Parity check code:

110101 → Even no of 1 gives the even code.

$C = \{(n-1) \text{ information} + 1 \text{ bit}\}$

C = encoding

information rate $r = \frac{n-1}{n}$

Error corrected = 1

$X = 1\ 1\ 0\ 1\ 0\ 1$

$Y = 1\ 0\ 1\ 0\ 1\ 0$

Hamming distance = 1+1+1+1+1 = 5

Formula for Hamming distance

$$\boxed{d_{min}(C) = \min_{\substack{x,y \\ x \neq y}} d_H(x,y)}$$

for repetition code :-

$X = \{0000\,0 \ldots 0_n,\ 1111 \ldots 1_n\}$

for parity check code

$X = x_0, x_1 \ldots x_{n-2}, x_{n-1}$

$Y = y_0, y_1 \ldots y_{n-2}, y_{n-1}$

distance = 2.

Huffman's coding (lossless Compression Technique)
(Source Coding)

Parameters - Freq (weight of code)

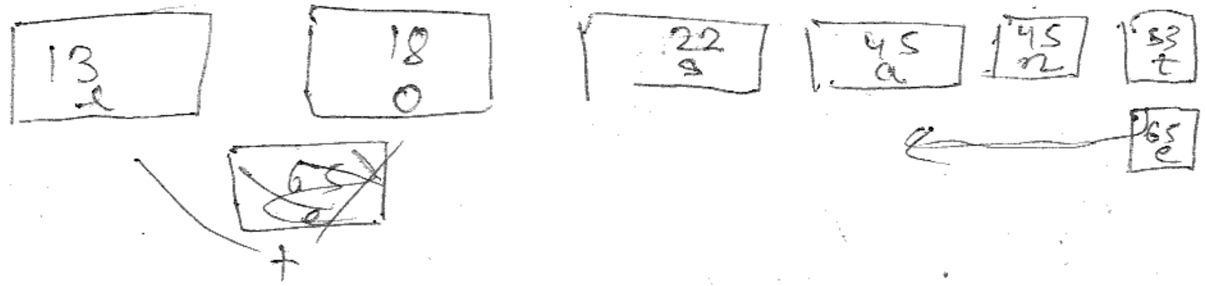
1) Information contains the following characteristics

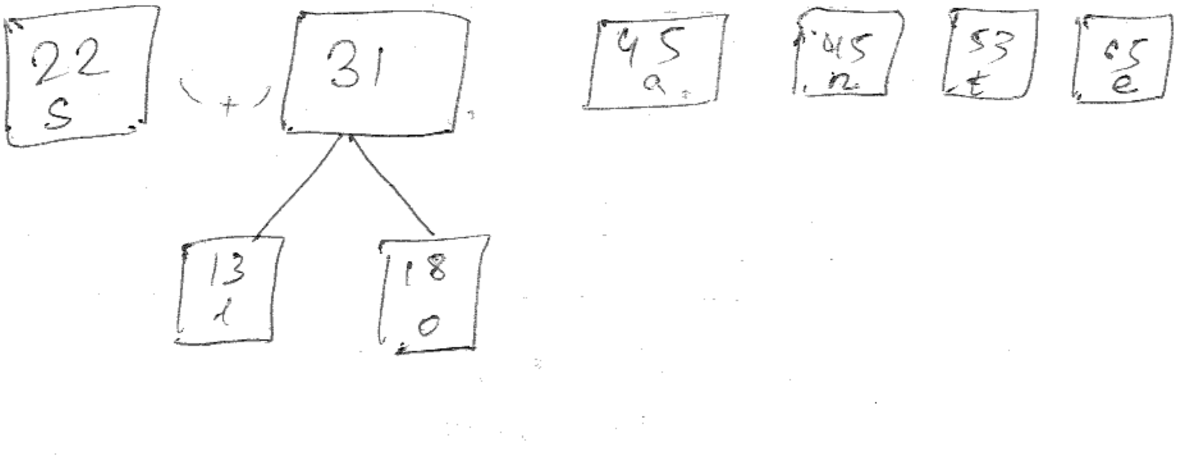
Symbol	a	e	l	n	o	s	t
Freq (weight)	45	65	13	45	18	22	53

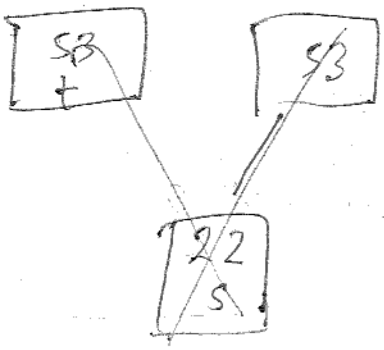
$\Rightarrow 8 \times 261 = 2088$
After samp. before comp (Total bits)

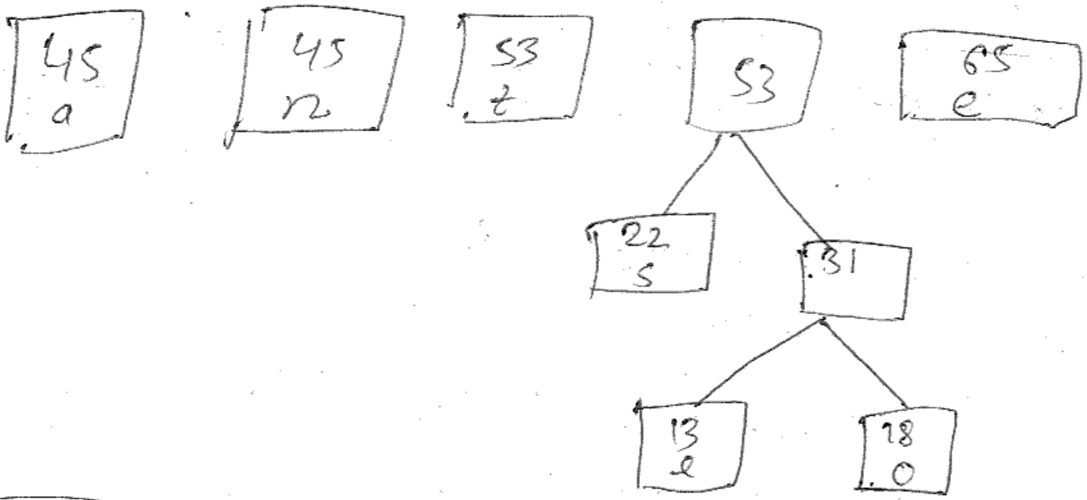
Use the huffman's coding to answer the following questions.

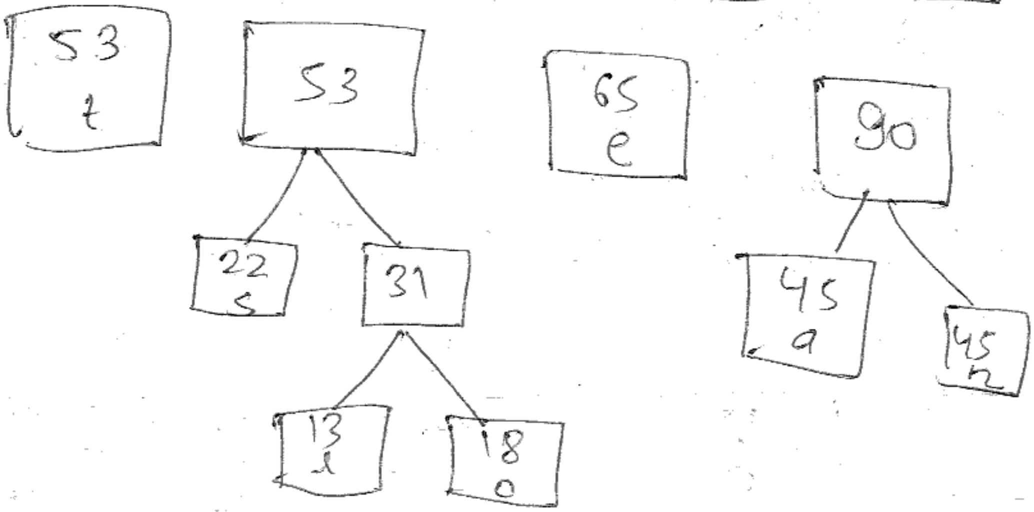
1. build huffman code tree
2. By using the tree find the code words.
3. Find Total no of bits to be transmitted and what is compressed ratio.
4. verify the prefix property

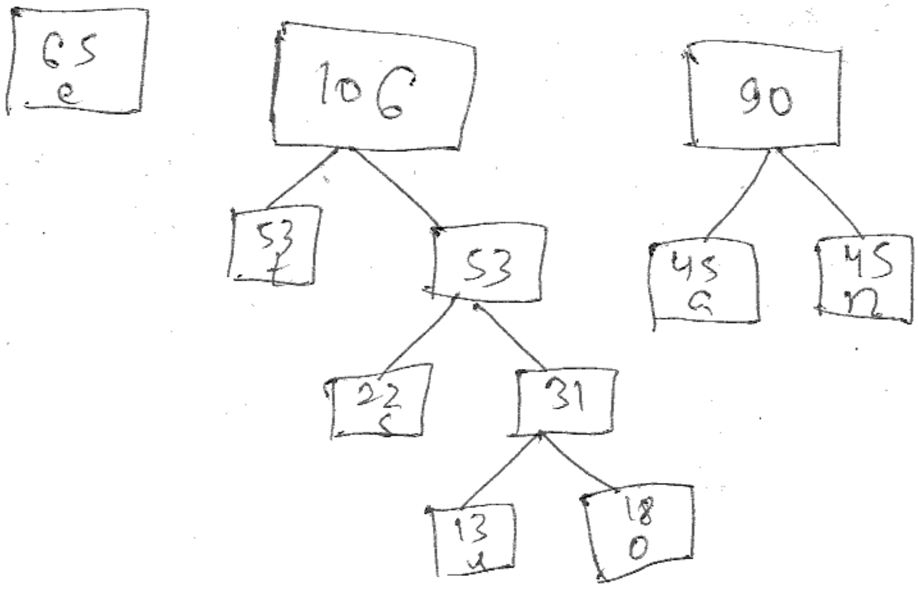


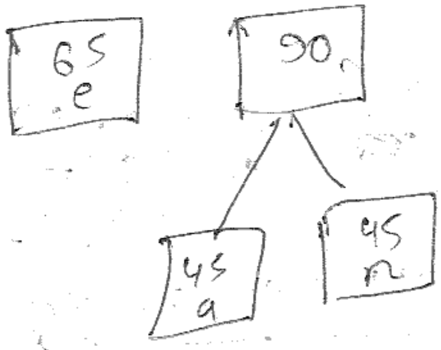



53 t
53
22 s

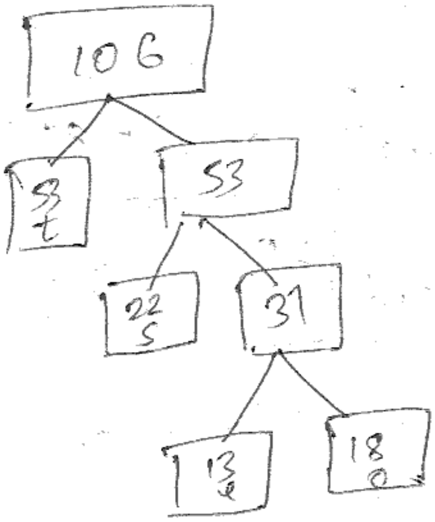

45 a
45 n
53 t
53
22 s
31
13 l
18 o
65 e


53 t
53
22 s
31
13 l
18 o
65 e
90
45 a
45 n

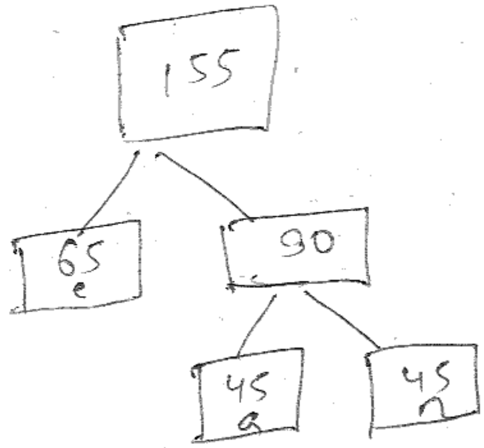

65 e
106
53 t
53
22 s
31
13 l
18 o
90
45 a
45 n



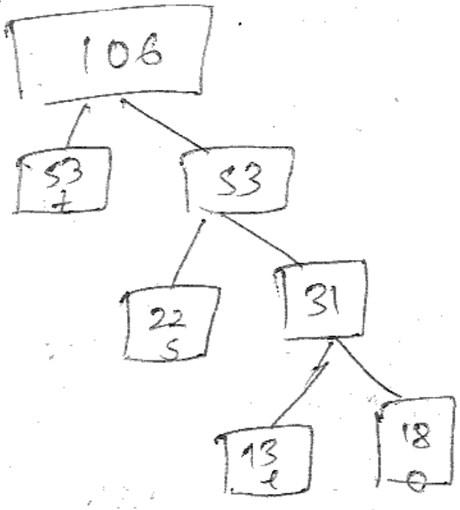
65
e
90
45
a
45
n



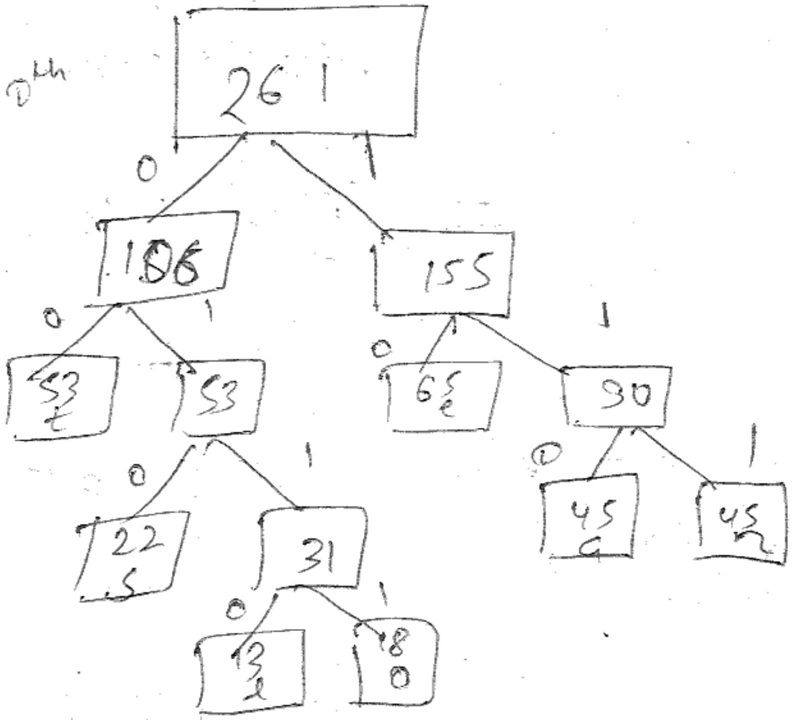
106
53
t
53
22
s
31
13
l
18
o



155
65
e
90
45
a
45
n



106
53
t
53
22
s
31
13
l
18
o



261
106
155
53
t
53
65
e
90
22
s
31
45
a
45
n
13
l
18
o

Symbol	a	e	l	m	o	s	t
code word	110	10	0110	111	0111	010	00
length	3	2	4	3	4	3	2
weight	45	65	13	45	18	22	53
length x weight	135	130	52	135	72	66	106

→ 696

Total no of bits for the transmitted file

Total bits = bits (7) + bits (character) + bits (code word) + bits (compressed msg)

(Entropy) (avg bits/sec)

$= \left(\frac{21}{7} = 3\right) + (7 \text{char} \times 8 \text{ bits}) + (7 \times 3) + 696$

Total bits after huffman coding = 776 bits

Compression ratio $= \frac{776}{2088}$ bits for ASCII Rep) no of bits transferrent.

$$= \frac{2088}{776} = 2.69$$

Percentage $= \frac{100}{2.69} = 37.17\,\%$ amount

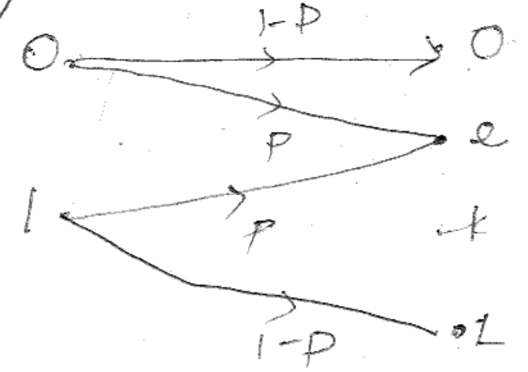
we have compressed our original information.

All codewords are not same, so it is a prefix code (Instantaneous code).

UNIT-III

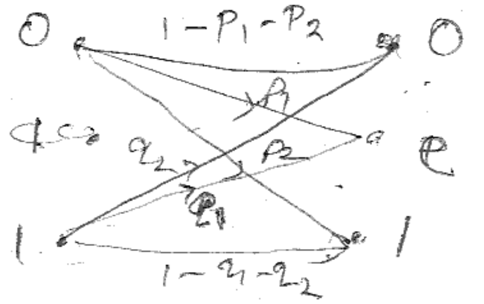
Channel Models

i) Binary Eraser channel



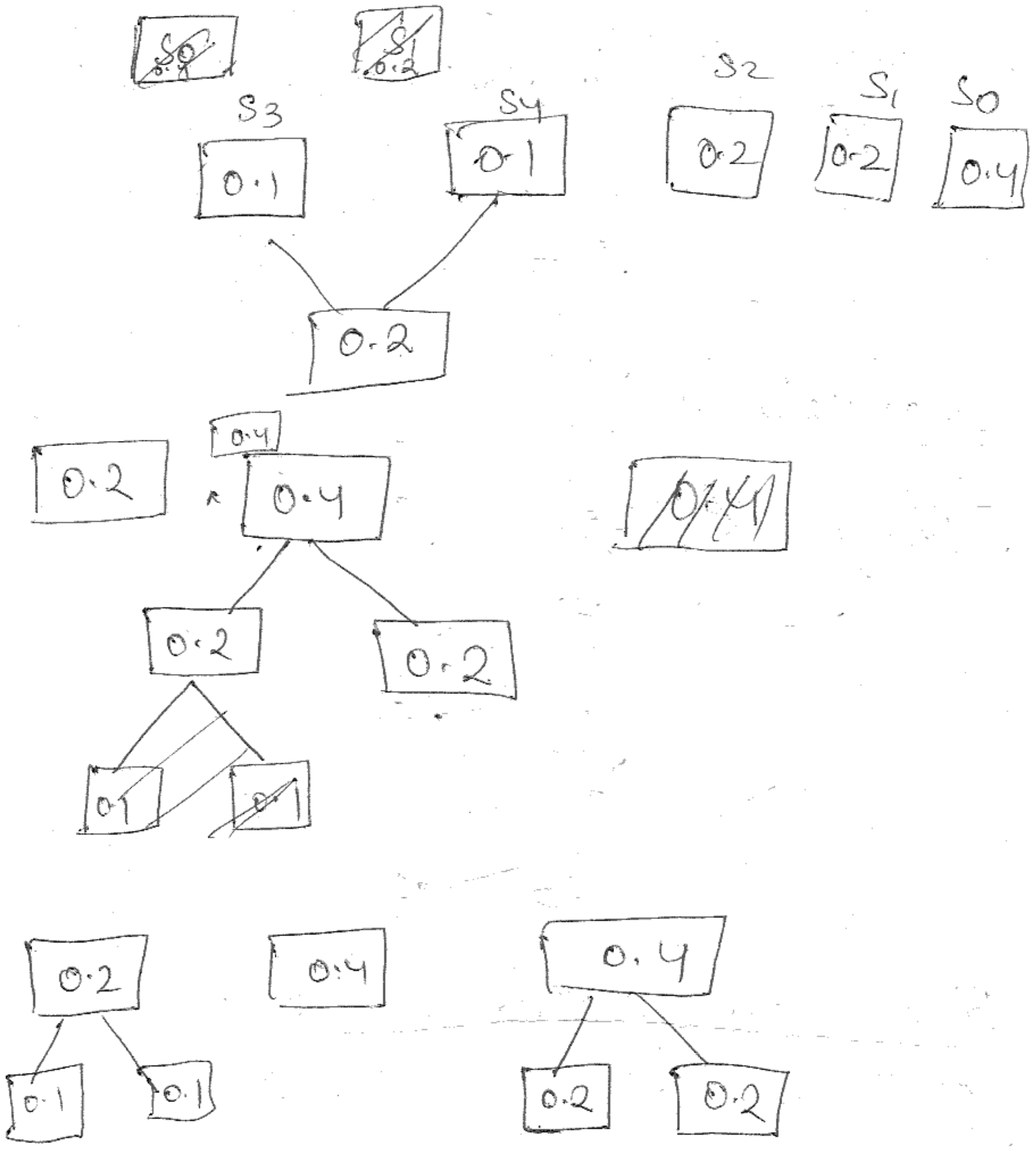
Binary Error & Eraser Channel

combination of Binary Eraser & Binary symmetric channel.

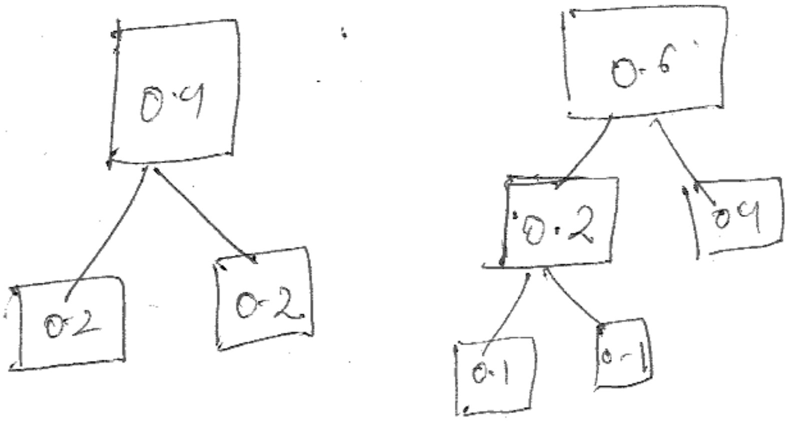


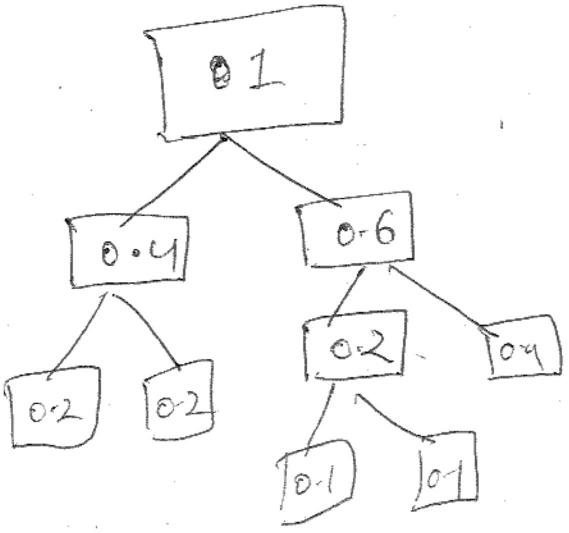
Q. Huffmann Coding.

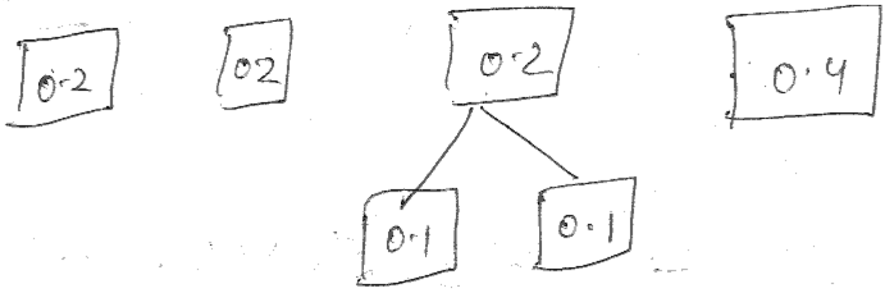
S_0	S_1	S_2	S_3	S_4
0.4	0.2	0.2	0.1	0.1

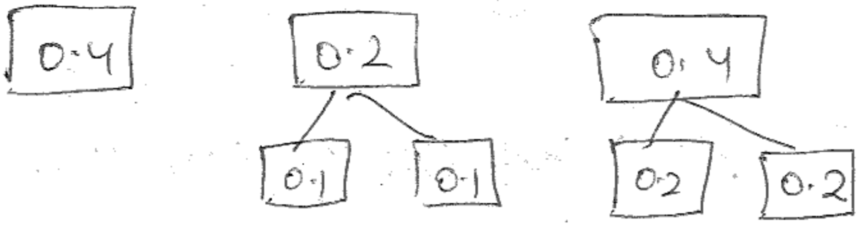


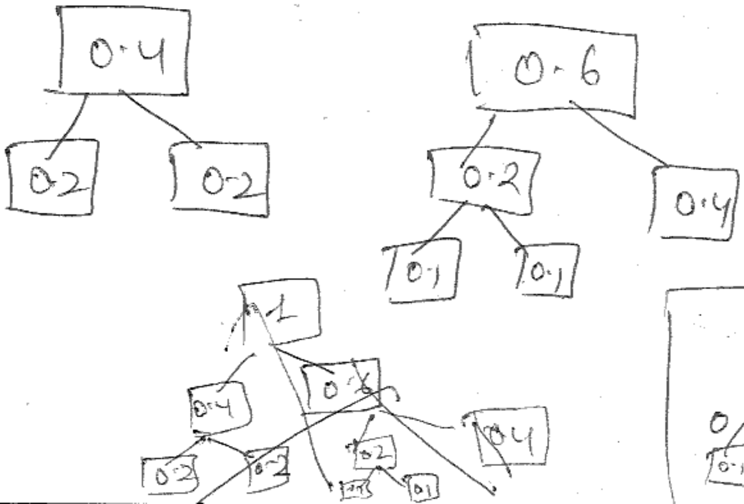
So
0.4

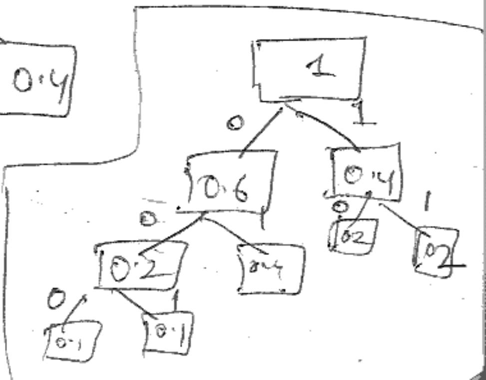

0.4
0.2
0.2
0.6
0.2
0.4
0.1
0.1


1
0.4
0.6
0.2
0.2
0.2
0.4
0.1
0.1


0.2
0.2
0.2
0.4
0.1
0.1


0.4
0.2
0.4
0.1
0.1
0.2
0.2


0.4
0.6
0.2
0.2
0.2
0.4
0.1
0.1
1


1
0
1
0.6
0.4
0
1
0
0.2
0.4
0.2
0.2
0
0.1
0.1

Symbol	s_0	S_1	S_2	S_3	S_4
Code word	01	10	11	000	001

$$H(X) = -\sum_{i=1}^{M} P(x)\log P(x)$$

$$= -\left[0.4 \log 0.4 + 0.2 \log 0.2 + 0.2 \log 0.2 + 0.1 \log 0.1 + 0.1 \log 0.1\right]$$

$$= 2.12,$$

~~Average~~ length $= \frac{12}{5} = 2.4$ bits/sec.

$$\text{length} = 0.4\times 2 + 0.2\times 2 + 0.2\times 2 + 0.1\times 3 + 0.1\times 3$$

$$= 0.8 + 0.4 + 0.4 + 0.3 + 0.3$$

$$= 1 + 1.2$$

$$\text{length} = 2.2$$

Information capacity Theorem

11/8/11

$$\boxed{C = \underset{\downarrow \text{B (Band width)}}{W} \log_2\left(1+\frac{S}{N}\right)}$$

Band limited Gaussian channel.

~~Let~~ let us assume $x(t)$, is a band limited continous signal with the frequency 'W' Hz.

let x_K ~~denotes the continous random~~ ~~var~~ is the no of samples.

$K = 1, 2, \ldots K$ (continuous random variable).

is obtained from the uniform sampling process.

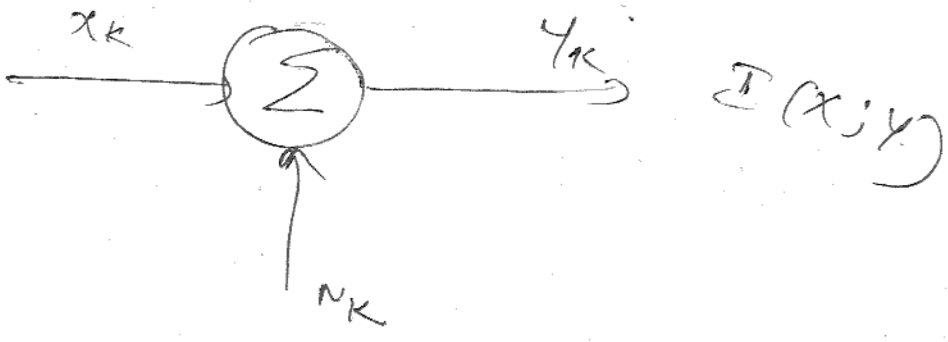
Nyquist Rate.

$$K = 2BT \quad — (1)$$

$T \rightarrow$ Time period

$B \rightarrow$ Bandwidth.

~~Assume~~ Additive white Gaussian N_K the same continous random variable



Received Sample is Y_K.

$$\boxed{Y_K = X_K + N_K.} \quad \text{—} (2)$$

Noise is having variances & means.

$\sigma^2 = N_0 B$ {pre calculated value}

(Estimated value)

$N_0 \rightarrow$ Noise

$B \rightarrow$ Band width.

$E(X_K^2) = P$ {for discrete channel}

Energy — Power (Avg transmitted power).

General equation of channel capacity.

$$C = \max [I(X;Y); E(X_K^2)]$$

$$I(X;Y) = H(Y) - H(Y/X)$$
$$= H(X) - H(X/Y)$$

$$H(Y) = \frac{1}{2}\log_2(2\pi e(P+\sigma^2))$$

$$)= H(Y_K/X_K) = \frac{1}{2}\log_2 2\pi e \sigma^2$$

$$\sigma^2 = N_0 B.$$

$$C = \max[I(X;Y)] = H(Y_K) - H(N_K)$$

$$= \frac{1}{2}\log_2 \left(2\pi e (P+\sigma^2)\right) - \frac{1}{2}\log_2 2\pi e \sigma^2$$

$$= \frac{1}{2} \log_2 \left[\frac{2\pi e (P+\sigma^2)}{2\pi e \sigma^2}\right]$$

$$= \frac{1}{2} \log_2 \left(\frac{P+\sigma^2}{\sigma^2}\right)$$

$$C = \frac{1}{2}\log_2 \left(1+\frac{P}{\sigma^2}\right)$$

We have to calculate the unit channel capacity

$$= \frac{K}{T}(C)$$

$$= \frac{2B\cancel{T}}{\cancel{T}}\left[\frac{1}{2}\log_2\left(1+\frac{P}{\sigma^2}\right)\right]$$

$$\boxed{C = B\log_2\left(1+\frac{P}{\sigma^2}\right)}$$

A/8/11

Data Compression

For lossless Compression algorithm

1) Data Duplication.
2) Run-length encoding.
3) Dictionary coder's
   (i) LZ77 & LZ78
   (ii) LZW
   iii) Statistical lempel-ziv
4) Burrow's wheeler transform
5) Entropy Encoding
   i) Huffman coding
   ii) Shannon-fano coding
6) Context Mixing.
7) Dynamic Markov Compress.
8) Prediction by partial matching.
9) Distributed source coding.

lossy Compression algorithm

(1) Discrete cosine Transform (DCT)

(2) Fractal compression.

i) Fractal transform.

(3) wavelet compression.

(4) vector quantization.

(5) Linear predictive code.

(6) Modulo - n code

(7) A- law compander

(8) Mu - law Compander

(9) wyner - Ziv code (WZ C).

Example implementation.

(Lossless)

1. DEFLATE (Algorithm)

combination of LZ77 ~~& LZ88 & Shannon-Coand~~ Huffman coding.

· ZIP, ~~i~~ gZIP, · PNG.

(GNU company develop this software)

2. LZMA → .7-ZIP
↓
(Modern code of LZ77)

3. BZIP2 (file name) - It is Combination of Burrow's wheeler & huffman Coding.

4. .PAQ → file name for context mixing

Example of lossy Compression.

i) JPEG - Join Picture Experts Group.
Discrete cosine transform for image Compression technique.
This JPEG having down sampling and Cosine transform.

ii) MPEG - combination of Audio & Video
(Moving Picture Expert Group)
→ Discrete Cosine Transform & Linear Pre-dictive code
i) MP3 ii) AAC (Recording of voice)
i) MP4, 3GP, AVI (videos audio both)

iii) JPEG 2000 :- wavelet compression algorithm.

Implication of Information capacity Theorem

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bits/second}$$

Average signal power, $S = E_b \cdot C$

$E_b \rightarrow$ transmitted Energy per bit

$$N = N_0 B$$

$$\frac{C}{B} = \log_2\left(1+\frac{E_b \cdot C}{N_0 B}\right)$$

$$\frac{C}{B} = \frac{\log_e\left(1+\frac{E_b \cdot C}{N_0 B}\right)}{\log_e 2}$$

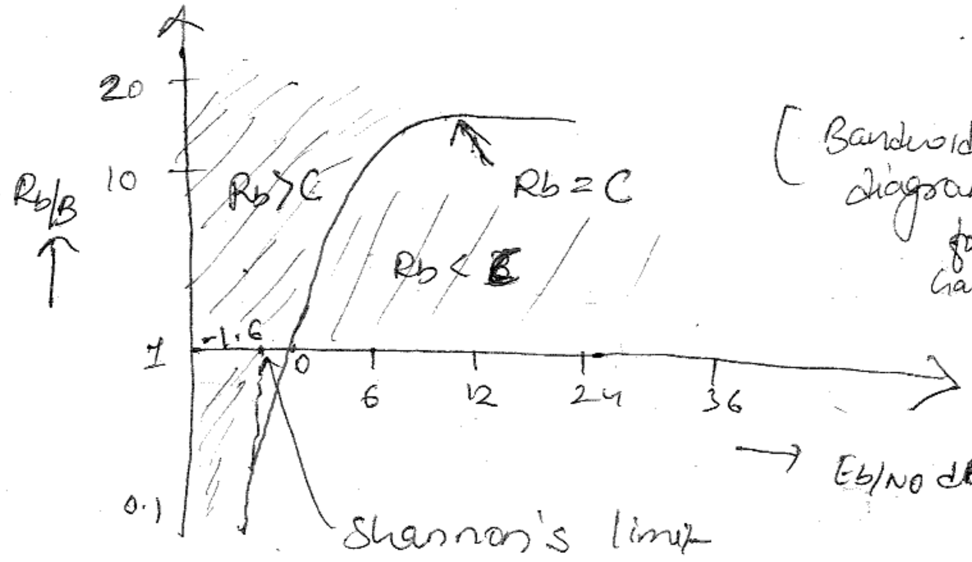
$$\frac{C}{B}\log_e 2 = \log_e\left(1+\frac{E_b}{N_0}\cdot\frac{C}{B}\right)$$

$$\log_e 2^{C/B} = \log_e\left(1+\frac{E_b}{N_0}\cdot\frac{C}{B}\right)$$

$$2^{C/B} = 1+\frac{E_b}{N_0}\cdot\frac{C}{B}$$

$$\frac{2^{C/B}-1}{C/B} = \frac{E_b}{N_0}$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{2^{C/B} - 1}{C/B}$$



(Bandwidth Efficiency diagram) for Gaussian channel.

For ideal s/m

$R_b = C$,

$$\frac{E_b}{N_0} = \frac{2^{R_b/B} - 1}{R_b/B}$$

Shanon limit says that it is noisy communication or noiseless communication.

$$\lim_{B \to \infty} \left(\frac{E_b}{N_0} \right) = \ln 2 = 0.693 = -1.6\,dB$$

$$P_e \begin{cases} 0, & R_b = C,\ R_b < C \\ 1, & R_b > C \end{cases}$$

$R_b \rightarrow$ Information.

Rate Distortion Theorem

→ Amount of Error is Present inside ~~the~~ information.

$R_t = I(X, Y)$

$R(D) =$ Distortion rate

i) $R < C$ information rate less than Channel capacity.

ii) $R > C$, information get lost.

$$R(D) = \min_{P(y_j/x_i) \in P_D} I(X; Y)$$

Consider $X = \{x_i / i = 1, 2, \dots M\}$

$Y = \{y_j / j = 1, 2, \dots N\}$.

i) ~~R > C~~ $R < C \rightarrow$ no distortion.

ii) $R > C$

Joint propability $P(x_i y_j) = \sum P(x_i) \cdot P\left(\frac{y_j}{x_i}\right)$

information * Error.

$$\bar{d} = \sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i) \cdot P\left(\frac{y_j}{x_i}\right) d(x_i, y_j)$$

Average distortion

Single letter distortion

$$\bar{d} > 1$$

$$P_D = \{ P(y_j/x_i) : \bar{d} \leq D \}.$$

$$R(D) = \min_{P(y_j/x_i) \in P_D} I(x, y)$$

$$I(x; y) = \sum \sum P(x_i, y_j) \cdot P(y_j/x_i)$$

Total probability Error

$$\sum_{i=1}^{M} \sum_{j=1}^{N} P(y_j/x_i) = 1$$

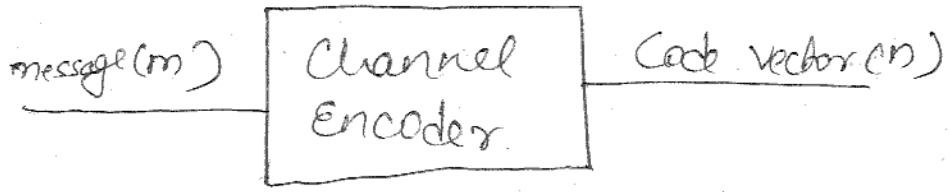
7/9/11

Linear Block codes

(Fig)

letter

General representation



m – Message with K bits.

n – Code vector (or) code word.

q – Redundancy with $n-K$ bits

let

$X = [m_1, m_2\ m_3 \dots\dots m_K,\ c_1, c_2\ c_3 \dots c_q]$

K – message bits

q – redundancy bits.

$X = [M | C]_{1 \times n}$

$C \rightarrow$ check bits.

$X = [M\ G]$

G – Generator Matrix.

(To generate redundancy)

$$[X] = [M]_{1\times k}\,[G]_{k\times n}$$

$$[G]_{k\times n} = \left[I_k : [P]_{k\times q} \right]_{k\times n}$$

$I_k \rightarrow$ Identity Matrix

$[P]_{k\times q} \rightarrow$ Submatrix

check vector

$$C = MP$$

$$[P]_{k\times q} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & & & \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}_{k\times q}$$

$$[C] = [M]_{1\times k}\,[P]_{k\times q}$$

$$= [m_1\ m_2 \ldots m_k] \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & \vdots & & \vdots \\ P_{k1} & P_{k2} & & P_{kq} \end{bmatrix}$$

$$C_1 = m_1 P_{11} \oplus m_2 P_{12} \oplus \cdots m_k P_{1q}$$

$$C_2 = m_1 P_{12} \oplus m_2 P_{12} \oplus \cdots m_k P_{2q}.$$

$$\vdots$$

Q. A Generator matrix G for a [6,3] ~~bbo~~ block code is given below. find all code vectors of this code.

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$$[G]_{K \times n} = \Big[I_K : [P]_{K \times q} \Big]_{K \times n}.$$

$$[P]_{K \times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

WKT

$K = 3$,

M_1	M_2	M_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$[C] = [M / P]$$

$$= [m_1\ m_2\ m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C_1 = m_1 \cdot 0 \oplus m_2 \oplus m_3$$

$$C_2 = m_1 \cdot 1 \oplus m_2 \cdot 0 \oplus m_3$$

$$C_3 = m_1 \cdot 1 \oplus m_2 \oplus m_3 (0).$$

m_1	m_2	m_3	C_1	C_2	C_3	code
0	0	0	0	0	0	000000
0	0	1	1	1	0	001110
0	1	0	1	0	1	010101
0	1	1	0	1	1	011011
1	0	0	0	1	1	100011
1	0	1	1	0	1	101101
1	1	0	1	1	0	110110
1	1	1	0	0	0	111000

8/7/11

Hamming codes :-

Hamming distance - How many error occurs

Parity Check matrix [H], to correct Error & used in Decoder side.

$$H = [P^T : I_q]$$

Four important condition for Hamming codes

1. Find Block length $n = 2^q - 1$
   $n = k + q$

2. Find Message bits $k = n - q$

3. Minimum distance d_{min} (Hamming distance).

$$\text{Code rate } (r) = \frac{k}{n}$$

$$= \frac{n-q}{n}$$

$$\boxed{r = 1 - \frac{q}{n}}$$

$$\boxed{r = 1 - \frac{q}{2^q - 1}}$$

3 2^q

$r \cancel{\approx} 1, \; q >> 1$

Q. The parity check matrix of a Particular 7,4 (n,K) Linear block code is given by

$$[H] = \left[\begin{array}{cccc:ccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

i) Find the Generator Matrix

ii) List all code vectors

iii) What is the ~~maximum~~ max distance b/w code vectors.

iv) How can be detected & corrected.

$$[H]_{q \times n} = \left[[P^T]_{q \times K} : I_K \right]$$

$$P = (P^T)^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}^{-1}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{k \times q}$$

$$G = \begin{bmatrix} I_k : [P]_{k \times q} \end{bmatrix}_{n \times q}$$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

ance

ocha.

$$C = MP$$

$$= [m_1 \; m_2 \; m_3 \; m_4] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$C_1 = M_1 \oplus m_2$

$C_2 = m_1 \oplus m_2 \oplus m_4$

$C_3 = m_1 \oplus m_3 \oplus m_4$

Max distance = 3

weight of code

000

⋮

111 – 3 (one's)

$M = 1010$

$\boxed{X = 1010 : 110}$

Error detection.

$d_{min} \geq S + 1$

How many Error.

If $d_{min} = 3$

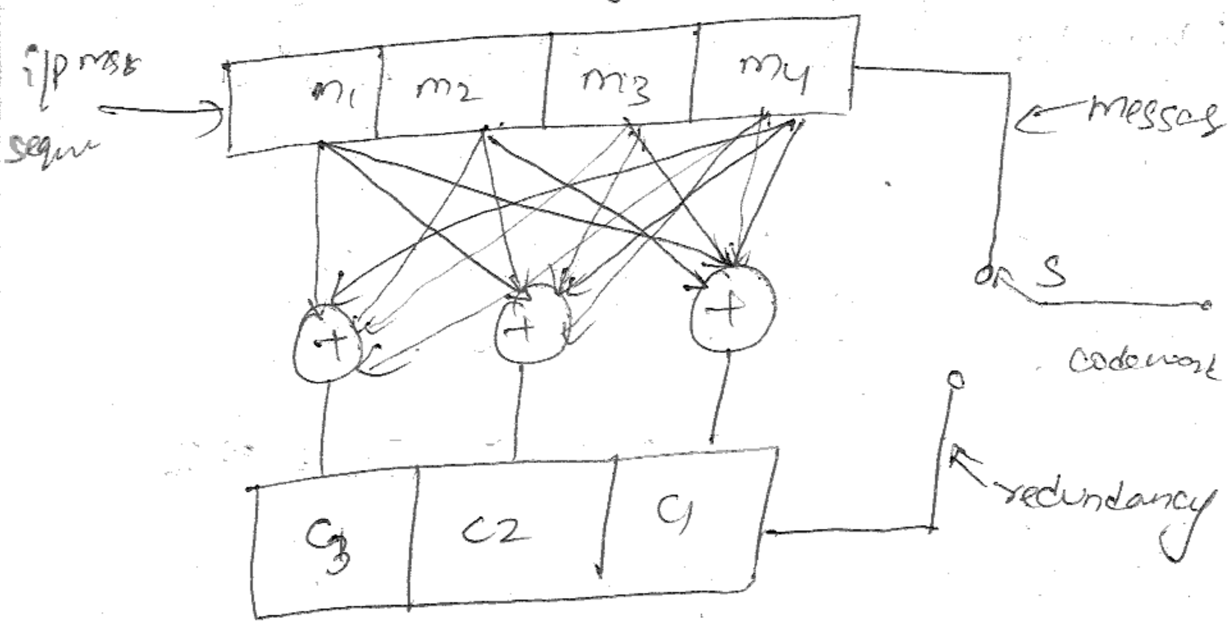
$3 \geq 2+1$

$\boxed{S \leq 2}$ → 2 error can be detected

ii) Error Correction

$d_{min} \geq 2t+1$ $\boxed{t \leq 1}$

Practical Hamming Encoder



Syndrome Decoding

$S = EH^T \rightarrow$ Correct the Error

$E \rightarrow$ Error ~~matrix~~ vector.

$\boxed{S = YH^T} \rightarrow$ Error detection

$\boxed{X = \cancel{Y} \oplus E}$

Q. The parity check matrix of [7, 4] hamming code is given below

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \vdots & 0 & 0 & 1 \end{bmatrix}_{3 \times 7}$$

i) Calculate the Syndrome vector for Single bit Error.

ii) $$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

$$S = EH^T$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = 1 \oplus 1 = 0$$

$$Y = 1\,0\,1\,1\,1\,1\,0$$

$$X = 1\,0\,0\,1\,1\,1\,0$$

$$S = YH^T$$

$$= \begin{bmatrix} 1011110 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = 110$$

$$1 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 0 +$$

$$Y = X \oplus E$$

$X = Y \oplus E$

$E = 0010000$

$X = Y \oplus E$

$= 1011110 \oplus 0010000$

$= 1001110$.

<u>cyclic code</u>

1. Non-systematic
2. Systematic

Q. The generator polynomial of (7, 4) cyclic code is $G(P) = P^3 + P + 1$. Find all the code for the code in non-systematic form.

Sol^n. $n = 7$

$k = 4$

$q = 3$

$$\begin{array}{r} 1011110 \\ 0010000 \\ \hline 1001110 \end{array}$$

$m = (m_3 m_2 m_1 m_0)$

$X = [M(P) \cdot G(P)]$

$X = (m_3 m_2 m_1 m_0)(P^3 + P + 1)$

$\boxed{m = 1011}$

$X = (P^3 + P + 1)(P^3 + P + 1)$

$X = P^6 + P^4 + P^3 + P^4 + P^2 + P + P^3 + P + 1$

$\cancel{X = P^6 + 2P^4 + 2P^3 + P^2 + 2P + 1}$

$X = P^6 + P^4(1 \oplus 1) + P^3(1 \oplus 1) + P(1 \oplus 1) + P^2 + 1$

$= P^6 + 0 + 0 + 1$

$\boxed{X = P^6 + 1}$

$X = 1000001$

$m = 0000$

$X = 0000$

$m = 0001$

$X = P^0(P^3 + P + 1) = P^3 + P + 1$

$= 1011$

$m = 0010$

$= P(P^3 + P + 1)$

$= P^4 + P^2 + P$

$= 10110$

m_3	m_2	m_1	m_0	x = codes $x^6\ x^5\ x^4\ x^3\ x^2\ x^1\ x^0$
0	0	0	0	x = 0 0 0 0 0 0
0	0	0	1	0 0 0 1 0 1 1
0	0	1	0	0 0 1 0 1 1 0

$x_2 = 0001011$

left shifting property

$= 1000101$

Systematic manner

Division property - I/P data / Generator Polynomial.

CRC (cyclic Redundancy check).

$X = m(P)$ (remainder)

$g(P)\ )\ m(P)\ ($

rem.

$$\text{Check bit} = \text{rem}\left(\frac{P^q m(P)}{G(P)}\right)$$

$n = 7, \; q = 3, \; K = 4$

$m = (m_4, m_3\, m_2\, m_1)$

$m(P) = m_4 P^3 + m_3 P^2 + m_2 P^1 + m_1 P^0$

Q1. The Generator polynomial of a (7, 4) cyclic code is $G(P) = P^3 + P + 1$. Find all the code vectors for the code in systematic form.

$m = (m_4\; m_3\; m_2\; m_1)$

$$\boxed{m(P) = m_4 P^3 + m_3 P^2 + m_2 P^1 + m_1 P^0}$$

0101

$m(P) = 0 + P^2 + 0 + 1$

$m(P) = P^2 + 1$

$$C = \text{rem}\left(\frac{P^3(P^2+1)}{P^3+P+1}\right)$$

$$= \text{rem}\left(\frac{P^5 + P^3}{P^3+P+1}\right)$$

$$p^3+p+1 \overline{\big)\, \cancel{p^5} + p \,}\big(\cancel{p^2-1}$$

$$\underline{\cancel{p^5} + p^3 + p^2}$$
$$- \quad - \quad -$$

$$-\cancel{p^3} - p^2 + p$$
$$-\cancel{p^3} \quad -p - 1$$
$$+ \qquad +$$

$$\cancel{-p^2 + 2p - 1}$$

$$p^3+p+1 \overline{\big)\, \cancel{p^5} + \cancel{p^3} \,}\big(p^2$$
$$\underline{\cancel{p^5} + \cancel{p^3} + p^2}$$
$$+ \quad + \quad +$$

$$+p^2$$

$$\boxed{C = p^2}$$

$q = 3$

$$c_2\, c_1\, c_0 = p^2 + 0p^1 + 0p^0$$

$$c_2\, c_1\, c_0 = 100$$

$$X = (0101 : 100)$$

Q1. Find out the possible generator polynomial of 7x4 cyclic codes. Find out the code vectors corresponding to generator polynomials

Generator Matrix and parity check matrix of cyclic codes

Ans. (7,4)

$n=7 \quad k=4, \quad q=3$

$G(P) = (P \oplus 1)(P^3 \oplus P \oplus 1)(P^3 \oplus P^2 \oplus 1)$

$$\boxed{P^i G(P) = P^{i+q} + g_{q-1} P^{i+q-1} + \cdots + g_1 P^{i+1} + P^i}$$

Q. Obtain the Generator matrix corresponding to $G(P) = P^3+P+1$ for a (7, 4) cyclic code

(Non-systematic)

$X = MG$

$i = k-1 \quad \boxed{i=3}$
$(4-1)$

$i \rightarrow$ row value $\quad i = 3, 2, 1, 0$

olynomial

nials

$P^3 G(P) = P^3 (P^3+P+1)$

$\Rightarrow P^6 + P^4 + P^3$

row 1 $= 1\,0\,1\,1\,0\,0\,0$

$i = 2$

$$p^2 G(p) = p^2(p^3 + p + 1)$$
$$= p^5 + p^3 + p^2$$
$$= 0101100$$

$i = 1$

$$p\,G(p) = p(p^3 + p + 1)$$
$$= p^4 + p^2 + p$$
$$= 0010110$$

$i = 0$

$$p^0 G(p) = p^3 + p + 1$$
$$= 0001011$$

$$G_{k \times n} = \begin{array}{c} \qquad I_k \qquad\qquad\quad P_q \\ \left[\begin{array}{cccc:ccc} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]_{k \times n} \end{array}$$

Parity check matrix :-

$$H = [P^T : I_q]_{n \times k}$$

$$G = [I_k : P_{k \times q}]$$

$$P_q = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = [P^T : I_q]$$

I_q = Identity matrix

$$H = \left[\begin{array}{cccc:ccc} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Q. Find out the generator matrix for a Systematic (7, 4) cyclic code if $G(P) = P^3 + P + 1$. Also find the parity check matrix.

kxn