25/06/11. State & prove kraft inequality

KRAFT - Mc-Millan Inequality :

- The condition for the existence of the cord that satisfies the prefix condition are given by kraft- inequality.

The necessary and sufficient condition for the existence of binary code which is uniquely ~~desiphera~~ decodable with code length $l_0, l_1, \ldots l_{k-1}$, satisfy the condition,

$$\boxed{\sum_{k=0}^{K-1} r^{-l_k} \leq 1}$$

where

r is radix (no. of symbols) of the alphabets of encoded symbols.

proof:

$$\sum_{k=0}^{K-1} \left(r^{-l_k}\right)^n = \left(r^{-l_0} + r^{-l_1} + \ldots + r^{-l_{k-1}}\right)^n$$

where, $n \to$ positive integer

when RHS is expanded, we have a sum of 'K^n' terms, each of which is in the form,

$$r^{-l_{k_1} - l_{k_2} - \ldots - l_{k_n}} = r^{-[l_{k_1} + l_{k_2} + l_{k_n} = i]}$$

$$\cancel{r^{-l_{k_1} - l_{k_2} - \ldots - l_{k_n}} = r^{-i}}$$

$$r^{-l_{k_1} - l_{k_2} - \ldots - l_{k_n}} = r^{-i}$$

Let the smallest of code lengths
$l_K \to$ unity, largest is l.

Let, $N_i \to$ No. of terms of the form r^{-i}

$$\therefore \left(\sum_{K=0}^{K-1} r^{-l_K}\right)^n = \sum_{i=n}^{nl} N_i r^{-i}$$

Integer i may take on a value of extending from n to nl.

$N_i \to$ Also no. of code symbols of length i.
Hence, if uniquely decodable, N_i cannot be greater than r^i, which is the no. of distinct sequences of length i in an alphabet of radix r.

$$\sum_{K=0}^{K-1} r^{-l_K} \leq \sum_{i=n}^{nl} r^i r^{-i} = nl - n + 1$$

$$\left[\because \sum_{n}^{nl} 1 = nl - n + 1\right]$$

$n \to 0$
$1 \to nl$

Accordingly we have,

$$\left(\sum_{K=0}^{K-1} r^{-l_K}\right)^n \leq nl_0 \quad \forall n$$

Taking n^{th} roots

$$\sum_{k=0}^{K-1} r^{-l_k} = (n\alpha)^{1/n} \quad , \forall n$$

we may choose n as large, go to limit, $n \to \infty$.

$$\lim_{n\to\infty} (n\alpha)^{1/n} = 1$$

$$\boxed{\sum_{k=0}^{K-1} r^{-l_k} \leq 1}$$

//.

i.

Entropy in Continuous Case

1D Random variable,

$$H(X) = -\int_{-\infty}^{\infty} f(x) \log f(x)\, dx$$

$$H(X,Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log f(x,y)\, dx\, dy$$

$$H(X/Y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2(x,y) \log \frac{f(x,y)}{f_2(y)}\, dx\, dy$$

$f(x,y)$ = joint density

$f_1(x)$ and $f_2(y) \to$ Marginal density.

$$H(Y/X) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \log \frac{f(x,y)}{f_1(x)}\, dx\, dy$$

For n-dimensional random variable,

$$H(X_1, X_2 \ldots X_n) = -\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} f(x_1, x_2, \ldots x_n) \cdot \log f(x_1, x_2, \ldots x_n)\, dx_1 \ldots dx_n$$

Approximate continuous density $W_x(x)$ by the relationship.

$$P(x_i) \simeq W_x(X_i)\, \Delta x_i$$

Where, $P(x_i)$ is approximately probability that the continuous random variable x with density $W_x(X)$ lies in the interval Δx_i, which includes x_i

~~$H(x) = \sum_i$~~

$$H(x) = -\sum_i P(x_i) \log P(x_i)$$

$$H(x) = \lim_{\Delta x \to 0} \left[-\sum W_x(X_i)\, \Delta X_i \log \left(W_x(X_i)\, \Delta X_i \right) \right]$$

$$= -\lim_{\Delta X \to 0} \sum W_x(X_i)\, \Delta X_i \log W_x(X_i) - \lim_{\Delta X \to 0} \sum_i W_x(X_i)\, \Delta X_i \log \Delta$$

First term of the expression may be considered as limiting form of integral

$$\int_{-\infty}^{\infty} W_x(x) \log W_x(x)\, dx$$

while the second term tends to infinity. Thus, entropy of continuous distribution $W_x(X)$ might be defined with the discrete

Case as,

$$\boxed{H(X) = \int_{-\infty}^{\infty} w_X(x) \log w_X(x)\, dx.}$$

//

29/06/11.

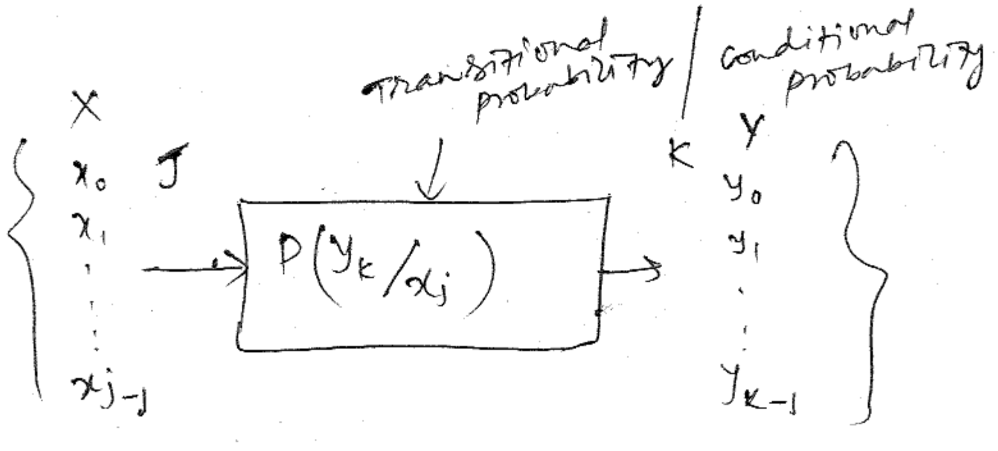
UNIT - II

Noise Coding

1) Discrete Memoryless Channel.

- A discrete memoryless channel is a statistical model that the i/p 'x' and o/p 'y' i.e. noisy portion of X_i. Every unit of time, the channel accepts an input symbol and in response emits o/p symbol.

→ A channel is said to be discrete when code i/p and o/p alphabets have finite size. It is said to be memoryless when the current o/p symbol depends only on current i/p symbol and not on previous one.



→ The channel in terms of i/p alphabet

$$X = \{x_0, x_1, \ldots x_{j-1}\}$$

and o/p alphabet as

$$Y = \{y_0, y_1, \ldots y_{k-1}\}$$

set of transition probabilities.

$$P\left(y_k/x_j\right) = P\left(Y = y_k / X = x_j\right) \text{ for all } j \text{ and } K.$$

$$\boxed{0 \leq P\left(y_k/x_j\right) \leq 1}, \ \forall \ j \,\&\, K$$

I/p alphabet and o/p alphabet need not ~~haif~~ have same size.

Eg. In channel coding, the size K of o/p alphabets may be larger than size 'j' of i/p alphabet.

Thus, $\boxed{K \geq J}$

→ $P(y_k/x_j)$ is a transition probability defined as conditional probability that the channel output $Y = y_k$ given that channel i/p $X = x_j$. when information transmit over the channel, there is probability of errors during transmition.

when $K = J$, $P(y_k/x_j)$ represents the conditional prob.% of correct reception.

when $K \neq J$, it represents the conditional probability of error.

Channel Matrix

$$P = \begin{array}{c} \\ x_0 \\ x_1 \\ \vdots \\ x_{j-1} \end{array} \begin{array}{c} \begin{array}{cccc} y_0 & y_1 & \cdots & y_{K-1} \end{array} \\ \left[\begin{array}{cccc} p(y_0/x_0) & p(y_1/x_0) & \cdots & p(y_{K-1}/x_0) \\ p(y_0/x_1) & p(y_1/x_1) & \cdots & p(y_{K-1}/x_1) \\ & & & \\ & & & \end{array} \right] \end{array}$$

Property:

$$\sum_{K=0}^{K-1} p(y_K/x_j) = 1, \ \forall j$$

sum of elements along any row of matrix is always 1.

Joint probability distribution,

$$p(x_j, y_K) = p(X = x_j, Y = y_K)$$

$$= p(Y = y_K / X = x_j)$$

$$= p(y_K/x_j) \cdot p(x_j)$$

Marginal probability distribution of o/p random variable is obtained by averaging out the dependence of $p(x_j, y_K)$ on x_j,

$$p(y_K) = p(Y = y_K)$$

$$\Rightarrow \sum_{j=0}^{J-1} p(Y = y_K / X = x_j) \, p(X = x_j)$$

For $J=K$, the average prob'l. of symbol error P_e is defined as prob'l. that o/p random variable y_K is different from i/p random variable x_j, averaged over all $K \neq j$

$$P_e = \sum_{\substack{K=0 \\ K\neq j}}^{K-1} P(Y = y_K)$$

$$= \sum_{j=0}^{J-1} \sum_{\substack{K=0, \\ K\neq j}}^{K-1} P(y_K/x_j)\, P(x_j)$$

The difference,

$\boxed{1 - P_e}$ → Average probability of correct reception.

The probability $P(x_j)$ is known as ~~priory~~ priori probabilities of various i/p symbols.

27/06/11.

(Discrete memoryless ~~ch~~ channel)

Channel Capacity of DMC

Channel Capacity has the maximum average mutual information in any single use of channel, where the maximization is overall possible i/p probability distribution.

The mutual information depends not only on the channel but also, on the way in which the channel is used.

channel capacity is denoted by 'C'

$$\boxed{C = \text{Max } I(X;Y)}$$

'C' is measured in bits/channel.

Types ~~Heights~~ of channel and its capacity

i) lossless Channel.

channel Capacity is obtained when source entropy is maximum.

$$\boxed{C = \text{Max } H(X) = \log M}$$

[∵ size of M - input L - o/p.]

- A channel is lossless if,

$\boxed{H(X/Y) = 0}$ for all i/p distributions.

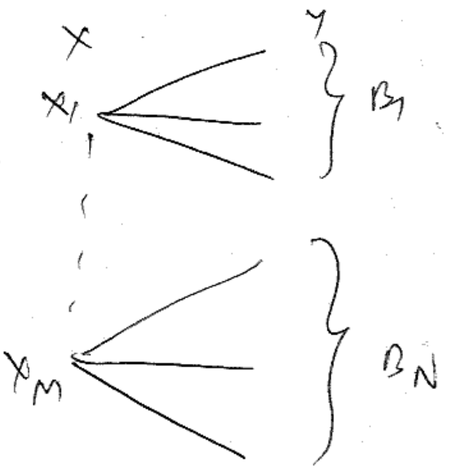
~~$I(X,Y) = H(X) -$~~ $H(Y/X)$

$$I(X,Y) = H(Y) - H(Y/X)$$

or $H(X) - H(X/Y)$

The values of Y must be partition into disjoint set $B_1, B_2, \ldots, B_N$ such that

$$P\{y \in B_i / x = x_i\} = 1$$



ii) Deterministic Channel:

Destination entropy is maximum.

$$\boxed{C = \text{Max } H(Y) = \log N}$$

The channel is deterministic if

$P(y_j/x_i) = 1$ or 0 for all j. ie, If y is determined by x or equivalently ~~Hx~~

$H(Y/X) = 0$ $\forall$ input distributions.

'x' alphabets of size 'M' may be partioned into 'N' disjoint set. Each set is uniquely associated with one and only member of destination alphabet y.

iii) Noiseless channel

- The channel is noiseless if it is so loseless and also deterministic.

$$\boxed{I(X,Y) = H(X) = H(Y)}$$

$$C = \text{Max } I(X,Y)$$

$$= \text{Max } H(X,Y) = \text{Max } H(Y)$$

$$\boxed{C \Rightarrow \log M = \log N}$$

$$\begin{array}{ccc} X & & Y \\ X_1 & \longrightarrow & Y_1 \\ X_2 & \longrightarrow & Y_2 \\ X_3 & \longrightarrow & Y_3 \\ \vdots & & \\ X_M & \longrightarrow & Y_N \end{array}$$

Each set A and B has only one member. the source & ~~dsta~~ destination alphabets are of same size.

$$\boxed{M = N}$$

iv) Useless Channel

A channel is useless or zero capacity channel if $I(X,Y) = 0$ for all i/p distributions.

$\therefore \boxed{C = 0}$

It can be characterized by the condition,

$H(X) = H(X/Y)$

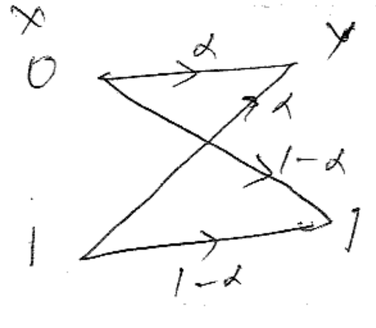
$I(X,Y) = H(X) - H(X/Y)$

$\boxed{I(X,Y) = 0}$

For all $P(x_i)$ or ~~also~~ alternatively x and y are independent for all $P(x_i)$.

Since, independent of x and y means that $P(y_j/x_i) = P(y_j) \ \forall \ i,j$.

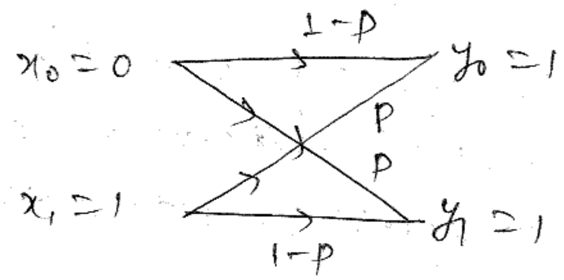
A channel is useless if and only if channel matrix has identical rows. It completely scrambles (destroys) the i/p messages.



(v) Binary Symmetric Channel

→ It is DMC with ~~J=K~~ $J = K = 2$.
The channel has two i/p symbols.
$x_0 = 0$, $x_1 = 1$ and two o/p symbols
$y_0 = 0$, $y_1 = 1$.

→ A channel is symmetric because a probability of receiving '1' if 0 is sent is same as the prob'y. of receiving 0 if 1 is sent. This conditional probability of error is denoted by p.



p - error
$1 - p$ = correct reception.

$p(x_0) = 1/2$

$p(x_1) = 1/2$

$p(y_0/x_0) = 1 - p$

$p(y_1/x_1) = 1 - p$

$p(y_0/x_1) = p = p(x_0/y_1)$

$C = 1 + p\log p + (1-p)\log_2(1-p)$

Entropy,

$$H(X) = -\left[p\log_2 p + (1-p)\log_2(1-p)\right]$$

$$\boxed{C = 1 - H(X)}$$

07/03/2011.

Case-I:

When a channel is noise free without the error, set $p=0$, C attains maxm value of 1 bit per channel use, which is exactly the information in each channel input. At this value of p, the entropy function attains minimum value of zero.

$$C = 1 + p\log p + (1-p)\log(1-p)$$

$$\boxed{C = 1} \quad [\because p=0,\ \log 1 = 0]$$

$\therefore$ channel capacity is maxm when $p=0$.

Case-II:

When a channel is noise producing conditional probability of error $p = 1/2$, C attains minimum value of zero whereas entropy function attains maxm value of unity.

$$C = 1 + \frac{1}{2}\log\frac{1}{2} + \left(1-\frac{1}{2}\right)\log\left(1-\frac{1}{2}\right)$$

$$= 1 + \frac{1}{2}\log\frac{1}{2} + \left(+\frac{1}{2}\right)\log\left(+\frac{1}{2}\right)$$

$$= 1 + 2\times\frac{1}{2}\log\frac{1}{2}$$

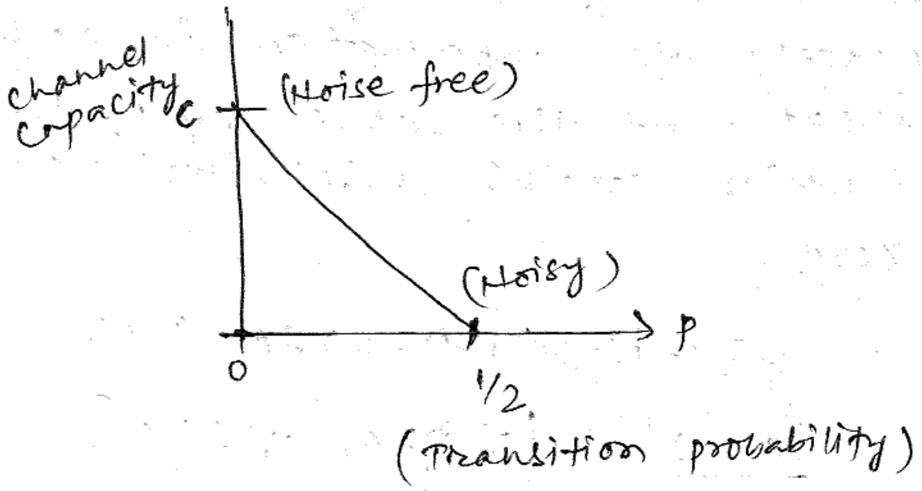
$$= 1 + \log 2^{-1}$$

$$= 1 - 1 \qquad [\log_2 2^{-1} = -1]$$

$$= 0$$

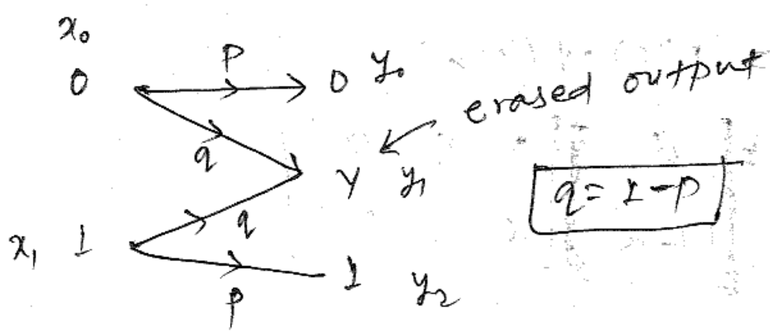
$$\therefore \boxed{C = 0}$$

Graphically Representation :



Binary Erasure Channel (BEC) :

consider two input symbols {0 & 1}, and three output symbols {0, y, 1}.



→ p is the probability of correct reception.
q is error.
Y is <u>output erased</u> and no deterministic can be made as wheather the transmitter letter was 1 or 0.

consider i/p probability as,

$$\left.\begin{array}{l} P(0) = \alpha \\ P(1) = 1-\alpha \end{array}\right\} \begin{array}{l} P(X) \\ \text{input probability.} \end{array}$$

$$P(Y/X) = \begin{array}{c c} & \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \end{array} & \begin{bmatrix} p & q & 0 \\ 0 & q & p \end{bmatrix} \end{array}$$

$C = \text{Max } I(X,Y)$ ← mutual information

$= \text{Max}\left[H(X) - H(X/Y)\right]$

or

$P(Y/X) = \text{Max}\left[H(Y) - H(Y/X)\right]$

$$P(X,Y) = P(X)\cdot P(Y/X)$$

$$= \begin{bmatrix} \alpha \\ (1-\alpha) \end{bmatrix} \begin{bmatrix} p & q & 0 \\ 0 & q & p \end{bmatrix}$$

$$\begin{array}{cccc} & y_0 & y_1 & y_2 \\ = & \alpha p & \alpha q & 0 \\ & 0 & q(1-\alpha) & p(1-\alpha) \end{array}$$

$P(Y)$ = Adding all the columns:

$P(Y_0) = \alpha p$

$P(Y_1) = \alpha q + (1-\cancel{\alpha})q = q$

$P(Y_2) = (1-\alpha)p$

$$P(X/Y) = \frac{P(X,Y)}{P(Y)}$$

$$\begin{array}{cccc} & P(y_0) & P(Y_1) & P(Y_2) \\ = & \alpha p & \alpha q & 0 \\ & 0 & (1-\alpha)q & (1-\alpha)p \\ \hline & \alpha p & q & (1-\alpha)p \end{array}$$

$$P(X/Y) = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1-\alpha & 1 \end{bmatrix}$$

$$H(X/Y) = -\sum_{i=1}^{M}\sum_{j=1}^{N} P(x_i, y_j) \log P(x/y)$$

$$= -\left[\alpha p \log 1 + \alpha q \log \alpha + q(1-\alpha)\log(1-\alpha) + p(1-\alpha)\log 1\right]$$

$$= -\left[\alpha q \log \alpha + q(1-\alpha)\log(1-\alpha)\right]$$

$$[\because \log 1 = 0]$$

$$= -q\left[\alpha \log \alpha + (1-\alpha)\log(1-\alpha)\right]$$

$$H(X/y) = -q\left[\alpha \log \alpha + (1-\alpha)\log(1-\alpha)\right]$$

Entropy of i/p $\{\alpha, 1-\alpha\}$

$$H(X) = -\left[\alpha \log \alpha + (1-\alpha)\log(1-\alpha)\right]$$

$$\therefore H(X/y) = -q H(X)$$

$$\{I(X,Y)\} = H(X) - H(X/Y)$$

$$= H(X) - q H(X)$$

Mutual information

$$= H(X)(1-q)$$

$$= p H(X) \quad \left[\because p+q=1, \; p = 1-q\right]$$

$$\therefore \boxed{C = \text{Max } I(X/Y)}$$

i.e. $\boxed{C = p}$

05/07/11.

Channel Capacity of unsymmetric Channel

Random variables

$$\boxed{C = \log_2\left[2^{Q_1} + 2^{Q_2} + 2^{Q_3} + \cdots\right]}$$

Q) Determine the Capacity of unsymmetric channel given as

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 2/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\log_2 = \frac{\log x}{\log 2}$

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 2/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 1/3\log_2 1/3 + 2/3\log_2 2/3 \\ 2/3\log 2/3 + 1/3\log_2 1/3 \\ \log 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 Q_1 + 2/3 Q_2 \\ 2/3 Q_1 + 1/3 Q_2 \\ Q_3 \end{bmatrix} = \frac{1}{0.3}\begin{bmatrix} -0.15 - 0.917 \\ -0.17 - 0.15 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2076/0.3 \\ 0.2076/0.3 \\ 0. \end{bmatrix}$$

$\log_2 1/3 \Rightarrow \dfrac{\log_{10} 1/3}{\log_{10} 2}$

=

$$\tfrac{1}{3}\theta_1 + \tfrac{2}{3}\theta_2 = -0.276 \times 3 \;]= 0.828$$

$$\tfrac{2}{3}\theta_1 + \tfrac{1}{3}\theta_2 = -0.276$$

$$\theta_1 + 2\theta_2 = -0.828$$

$$\theta_1 + \tfrac{1}{2}\theta_2 = -0.414$$

$$- \quad\quad - \quad\quad\quad +$$

$$2\theta_2 - \tfrac{1}{2}\theta_2 = -0.414$$

$$\frac{3\theta_2}{2} = -0.414$$

lly $\boxed{\begin{matrix}\theta_2 = 0.276 \\ \theta_1 = 0.276\end{matrix}}$

$$\theta_3 = 0$$

$$C = \log_2 \left[2^{0.276} + 2^{0.276} + 1\right]$$

$$= \log_2 (1.219 + 1.219 + 1)$$

$$= \log_2 (3.42)$$

$$= \frac{\log (3.42)}{\log_{10} 2}$$

$$\boxed{C = 1.7 \text{ bits/symbol.}}$$ //

[$\therefore$ 1.322 bits/symbol]

2)

$$\begin{bmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/4 & 0 & 1/4 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/4 & 0 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1/2 \log 1/2 + 1/4 \log 1/4 + 1/4 \log 1/4 \\ 1 \log 1 \\ 1 \log 1 \\ 1/4 \log 1/4 + 1/4 \log 1/4 + 1/2 \log 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2\theta_1 + 1/4\theta_2 + 1/4\theta_4 \\ \theta_2 \\ \theta_3 \\ 1/4\theta_1 + 1/4\theta_3 + 1/2\theta_4 \end{bmatrix} = \begin{bmatrix} -0.15 - 0.3 \\ 0 \\ 0 \\ -0.3 - 0.15 \end{bmatrix}$$

$$\begin{pmatrix} 1/2\theta_1 + 1/4\theta_2 + 1/4\theta_4 = -0.45 \\ \theta_2 = 0 \\ \theta_3 = 0 \\ 1/4\theta_1 \cancel{\theta_3} + 1/4\theta_3 + 1/2\theta_4 = -0.45 \end{pmatrix}$$

bits/symbol]

$1/2\, \cancel{\theta_1} + \frac{1}{4}\theta_4 = -0.45$

$2\times\frac{1}{4}\cancel{\theta_1} + \frac{1}{2}\times 2\,\theta_4 = -0.45 \times 2$

$\cancel{\frac{1}{4}\theta_1 + \frac{1}{8}\theta_4} =$

$\frac{1}{4}\theta_4 - \theta_4 = \cancel{0.90}\ 0.45$

$\theta_4 - 4\theta_4 = \cancel{3.6}\ 0.45$

$-\frac{3\theta_4}{4} = \cancel{3.6}\ 0.45$

$\cancel{\theta_4 = \frac{-1.2\times 4}{3}}$

$\theta_4 = \frac{-0.45\times 4}{3}$

$\boxed{\theta_4 = 0.6}$

$1/2\,\theta_1 + 1/4(0.6) = -0.45$.

$1/2\,\theta_1 = -0.45 + 0.15$

$1/2\,\theta_1 = -0.3$

$\boxed{\theta_1 = 0.6}$

$C = \log_2\left[2^{0.6} + 2^0 + 2^0 + 2^{0.6}\right]$

$= \log_2\left[1 + 1 + 1.52 + 1.52\right]$

$= \frac{\log_{10}(5.817)}{\log_{10}2} =$

$\log 1/4 + 1/4 \log 1/4$

$\log 1/4 + 1/2 \log 1/2$

Q13) Find the channel capacity for the

$$\begin{bmatrix} 1/3 & 1/3 & 1/6 & 1/6 \\ 1/3 & 1/6 & 1/6 & 1/3 \end{bmatrix}$$

4 variables

$\boxed{C = \log m - h} \leftarrow H(Y/X)$

$$P(Y/X) = \begin{array}{c} x_1 \\ x_2 \end{array} \begin{bmatrix} 1/3 & 1/3 & 1/6 & 1/6 \\ 1/3 & 1/6 & 1/6 & 1/3 \end{bmatrix}$$

$H(Y/X) = p(y) - p(y/x)$

$H(Y/X) = \sum\sum p(x,y) \log p(y/x) \quad h \to H(Y/x)$

$p(X,Y) = p(X)\, p(Y/X)$

$P(X) = \{1/2, 1/2\}$

$$p(X,Y) = \begin{array}{c} P(x_1) \\ P(x_2) \end{array} \begin{bmatrix} 1/3 & 1/3 & 1/6 & 1/6 \\ 1/3 & 1/6 & 1/6 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/6 & 1/6 & 1/12 & 1/12 \\ \frac{1}{6} & 1/12 & 1/12 & 1/6 \end{bmatrix}$$

$$H(Y/X) = -\frac{1}{0.3}\Big[\frac{1}{6}\log 1/3 + \frac{1}{6}\log 1/3 + \frac{1}{12}\log 1/6 + \frac{1}{12}\log\frac{1}{6} + \frac{1}{6}\log 1/3 + \frac{1}{12}\log 1/6 + \frac{1}{12}\log 1/6 + \frac{1}{6}\log 1/3\Big]$$

$$= -\frac{1}{0.3}\big[-0.079 - 0.079 - 0.0648 - 0.0648 - 0.079 - 0.0648 - 0.0648 - 0.079\big]$$

$$= \frac{0.575}{0.3}$$

$$\boxed{H(Y/X) = 1.917 = h}$$

$$\therefore C = \log m - h$$
$$= \log 4 - 1.917$$
$$=$$

8)

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

[C = 0.49 bits/symbol]

Being Unsymmetric channel,

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 1/2 \log 1/2 + 1/2 \log 1/2 \\ 1/4 \log 1/4 + 3/4 \log 3/4 \end{bmatrix}$$

$1/2\, Q_1 + 1/2\, Q_2 = -0.3$

$1/4\, Q_2 + 3/4\, Q_2 = -0.2437 \;]\times 2$

$-\quad - \quad +$

$1/2\, Q_2 - 3/2\, Q_2 = 0.1874$

$-Q_2 = 0.1874$

$Q_2 = -0.1874$

$\boxed{Q_2 = 0.1874}$ //

$1/2\, Q_1 + 1/2(-0.1874) = -0.3$

$1/2\, Q_1 = -0.3 + 0.0937$

$Q_1 = -0.2063 \times 2$

$Q_1 = -0.4126$

$\boxed{Q_1 = 0.4126}$ //

$C = \log_2 \left[2^{0.4126} + 2^{0.1874}\right] = \log_2 (1.33 + 1.13)$

$= \dfrac{\log_{10}(2.4687)}{\log_{10} 2} = \dfrac{0.38}{0.3} = 1.20$ bits/symbol.

05/07/17.

Decoding scheme:

To achieve realiability accuracy should be more consider input alphabet of a channel.

$x_1, x_2 ------ x_n$

$y_1, y_2, ----- y_N$

Channel matrix $p(y_j/x_i)$

Decoder or decision scheme is an assignment to every o/p symbol y_j of an input symbol x_i^*. from the alphabet ~~x_i~~ $x_1, x_2, --- x_n$.

If y_j