08/07/11

Capacity of Band limited Gaussian channel.

Consider the zero mean stationary process x(t) that is band limited to B Hertz.

Let X_K is equal $= 1, 2, \ldots n$ denotes a continuous random variable obtained by uniform sampling of process at the rate of 2B samples/second.

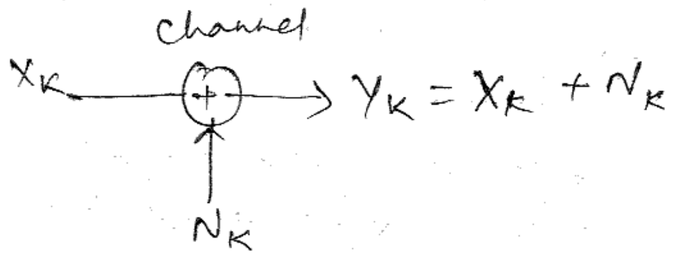
(If random process x(t), limited to the frequency internal $-B \leq f \leq B$ samples at the rate equal to or greater than 2B samples/sec; then the original process can be reconstructed from its samples with zero mean square error). These samples are transmitted in T sec. over a noisy channel, also band limited to B Hz.

The no. of samples n is given by

$n = 2BT$ —(1)

S_K is sample of transmitter signal. The channel o/p is affected by ~~att~~ additive by white gaussian noise of zero mean and power spectral density $N_0/2$. The noise is band limited to BHz. Let

Y_K denote samples of received signal.



$$Y_K = X_K + N_K \quad \text{--- (2)}$$

$$K = 1, 2, \ldots n$$

The noise sample N_K is gaussian with zero mean and variance.

$$\sigma^2 = N_0 B \quad \text{--- (3)}$$

Assume samples Y_K are statistically independent. A channel for which the noise and receive signal are as described, as known as discrete time, memoryless Gaussian channel. Typically, a transmitter is power limited.

$$P = E[X_K^2] \quad \text{--- (4)}$$

channel capacity

$$C = \max \{ I(X_K, Y_K) : E[X_K^2] = P \} \quad \text{--- (5)}$$

where,

$I(X_K, Y_K)$ is average mutual information b/w transmitter signal and receive signal.

$f[X_K(n)]$ is probability density function of X_K

$I(X_K \cdot Y_K) = H(Y_K) - H(Y_K/X_K)$ — 6

Since, X_K and Y_K are independent random variables and their sum equal Y_K.

$H(Y_K/X_K) = H(N_K)$ — 7

conditional entropy of Y_K given as X_K is equal to entropy of $H(N_K)$

~~I XK~~

$I(X_K, Y_K) = H(Y_K) - H(N_K)$ — 8

The variance of samples Y_K of received signal is equal to ~~var(Yk)~~

$Var(Y_K) = P + \sigma^2$.

Hence,

$H(Y_K) = \frac{1}{2} \log_2 (2\pi e (P+\sigma^2))$ — 9

variance of noise sample

$N_K = \sigma^2$

Differential entropy of N_K is given as

$h(N_K) = \frac{1}{2} \log_2 (2\pi e \sigma^2)$ — 10

sub. 10 & 9 in 6

$C = M(I(X_K, Y_K))$

$= H[Y_K - H(N_K)]$

$$C = \tfrac{1}{2}\log_2 2\pi e(P+\sigma^2) - \tfrac{1}{2}\log_2 2\pi e\sigma^2$$

$$= \tfrac{1}{2}\log_2 \left[\frac{2\pi e(P+\sigma^2)}{2\pi e\sigma^2}\right]$$

$$= \tfrac{1}{2}\log_2 \left[P/\sigma^2 + 1\right]$$

$$\boxed{C = \tfrac{1}{2}\log\left[1 + P/N_0B\right] \text{ bits/channel.}}$$

If the channel used n times for the transmission of n channel samples of the process $X(t)$ in 'T' sec, we find that the channel capacity per unit time $\frac{n}{T}$ times the result.

$$C = \frac{2BT}{T} \cdot \tfrac{1}{2}\log_2\left(1 + P/N_0B\right)$$

$$C = B\log_2\left[1 + \frac{P}{N_0B}\right] \text{ bits/}\cancel{\text{symbol}}\text{ sec.}$$

$$\therefore \boxed{C = B\log_2\left[1 + \frac{P}{N_0B}\right] \text{bits/sec}}$$

∴ This is known as Shanon's third theorem.

12/07/11.

Gaussian Distribution

Random X and Y.

pdf f(x). $f_X(x)$

From fundamental inequality.

$$\int_{-\infty}^{\infty} f_X(x) \log_2 \frac{f_X(x)}{f_Y(x)} dx \le 0$$

$$-\int_{-\infty}^{\infty} f_X(x) \log f_X(x) = -\int_{-\infty}^{\infty} f_Y(x) \log_2 f_X(x)$$

X and Y have same mean and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$$

$$h(Y) \le -\int_{-\infty}^{\infty} f_Y(x) \log_2 \left\{ \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \right\}$$

$$\le \frac{1}{\ln 2} \left\{ \int_{-\infty}^{\infty} f_Y(x) \ln\left[\frac{1}{\sqrt{2\pi}\sigma}\right] - \int_{-\infty}^{\infty} f_Y(x) \ln \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \right\}$$

$$\Rightarrow \log_2 e \left\{ \int_{-\infty}^{\infty} f_Y(x) \left[-\ln\sqrt{2\pi}\sigma\right] - f_Y(x)\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \right\}$$

properties:

$$\int_{-\infty}^{\infty} f_Y(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} (x-\mu)^2 f_Y(x) dx = (x-\mu)^2 = \sigma^2$$

$$h(y) \le -\log_2 e \left\{ \int_{-\infty}^{\infty} f_y(x) \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx - \int_{-\infty}^{\infty} f_y(x) \ln \sqrt{2\pi}\,\sigma \, dx \right.$$

$$= -\log_2 e \left[-\frac{\sigma^2}{2\sigma^2} \right] + \log_2 e \ln \sqrt{2\pi}\,\sigma$$

$$\Rightarrow \frac{\log_2 e}{2} + \log \sqrt{2\pi}\,\sigma^2 \Rightarrow \frac{1}{2} \log (2\pi\sigma^2)$$

$$h(y) \Rightarrow \frac{1}{2} \log (2\pi e \sigma^2)$$

$$\therefore \boxed{h(y) \le \frac{1}{2} \log 2\pi e \sigma^2}$$

8] Data has to be transmitted at the rate of 10,000 bits/sec. over a channel having bandwidth 3000 Hz. Determine the signal to noise ratio required. If bandwidth is increased to 10,000 Hz, then determine the signal to noise ratio.

Soln:

$B.W = 3000\,Hz$

$C = 10{,}000$ bits/sec.

$$S/N = B \log_2 \left[1 + P/N_0 B \right]$$

where,

$P \rightarrow$ Signal power

$N_0 B \rightarrow$ Noise power

$C = B\log_2(1+S/N)$

$10{,}\cancel{000} = 3\cancel{000}\log_2(1+S/N)$

$3.3 = \log_2(1+S/N)$

$$1 + S/N = 2^{3.3} \qquad \left[\because c = \log_b a \quad a = b^c\right]$$

$S/N = 2^{3.3} - 1 = 9$

$$\boxed{\begin{array}{ll} B/W = 3000, & S/N = 9 \\ B/W = 1000, & S/N = 1 \end{array}}$$

13/07/11.

Unsymmetric channel

If the symmetry of the channel matrix is not present, the channel capacity cannot be used. Consider an unsymmetric binary channel with

$$P(Y/X) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

$P(X_1) = P_1, \quad P(X_2) = P_2$

$P(Y_1) = P_1', \quad P(Y_2) = P_2'$

New variables Q_1 and Q_2

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} P_{11}\log P_{11} & P_{12}\log P_{12} \\ P_{21}\log P_{21} & P_{22}\log P_{22} \end{bmatrix}$$

Let, $R = I(X,Y) = H(Y) - H(Y/X)$

(or) $H(X) - H(X/Y)$

$$P(X,Y) = P(X)\cdot P(Y/X)$$

$$= \begin{bmatrix} P_1 P_{11} & P_1 P_{12} \\ P_2 P_{21} & P_2 P_{22} \end{bmatrix}$$

$$P(Y_1) = P_1 P_{11} + P_2 P_{21} = P_1'$$

$$P(Y_2) = P_1 P_{12} + P_2 P_{22} = P_2'$$

$$H(Y/X) = -\sum\sum P(X,Y)\log P(Y/X)$$

$$= -\left[P_1 P_{11}\log P_{11} + P_1 P_{12}\log P_{12} + P_2 P_{21}\log P_{21} + P_2 P_{22}\log P_{22}\right]$$

$$H(Y) = -\left[P_1'\log P_1' + P_2'\log P_2'\right]$$

$$\left.\begin{aligned} Q_1 &= \log P_{11} \\ &= \log P_{21} \\ Q_2 &= \log P_{12} \\ &= \log P_{22} \end{aligned}\right\} \text{Substitn also } H(Y).$$

Sub in $H(Y/X)$

$$\therefore H(Y/X) = -\left[P_1 P_{11} Q_1 + P_1 P_{12} Q_2 + P_2 P_{21} Q_1 + P_2 P_{22} Q_2\right]$$

$$= -\left\{Q_1 (P_1 P_{11} + P_2 P_{21}) + Q_2 (P_1 P_{12} + P_2 P_{22})\right\}$$

$$= -\left[Q_1 P_1' + Q_2 P_2'\right]$$

Sub $H(Y)$ and $H(Y/X)$ in $I(X,Y)$.

$$\therefore I(X,Y) = H(Y) - H(Y/X)$$

$$\boxed{I(X,Y) = -P_1' \log P_1' - P_2' \log P_2' + Q_1 P_1' + Q_2 P_2'}$$ //

To maximize $I(X,Y)$, use the calculus of variations (method of Lagrangian multiplies).

$$F = I(X,Y) + \lambda (P_1' + P_2')$$

$$\frac{\partial F}{\partial P_1'} = \frac{d}{dP_1'}\left[-P_1' \log P_1' - P_2' \log P_2' + Q_1 P_1' + Q_2 P_2'\right]$$

$$= -\left(P_1' \frac{1}{P_1'} + \log P_1'\right) + Q_1 + \lambda$$

$$= -(1 + \log P_1') + Q_1 + \lambda = 0$$

$$\frac{dF}{dP_2'} = -\left[1 + \log P_2'\right] + Q_2 + \lambda = 0$$

$$\Rightarrow \cancel{-1} - \log P_1' + Q_1 + \cancel{\lambda} = \cancel{-1} - \log P_2' + Q_2 + \cancel{\lambda}$$

$$Q_1 = \log P_1' - \log P_2' + Q_2$$

Sub. θ_1 in $I(X,Y)$,

$C = h_{max} = I(X,Y) =$

$= -p_1' \log p_1' - p_2' \log p_2' + (\log p_1' - \log p_2' + \theta_2) p_1' + \theta_2 p_2'$

$= -p_1' \cancel{\log p_1'} - p_2' \log p_2' + p_1' \cancel{\log p_1'} - p_1' \log p_2' + p_1' \theta_2 + \theta_2 p_2'$

$= -\log p_2' (p_1' + p_2') + \theta_2 p_1' + \theta_2 p_2'$

$= -\log p_2' + \theta_2 (p_1' + p_2')$

$= -\log p_2' + \theta_2$ $\qquad [\because p_1' + p_2' = 1]$

$\therefore \boxed{I(X,Y) = -\log p_2' + \theta_2}$ //

$\theta_2 = \theta_1 - \log p_1' + \log p_2'$ — (*)

$I(X,Y) = -p_1' \log p_1' - p_2' \log p_2' + \theta_1 p_1'$
$+ (\theta_1 - \log p_1' + \log p_2') p_2'$

$= -p_1' \log p_1' - p_2' \cancel{\log p_2'} + \theta_1 p_1'$
$+ \theta_1 p_2' - p_2' \log p_1' + p_2' \cancel{\log p_2'}$

$= -\log p_1' (p_1' + p_2') + \theta_1 (p_1' + p_2')$

$= -\log p_1' + \theta_1$

$\therefore \boxed{I(X,Y) = \theta_1 - \log p_1'}$ //

$c = \theta_2 - \log p_2'$ $\quad\quad$ $c = \theta_1 - \log p_1'$

$\log p_2' = \theta_2 - c$ $\quad\quad$ $\log p_1' = \theta_1 - c$

$p_2' = \exp(\theta_2 - c)$ $\quad\quad$ $p_1' = \exp(\theta_1 - c)$

$\theta_2 p_2'$

Since, $p' + p_2' = 0$

$$e^{\theta_1 - c} + e^{\theta_2 - c} = 1$$

$$\frac{e^{\theta_1}}{e^c} + \frac{e^{\theta_2}}{e^c} = 1$$

$$e^{\theta_1} + e^{\theta_2} = e^c$$

$$\boxed{c = \log\left[2^{\theta_1} + 2^{\theta_2}\right]}$$

Q: For an additive awgn Guassian channel with 4 KHz B.W. and noise ~~from~~ power spectral density 10^{-12} W/Hertz., the signal power required at the receiver is 0.1mW (10^{-3}W). Compute channel capacity.

Sol:-

$B \cdot W = 4\,KHz = 4 \times 10^3\,Hz$

$\frac{N_0}{2} = 10^{-12}\,W/Hz$

$P = S = 10^{-3}\,W$

$C = B\log(1 + S/N)$ (or)

$= B\log(1 + P/N_0 B)$

$\boxed{N = N_0 B}$

$$C = 4\times10^3 \log_2\left(1 + \frac{10^{-3}}{2\times10^{-12}\times4\times10^3}\right)$$

$$= 4\times10^3 \log_2\left[1 + \frac{10^6}{8}\right]$$

$$= 4\times10^3 \log$$

15/07/11 · UNIT - II

$R_s < C$ (error free signal)

What is Shanon's

Channel coding - Shanon's second.

$$\underset{\text{Rate}}{\frac{H(X)}{TS}} \le \frac{C}{T_C}$$

Channel Coding Theorem: (2nd Theorem)

The accurate reconstruction of original source sequence at the destination requires the average probability of symbol error is low.

Assume the source emits symbol once every T_s seconds. Hence, average information rate of source is $\frac{H(X)}{TS}$ bits/sec.

The decoder delivers delivers decoded symbol to the destination from the source alphabet 'X' and at the same source rate of one symbol every Ts second.

The DMC has channel capacity equal to 'C' bits per use of channel.

channel is used once every T_c second. Channel Capacity where unit time is $\frac{C}{T_s}$ bits per second, which represents the maximum rate of information transfer over the channel. ~~The~~

The theorem states that,"

" Discrete Memoryless Source with an alphabet 'x' have entropy $H(x)$, then

$$\frac{H(X)}{TS} \leq \frac{C}{T_c} \text{ "}$$

The parameter $\frac{C}{T_c}$ is called critical rate. When, above eqn ~~stat~~ satisfies with the equality sign then, the system is said to be critical condition.

$$\frac{H(X)}{TS} < \frac{C}{T_e} \text{ (error)}$$

Conversely, If $\frac{H(X)}{T_s} > \frac{C}{T_c}$, it is not possible to transmit the information over the channel and reconstructed it with an arbitary small probability of error.

Information rate→R

Application of Channel Coding Theorem to BSC

(Binary symmetric channel)

Consider a DMS that emits equally, likely, binary symbols (0 & 1) once every T_s second. with source entropy ($H(x)$) is equal to 1 bit / source

ie $H(X) = 1$.

Information rate $= \frac{1}{T_s}$ bits/sec.

The source sequence is applied to channel encoder with Code rate 'r'

The channel encoder produces a symbol once every T_c second. Hence, the encoder symbol transmission rate is $1/T_c$ symbols/second.

The channel encoder engages the binary symmetric channel once every T_c second. Hence, channel capacity per unit time is $\frac{C}{T_c}$ bits/sec.

If $\frac{1}{T_s} \le \frac{C}{T_c}$, probability of error can be made low by the use of suitable channel encoding scheme.

But, the ratio $\frac{T_c}{T_s} = r$ (Code rate of encoder)

i.e. $\boxed{r \leq C}$, that exists Code Capable of achieving low probability of error.

Trade off b/w B.W and SNR

$$\boxed{C = B\log_2 (1+S/N)}$$ 3rd theorem

↓

B.W

For Noiseless channel, $N=0$

$S/N = \infty$

$$\boxed{C = B\log_2(1+\infty) = \infty \text{ [Infinitive Capacity]}}$$

<u>Infinite Bandwith Channel has limited Capacity:</u>

As BW ↑s, N ↑s

Noise power is given as, $N_0 B$

Signal power P is given as,

$$P = \int_{-B}^{B} \text{power spectral density}$$

Noise power, $N = \int_{-B}^{B} \frac{N_0}{2} \, df$

(Gaussian noise)

S → signal power

N → Noise power

$\boxed{N = N_0 B}$,

$$= \left[\frac{N_0}{2}\right] [f]_{-B}^{B}$$

$$\Rightarrow N_0 B$$

when Noise power $\uparrow$, SNR $\downarrow$.

Hence, If $B \to \infty$, C approaches upper limit is,

$$\boxed{C_\infty = \lim_{B \to \infty} C = 1.44\, S/N_0}$$

proof:

$$C = B \log_2 (1 + S/N)$$

$$C = B \log_2 \left(1 + \frac{S}{N_0 B}\right)$$

$$C = \frac{S}{N_0} \cdot \frac{N_0}{S} B \log_2 \left(1 + S/N_0 B\right)$$

$$= \frac{S}{N_0} \cdot \log_2 \left(1 + S/N_0 B\right)^{\frac{N_0 B}{S}}$$

$$= \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B}\right)^{\frac{1}{S/N_0 B}}$$

As $B \to \infty$,

$$C_\infty = \lim_{B \to \infty} C$$

$$= \lim_{B \to \infty} \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B}\right)^{\frac{1}{S/N_0 B}}$$

Let, $x = \frac{S}{N_0 B}$

As $B \to \infty$

$x \to 0$

$$C_\infty = \frac{S}{N_0} \lim_{x \to 0} \log_2 (1+x)^{1/x}$$

$$\left[\lim_{x \to 0} (1+x)^{1/x} = e \right]$$

$$C_\infty = S/N_0 \log_2 e$$

$$\boxed{C_\infty = 1.44\, S/N_0}$$

19/07/11 ~~Implementation of~~

Implication of channel capacity Theorem

for an ideal system, data is transmitted at a bit rate R_b, equal to information capacity 'C'. Let, transmitted power 'B',

$$S = E_b C$$

where, $E_b \rightarrow$ Transmitted energy per bit.

The ideal system is defined by an equation,

$$\frac{C}{B} = \log_2 \left[1 + \frac{E_b C}{N_0 B} \right]$$

The signal energy per bit to noise power ~~ratio~~ spectral density ratio

$\frac{E_b}{N_o}$ in terms of band width efficiency

$\frac{C}{B}$ for the ideal system.

$$\frac{C}{B} = \log_2\left(1 + \frac{E_b C}{N_o B}\right)$$

$$2^{C/B} = 1 + \frac{E_b C}{N_o B}$$

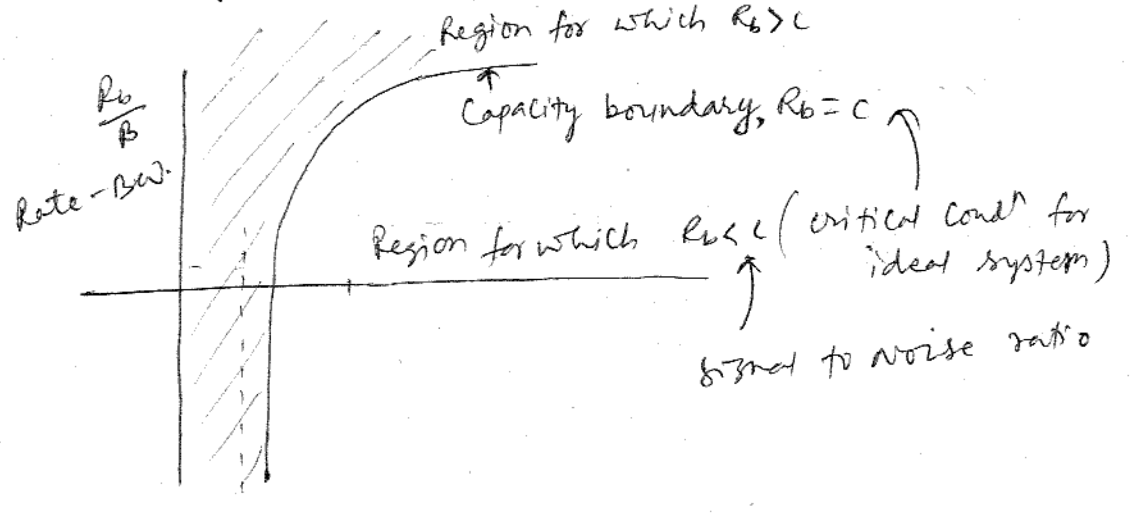
$$2^{C/B} - 1 = \frac{E_b C}{N_o B}$$

$$2^{C/B} - 1 = \frac{E_b}{N_o} \cdot \frac{C}{B}$$

Bandwidth efficiency

$$\boxed{\frac{E_b}{N_o} = \left(2^{C/B} - 1\right) B/C}$$

A plot of bandwidth efficiency $\frac{R_b}{B}$ Vs $\frac{E_b}{N_o}$ is known as bandwidth efficiency diagram.



Case-1:

For the infinite B.W, the ratio E_b/N_o approaches the limiting value.

Case

$$\left(\frac{E_b}{N_o}\right)_\infty = \lim_{B \to \infty} \frac{E_b}{N_o} \Rightarrow 0.693$$

$$C_\infty = 1.44\ S/N_o$$

$$\cancel{S_\infty} = \frac{1.44\ E_b C_\infty}{N_o}$$

$$\frac{E_b}{N_o} = \cancel{\phi}\ \frac{1}{1.44}$$

$$\frac{E_b}{N_o} = 0.693$$

$$\boxed{\frac{E_b}{N_o} = -1.6\,dB}$$ //

This value is called Shanon limit.

Case-II

The capacity boundary defined by the curve for the critical bit rate $R_b = C$, separates combinations of system parameters that have potential for supporting error-free transmission from those for which, error free transmission is not possible.

The diagram highlights potential trade of among E_b/N_o, R_b/B, and probability of symbol error 'pe'.

$$P_e = \begin{cases} 0, & E_b/N_0 \geq \ln 2 \quad 1, \; R_b \geq C \\ 1, & E_b/N_0 < \ln 2 \quad 0; \; R_b < C \end{cases}$$

~~Huffman~~

Construct a Huffman code

Symbols	x_1	x_2	x_3	x_4	x_5	x_6
probability	0.3	0.25	0.2	0.1	0.1	0.05

Encoding alphabet, $D = 2$

$D = \{0, 1\}$

Arranging in descending order.

Symbol	Prob.	Stage-I	Stage-II	Stage-III	Stage-IV
	0.3	0.3 00	0.3 00	0.45 1	0.55 0
	0.25	0.25 10	0.25 01	0.3 00	0.45 1
	0.2	0.2 11	0.25 10	0.25 01	~~0.3~~
	0.1	0.15 010	0.2 11		
	0.1	0.1 010			
	0.05				

$00 \to 2$
$10 \to 2$
$11 \to 2$
$011 \to 3$
$0100 - 4$
$010 - 4$

$$\eta = \frac{H(x)}{\bar{l}} \quad - (*)$$

$\bar{l}$ = prob (length)

$$= 0.3(2) + 0.25(2) + 0.2(2) + 0.1(3) + 0.1(4) + 0.05(4)$$

$$= 0.6 + 0.5 + 0.4 + 0.3 + 0.4 + 0.2$$

$$= 2.4$$

$$H(x) = -\sum p_K \log p_K$$

$$= -\left[0.3\log_2 0.3 + 0.25\log_2 0.25 + 0.2\log 0.2 + 0.3\log_2 0.3 + 0.4\log_2 0.4 + 0.05\log_2 0.05\right]$$

$$= \frac{-1}{\log_{10} 2}\left[0.156 + 0.15 + 0.13 + 0.1 + 0.1 + 0.065\right]$$

$$= \frac{0.701}{0.30} = 2.336$$

$$\eta = \frac{2.336}{2.4} = 0.95$$

Hence $\boxed{\eta = 95\%}$

Symbol	Prob.	Stage-I	Stage-II
x_1	0.18	0.18	0.18
x_2	0.17	0.17	0.17
x_3	0.16	0.16	0.16
x_4	0.15	0.16	0.16
x_5	0.1	0.1	0.15
x_6	0.08	0.08	0.1
x_7	0.05	0.16	0.08
x_8	0.05	0.05	
x_9	0.04	0.05	
x_{10}	0.02		

Encoding alphabet $D = 4$ . . $D = \{0, 1, 2, 3\}$

$D = \{00, 01, 10, 11\}$

length ~~Code~~	Code ~~length~~
1	1
1	2
2	00
2	01
2	02
2	03
2	30
2	31
2	32
2	33

$H(X) = -\sum P_K \log_4 P_K$

Q.

symbol	prob.		Stage-I		Stage-II	
x_1	0.18	1	0.18	1	0.49	0
x_2	0.17	2	0.17	2	0.18	1
x_3	0.16	00	0.16	3	0.17	2
x_4	0.15	01	0.16	00	0.16	3
x_5	0.1	02	0.15	01		
x_6	0.08	03	0.1	02		
x_7	0.05	30	0.08	03		
x_8	0.05	31				
x_9	0.04	32				
x_{10}	0.02	33				

Encoding alphabet $D = 4$ ie
$D = \{0, 1, 2, 3\}$

code	length
1	1
2	1
00	2
01	2
02	2
03	2
30	2
31	2
32	2
33	2

$\eta = \frac{H(X)}{\bar{L}}$

$\bar{L} = \text{prob} \times \text{length}$

$= \cancel{0.18 \log 1 + 0.17 \log 1 + 0.16 \log 2}$

$= 0.18 \times 1 + 0.17(1) + 0.16 \times 2 + 0.15 \times 2 + 0.1 \times 2$

$+ 0.08 \times 2 + 0.05 \times 2 + 0.05 \times 2 + 0.04 \times 2$

$+ 0.02 \times 2$

$= 0.18 + 0.17 + 0.32 + 0.30 + 0.2 + 0.16 + 0.1 +$
$0.1 + \cancel{0.8}\, 0.08 + 0.04$

$= 1.56$

$\cancel{H(X) = -\sum P_4 \log_4 P_4} \times p.$

$H(X) = -\sum P_k \log_4 P_k$

$= -[0.18 \log_4 0.18 + 0.17 \log_4 0.17 + 0.16 \log_4 0.16$
$+ 0.15 \log_4 0.15 + 0.1 \log_4 0.1 +$
$0.08 \log_4 0.08 + 0.05 \log_4 0.05$
$+ 0.05 \log_4 0.05 + 0.04 \log_4 0.04$
$+ 0.02 \log_4 0.02]$

$= \frac{-1}{\log_{10} 4} [-0.134 - 0.13 - 0.127 - 0.123 - 0.1$
$- 0.087 - 0.065 - 0.065 - 0.055$
$- 0.033]$

$= \frac{0.919}{0.6}$

$= 1.53$

$\eta = \frac{1.53}{1.56} \times 100\%$

$\boxed{\eta = 98\%}$