20.7.11

Q- Construct a Hoffmann code given below

x_1	0.3	0.3	0.3	0.45	1 0.55
x_2	0.25	0.25	0.25	0.3	0.45 0
x_3	0.2	0.2	0.25	0.25	
x_4	0.1	0.15	0.2		
x_5	0.1	0.1			
x_6	0.05				

code word

11 (written in reverse

10 order]

00

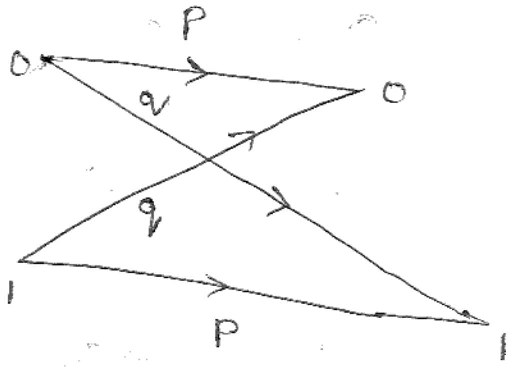
010

0111

0110

* Calculation of ~~entropy~~ Channel capacity

- Binary symmetric channel



$$P(0) = \alpha$$

$$P(1) = 1-\alpha$$

$$P(Y/X) = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & p & q \\ 1 & q & p \end{array}$$

$$P(x,y) = \begin{vmatrix} p\alpha & q\alpha \\ q(1-\alpha) & p(1-\alpha) \end{vmatrix}$$

$$H(Y/X) = -p\alpha \log p - q\alpha \log q - q(1-\alpha)\log q - p(1-\alpha)\log p$$

$$H(Y/X) = -\left(q \log_2 q + p \log_2 p\right)$$

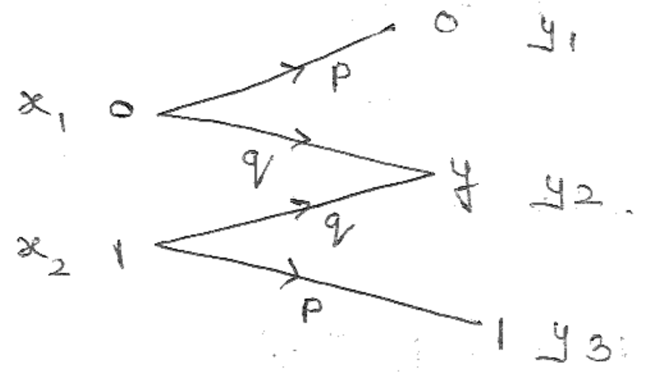
$$I(X,Y) = H(Y) - H(Y/X)$$

$$= H(Y) + q \log q + p \log p$$

$$\max H(Y) = \log_2 2 = 1$$

$$\therefore \boxed{I(X,Y)_{max} = 1 + q \log q + p \log p}$$

- Binary erasure ~~Coding~~ channel



$P(0) = \alpha$

$P(1) = 1-\alpha$

$$H(X) = -(\alpha \log \alpha + (1-\alpha)\log(1-\alpha))$$

$$P(Y/X) = \begin{array}{c|ccc} & 0 & y & 1 \\ \hline x_1 = 0 & p & q & 0 \\ x_2 = 1 & 0 & q & p \end{array}$$

$$P(X,Y) = \begin{vmatrix} p\alpha & q\alpha & 0 \\ 0 & q(1-\alpha) & p(1-\alpha) \end{vmatrix}$$

$p(y_1) = p\alpha$ $\qquad$ $P(y_3) = p(1-\alpha)$

$$p(y_2) = \cancel{q\alpha} + q \cancel{-q\alpha}$$
$$= q$$

$$P(X/Y) = \frac{P(X,Y)}{P(Y)} = \begin{vmatrix} 1 & \alpha & 0 \\ 0 & 1-\alpha & 1 \end{vmatrix}$$

$$H(X/Y) = -[p\alpha \log 1 + q\alpha \log \alpha + q(1-\alpha)\log(1-\alpha) + p(1-\alpha)\log 1]$$

$$H(X/Y) = qH(X)$$

$$I(X,Y) = H(X) - H(X/Y)$$

$$= H(X) - qH(X)$$

$$= H(X)[1-q]$$

$$= (1-q)$$

$$\boxed{I(X,Y)_{max} = P}$$

~~20~~ 21.7.11

Q- Apply haffmann encoding procedure and determine avg. length of encoded message.

$D = 0, 1, 2, 3$

x_1	0.18		0.18		0.49	0
x_2	0.17		0.17		0.18	1
x_3	0.16		0.16		0.17	2
x_4	0.15		0.16	0	0.16	3
x_5	0.10		0.15	1		
x_6	0.08		0.10	2		
x_7	0.05	0	0.08	3		
x_8	0.05	1				
x_9	0.04	2				
x_{10}	0.02	3				

code word

1	0.18
2	0.17
3	0.15
01	0.10
02	0.08
03	0.05
30	0.05
31	0.04
32	
33	0.02

$$\bar{L} = \sum_{i=1}^{10} P_i n_i$$

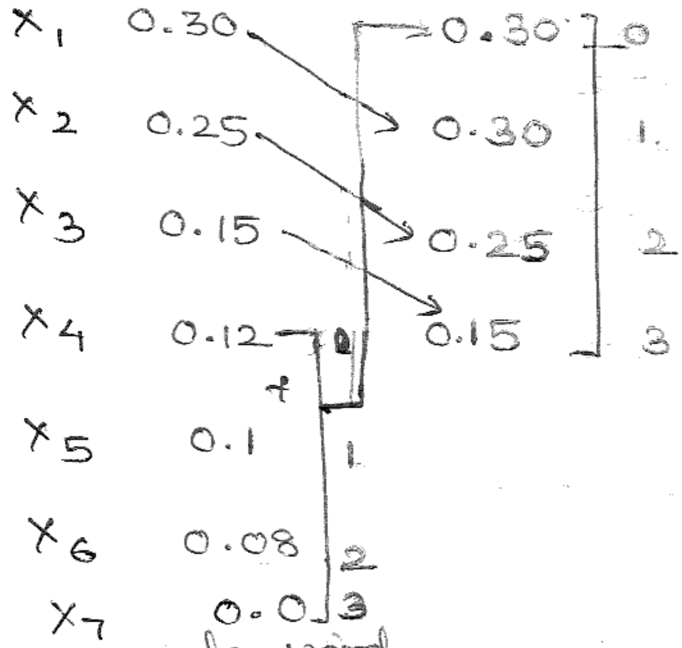
$$\cancel{= 1\times1 + 2\times1 +}$$

$$= 0.18 + 0.17 + 2\times0.16 + 2\times0.15 + 2\times0.10 + 2\times0.08 + 2\times0.05 + 2\times 0.05 + 2\times0.04 + 2\times0.02$$

$$\bar{L} = 1.65$$

Q- Construct the ~~four-array~~ haffmann code using 4-ary. Find avg code length, efficiency and redundancy.

$\rightarrow D = 0,1,2,3$



code word

$x_1 = 1$ $\quad$ $x_4 = 00$ $\quad$ $x_7 = 03$

$x_2 = 2$ $\quad$ $x_5 = 01$

$x_3 = 3$ $\quad$ $x_6 = 02$

$$\bar{L} = \sum_{i=1}^{7} P_i n_i$$

$$= 0.30 \times 1 + 0.25 \times 1 + 0.15 \times 1 + 0.12 \times 2 + 0.1 \times 2 + 0.08 \times 2 + 0 \times 2$$

$$\bar{L} = 1.3$$

$$H(x) = \sum_{i=1}^{7} -P_i \log P_i$$

$$= -0.30 \log_4 0.30 - 0.25 \log 0.25 - 0.15 \log 0.15 - 0.12 \log 0.12 - 0.1 \log 0.1 - 0.08 \log 0.08$$

$$= 0.26 + 0.25 + 0.20 + 0.18 + 0.16 + 0.14$$

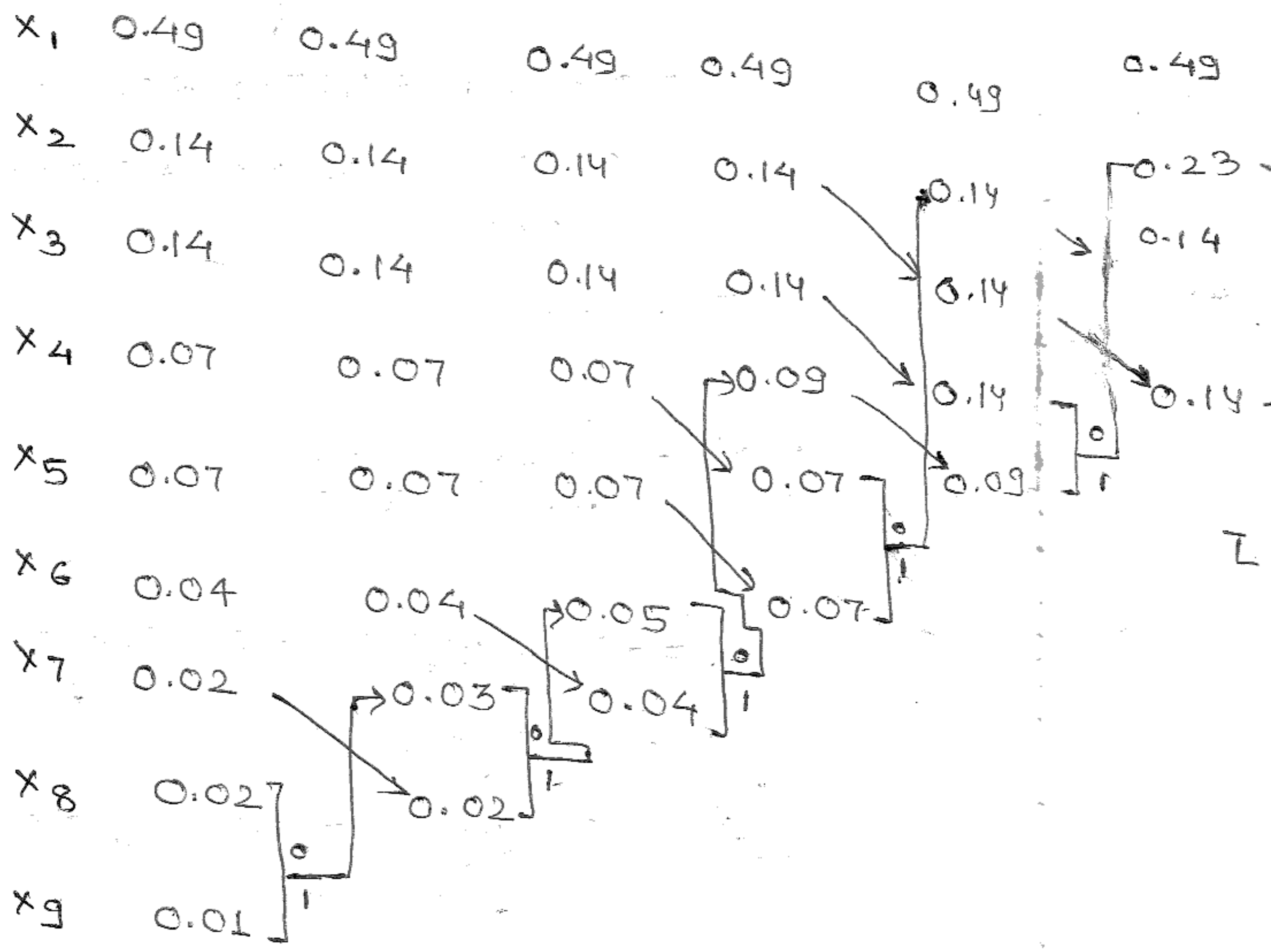
$$= 1.209$$

$$\text{Efficiency} = \frac{H(x)}{\bar{L}} = \eta = \frac{1.209}{1.3} = 0.93$$

$$R_c = 1 - \eta = 1 - 0.93 = 0.07$$

22.7.11

Q-



<u>code word</u>

x_1 = 0 1

x_2 = 100 3

x_3 = 101 3

x_4 = 0000 4

x_5 = 0010 4

x_6 = 0111 4

x_7 = 10100 5

x_8 = 0 11000 6

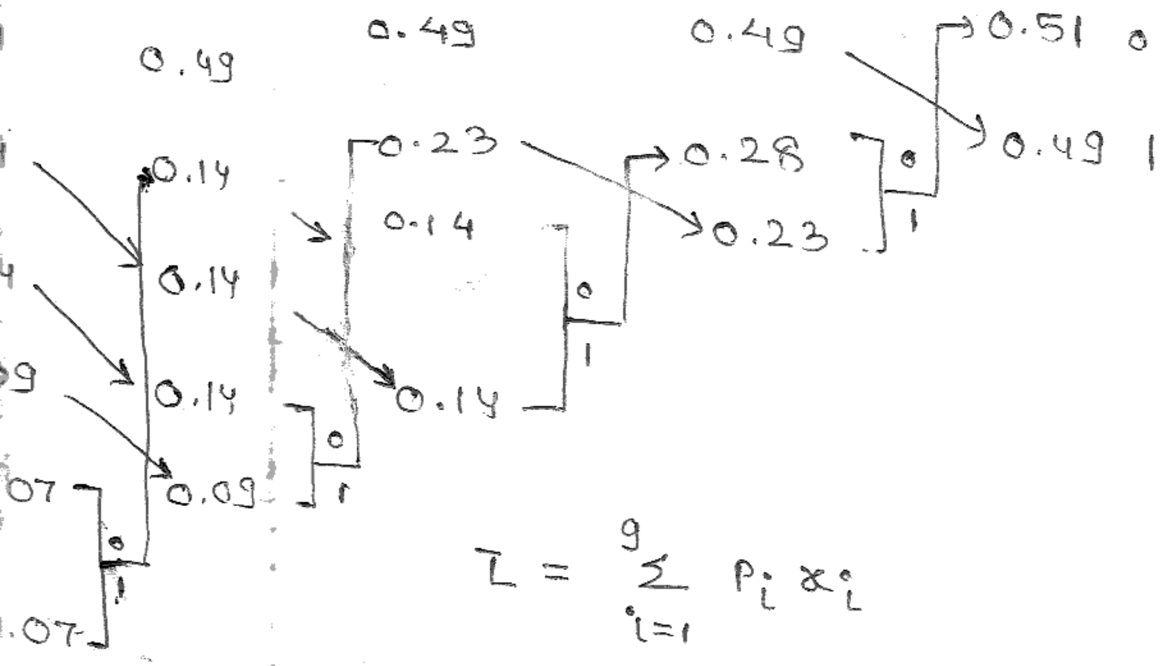
x_9 = 011001 6

L

L

H(

H(



$$\bar{L} = \sum_{i=1}^{9} P_i x_i$$

$$= 0.49 \times 1 + 0.14 \times 3 + 0.14 \times 3 + 0.07 \times 4 + 0.07 \times 4 + 0.04 \times 4 + 0.02 \times 5 + 0.02 \times 6 + 0.01 \times 6$$

$$\bar{L} = 2.33$$

D 5

11000 6

11001 6

$$H(X) = \sum_{i=1}^{9} -P_i \log P_i$$

$$= -0.49 \log 0.49 - 0.14 \log 0.14 \times 2 - 0.07 \log 0.07 \times 2 - 0.04 \log 0.04 - 0.02 \log 0.02 \times 2 - 0.01 \log 0.01$$

$$H(X) = 0.65$$

$$\eta = \frac{H(x)}{L} = \frac{0.65}{2.33} =$$