★ Unique Decipherblelity And Instantaneous Code

Symbol	code A	Code B	Code C	Code D
x_1	0	0	0	0
x_2	1	10	01	10
x_3	00	110	011	110
x_4	11	111	0111	1110

Q- Determine whether or not the following code is uniquly decipherable. ~~or not.~~

	S_0	S_1	S_2	S_3	S_4	S_5	S_6
x_1	abc	d	ba	ce	ac	c	eac
x_2	abcd	abd			ab	cd	eab
x_3	e						
x_4	dba						
x_5	bace						
x_6	ceac						
x_7	ceab						
x_8	eabd						

stantaneous

code D

0

10

110

1110

ollowing

~~not.~~

S_5	S_6
c	eac
cd	ea

S_7	S_8	S_9	S_{10}
ac	ba	ce	ac
ab	c	eac	ab
d	cd	eab	d

Here,

$$S_7 = S_{10}$$

No, code is equal to S_0 value. So, it is uniquely decipherable.

x_1	0	ab
x_2	10	bd
x_3	11	ace
x_4	111	abcd

$D = 2$

$w_1 = 1$ $\quad n_1 = 1$

$w_2 = 2$ $\quad n_2 = 2$

$w_3 = 1$ $\quad n_3 = 2$

$n_4 = 3$

$D = 5$

$w_1 = 0$ $\quad n_1 = 2$

$w_2 = 2$ $\quad n_2 = 2$

$w_3 = 1$ $\quad n_3 = 1$

$w_4 = 1$ $\quad n_4 = 1$

5.7.11

Kraft Inequality

The necessary and sufficient condition for existance of an irreducible noiseless encoding procedure with specified word length $\{n_1, n_2 \dots n_N\}$ is a set of +ve integer $\{n_1, n_2, \dots n_N\}$ can be found such that

$$\sum_{i=1}^{N} D^{-n_i} \leq 1$$

$$\Rightarrow W_1 \leq D$$

$$\rightarrow W_2 \leq (D - W_1) D$$

$$\leq D^2 - W_1 D$$

$$\rightarrow W_3 \leq [(D - W_1) D - W_2] D$$

$$\leq D^3 - W_1 D^2 - W_2 D$$

$$\rightarrow W_m \leq D^m - W_1 D^{m-1} - W_2 D^{m-2} \dots \dots \dots W_{m-1} D$$

condition
ceble
with
$\dots r_N\}$
$, r_2, \dots r_N$

$$0 \leq D^m - n_1 D^{m-1} - n_2 D^{m-2} \dots n_{m-1} D - n_m$$

Dividing both side by D^m

$$0 \leq 1 - n_1 D^{-1} - n_2 D^{-2} \dots n_{m-1} D^{1-m} - n_m D^{-m}$$

$$n_1 D^{-1} + n_2 D^{-2} \dots + n_{m-1} D^{1-m} + n_m D^{-m} \leq 1$$

$$\boxed{\sum_{i=1}^{N} n_i D^{-i} \leq 1}$$

Q. $n_1 = 2$, $n_2 = 2$, $n_3 = 3$, $n_4 = 3$, $n_5 = 3$, $n_6 = 4$, $n_7 = 5$. Find W.

Sol- $n_1 = 0$; $n_2 = 2$; $n_3 = 3$; $n_4 = 1$; $n_5 = 1$; $n_6 = 0$; $n_7 = 0$

$$\therefore \sum_{i=1}^{N} n_i D^{-i} \leq 1$$

$m-2$

$$0 + 2D^{-2} + 3D^{-3} + 1D^{-4} + 1D^{-5} + 0 + 0 \leq 1$$

$$\frac{2}{D^2} + \frac{3}{D^3} + \frac{1}{D^4} + \frac{1}{D^5} \leq 1 \quad \text{—} (1)$$

For

$$\sum_{i=1}^{N} D^{-n_i} \leq 1$$

$$\frac{1}{D^2} + \frac{1}{D^2} + \frac{1}{D^3} + \frac{1}{D^3} + \frac{1}{D^4} + \frac{1}{D^5} \leq 1$$

$$\frac{2}{D^2} + \frac{3}{D^3} + \frac{1}{D^4} + \frac{1}{D^5} \leq 1 \quad \text{— (ii)}$$

As (i) and (ii) are equal. So, the theorem is equal.

Q- Code A - 0

1

00

11

$n_1 = 1$; $n_2 = 1$; $n_3 = 2$; $n_4 = 2$

$D = 2$

$$\therefore \sum_{i=1}^{4} D^{-n_i} \leq 1$$

$$2^{-1} + 2^{-1} + 2^{-2} + 2^{-2} \leq 1$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \leq 1$$

$1.5 \leq 1$

Here, it is greater than 1. So, it is not instantaneous and not decipheriable.

$\frac{1}{D^5} \leq 1$

Q- Code B- 0 $\quad n_1 = 1$

10 $\quad n_2 = 2$

— (11)

110 $\quad n_3 = 3$

the

111 $\quad n_4 = 3$

$$\therefore \sum_{i=1}^{4} D^{-n_i} \leq 1$$

$$2^{-1} + 2^{-2} + 2^{-3} + 2^{-3} \leq 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8\cancel{16}} \leq 1$$

$$1 \leq 1$$

$_4 = 2$

Here, they are equal. So, it is instantaneous and decipheriable.

Q- Code C- 0 $\quad n_1 = 1$

01 $\quad n_2 = 2$

011 $\quad n_3 = 3$

0111 $\quad n_4 = 4$

$$\therefore \sum_{i=1}^{4} D^{-n_i} \leq 1$$

$$2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} \leq 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \leq 1$$

$$0.93 \leq 1$$

Here, the code 1 is prefix for 2nd code. So, it is not instantaneous.

Q- Code D -

0	$n_1 = 1$
10	$n_2 = 2$
110	$n_3 = 3$
1110	$n_4 = 4$

$$\therefore \sum_{i=1}^{4} D^{-n_i} \leq 1$$

$$2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} \leq 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \leq 1$$

$$0.93 \leq 1$$

It is instantaneous.

Q- The o/p of a discrete source X =

$$[X] = \{x_1, x_2, x_3, x_4, x_5, x_6\}$$

$$P[X] = \{2^{-1}, 2^{-2}, 2^{-4}, 2^{-4}, 2^{-4}, 2^{-4}\}$$

Its encoder is in the following six ways.

	C_1	C_2	C_3	C_4	C_5	C_6
x_1	0	1	0	111	1	0
x_2	10	011	10	110	01	01
x_3	110	010	110	101	0011	011
x_4	1110	001	1110	100	0010	0111
x_5	1011	000	11110	011	0001	01111
x_6	1101	110	111110	010	0000	011111

a) Determine which of the codes are uniquely decipherable.

b) Determine which have the prefix property

c) Find the average length of each uniquely decipherable code.

d) ~~Thy~~ Thus any of the above code give minimum avg length.

Sol a)

	C_1	S_1	S_2
x_1	0	11	0
x_2	10	1	10
x_3	110		01
x_4	1110		110
x_5	1011		011
x_6	1101		101

Since, S_1 contains 0 So, it is not uniquely decipherable.

	C_2	S_2	S_3	S_3	S_3	S_4
x_1	$\boxed{1}$	10	11	0	11	0
x_2	011	$\cancel{0}$	10		10	0
x_3	010		01		01	1
x_4	001		00		00	$\boxed{1}$
x_5	000				10	0
x_6	110					

$S_2 = S_3 = 10$ Same code. So not uniquely decipherable.

	C_3	C_4	C_5
x_1	0		
x_2	10		
x_3	110		
x_4	1110		
x_5	11110		
x_6	111110		

uniquely decipherable

So, C_3, C_4 and C_5 are uniquely decipherable and C_6 also.

b) C_6 have the prefix property.

c)

	C_3	Length
x_1	0	1
x_2	10	2
x_3	110	3
x_4	1110	4
x_5	11110	5
x_6	111110	6

$$\bar{L} = P_1L_1 + P_2L_2 + P_3L_3 + P_4L_4 + P_5L_5 + P_6L_6$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{16} + \frac{4}{16} + \frac{5}{16} + \frac{6}{16}$$

$$= 2.125$$

$$\bar{L}_4 = P_1L_1 + P_2L_2 + P_3L_3 + P_4L_4 + P_5L_5 + P_6L_6$$

$$= \frac{3}{2} + \frac{3}{4} + \frac{3}{16} + \frac{3}{16} + \frac{3}{16} + \frac{3}{16}$$

$$= 3$$

$$\bar{L}_5 = \frac{1}{2^2} + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2}$$

$$= \cancel{4}\ 1.75$$

$$\bar{L}_6 = \{1, 2, 3, 4, 5, 6\}$$

$$= 2.125$$