,1\,1\,0\,1]\,[0\,0\,0\,0\,0\,1]$$

$$\boxed{X = [1\,0\,1\,1\,0\,0]} \quad -(2)$$

Hence, it is the ~~message bit~~ transmitted vector. ~~sequence~~.

$$X = m_1 m_2 m_3 \; c_1 c_2 c_3 \quad -(3)$$

Comparing (2) & (3),

we get, $m_1 m_2 m_3 = 101$.

Hence,

$$\boxed{\text{The message bits are } m_1 m_2 m_3 = 101}$$

6) How is syndrome calculated in cyclic codes?

→ Even in cyclic codes, some errors may occur during transmission. syndrome decoding can be used to correct those errors.

corrected code vector $(X) = Y \oplus E$ — ①

$Y = X \oplus E$ — ②

we can write so, because it is mod-2 addition,

In the polynomial form, the above eqn can be written as,

$Y(P) = X(P) + E(P)$

$Y(P) = M(P)G(P)$

$Y(P) = M(P)\,G(P) + E(P)$ — (*)

$$\frac{Y(P)}{G(P)} = \text{Quotient} + \frac{\text{Remainder}}{G(P)} \quad -③$$

In the above eqn if, $Y(P) = X(P)$ i.e. if it does not contain any error then,

$$\frac{X(P)}{G(P)} = \text{Quotient} + \frac{\text{Remainder}}{G(P)}$$

$$\frac{X(P)}{G(P)} = Q(P) + \frac{R(P)}{G(P)}$$

∵ $X(P) = M(P)\,G(P)$, Quotient will be equal to $M(P)$ & remainder will be zero.

This shows that there is no error, then remainder will be zero. Hence, $G(P)$ is a factor of vector polynomial.

$$\frac{Y(P)}{G(P)} = Q(P) + \frac{R(P)}{G(P)} \quad - (R)$$

$Y(P) = Q(P)\,G(P) + R(P)$ — (4) [∵ by multiplying with $G(P)$]

Equations (*) & (4)

$$M(P)\,G(P) \oplus E(P) = Q(P)\,G(P) \oplus R(P)$$

$$E(P) = M(P)\,G(P) \oplus Q(P)\,G(P) \oplus R(P)$$

Because of mod-2 additions, subtraction & addition is same

$$\therefore\; E(P) = [M(P) + Q(P)]\,G(P) \oplus R(P).$$

This eqn shows that for a fixed msg. vector and generator polynomial, an error pattern or error vector 'E' depends on remainder 'R'.

For every remainder 'R', there will be specific error vector.

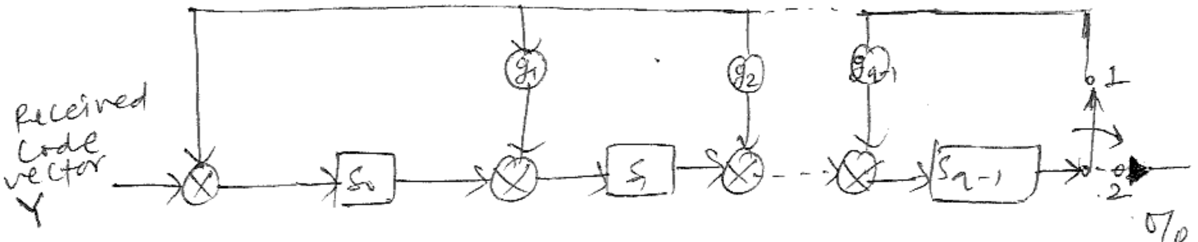
∴ we can call the remainder vector 'R' as syndrome vector S, or $R(P) = S(P)$.

∴ The ~~above~~ eqn (4) will be,

$$\frac{Y(P)}{G(P)} = Q(P) + \frac{S(P)}{G(P)}$$

$$\therefore\; \boxed{S(P) = \text{rem}\left[\frac{Y(P)}{G(P)}\right]}$$

<u>Block diagram of Syndrome Calculator</u>.



Computation of syndrom for an (n,k) cyclic code.

(3) Soln:

Given $G(p) = p^3 + p + 1$

$n = 7,\ k = 4, \qquad q = n - k = 3$

There will be total $2^k = 2^4 = 16$ message vectors of 7 bits each.

Let us consider a msg vector,

$M = (m_3\ m_2\ m_1\ m_0) = (0\ 0\ 0\ 1)$

Here $k = 4$.

$\therefore$ The message polynomial, $M(p) = m_3 p^3 + m_2 p^2 + m_1 p + m_0 p^0$

$= 1$

Given $G(p) = p^3 + p + 1$

~~The~~ the non-systematic code vectors is given by

$X_1(p) = M(p)\, G(p)$

$= p^3 + p + 1$

$\boxed{X_1(p) = 1\ 0\ 1\ 1}$

If $M = (m_3\ m_2\ m_1\ m_0) = (0\ 0\ 1\ 0)$

$M(p) = p$

$\therefore X_2(p) = p\,(p^3 + p + 1)$

$= p^4 + p^2 + p$

$\boxed{X_2(p) = 1\ 0\ 1\ 1\ 0}$

If $(m_3\ m_2\ m_1\ m_0) = (0\ 0\ 1\ 1)$

$M(p) = p + 1$

$\therefore X_3(p) = (p+1)(p^3 + p + 1)$

$= p^4 + p^2 + p + p^3 + p + 1$

$= p^4 + p^3 + p^2 + 1$

$X_3(p) = 1\ 1\ 1\ 0\ 1$

If $(m_3 m_2 m_1 m_0) = (0100)$

$$M(p) = p^2$$

$$\therefore X_4(p) = p^2 (p^3 + p + 1)$$

$$= p^5 + p^3 + p^2$$

ectors

$$\boxed{X_4(p) = 0101100}$$

Similarly, all the codevectors are obtained using the same procedure.

$^2 = m_1 p + m_0 p^0$

given by

Sr. No.	Message bits, $M = m_3$	m_2	m_1	m_0	Non-systematic code vectors $X = x_6$	x_5	x_4	x_3	x_2	x_1	x_0
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	1	0	1	1
3	0	0	1	0	0	0	1	0	1	1	0
4	0	0	1	1	0	0	1	1	1	0	1
5	0	1	0	0	0	1	0	1	1	0	0
6	0	1	0	1	0	1	0	0	1	1	1
7	0	1	1	0	0	1	1	1	0	1	0
8	0	1	1	1	0	1	1	0	0	0	1
9	1	0	0	0	1	0	1	1	0	0	0
10	1	0	0	1	1	0	1	0	0	1	1
11	1	0	1	0	1	0	0	1	1	1	0
12	1	0	1	1	1	0	0	0	1	0	1
13	1	1	0	0	1	1	1	0	1	0	0
14	1	1	0	1	1	1	1	1	1	1	1
15	1	1	1	0	1	1	0	0	0	1	0
16	1	1	1	1	1	1	0	1	0	0	1

Fig. Code vectors of a (7, 4) cyclic code for $G(p) = p^3 + p + 1$

To check whether cyclic property is satisfied

Let us consider code vector X_9 which is given in above table as,

$$X_9 = (1011000)$$

Let us shift this code vector cyclically to left side by 1 bit position. Then we get,

$$X' = 0110001$$

From the table, $X' = X_8 = (0110001)$

Thus, cyclic shift of X_9 produces X_8. This can be verified for other code vector also.

(9) Soln:

$G(p) = p^3 + p + 1$ — (1)

& $(7,4) = (n, k)$

$n = 7, k = 4, q = 3.$

There are total $2^k = 2^4 = 16$ code vectors.

Consider any message vector i.e.

$M_1 = (m_3 m_2 m_1 m_0) = (0\ 0\ 0\ 1)$

Then the message polynomial will be,

$M_1(p) = m_3 p^3 + m_2 p^2 + m_1 p + m_0 p^0$

$= 1.$ — (2)

& $G(p) = p^3 + p + 1$

To obtain, $p^q M(p) = p^3 \times 1 = p^3$

& $G(p) = p^3 + p + 1$

Now, To perform the division, $\frac{p^q M(p)}{G(p)}$,

$$\begin{array}{r|l|l} p^3+p+1 & p^3 & 1 \\ & p^3+p+1 & \\ & \oplus \quad \oplus \quad \oplus & \\ \hline & p+1 & \end{array}$$

$$C(p) = \text{rem}\left[\frac{p^q M(p)}{G(p)}\right] = p+1$$

∴ The check bits are,

$C(p) = c_1 p^1 + c_0 p^0$

∴ $\boxed{C_1(P) = (c_1, c_0) = (1, 1)}$

$M_2 = (m_3 m_2 m_1 m_0) = (0\,0\,1\,0)$

$M_2(p) = m_1 p^1 + m_0 p^0$

$= p$

$\therefore \quad C_2(p) = \text{rem}\left[\dfrac{p^q M(p)}{G(p)} \right]$

$= \text{rem}\left[\dfrac{p^4}{p^3+p+1} \right]$

$\therefore$

$$\begin{array}{r|l} & p \\ p^3+p+1 & p^4 \\ & p^4 + p^2 + p \\ \hline & p^2 + p \end{array}$$

$C_2(p) = m_2 p^2 + m_1 p + m_0 p^0$

$$\boxed{C(p) = (c_2 c_1 c_0) = (1\,1\,0)}$$

$M_3 = (m_3 m_2 m_1 m_0) = (0\,0\,1\,1)$

$M(p) = \cancel{m_1 p^1 + m_0 p^0} \quad m_3 p^3 + m_2 p^2 + m_1 p + m_0$

$= 0p^3 + 0p^2 + 1p + 1$

$= p + 1$

$\therefore \quad p^q M(p) = p^3(p+1) = p^4 + p^3$

$\therefore \quad C(p) = \text{rem}\left[\dfrac{p^4+p^3}{p^3+p+1} \right]$

$$\begin{array}{r|l} & p + 1 \\ p^3+p+1 & p^4 + p^3 \\ & p^4 + p^2 + p \\ \hline & p^3 + p^2 + p \\ & p^3 + p + 1 \\ \hline & p^2 + 1 \end{array}$$

i.e. $\boxed{C(p) = (1\,0\,1)}$ $\quad$ i.e. $C(p) = c_2 p^2 + c_1 p^1 + c_0 p^0$

Similarly, the remaining code vectors ~~can be~~ is are obtained as the above procedure..

Sr-No	Message bits				Systematic code vectors						
	$M = m_3$	m_2	m_1	m_0	$X = m_3$	m_2	m_1	m_0	c_2	c_1	c_0
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	1	0	1	1
3	0	0	1	0	0	0	1	0	1	1	0
4	0	0	1	1	0	0	1	1	1	0	1
5	0	1	0	0	0	1	0	0	1	1	1
6	0	1	0	~~0~~1	0	1	0	~~0~~1	1	0	0
7	0	1	1	0	0	1	1	0	0	0	1
8	0	1	1	1	0	1	1	1	0	1	0
9	1	0	0	0	1	0	0	0	1	0	1
10	1	0	0	1	1	0	0	1	1	1	0
11	1	0	1	0	1	0	1	0	0	1	1
12	1	0	1	1	1	0	1	1	0	0	0
13	1	1	0	0	1	1	0	0	0	1	0
14	1	1	0	1	1	1	0	1	0	0	1
15	1	1	1	0	1	1	1	0	1	0	0
16	1	1	1	1	1	1	1	1	1	1	1

10) soln:

Given,

$g_1 = (1\ 0\ 1)$,

$g_2 = (1\ 1\ 0)$

$g_3 = (1\ 1\ 1)$

constraint length $(K) = 3$

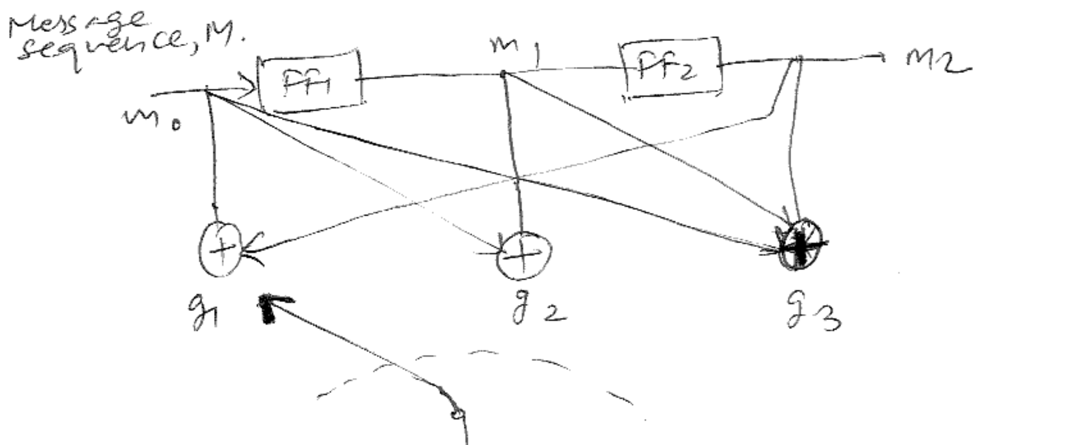


fig- Block diagram of the convolutional encoder.

above

vectors.

c_2	c_1	c_0
0	0	0
0	1	1
1	1	0
1	0	1
1	1	1
1	0	0
0	0	1
0	1	0
1	0	1
1	1	0
0	1	1
0	0	0
0	1	0
0	0	1
1	0	0
1	1	1

To obtain o/p sequence

By writing the generator polynomials of g_1, g_2 and g_3 can be written as,

$$g_1 = (101) \Rightarrow g_1(P) = 1 + p^2$$

$$g_2 = (110) \Rightarrow g_2(P) = 1 + p$$

$$g_3 = (111) \Rightarrow g_3(P) = 1 + p + p^2$$

Again,

By writing the generator polynomial of message sequence.

i.e. $m = (10111) = m(p) = 1 + p^2 + p^3 + p^4$

o/p sequence due to g_1

$$x_1(P) = g_1(P)\, m(P)$$

$$= (1+p^2)(1+p^2+p^3+p^4)$$

$$= 1 + p^2 + p^3 + p^4 + p^2 + p^4 + p^5 + p^6$$

$$= 1 + p^3 + p^5 + p^6$$

$$= 1001011$$

$$x_2(P) = g_2(P)\, m(P)$$

$$= (1+P)\,\cancel{(1+p+p^2)} = 1 + p + p^2 + p + p^2 + p^3$$

$(1+p^2+p^3+p^4)$

$$= 1 + p^3$$

$= (1001000)$ $\Rightarrow (111001)$ Six bit

$$x_3(P) = g_3(P)\, m(P)$$

$$= (1+p+p^2)(1+p^2+p^3+p^4)$$

$$= 1 + p^2 + p^3 + p^4 + p + p^3 + p^4 + p^5 + p^2 + p^4 + p^5 + p^6$$

$$= 1 + p + p^4 + p^6$$

$$= (1100101)$$

Hence, The multiplexer will multiplex the bits of x_1, x_2 & x_3 as follows:

$\boxed{\text{o/p sequence} = (111001000110001100101)}$

onal