Second Scheme is Efficient for Encoding, because its Avg length is less.

J.S.
06/07/11

UNIT-II

Noise Coding

Channel capacity $C_x = \text{Max}\left[I(X,Y) \right]$

$$I(X,Y) = H(X) - H(X/Y)$$

$H(X)$ → Total Entropy.

$H(X/Y)$ → Error or conditional Entrop

Types of channel

1) lossless channel. $H(X/Y) = 0,\ P(X,Y) = 1$
2) useless channel.
3) Deterministic channel.
4) Symmetric channel.
5) Noise less channel.

1. lossless channel → Ex - one source multiple receiver. (Internet).

$$C = \text{Max}\left[H(X) \right]$$

2. Deterministic channel :- In the receiver side we can easily determine the information

$$C = \mathrm{Max}\,[H(Y)]$$

3. Useless channel :-

$$H(X) = H(Y) = 0$$

No channel capacity

~~This is called~~

4. <u>Noise less channel</u> :

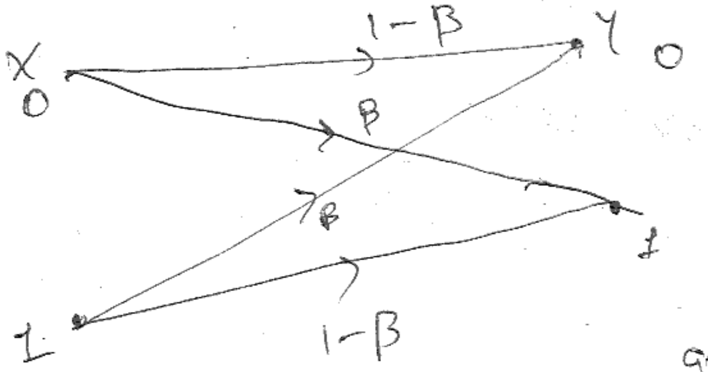
$$H(X) = H(Y)$$

Channel capacity $C = \mathrm{Max}\,[H(X)] = \mathrm{Max}\,[H(Y)]$

Combinationation of lossless and deterministic channel is called Noise less channel

5. Symmetric channel :-

Now a days we are using symmetric channel.



Symmetric channel
1−β, β, β, 1−β are probabilities.

Channel capacity, $\mathbb{C} = Max[H(x)] - Max[H(x/y)]$

Q. A Binary channel has the following noise characteristics

$$P(Y/X) = \begin{array}{c c} & \begin{array}{cc} y_1\ 0 & y_2\ 1 \end{array} \\ \begin{array}{c} x_1\ 0 \\ x_2\ 1 \end{array} & \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \end{array}$$

Find

i) If the input symbols are transmitted with respective probabilities of 3/4 & 1/4.

~~P(x)~~ $P(0) = \frac{3}{4}$, $P(1) = \frac{1}{4}$

ii) Find ~~the also~~ $H(X)$; $H(Y)$, $H(Y/X)$, $H(X/Y)$, $I(X;Y)$

iii) Find channel capacity

<u>Soln</u>

~~H(X) =~~

$P(x) = \frac{3}{4} = P(0)$

$P(y) = \frac{1}{4} = P(1)$

$P(x_1, y_1) = P(y_1/x_1) \cdot P(x)$

$= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

$$P(x_2, y_1) = P(y_1 / x_2)\, P(x)$$

$$= \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$

$$P(X, Y) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & \frac{1}{2} & \frac{1}{12} \\ x_2 & \frac{1}{4} & \frac{1}{6} \end{array}$$

$\frac{1}{2} + \frac{1}{4}$ $\frac{3}{4}$

$= \frac{2+1}{4} = \frac{3}{4}$

$\frac{1}{2} + \frac{1}{12}$

$= \frac{6+1}{12} = \frac{7}{12}$

$$H(X) = -\sum_{i=1}^{M} P(x_i) \log P(x_i)$$

$$= -\sum_{i=1}^{2} P(x_i) \log P(x_i)$$

$$= -P(x_1) \log P(x_1) - P(x_2) \log P(x_2)$$

$$= -\frac{3}{4} \log\left(\frac{1}{2}\right) - \frac{1}{4} \log\left(\frac{1}{4}\right)$$

$$= 0.30$$

$$= -\frac{7}{12} \log \frac{7}{12}$$

$$H(X, Y) = -\sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i y_j) \log P(x_i y_j)$$

$$= -\sum_{i=1}^{2} \sum_{j=1}^{2} P(x_i y_j) \log P(x_i y_j)$$

(-0.215)

$$= -\Big[P(x_1, y_1) \log P(x_1, y_1) + P(x_2, y_1) \log P(x_2, y_1) + P(x_1, y_2) \log P(x_1, y_2) + P(x_2, y_2) \log P(x_2, y_2) \Big]$$

$$= -\Big[\frac{1}{2} \log\left(\frac{1}{2}\right) + \frac{1}{4} \log\left(\frac{1}{4}\right) + \frac{1}{12} \log\left(\frac{1}{12}\right) + \frac{1}{6} \log\left(\frac{1}{6}\right) \Big]$$

$$= 0.52$$

$$P(X, Y) = \begin{array}{cc} & \begin{array}{cc} y_1 & y_2 \\ 0 & 1 \end{array} \\ \begin{array}{c} x_1\ 0 \\ x_2\ 1 \end{array} & \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{6} \end{bmatrix} \end{array}$$

$P(x_2)$

$$P(x_1) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$P(x_2) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$$

$$P(y_1) = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$$

$$P(y_2) = \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$$

$x_i y_i)$

$$H(X) = -\sum_{i=1}^{2} P(x_i) \log P(x_i)$$

$$= -\Big[\frac{3}{4} \log\left(\frac{3}{4}\right) + \frac{1}{4} \log\left(\frac{1}{4}\right) \Big]$$

$$= 0.8[illegible]$$

$$H(Y) = -\left[\frac{7}{12}\log_2\frac{7}{12} + \frac{5}{12}\log_2\frac{5}{12}\right]$$

$$= 0.98 \text{ bits/symbol.}$$

$$H(Y/X) = -\sum_{i=1}^{2}\sum_{j=1}^{2} P(x_i y_j)\log P\left(\frac{y_j}{x_i}\right)$$

$$H(Y/X) = -\left[\frac{1}{2}\log\left(\frac{2}{3}\right) + \frac{1}{4}\log\left(\frac{1}{3}\right) + \frac{1}{2}\log\left(\frac{1}{3}\right) + \frac{1}{6}\log\left(\frac{2}{3}\right)\right]$$

$$= 0.92$$

$$I(X;Y) = H(Y) - H(Y/X)$$

$$= 0.06$$

Channel capacity $\cancel{C = \max[H(x)] - \max[H(Y/X)]}$

(Symmetric) $C = \max[H(X) - H(X/Y)]$

$$= \log m - h$$

$$= 1 + \left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right]$$

Total Entropy = 1

7/7/11

Decoding scheme

$$H(E) > H(X/Y)$$

$$H(E) > H(Y/X)$$

$$\boxed{H(X/Y) \leq H(e) + P(e)\log r}$$ ~~Pte~~ Fano's Inequality.

Total ~~probabilit~~ Entropy of error signal

$P(e) \rightarrow$ Total probability of error signal

$r \rightarrow$ length of the signal.

Proof :-

$$H(X/Y) = -\sum_{i,j} P(x_i, y_j) \log P(x_i/y_j)$$

$$P(e) = \sum_{i \neq j} P\left(\frac{y_i}{x_i}\right)$$

Binary Entropy

$$H(e) = -P(e)\log P(e) - (1-P(e))\log(1-P(e))$$

$$P(e) = \sum_{i \neq j} P(x_i/y_j)$$

$$1 - P(e) = P(x_i/y_j)$$

Binary Entropy of ~~error pro~~ the Source error probability.

$$H(X) = H(E) + P(e)\log P(e) - P(e)\sum_{i\neq j} P(x_i/y_j)\log P(x_i/y_j)$$

$$= H(E) - P(e)\sum_{i\neq j} P(x_i/y_j)\log P(x_i/y_j)$$

$$H(X) = H(E) + P(e)H(X_{i\neq j})$$

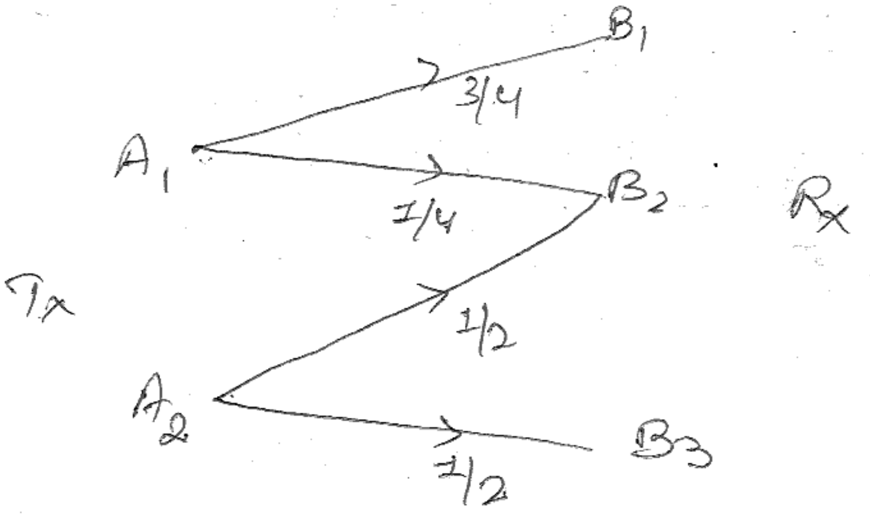
$$H(X/Y) \leq H(E) + P(e)\log \cancel{\text{}}$$

Proved.

Shanon's theorem - we can calculate the original signal.

12/7/11

Q. Given the noise characteristics of a communication channel. Evaluate the rate of ~~information~~ transmission through channel



$P(A_1) = \frac{1}{2}$

$P(A_2) = \frac{1}{2}$

$$P(Y/X) = \begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \end{array}$$

$$P(X,Y) = P(X) \cdot P(Y/x)$$

$$= \left[\tfrac{1}{2}, \tfrac{1}{2}\right] \begin{bmatrix} 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P(X,Y) \left[\cancel{1/2 \times \tfrac{3}{4} + 1/4 \times \tfrac{1}{2} + 0} \right] = \begin{bmatrix} 3/8 & 1/8 & 0 \\ 0 & 1/4 & 1/4 \end{bmatrix}$$

$P(Y) = ?$

$$P(Y) = \left[\frac{3}{8}, \frac{3}{8}, \frac{1}{4}\right]$$

$$H(X) = -\sum_{i=1}^{m} P(x_i) \log P(x_i)$$

$$= -\left(\sum_{i=1}^{2} - \left[\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log 1/2\right]\right.$$

$$= 1$$

$$H(Y) = -\left(\frac{3}{8}\log\frac{3}{8} + \frac{3}{8}\log 3/8 + \frac{1}{4}\log\frac{1}{4}\right)$$

$$H(Y) = 1.56$$

$$H(Y|X) = \cancel{\sum_{i=1}^{M}\sum_{j=1}^{N}} P(x,y) \log P(Y/x)$$

$$= \cancel{\sum_{i=1}^{2}\sum_{j=1}^{3}}$$

$$H(Y/X) = -\left[\frac{3}{8}\log\frac{3}{4} + \frac{1}{8}\log\frac{1}{4} + \frac{1}{4}\log 1/2 + \frac{1}{4}\log 1/2\right]$$

$$= 0.9078$$

$$I(X;Y) = H(Y) - H(Y/x)$$

(channel capacity) $= 1.566 - 0.9078$

$= 0.658$ (original info.)

Assignment

Q. A signal is Bandlimited to 5 KHz sampled at a rate of 10,000 samples per second and quantized to Four levels such that the sampled value can take values 0, 1, 2, 3 with probability P, P, $(\frac{1}{2}-P)$, $(\frac{1}{2}-P)$ respectively and the channel is characterized by the transmission Probability matrix.

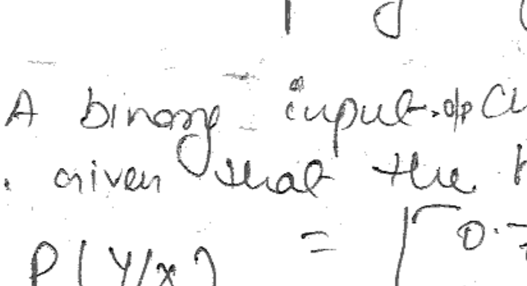
$$P = P(X=x_i / Y=y_j)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

Q. Find the capacity of channel.

Q2. A binary input-o/p channel is shown in fig. given that the transition probability matrix is

$$P(Y/X) = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$



The i/p prob. are $P(X_1) = 0.5$, $P(X_2) = 0.5$

Calculate the o/p prob matrix $P(Y)$ & joint Prob matrix $P(X, Y)$.

Calculation of Channel capacity for Bandlimited Gaussian channel

Given by Gaussian

$$= \frac{1}{\sqrt{2\pi}\, \sigma_n} \cdot e^{\frac{-n^2}{\sigma_n^2}}$$

$$N = \sigma_n^2$$

$$\boxed{C = \omega \log_2 \left(1 + \frac{S}{N}\right)} \text{ bits/}\cancel{\text{symb}}\text{ second}$$

$\omega \to$ bandwidth of Gaussian channel

$S/N \to$ signal to noise ratio.

Q. Calculate the capacity of low pass channel with a usable bandwidth of 3 KHZ and $\frac{S}{N} = 10^3$ at the channel O/P. assume the channel noise to be Gaussian white noise

$$C = 3000 \log_2 (1 + 10^3)$$

$$C = \frac{29.97 \times}{1\cancel{0}\cancel{0}} \text{ bits / second.}$$ $100\cancel{0}$

$$C = 29{,}974 \text{ bits/second}$$

$$C = 30{,}000 \text{ bits/second.}$$

imited

channel
and
sume
ite
noise

Q. An ideal communication system with avg power limitation and Perturbed by white Gaussian noise has a bandwidth of 1 MHZ and a signal to Noise ratio of 10. Calculate

i) channel capacity in bits/second

ii) If the S/N drops to 5. what bandwidth is required for the same channel capacity.

iii) If the bandwidth is decreased to 0.5 MHZ - what S/N ratio is required to maintain the same channel capacity

As.

1) $C = w \log_2 (1 + \frac{S}{N})$

$= 1 \times 10^6 \log_2 (1+10)$

$= 10^6 \times 3.33 \times \log 11$

$= 3.46 \times 10^6$ bits/sec

2) $3.46 \times 10^6 = w \log_2 (1+5)$

$3.46 \times 10^6 = w \times 2.59$

$w = \frac{3.46 \times 10^6}{2.59} = 1.33 \times 10^6$

3) $3.46\times10^{6} = 0.5\times10^{6}\log\left(1+\frac{S}{N}\right)$

$$6.92 = \log\left(1+\frac{S}{N}\right)$$

$$2.078 = \log\left(1+\frac{S}{N}\right)$$

$$e^{+2.078} = 1+\frac{S}{N}$$

$$7.98 = 1+\frac{S}{N}$$

$$6.98 = \frac{S}{N}$$

$$e^{6.92} = 1+\frac{S}{N}$$

$$\boxed{\frac{S}{N} = 118.42}$$

$$1+\frac{S}{N} = 2^{6.92}$$

$$1+\frac{S}{N} = 119.42$$

$$\frac{S}{N} = 119.42-1$$

$$\frac{S}{N} = 118.42$$

Q. A source is transmitting two Symbals A and B with $P(A) = 1/16$ and $P(B) = 15/16$. Design the code which would provide a transmission Efficiency of around 70 %.

$A \to 0$, $B \to 1$

$P(A) = \frac{1}{16}$, $P(B) = 15/16$

$$\text{Eff}(\eta) = \frac{H(x)}{\bar{L} \log D}$$

$$H(x) = -\sum_{i=1}^{2} P(A)\log P(A) + P(B)\log P(B)$$

$$= -\left[\frac{1}{16}\log\left(\frac{1}{16}\right) + \frac{15}{16}\log\left(\frac{15}{16}\right)\right]$$

$$= 0.101 \times 3.33$$

$$= 0.336 \text{ bits / symbol}$$

$$\eta = \frac{0.33}{1} \qquad \log_2 D = 1$$

$$\eta = 33\ \%$$

To improve our efficiency we are moving to two bit transmission →

$AA \to 111 \to 3$

$AB \to 110 \to 3$

$BA \to 100 \to 2$

$BB \to 000 \to 1$

$$P(AA) = \frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$$

$$P(AB) = \frac{1}{16} \times \frac{15}{16} = \frac{15}{256}$$

$$P(BA) = \frac{15}{16} \times \frac{1}{16} = \frac{15}{256}$$

$$P(BB) = \frac{15 \times 15}{16 \times 16} = \frac{225}{256}$$

$$\boxed{\begin{aligned} H(X) &= -\sum \Big[\frac{1}{256} \log\left(\frac{1}{256}\right) + \frac{15}{256} \log\left(\frac{15}{256}\right) \\ &\quad + \frac{15}{256} \log\left(\frac{15}{256}\right) + \frac{225}{256} \log\left(\frac{225}{256}\right) \Big] \\ H(X) &= \cancel{0.676} \end{aligned}}$$

$$H'(X) = 0.325$$

$$\bar{L} = \sum_{i=1}^{m} P(x_i) \cdot n_i$$

$$\bar{L} = \frac{1}{256} \times 3 + \frac{15}{256} \times 2 + \frac{15}{256} \times 2 + \frac{225}{256} \times 1$$

$$\bar{L} = \cancel{2.93}$$

$$\bar{L} = \frac{303}{256} = 1.18 \text{ bits/pair of symbol.}$$

$$\bar{L} = 0.59 \text{ bits/symbol.}$$

$$\cancel{\bar{L}\log D = 1.18\times 2 = 2.36}$$

$$\cancel{\eta = \frac{H}{\bar{L}\log D} = \frac{0.325}{2.36} = 0.137}$$

$$\eta = \frac{H(X)}{\bar{L}\log D} = \frac{0.325}{0.59}$$

$$= 0.55$$

$$\eta = 55\%$$

$$1111110 \leftarrow AAA - \frac{1}{16}\times\frac{1}{16}\times\frac{1}{16} = \frac{1}{4096}$$

$$11110 \leftarrow AAB - \frac{1}{16}\times\frac{1}{16}\times\frac{15}{16} = \frac{15}{4096}$$

$$11110 \leftarrow ABA - \frac{1}{16}\times\frac{1}{16}\times\frac{15}{16} = \frac{15}{4096}$$

$$1110 \leftarrow ABB - \frac{1}{16}\times\frac{15}{16}\times\frac{15}{16} = \frac{225}{4096}$$

$$0 \leftarrow BBB - \frac{15}{16}\times\frac{15}{16}\times\frac{15}{16} = \frac{3375}{4096}$$

$$10 \leftarrow BBA - \frac{15}{16}\times\frac{15}{16}\times\frac{1}{16} = \frac{225}{4096}$$

$$11110 \leftarrow BAA - \frac{15}{16}\times\frac{1}{16}\times\frac{1}{16} = \frac{15}{4096}$$

$$110 \leftarrow BAB - \frac{15}{16}\times\frac{1}{16}\times\frac{15}{16} = \frac{225}{4096}$$

$$\bar{L} = \sum_{i=1}^{n} P(x_i)\, n_i$$

$$= \frac{3375}{4096}\times 1 + \frac{225}{4096}\times 2 + \frac{225}{4096}\times 3 + \frac{225}{4096}\times 4 + \frac{5\times 15}{4096} + \frac{15}{4096}\times 6 + \frac{15}{4096}\times 7 + \frac{1}{4096}\times 7$$

0.296

log(15/256)

(225/256)

×1

air of

l.

$$= \frac{1}{4096}\left[3375 + 450 + 675 + 900 + 75 + 90 + 105 + 7\right]$$

$$= 1.385 \text{ bits / trisymbol}$$

$$\bar{L} = 0.46 \text{ bits / Symbol.}$$

$$\eta = \frac{H(x)}{\bar{L}} = \frac{0.325}{0.46}$$

$$= 0.70$$

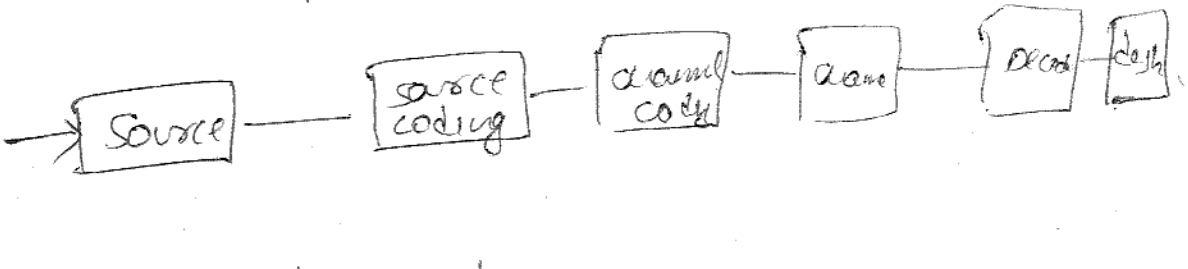
$$\eta = 70\%.$$

UNIT - III

CHANNEL CODING

1. Repetition code.
2. Parity check code.

before coding we have to calculate channel capacity.



1. Repetition code

$c = \{0, 1\}$

$C = \{0000\ldots 0_n,\ 1111\ldots 1_n\}$

→ Redundancy bits are null bit they carry the original information.

rate of info $= \frac{1}{n}$

no of errors removed =