$$\eta = \frac{H(X)}{L} = \frac{0.65}{2.33} =$$

J.S.
22/07/11

27.7.11

Ans.

Q- Determine the capacity of the unsymmetric channel.

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 2/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 \log 1/3 + 2/3 \log 2/3 & 0 \\ 2/3 \log 2/3 + 1/3 \log 1/3 & 0 \\ 0 + 0 + 1 \log 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.276 \\ 0.276 \\ 0 \end{bmatrix}$$

#

$$\Rightarrow \begin{bmatrix} 1/3\,\phi_1 + \frac{2}{3}\phi_2 + 0 \\ 2/3\,\phi_1 + \frac{1}{3}\phi_2 + 0 \\ 0 + 0 + \phi_3 \end{bmatrix} = \begin{bmatrix} 0.276 \\ 0.276 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{3}\varphi_1 + \frac{2}{3}\varphi_2 = 0.276$$

$$\Rightarrow \varphi_1 + 2\varphi_2 = -0.828 \quad \text{—} \ (1)$$

$$2\varphi_1 + \varphi_2 = -0.828 \quad \text{—} \ (2)$$

$$\varphi_3 = 0 \quad \text{—} \ (3)$$

$$(1)\times 2 \Rightarrow 2\varphi_1 + 4\varphi_2 = -1.656$$

$$(11) \ \cancel{\otimes} \Rightarrow 2\varphi_1 + \varphi_2 = -0.828$$

$$3\varphi_2 = -0.828$$

$$\varphi_2 = -0.246$$

$$\therefore 2\varphi_1 - 0.276 = -0.828$$

$$\varphi_1 = -0.276$$

$$C = \log_2 \{ 2^{\varphi_1} + 2^{\varphi_2} + 2^{\varphi_3}] \text{ bits/symbol}$$

$$= \log_2 \{ 2^{-0.276} + 2^{-0.276} + 1]$$

$$= 1.406 \text{ bits/symbol}$$

Capacity of band limited gaussian channel

The rate of transmission for a continuous channel is defined as

$$1)\ I(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x,Y) \log \frac{P(x,Y)}{P(x).P(y)}\, dx\, dy$$

$$\text{Apply} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x,y) \left[\log P(x,Y) - \log P(x).P(y) \right] dx\, dy$$

$$I(X,Y) = -H(X,Y) + H(X) + H(Y)$$

- Generalised formula

ii) $$I(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \log \frac{P(X/Y)}{P(X)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \left[\log P(X/Y) - \log P(x)\right] dx\, dy$$

$$= -I(X,Y) = -H(X/Y) + H(X)$$

Sim

$$I(X,Y) = -H(Y/X) + H(X)$$

Again

$$I(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \log \frac{P(X,Y)}{P(X)}\, dx\, dy$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \log \frac{P(X)}{P(X/Y)}\, dx\, dy$$

lemma $\ln x \leq (x-1) \Rightarrow \log x = \ln x \times \log e$

$$\therefore I(X,Y) \geq -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \left[\frac{P(x)}{P(X/Y)} - 1\right] \log e\, dx\, dy$$

$$P(X,Y)/P(X/Y) = P(x)$$

$$\therefore I(X,Y) \geq -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{P(X,Y)\,P(x)}{P(X/Y)} \log e\, dx\, dy$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(X,Y) \log_e dx\, dy$$

$$\therefore I(X,Y) \geq -\cancel{\log e} + \cancel{\log e} \geq 0$$

28.7.11

If x is a transmitter random variable then y is a received random variable. The y have the effect of channel noise.

If the noise is additive and channel dos not phase and attenuation we must add $y = x + n$. n is the channel noise random variable.

If x is L constant at x_0 then

$$P[y_1 < y \leq y_2 / x = x_0] = P[n_1 < n \leq n_2 / x = x_0]$$

$$y_1 = x_0 + n_1$$

$$y_2 = x_0 + n_2$$

$$\int_{y_1}^{y_2} P(y/x)\, dy = \int_{n_1}^{n_2} P(n/x)\, dn$$

$$P(n/x) = P(n)$$

$$\therefore \int_{y_1}^{y_2} P(y/x)\, dy = \int_{n_1}^{n_2} P(n)\, dn$$

$$n = y - x_0$$

$$\int_{y_1}^{y_2} P(y/x)\,dy = \int_{n_1}^{n_2} P(y-x_0)\,dy$$

Here,

$$n = y - x_0$$

$$dn = dy$$

$$\therefore \int_{y_1}^{y_2} P(y/x)\,dy = \int_{y_1}^{y_2} P(y-x_0)\,dy$$

$$\therefore P(y/x) = P(y-x_0) = P(n)$$

$$I(X,Y) = H(Y) - H(Y/X)$$

$$= H(Y) - H(n)$$

$$= H(Y) + \int_{-\infty}^{\infty} P(n) \log P(n)\,dn$$

Let us now consider the calculation of channel capacity in the presence of additive gaussian noise with specified transmitter and noise power.

$$\boxed{P(n) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-n^2/2\sigma_n^2}}$$

Here,

$$N = \sigma_n^2$$

$$C = \max I(X, Y)$$

To find the value of $H(n)$

$$H(n) = -\int_{-\infty}^{\infty} P(n) \log P(n)$$

$$H(n) = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-n^2/2\sigma_n^2} \log \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-n^2/2\sigma_n^2} dn$$

$$\therefore H(n) = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-n^2}{2\sigma_n^2}} \ln\left(\frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-n^2}{2\sigma_n^2}}\right) dn \log e$$

$$= -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-n^2}{2\sigma_n^2}} \left[\ln\left(\frac{1}{\sqrt{2\pi}\,\sigma_n}\right) + \ln\left(e^{\frac{-n^2}{2\sigma_n^2}}\right)\right]$$

$$\log e \, dn$$

$$H(n) = -\log e \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-n^2}{2\sigma_n^2}} \left[-\ln \sqrt{2\pi}\,\sigma_n - \frac{n^2}{2\sigma_n^2}\right] dn$$

$$H(n) = -\log e \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-n^2}{2\sigma_n^2}} dn}_{1} \left[-\ln \sqrt{2\pi}\,\sigma_n - \frac{n^2}{2\sigma_n^2}\right]$$

$$H(n) = \log e \left[\ln \sqrt{2\pi}\, \sigma_n + \frac{1}{2} \right]$$

$$H(n) = \log e \frac{1}{2} + \log \sqrt{2\pi}\, \sigma_n$$

$$\boxed{\begin{array}{l} \text{Note :-} \\ \log x = \ln x \times \log e \end{array}}$$

$$H(n) = \frac{1}{2} \log e \cancel{\frac{1}{2}} + \frac{1}{2} \log (2\pi \sigma_n^2)$$

$$H(n) = \frac{1}{2} \left[\log 2\pi e \sigma_n^2 \right]$$

$$H(n) = \log_2 \sqrt{2\pi}\, e \sigma_n \text{ bits/symbol}$$

$$C = H(y)_{max} - \log_2 \sqrt{2\pi}\, e \sigma_n$$

Thus, y is a random variable with a specified variance and hence H(y) is maximum ~~with~~ when y as a normal distribution.

Then,

$$H(y)_{max} = \ln \sqrt{2\pi}\, e \sigma_y$$

$$\therefore C = \ln \sqrt{2\pi}\, e \sigma_y - \ln \sqrt{2\pi}\, e \sigma_n$$

$$c = \ln\sqrt{2\pi e \sigma_y^2} - \ln\sqrt{2\pi e \sigma_n^2}$$

$$c = \ln\sqrt{\frac{2\pi e \sigma_y^2}{2\pi e \sigma_n^2}}$$

$$c = \ln\sqrt{\frac{\sigma_y^2}{\sigma_n^2}}$$

$$\boxed{c = \ln\left(\frac{\sigma_y^2}{\sigma_n^2}\right)^{1/2}}$$

$$c = \ln\left(\frac{\sigma_x^2}{\sigma_n^2} + 1\right)^{1/2} dn$$

$$c = \ln\left(1 + \frac{S}{N}\right)^{1/2} \text{ nats/symbol}$$

or,

$$c = \log\left(1 + \frac{S}{N}\right)^{1/2} \text{ bits/symbol}$$

$$c = \frac{1}{\cancel{2}} \log\left(1 + \frac{S}{N}\right) \times \cancel{2} w$$

$$c = w \log\left(1 + \frac{S}{N}\right)$$

Unit - 3

8.8.11

* Information Capacity Theorem

To formulate the information capacity for a band limited powerlimited gaussian channel.

Consider a zero mean stationary proces $x(t)$ i.e. bandlimited to B Hz.

Let x_K where $K = 1, 2, 3, \ldots$

K denotes the continuous random variab obtained by uniform sampling process of $x(t)$ at nyquest rate of 2B samples / second. These samples are transmitted in T sec over a noisy channel also ~~Let~~ X_K band limited to B Hz.

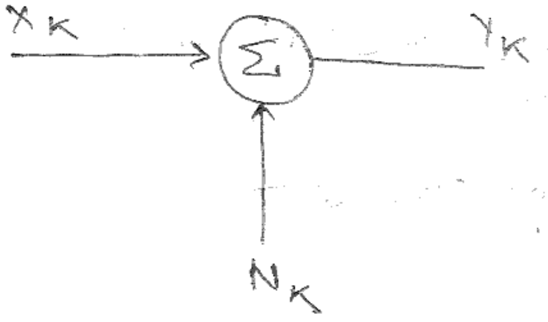
K denotes $K = 2BT$

We refer to X_K as a sample of transmitter signal.

The channel o/p is additive wide gausian noise (AWGN) whose mean is zero and power spectral density is $\frac{N_o}{2}$ also band limited to B Hz.

$$Y_K = X_K + N_K$$

$$E[X_K^2] = P$$



channel capacity $C = \max I(X_K, Y_K)$

$$I(X_K, Y_K) = H(Y) - H(Y/X)$$

$$I(X_K, Y_K) = H(Y_K) - H(Y_K/X_K)$$

$$H\left(\frac{Y_K}{X_K}\right) = H(N_K)$$

Then variance of Y_K of received signal equals to $P + \sigma^2$

Variance of $N_K = \sigma^2$

The differental entropy of a gausian random variable X is

$$H(X) = \frac{1}{2}\log_2(2\pi e\sigma^2)$$

~~Diff ence entropy of $Y_K = P + \sigma^2$~~

$$N_K = \sigma^2$$

$$H(Y_K) = \frac{1}{2}\log_2 2\pi e(P+\sigma^2)$$

$$H(N_K) = \frac{1}{2}\log 2\pi e\sigma^2$$

$$I_{max}(X_K, Y_K) = H(Y_K) - H(N_K)$$

$$= \cancel{\frac{1}{2}\log_2 2\pi e\left[P+\sigma^2-\sigma^2\right]}$$

$$= \cancel{\frac{1}{2}\log_2 2\pi e P}$$

$$= \frac{1}{2}\log_2\left(\frac{P+\sigma^2}{\sigma^2}\right)$$

$$= \frac{1}{2}\log_2\left[1+\frac{P}{\sigma^2}\right]$$

If the channel uses K times for the transmission of K ~~channel~~ samples of the process $x(t)$ in T sec we find that the information capacity per unit time is $\frac{K}{T}$ times of the result.

$$C = B \log_2 \left(\frac{P + \sigma^2}{\sigma^2} \right)$$

$$C = B \log_2 \left(1 + \frac{P}{\sigma^2} \right)$$

$$C = B \log_2 \left(1 + \frac{P}{N_0 B} \right)$$

where

$$\sigma^2 = N_0 B$$

Thus, information capacity of continuous channel of bw B Hz perturbed by additive ~~by~~ wide gausian of power ~~sep~~ spectral density of $N_0/2$ and limited in band width 2B is given by

$$C = B \log_2 \left(1 + \frac{P}{N_0 B} \right) \text{ bits/symbol}$$

10.8.11

★ Implication of information capacity theorm

An ideal system is defined as one that transmit data at a bit rate R_b equal to information capacity C.

The average transmitted power

$$P = E_b C$$

where

E_b = transmitted ~~p~~ energy per bit

For ideal system

$$\frac{C}{B} = \log_2\left(1 + \frac{E_b C}{N_o B}\right)$$

$$1 + \frac{E_b C}{N_o B} = 2^{C/B}$$

$$\frac{E_b C}{N_o B} = 2^{C/B} - 1$$

$$\boxed{\frac{E_b}{N_o} = \frac{(2^{C/B} - 1)}{C/B}}$$

where,

$\frac{E_b}{N_o}$ is the ratio of signal energy per bit to the noise power spectral density.

Again,

$$C = B \log_2\left(1 + \frac{P}{N_o B}\right)$$

$$C = \frac{P}{N_0} \cdot \frac{N_0}{P} B \log_2 \left(1 + \frac{P}{N_0 B}\right)$$

$$C = \frac{P}{N_0} \log_2 \left(1 + \frac{P}{N_0 B}\right)^{N_0 B / P}$$

$$C = \frac{P}{N_0} \log_2 \left(1 + \frac{P}{N_0 B}\right)^{\frac{1}{\frac{P}{N_0 B}}}$$

$$C_\infty = \lim_{B \to \infty} C = \lim_{B \to \infty} \frac{P}{N_0} \log_2 \left(1 + \frac{P}{N_0 B}\right)^{1/P/N_0 B}$$

take

$$x = \frac{P}{N_0 B}$$

$$x \to 0$$

$$C_\infty = \lim_{x \to 0} \frac{P}{N_0} \log_2 (1 + x)^{1/x}$$

Here,

$$\boxed{\lim_{x \to 0} (1+x)^{1/x} = e}$$

$$C_\infty = \frac{P}{N_0} \log_2 e$$

$$C_\infty = \frac{P}{N} \frac{\ln e}{\ln 2}$$

$$C_\infty = 1.44 \frac{P}{N_0}$$

$$\therefore C/_\infty = 1.44 \frac{E_b \, C/_\infty}{N_0}$$

$$\boxed{\frac{E_b}{N_0} = \frac{1}{1.44}}$$

$$\frac{E_b}{N_0} = 0.69$$

$$\left(\frac{E_b}{N_0}\right)_{dB} = 10 \log 0.693$$

$$\boxed{\left(\frac{E_b}{N_0}\right)_{dB} = -1.6\,dB}$$

This value of $\frac{E_b}{N_0}$ is called the Shannon's limit.

For $R_b < C$ error free transmission is possible.

$R_b > C$ error free transmission is not possible.

$R_b = C$ is called capacitive boundary.

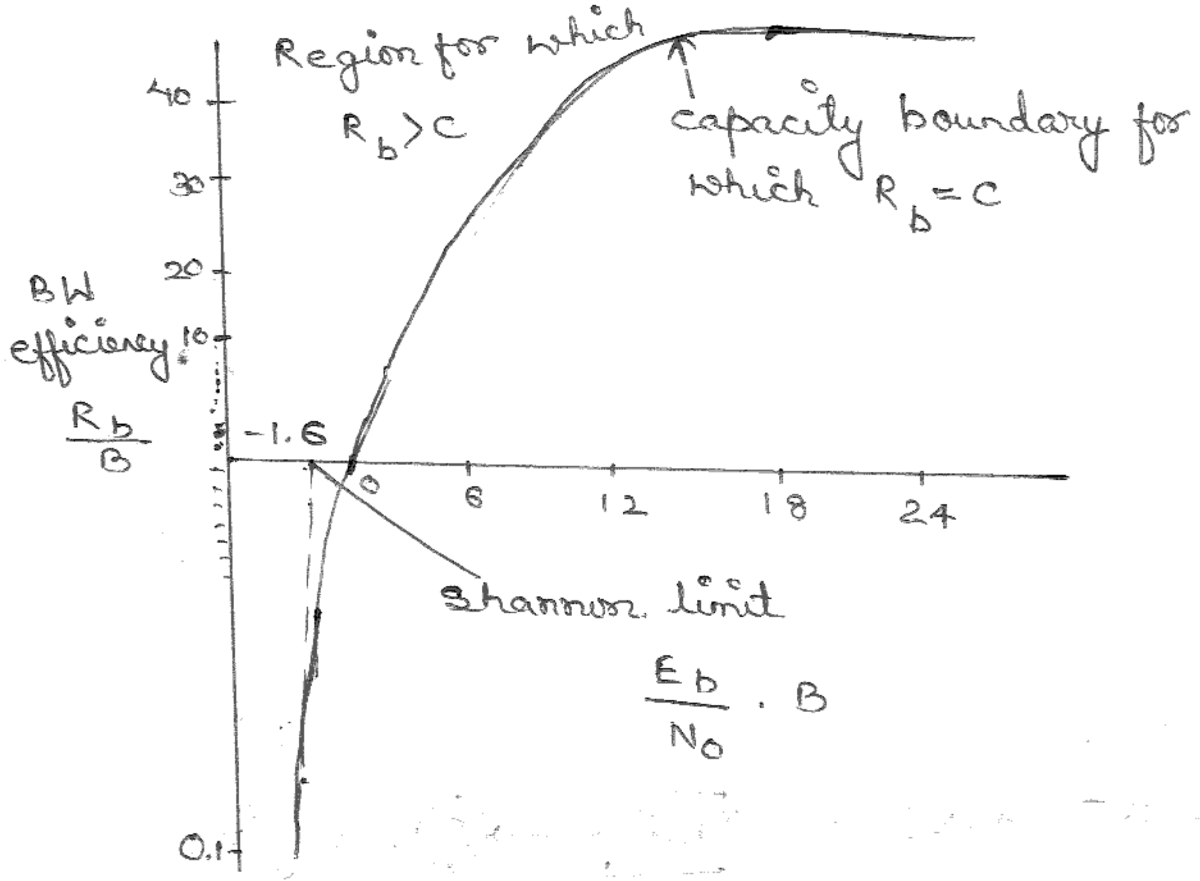
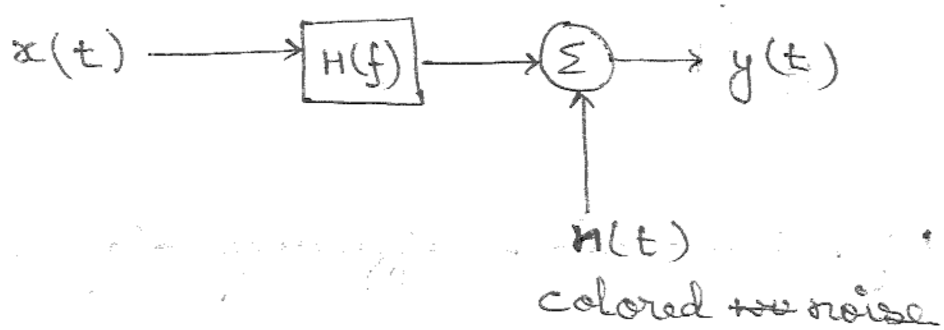


Fig. Band width efficiency diagram

★ Information Capacity Of Colour Noise Channel

Sol-



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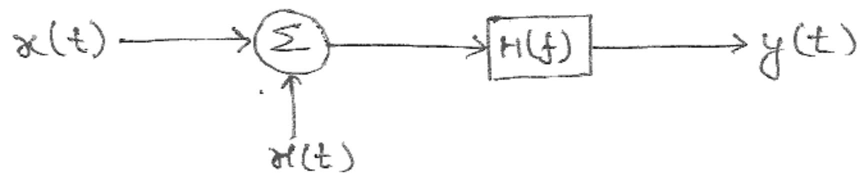


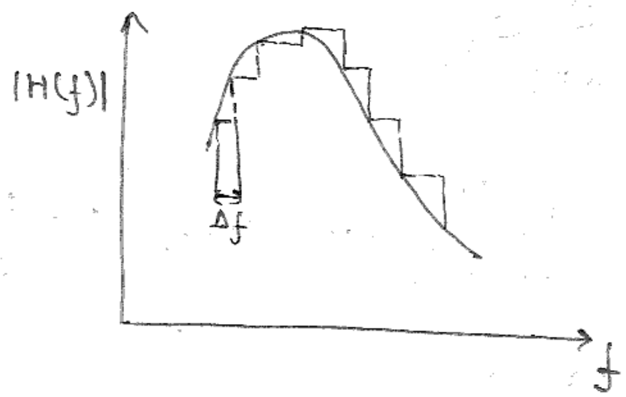
Power spectral density of noise $n'(t)$ i.e.

$$S_N'(f) = \frac{S_N(f)}{|H(f)|^2}$$

where

$|H(f)|$ = mag. response of channel.

Now, the channel is divided into large no. of adjoining frequency slots. The smallel frequency travelled is denoted as Δf.



The original model is replaced by the parallel combination of finite no. of sub channels(N). Each of which is computed essentially by band limited white gaussian channel.

Consider $Y_K(t) = x_K(t) + n_K(t)$

The average power of the signal component $x_K(t)$ is

$$P_K = S_x(f_K)\,\Delta f \quad \text{——— ①}$$

&

$$\sigma_K^2 = \frac{S_\gamma(f_K)}{|H(f_K)|^2} \quad \text{——— ②}$$

$$C = \frac{1}{2}\,\Delta f \,\log_2\left(1 + \frac{P_K}{\sigma_K^2}\right)$$

All the N channels are independent of one another. The overall channel capacity is approx. given by summation.

$$C = \sum_{K=1}^{N} C_K$$

$$\therefore \quad C = \frac{1}{2}\sum_{K=1}^{N} \Delta f \,\log_2\left(1 + \frac{P_K}{\sigma_K^2}\right)$$

As we know

$$\sum_{K=1}^{N} P_K = P = \text{constant}$$

Using Lagrangian multiplier

$$J = \underset{\downarrow}{C} + \lambda\underbrace{\left(P - \sum_{K=1}^{N} P_K\right)}_{\phi(x)=0}$$

$$f(x)$$

$$\therefore \frac{\partial}{\partial x} f(x) + \lambda(\phi(x)) = 0$$

$\therefore J = C + \lambda \left(P - \sum_{K=1}^{N} P_K \right)$

$$J = \frac{1}{2} \sum_{K=1}^{N} \Delta f \log_2 \left[1 + \frac{P_K}{\sigma_K^2} \right] + \lambda \left[P - \sum_{K=1}^{N} P_K \right]$$

diff w r t P_K with P_K and equating to zero.

Note :-

$$\boxed{\log_2 x = \frac{\log_e x}{\log_e 2} \qquad \text{And}, \ \frac{1}{\log_e 2} = \log_2 e}$$

$$J = \frac{1/2 \, \Delta f \log_2 e}{P_K + \sigma_K^2} - \lambda = 0$$

$$\frac{\frac{1}{2} \Delta f \log_2 e}{P_K + \sigma_K^2} - \lambda = 0$$

Condition is

$$P_K + \sigma_K^2 = K \Delta f$$

$$P_K = S_X (f_K) \Delta f$$

$$S_X(f_K)\,\Delta f + \sigma_K^2 = K\Delta f$$

As we know

$$\sigma_K^2 = \frac{S_N(f_K)}{|H(f_K)|^2}\,\Delta f$$

$$S_X(f_K)\,\Delta f + \left[\frac{S_N(f_K)}{|H(f_K)|^2}\,\Delta f\right] = K\Delta f$$

$$S_X(f_K) + \frac{S_N(f_K)}{|H(f_K)|^2} = K$$

$$S_X(f_K) = K - \frac{S_N(f_K)}{|H(f_K)|^2}$$

For this to be +ve

$$K \geqslant \frac{S_N(f_K)}{|H(f_K)|^2}$$

$$\therefore\ S_X(f_K) = \begin{cases} K - \dfrac{S_N(f)}{|H(f)|^2}, & f \in f_A \\ 0, & \text{otherwise} \end{cases}$$

$$P = \int_{f \in f_A} K - \frac{S_N(f)}{|H(f)|^2} \, df$$

$$\therefore \; C = \frac{1}{2} \sum_{K=1}^{N} \Delta f \log_2 \left(1 + \frac{P_K}{\sigma_K^2} \right)$$

$$= \frac{1}{2} \sum_{K=1}^{N} \Delta f \log_2 \left(\frac{K \Delta f}{\sigma_K^2} \right)$$

$$\left[\sigma_K^2 + P_K = K \Delta f \right]$$

$$C = \frac{1}{2} \sum_{K=1}^{N} \Delta f \log_2 \left[\frac{K \cancel{\Delta f} \times |H(f_K)|^2}{S_N(f_K) \cancel{\Delta f}} \right]$$

$$C = \frac{1}{2} \sum_{K=1}^{N} \Delta f \log_2 \left[\frac{K |H(f_K)|^2}{S_N(f_K)} \right]$$

Here, $\Delta f \to 0$

$$C = \frac{1}{2} \int_{-\infty}^{\infty} \log_2 \left[\frac{K |H(f_K)|^2}{S_N(f_K)} \right] df$$

A

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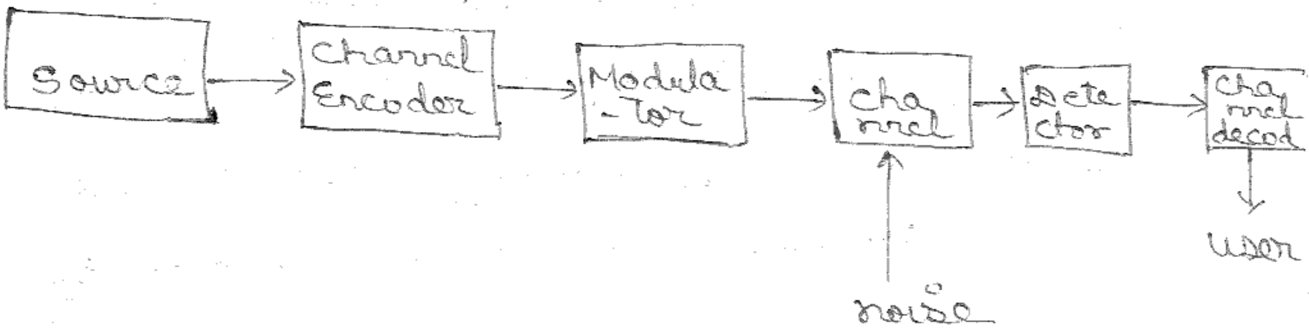
(Ans)

* Rate Distortion Theory

For a discrete memoryless source, the Source coding theorem is introduced where the average code word length must be atleast as large as a Source entropy for perfect coding. In many practical situation, there are constrain that force the coding to be imperfect.
There by resulting in unavoidable distortion.

This problem is refer to as source coding with a fedility criterion. The branch of information theory that deal with it is called rate distortion theory

Unit. 04. Error Control Coding



* Linear Block Codes

A code is said to be linear if any two code words in the code can be added in modulo to arithmetic to produce a 3rd code word in a code.

Consider an (n, K) linear block code in which K bits of the n code bits are always identical to the message sequence. The (n-K) bits are referred to as generalised parity. Block codes in which the message bits are transmitted in unaltered form are called systematic codes.

Let $m_0, m_1, \ldots, m_{k-1}$ constitute a block of k message bits. There are 2^k message bits. There are 2^k different message blocks. Let this sequence of message bits be applied to a linear block encoder producing a n bits code word. whose elements are denoted by $c_0, c_1 \ldots c_{n-1}$.

Let $b_0, b_1, \ldots, b_{n-k-1}$ denotes the $n-k$ parity bits in the code words.

$$c_j = \begin{cases} b_j & J = 0, 1, \ldots n-k-1 \\ m_{j+k-1} & j = n-k, n-k+1, \ldots n-1 \end{cases}$$

where

$$b_j = P_{0j} m_0 + P_{1j} m_1 + \ldots + P_{(k-1)j} m_{k-1}$$

$$P_{ij} = \begin{cases} 1 & b_j \text{ depends on } m_i \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{b_0, b_1, \ldots, b_{n-k-1} \;/\; m_0, m_1, \ldots, m_{k-1}}$$

$$m = m_0, m_1, \ldots, m_{k-1}$$

$$b = b_0, b_1, \ldots, b_{n-k-1}$$

$$c = c_0, c_1, \ldots, c_{n-1}$$

$$\boxed{b = mp}$$

$$p = \begin{bmatrix} p_{00}, & p_{01} & \cdots & p_{0\,n-k-1} \\ p_{10} & & & \\ \vdots & & & \\ p_{k-1} & \cdots & \cdots & p_{k-1,\,n-k-1} \end{bmatrix}$$

$$\cancel{c = [b : m]}$$

$$\cancel{c = [m : b]}$$

$$c = [b : m]$$

$$c = [mp : m]$$

$$c = m[P : I_k]$$

$$[P : I_k] = G$$

Then,

$$c = mG$$

$$c_i + c_j = m_i G + m_j G$$

$$\boxed{c_i + c_j = (m_i + m_j) G}$$

Parity checked matrix

$$H = \left[I_{(n-k)} \; \vdots \; P^T \right]$$

$$H G^T = \left[I_{(n-k)} \; \vdots \; P^T \right] \begin{bmatrix} P^T \\ \hdashline I_K \end{bmatrix}$$

$$= P^T \oplus P^T$$

$$= 0$$

lly

$$G H^T = 0$$

$$H^T c = m G H^T$$

$$H^T c = 0$$

$$m \rightarrow \boxed{G} \rightarrow c \qquad c \rightarrow \boxed{H^T} \rightarrow 0$$