Putting for P_{e1} from equation 5.13.32

$$P_e = 2\times\frac{1}{2}erfc\sqrt{\frac{E_b}{2N_0}}$$

$$= erfc\sqrt{\frac{E_b}{2N_0}}$$

Thus,

$$\textbf{Error Probability of QPSK, } P_e = erfc\sqrt{\frac{E_b}{2N_0}} \quad \text{... (5.13.33)}$$

Here E_b is the energy of one bit. We know that $E_b = \frac{1}{2}A^2T_b$. Similarly symbol energy is given as,

$$E_s = \frac{1}{2}A^2T_s$$

Since $T_s = 2T_b$,

$$E_s = \frac{1}{2}A^2.2T_b = 2\left[\frac{1}{2}A^2T_b\right] = 2E_b$$

or

$$E_b = \frac{E_s}{2}$$

Hence equation 5.13.33 becomes,

$$P_e = erfc\sqrt{\frac{E_s}{4N_0}}$$

Above equation gives symbol error probability.

Example 5.13.7 : *Binary data are transmitted over a microwave link at the rate of 1 Mbps and the PSD of the noise at the receiver input is 10^{-10} W/Hz. For each of the following pairs, determine which one requires more power than the other. Determine the extra average signal power required by the more power consuming scheme so that an average probability of error of 10^{-4} is always maintained.* **Nov./Dec.-2008, 16 Marks**

i) Coherent PSK and DPSK.

ii) Coherent PSK and QPSK.

iii) Coherent FSK and non-coherent FSK.

iv) Coherent FSK and coherent MSK.

Solution : Here $f_b = 1 \text{ Mbps} = 1\times 10^6 \text{ bps}$

$$\therefore \quad T_b = \frac{1}{f_b} = \frac{1}{1\times10^6} = 1\times10^{-6} \text{ sec}$$

$$\frac{N_0}{2} = 10^{-10} \text{ W/Hz}, \quad \text{hence } N_0 = 2\times10^{-10}$$

$$P_e = 10^{-4}$$

i) Coherent PSK and DPSK

For PSK, $$P_e = \frac{1}{2} erfc\sqrt{\frac{E}{N_0}}$$

$$\therefore \quad 10^{-4} = \frac{1}{2} erfc\sqrt{\frac{E}{N_0}}$$

$$\therefore \quad 2\times10^{-4} = erfc\sqrt{\frac{E}{N_0}}$$

since $$erfc(x) = 1-erf(x)$$

$$1-2\times10^{-4} = erf\sqrt{\frac{E}{N_0}}$$

or $$0.9998 = erf\sqrt{\frac{E}{N_0}}$$

From appendix G, table G - 1, $erf(2.5) = 0.99959$ and $erf(3.0) = 0.99998$. Hence reasonably we can consider $erf(2.8) \approx 0.9998$.

i.e. $$erf(2.8) = erf\sqrt{\frac{E}{N_0}}$$

or $$\sqrt{\frac{E}{N_0}} = 2.8 \Rightarrow \frac{E}{N_o} = 2.8^2 = 7.84$$

$$\therefore \quad E = 7.84\times N_0 = 7.84\times2\times10^{-10} = 1.568\times10^{-9} \text{ J}$$

Since $$E = PT_b,$$

$$P = \frac{E}{T_b} = \frac{1.568\times10^{-9}}{1\times10^{-6}} = 1.568 \text{ mW}$$

For DPSK, $$P_e = \frac{1}{2}e^{-E/N_0}$$

$$\therefore \quad 10^{-4} = \frac{1}{2}e^{-E/N_0}$$

$$\therefore \quad 2\times10^{-4} = e^{-E/N_0}$$

$$\text{or} \quad -\frac{E}{N_0} = ln(2\times10^{-4})$$

$$\therefore \quad \frac{E}{N_0} = 8.5172$$

$$\therefore \quad E = 8.5172\times N_0 = 8.5172\times2\times10^{-10} = 1.703\times10^{-9}\text{ J}$$

$$\therefore \quad P = \frac{E}{T_b} = \frac{1.703\times10^{-9}}{1\times10^{-6}} = 1.703\text{ mW}$$

ii) Coherent PSK and QPSK

In part (i) we have obtained that for coherent PSK,

$$P = 1.568\text{ mW.}$$

For QPSK, $$P_e = erfc\sqrt{\frac{E_b}{2N_0}}$$

$$\therefore \quad 10^{-4} = erfc\sqrt{\frac{E_b}{2N_0}}$$

$$\text{or} \quad 1-10^{-4} = erf\sqrt{\frac{E_b}{2N_0}}$$

$$\text{i.e.} \quad 0.9999 = erf\sqrt{\frac{E_b}{2N_0}}$$

From Table G - 1 (appendix - G), approximately, $erf(2.9) = 0.9999$.

$$\therefore \quad \sqrt{\frac{E_b}{2N_o}} = 2.9 \Rightarrow \frac{E_b}{2N_0} = 2.9^2 = 8.41$$

$$\therefore \quad E_b = 8.41\times2\times N_0 = 8.41\times2\times2\times10^{-10} = 3.364\times10^{-9}\text{ J}$$

$$\therefore \quad P = \frac{E_b}{T_b} = \frac{3.364\times10^{-9}}{1\times10^{-6}} = 3.364\text{ mW}$$

iii) Coherent FSK and non-coherent FSK

For coherent FSK,

$$P_e = \frac{1}{2}erfc\sqrt{\frac{0.6E}{N_0}}$$

$$10^{-4} = \frac{1}{2} erfc\sqrt{\frac{0.6E}{N_0}}$$

$$\therefore \quad 2\times10^{-4} = erfc\sqrt{\frac{0.6E}{N_0}}$$

$$\therefore \quad 1-2\times10^{-4} = erf\sqrt{\frac{0.6E}{N_0}}$$

i.e. $$0.9998 = erf\sqrt{\frac{0.6E}{N_0}}$$

Approximately $erf(2.8) = 0.9998$

$$\sqrt{\frac{0.6E}{N_0}} = 2.8 \Rightarrow \frac{0.6E}{N_0} = 2.8^2 = 7.84$$

$$\therefore \quad E = \frac{7.84\times N_0}{0.6} = \frac{7.84\times2\times10^{-10}}{0.6} = 2.613\times10^{-9} \text{ J}$$

$$\therefore \quad P = \frac{E}{T_b} = \frac{2.613\times10^{-9}}{1\times10^{-6}} = 2.613 \text{ mW}$$

For non-coherent FSK,

$$P_e = \frac{1}{2}e^{-\frac{E}{2N_0}}$$

$$10^{-4} = \frac{1}{2}e^{-\frac{E}{2N_0}}$$

$$\therefore \quad 2\times10^{-4} = e^{-\frac{E}{2N_0}}$$

$$\therefore \quad \frac{E}{2N_0} = 8.5172$$

$$\therefore \quad E = 8.5172\times2\times N_0$$

$$= 8.5172\times2\times2\times10^{-10}$$

$$= 3.407\times10^{-9}$$

$$\therefore \quad P = \frac{E}{T_b} = \frac{3.407\times10^{-9}}{1\times10^{-6}} = 3.407 \text{ mW}$$

iv) Coherent FSK and coherent MSK

In part (iii) we have obtained that for coherent FSK.

$$P = 2.613 \text{ mW}$$

For MSK, $$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

This is similar to coherent binary PSK. Hence from part (i), $P = 1.568$ mW.

Results

The results are given in the following Table.

Sr.No.	Modulation schemes	Power required	Difference in powers
(i)	Coherent PSK	1.568 mW	0.135 mW
	DPSK	1.703 mW	
(ii)	Coherent PSK	1.568 mW	1.796 mW
	QPSK	3.364 mW	
(iii)	Coherent FSK	2.613 mW	0.794 mW
	Non-coherent FSK	3.407 mW	
(iv)	Coherent FSK	2.613 mW	1.045 mW
	Coherent MSK	1.568 mW	

Table 5.13.1 Comparison between powers required by various schemes

Example 5.13.8 : *A communication system operates at a bit rate of 10 kbps with a transmitter power of 50 watts with a BER of 10^{-7} using BPSK (uncoded). If a channel coder of coding gain 3 dB is incorporated into the system, calculate the transmitter power required to maintain the same BER. (Assume the modulation scheme remains the same). State the disadvantages of including the channel coder, if there is any.*

April/May-2008, 4 Marks

Solution : For BPSK, the error probability with uncoding is given as,

$$P_{e(uncoding)} = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}} \quad \ldots(5.13.34)$$

When the signal is coded by rate $\frac{k}{n}$ convolutional coder, the error probability with coding will be

$$P_{e(coding)} = \frac{1}{2} erfc \sqrt{\frac{\frac{k}{n} E_b}{N_0}} \quad \ldots(5.13.35)$$

Thus energy with coding is $\frac{k}{n}$ times of uncoding energy. Here $\frac{k}{n}$ can be expressed in dB and it becomes coding gain. Thus,

Coding gain in dB = $10 \log_{10} \frac{k}{n}$

The coding gain is given as 3 dB. It should be taken as – 3 dB. Since $\frac{k}{n}$ in less than 1 i.e.

$$-3 \text{ dB} = 10 \log_{10} \frac{k}{n} \Rightarrow \frac{k}{n} = 0.5$$

Putting for $\frac{k}{n}$ in equation (5.13.34),

$$P_{e(coding)} = \frac{1}{2} erfc \sqrt{\frac{0.5 E_b}{N_0}}$$

The bit error rates (BER) given by above equation and equation (5.13.34) will be equal only if E_b is doubled in above equation. This means bit energy with coding will be,

$$E_{b(coding)} = 2\, E_{b(uncoding)}$$

$$P_{(coding)} T_b = 2 P_{(uncoding)} T_b, \quad \text{Here bit rate remains same, and } E = PT_b$$

$$\therefore \quad P_{(coding)} = 2 \times 50 \text{ W}, \quad \text{Since } P_{(uncoding)} = 50 \text{ W}$$

$$= 100 \text{ W}$$

Disadvantage : By inclusion of channel coder, output bit rate is increased. To maintain same BER, it is necessary to increase the transmitted power.

Unsolved Examples

1. *An FSK system transmits binary data at the rate of* 2.5×10^6 *bits per second. During the course of transmission, the white noise of zero mean and power spectral density* 10^{-20} *watts/Hz is added to the signal. In absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1 microvolt. Determine the average probability of symbol error for current binary FSK system.* **[Ans : $P_e = \frac{1}{2} erfc \sqrt{6}$]**

5.14 Symbol Error Performance of M-ary Systems

In the previous section we studied the bit error probabilities of binary systems. Now let us consider the error probabilities of M-ary systems for M>2. Such error probabilities are represented in terms of symbol energies, since more than one bit represent symbol. For example N=2 bits represent $M = 2^N = 2^2 = 4$ distinct symbol. Probabilities of M-ary systems can be represented with the help of union bound approximation. It is given as,

$$P_e \le \frac{1}{2} erfc \sqrt{\frac{d_k^2}{4N_0}} \qquad \text{... (5.14.1)}$$

Observe that RHS of above equation puts higher bound on error probability 'd_k' is the distance between nearest signal points.

5.14.1 Probability of Symbol Error for M-ary PSK

Consider the signal space diagram of M-ary PSK, shown in Fig. 5.5.1 and Fig. 5.5.3. The distance between nearest two symbols is,

$$d_{12} = d_{18} = 2\sqrt{E_s} \sin \frac{\pi}{M}$$

Consider that symbol s_1 is transmitted. Refer the signal space diagram shown in Fig. 5.5.3 observe that due to noise, either s_2 or s_8 will be detected. Hence considering only two nearest symbols in equation 5.14.1,

$$P_e \le \frac{1}{2} erfc \sqrt{\frac{d_{12}^2}{4N_0}} + \frac{1}{2} erfc \sqrt{\frac{d_{18}^2}{4N_0}}$$

$$\le erfc \sqrt{\frac{d_{12}^2}{4N_0}} \qquad \text{Since } d_{12} = d_{18} = \text{same for all symbols}$$

$$\le erfc \left[\frac{4E_s \sin^2 \frac{\pi}{M}}{4N_0} \right]^{\frac{1}{2}}$$

$$\le erfc \left(\sqrt{\frac{E_s}{N_0}} \sin \frac{\pi}{M} \right) \qquad \text{... (5.14.2)}$$

This is the probability of symbol error for M-ary PSK. Here $E_s = \sqrt{P_s T_s}$ is the symbol energy.

5.14.2 Probability of Symbol Error for M-ary FSK

The signal space diagram for M = 3 is shown in Fig. 5.5.1. Observe that all the points have same distance from each other. The distance between any two signal points is $d_k = \sqrt{2E_s}$. Hence one symbol can be detected as any of the rest of (M–1) symbols. Hence we have to take summation of (M–1) symbols in equation 5.14.1. ie.,

$$P_e \le \frac{1}{2} erfc \sqrt{\frac{2E_s}{4N_0}} + \frac{1}{2} erfc \sqrt{\frac{2E_s}{4N_0}} + \ldots (M-1) \; times$$

$$\le (M-1) \frac{1}{2} erfc \sqrt{\frac{2E_s}{4N_0}}$$

i.e. $$P_e \leq \frac{M-1}{2} erfc\sqrt{\frac{E_s}{2N_0}} \qquad \text{... (5.14.3)}$$

This is an expression for symbol error probability of coherently detected M-ary FSK system. For noncoherently detected M-ary FSK, the symbol error probability is given as,

$$P_e \leq \frac{M-1}{2} e^{-\frac{E_s}{2N_0}} \qquad \text{... (5.14.4)}$$

5.14.3 Bit Error Probability Versus Symbol Error Probability

For binary transmission, one bit forms one symbol. Hence bit error probability and symbol error probability are same for binary transmission. For M-ary transmission they are related to each other. This relationship can be understood with the help of QPSK. In QPSK, two bits represent four symbols. Let one symbol be transmitted, Then there can be error in first bit, it will be detected as second symbol. There can be error in second bit, then it will be detected as third symbol. And fourth symbol will be due to error in both the bits. The probability of bit error is then weighted average of probabilities of detecting 2^{nd}, 3^{rd} and 4^{th} symbols.

i.e. $$P_e(bit) = \frac{\frac{1}{2}P(1^{st}\text{ bit error}) + \frac{1}{2}P(2^{nd}\text{ bit error}) + \frac{2}{2}P(\text{both bits error})}{3}$$

In above observe that P(both bits error) is multiplied by $\frac{2}{2}$, since there are two bits in error out of available two bits. Note that,

P (1^{st} bit error) = P (2^{nd} bit error) = P (both bits error) = Symbol error probability P_e

Hence

$$P_e(bit) = \frac{\frac{1}{2}P_e + \frac{1}{2}P_e + \frac{2}{2}P_e}{3}$$

or $$P_e(bit) = \frac{2}{3}P_e \qquad \text{Here } P_e \text{ is symbol error probability ... (5.14.5)}$$

This relation can be generalized as,

$$P_e(bit) = \frac{M/2}{M-1} P_e(symbol) \qquad \text{... (5.14.6)}$$

Here M = 4 in equation 5.14.5.

When number of symbols are large, then above equation becomes,

$$P_e(bit) = \frac{1}{2}P_e(symbol) \qquad \text{... (5.14.7)}$$

5.14.4 Comparison of Digital Modulation Systems using a Single Carrier

April/May-2004

We studied the bit error rates (BER) of various schemes. Following table lists the bit error rates :

Sr. No.	Name of the scheme	Bit error rate (P_e)
1.	Binary ASK	$\frac{1}{2}\,erfc\sqrt{\frac{E}{4N_0}}$
2.	Binary coherent PSK	$\frac{1}{2}\,erfc\sqrt{\frac{E}{N_0}}$
3.	Binary coherent FSK	$\frac{1}{2}\,erfc\sqrt{\frac{0.6\,E}{4N_0}}$
4.	Binary non coherent FSK	$\frac{1}{2}\,e^{-\frac{E}{2N_0}}$
5.	Binary DPSK	$\frac{1}{2}\,e^{-\frac{E_b}{N_0}}$
6.	QPSK	$erfc\sqrt{\frac{E_b}{2N_0}}$

Table 5.14.1 Bit error rates of digital modulation systems

Fig. 5.14.1 shows the plot of BER as a function of $\frac{E_b}{N_0}$.

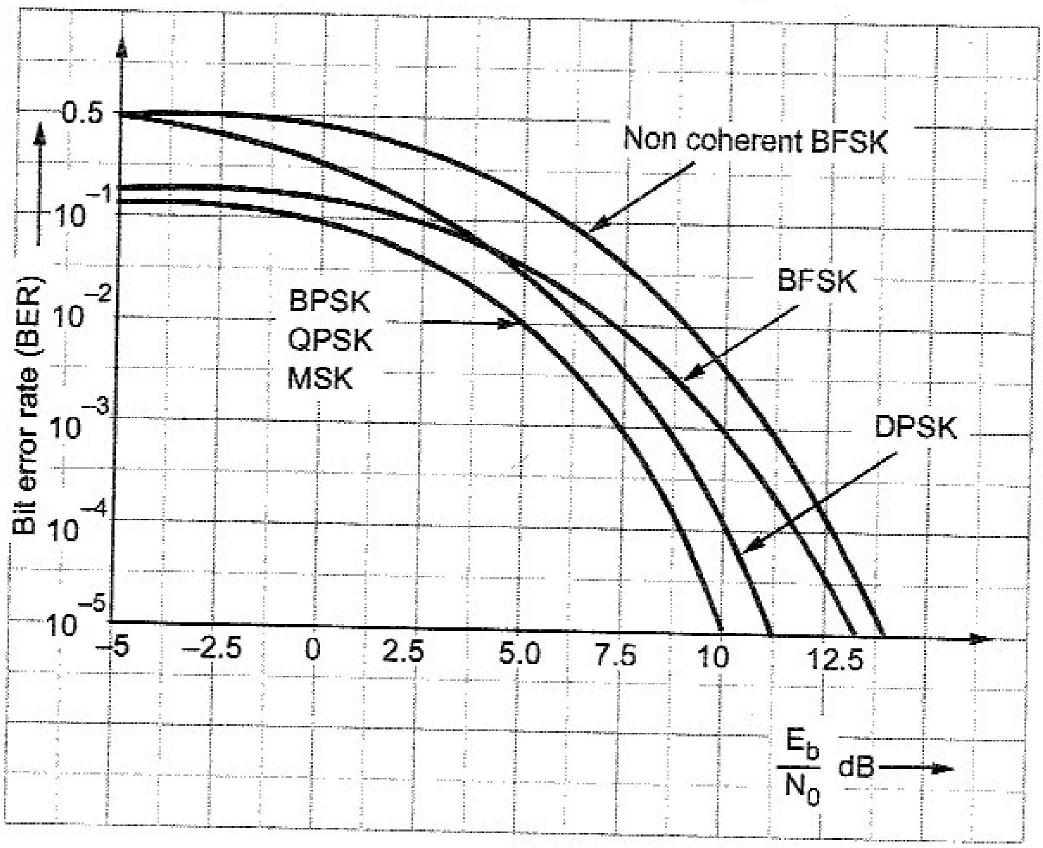

Fig. 5.14.1 Comparison of ***BER Vs*** $\frac{E_b}{N_0}$

Conclusions from above plot

1. The bit error rates for all the system decrease monotonically with increase in $\frac{E_b}{N_0}$.
2. For the given value of $\frac{E_b}{N_0}$, BER of BPSK, QPSK and MSK is smaller than other schemes.
3. For the given BER, the $\frac{E_b}{N_0}$ of BPSK is 3 dB less compared to that of BFSK.
4. For higher values of $\frac{E_b}{N_0}$, the performance of noncoherent BFSK is similar to that of BPSK with 1 dB difference.

Review Questions

1. *Explain the error performance of M-ary systems.*
2. *Compare error performance of M-ary FSK and PSK.*

5.15 Short Answered Questions

Q.1 *Mention the need of optimum transmitting and receiving filter in baseband data transmission.* **Madras Univ., April 97, Nov 97, 2-Marks**

Ans. : When binary data is transmitted over the baseband channel, noise interferes with it. Because of this noise interference, errors are introduced in signal detection. Optimum filter performs two functions while receiving the noisy signal :

i) Optimum filter integrates the signal during the bit interval and checks the output at the time instant where signal to noise ratio is maximum.

ii) Transfer function of the optimum filter is selected so as to maximize signal to noise ratio.

iii) Optimum filter minimizes the probability of error.

Q.2 *Define ASK.* **Madras Univ., April-97, 98, 2-Marks**

Ans. : In ASK, carrier is switched on when binary '1' is to be transmitted and it is switched off when binary 'D' is to be transmitted ASK is also called on-off keying.

Q.3 *What is meant by DPSK ?* **Madras Univ., April-98, 2 Marks**

Ans. : In DPSK, the input sequence is modified. Let input sequence be $d(t)$ and output sequence be $b(t)$. Fig. 5.3.1 shows how $b(t)$ is obtained. Sequence $b(t)$ changes level at the beginning of each interval in which $d(t) = 1$ and it does not changes level when $d(t) = 0$.

When $b(t)$ changes level, phase of the carrier is changed. And as stated above, $b(t)$ changes its level only when $d(t) = 1$. This means phase of the carrier is changed only if $d(t) = 1$. Hence the technique is called Differential PSK.

Q.4 *Explain coherent detection ?* **Madras Univ., Nov-97, 2 Marks; April/May-2004, 2 Marks**

Ans. : In coherent detecton, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. The detection is done by correlating received noisy signal and locally generated carrier. The coherent detection is a synchronous detection.

Q.5 *What is the difference between PSK and FSK ?* **Madras Univ., April-97, 2-Marks**

Ans. : In PSK, *phase* of the carrier is switched according to input bit sequence. In FSK, *frequency* of the carrier is switch according to input bit sequence. FSK needs double of the bandwidth of PSK.

Q.6 *What is meant by coherent ASK ?* **Madras Univ., Oct 98, 2 Marks**

Ans. : In coherent ASK, correlation receiver is used to detect the signal. Locally generated carrier is correlated with incomming ASK signal. The locally generated carrier is in exact phase with the transmitted carrier. Coherent ASK is also called synchronous ASK.

Q.7 *What is the major advantage of coherent PSK over coherent ASK ?*

Madras Univ., Oct-98, 2 Marks

Ans. : ASK is on-off signaling, where as the modulated carrier is continuously transmitted in PSK. Hence peak power requirement is more in ASK, whereas it is reduced in case of PSK.

Q.8 *Explain the model of bandpass digital data transmission system ?*

Ans. : The bandpass digital data transmission system consists of source, encoder and modulator in the transmitter. Similarly receiver, decoder and destination forms the transmitter.

Q.9 *Which digital modulation technique gives better error probability ?*

Ans. : Binary PSK gives reduced error probability compared to ASK and FSK. It is given as

$$P_e = \frac{1}{2} erfc \sqrt{\frac{E}{N_0}}$$

Q.10 *In minimum shift keying what is the relation between the signal frequencies and bit rate?* **April/May-2004, 2 Marks**

Ans. : Let the bit rate be f_b and frequency of carrier be f_0. Then higher and lower MSK signal frequencies are given as,

$$f_H = f_0 + \frac{f_b}{4}$$

and

$$f_L = f_0 - \frac{f_b}{4}$$

Q.11 *What do you understand by coherent detection?* **April/May-2004, 2 Marks**

Ans. : Refer answer of Q.4.

Q.12 *Write the expression for bit error rate for coherent binary FSK.* **Nov./Dec.-2004, 2 Marks**

Ans. : The bit error rate of coherent binary FSK is given as,

$$P_e = \frac{1}{2} erfc\sqrt{\frac{0.6E}{N_0}}$$

Q.13 *Bring out the difference between coherent and noncoherent binary modulation schemes.* **April/May-2005, Nov./Dec.-2009, 2 Marks**

Ans. : Refer section 5.1.2.

Q.14 *What is the error probability of MSK and DPSK?* **April/May-2005, 2 Marks**

Ans. :

Error probability of MSK : $$P_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

Error probability of DPSK : $$P_e = \frac{1}{2} e^{-\frac{E_b}{N_0}}$$

Q.15 *Highlight the major difference between a QPSK signal and a MSK signal.* **Nov./Dec.-2005, 2 Marks**

Ans. : MSK signal have continuous phase in all the cases, whereas QPSK signal has abrupt phase shift of $\frac{\pi}{2}$ or π.

Q.16 *Compare the probability of error of PSK with that of FSK.* **May/June-2006, 2 Marks**

Ans. : BPSK : $p_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$

BFSK : $P_e = \frac{1}{2} erfc\sqrt{\frac{0.6E_b}{4N_0}}$

- For the fixed value of $\frac{E_b}{N_0}$, error probability of BPSK is less than BFSK.

- For the given probability of error, the $\frac{E_b}{N_0}$ of BPSK is 3 dB less compared to that of BFSK.

Q.17 *Sate the difference between coherent and noncoherent binary modulation techniques.* **May/June-2006, 2 Marks**

Ans. : Refer section 5.1.2.

Q.18 *Compare the probability of error of PSK with that of FSK.* **Nov./Dec.-2006, 2 Marks**

Ans. :

Sr. No.	Error probability of PSK	Error probability of FSK
1.	$P_e = \frac{1}{2} erfc \sqrt{\frac{E}{N_0}}$	$P_e = \frac{1}{2} erfc \sqrt{\frac{0.6E}{N_0}}$
2.	For same $\frac{E}{N_0}$, error probability is less.	For same $\frac{E}{N_0}$, error probability is high.

Q.19 *Differentiate coherent and non coherent receivers.* **May/June-2007, 2 Marks**

Ans. : Refer answer of Q.13.

Q.20 *What do you understand by continuous phase frequency shift keying ?* **May/June-2007, 2 Marks**

Ans. : CPFSK : In FSK, when the phase change is gradual at the bit transition times, the signal appears to be continuous in phase. This is called continuous phase FSK or CPFSK. To have phase continuity, the two FSK frequencies f_H and f_L must differ by a bit rate of f_b or $\frac{1}{T_b}$.

Q.21 *Draw the waveform for the binary data sequence 101100 modulated under*

a) FSK b) PSK. **Nov./Dec.-2007, 2 Marks**

Ans. : Fig. 5.15.1 shows the FSK and PSK waveforms.

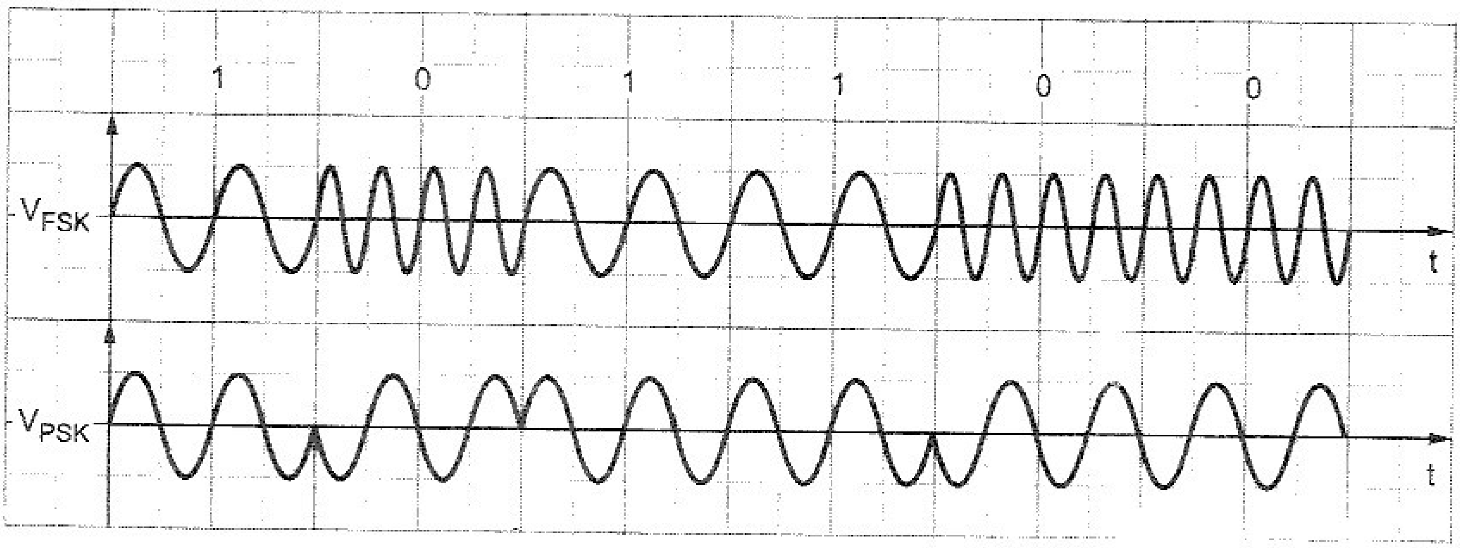

Fig. 5.15.1 FSK and PSK waveforms

Q.22 *What are the advantages and disadvantages of differential phase shift keying ?*

Nov./Dec.-2007, 2 Marks

Ans. : Refer section 5.3.3.

Q.23 *Plot the power spectrum of a BPSK signal operated with a carrier frequency of 140 MHz, modulated by data bits at a rate of 2400 bits/sec. What is the bandwidth requirement ?*

April/May-2008, 2 Marks

Ans. : Here f_0 = 140 MHz, f_b = 2400 bits/sec

Fig. 5.15.2 shows the power spectrum.

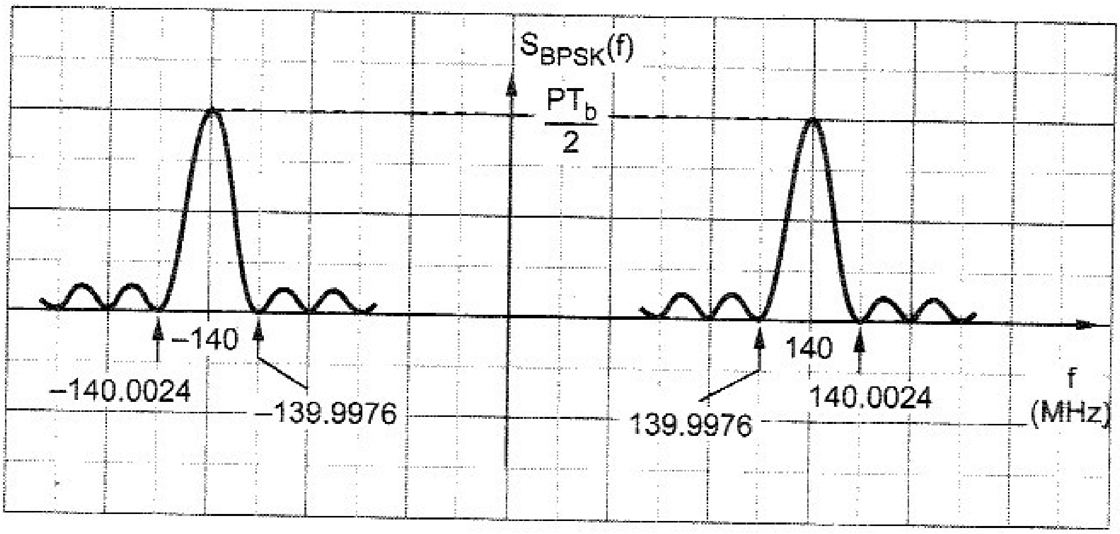

Fig. 5.15.2 Power spectrum of BPSK

Bandwidth is given as,

$$BW = 2 f_b$$

$$= 2 \times 2400 = 4.8 \text{ kHz}$$

Q.24 *Give the signal space representation of QPSK. How is the performance of the system related to the distance between the symbols in the signal space ?*

April/May-2008, 2 Marks

Ans. : Refer Fig. 3.4.6. It shows the signal space representation. The error probability of the system is given as,

$$P_e \leq \sum_{k=2}^{M} \frac{1}{2} erfc \sqrt{\frac{d_{k1}^2}{4N_0}}$$

Here 'M' are the number of signal points, and d_{k1} is the distance between s_1 and s_k in the signal space.

Q.25 *Compare : Coherent and Non-coherent detection.* **Nov./Dec.-2008, 2 Marks**

Ans. : Refer answer of Q.13.

Q.26 *What is signal constellation diagram ?* **Nov./Dec.-2008, 2 Marks**

Ans. : The signal constellation diagram is similar to the phasor diagram but the entire phasor is not drawn. The signal constellation diagram shows only relative positions of the peaks of the phasors. The signal constellation diagram is also called state space diagram.

Q.27 *Draw the functional model of pass band data transmission.* **May/June-2009, 2 Marks**

Ans. : Refer Fig. 5.1.2.

Q.28 *Distinguish between coherent and noncoherent receivers.* **May/June-2009, 2 Marks**

Ans. : Refer answer of Q.13.

Q.29 *Define QPSK.* **Nov./Dec.-2009, 2 Marks**

Ans. :

- In QPSK two successive bits in the data sequence are grouped together. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the phase of the carrier is changed by $45°\left(\text{or } \frac{\pi}{4}\right)$.
- Became of combination of two bits there will be four symbols. Hence the phase shift will be $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$.
- QPSK reduces amplitude variations and required transmission bandwidth.

Q.30 *Differentiate baseband transmission from passband transmission.*

April/May-2010, 2 Marks

Ans. :

Sr. No.	Baseband transmission	Passband transmission
1.	Signal is transmitted without any modulation of high frequency carrier.	The signal modulates high frequency carrier.
2.	Used for short distance transmission.	Used for long distance transmission.
3.	Used for LANs, printers, short distance links.	Used for transmission of digital video, data and speech.

Q.31 *Define MSK.* **April/May-2010, 2 Marks**

Ans. :

- MSK uses quadrature carriers which are orthogonal and difference between them is minimum.

- There are no abrupt changes in phase of MSK signal and it appears to be continuous.

Q.32 *Define QAM and draw its constellation diagram.* **Nov./Dec.-2010, 2 Marks**

Ans. : QAM : The phase as well as amplitude of the quadrature carriers is modulated. Hence it is called Quadrature Amplitude Phase shift keying or simply QAM.

Refer Fig. 5.6.1 for constellation diagram of QAM.

Q.33 *A binary frequency shift keying system employs two signaling frequencies f_1 and f_2. The lower frequency f_1 is 1200 Hz and signaling rate is 500 baud. Calculate f_2.*

Nov./Dec.-2010, 2 Marks

Ans. : For binary FSK,

$$\text{Baud} = f_b$$

$$\therefore \quad f_b = 500 \text{ Hz}$$

Considering the FM modulation index (h) of '1' in FSK,

$$\frac{|f_m - f_s|}{f_b} = h = 1 \qquad \text{(Here } h = 1\text{)}$$

$$|f_m - f_s| = f_b$$

Since $f_s = f_1 = 1200$ Hz,

$$f_m - 1200 \text{ Hz} = 500 \text{ Hz}$$

$$f_m = 1700 \text{ Hz}$$

Thus $f_2 = f_m = 1700$ Hz

University Questions with Answers

Q.1 *Draw the block diagram of QPSK transmitter and coherent QPSK receiver and explain their operation.* **April/May-2004, 10 Marks; May/June-2006, 8 Marks**

Ans. : Refer sections 5.4.1.1, 5.4.1.3.

Q.2 *Compare the BER of coherent PSK, coherent QPSK and coherent FSK.*

April/May-2004, 6 Marks

Ans. : Refer section 5.14.4.

Q.3 *Draw the block diagram of MSK transmitter and explain the function of each block.*

Nov./Dec.-2004, 8 Marks

Ans. : Refer section 5.9.4.1.

Q.4 *Explain how MSK signal is obtained from CPFSK signal*

Nov./Dec.-2004, 8 Marks

Ans. : Refer section 5.9.

Q.5 *With necessary equations and signal space diagram, obtain the probability of error for coherent binary FSK systems.* **April/May-2005, 12 Marks**

Ans. : Refer section 5.13.3.

Q.6 *Draw the block diagram of a QPSK receiver and explain its working.*

April/May-2005, 4 Marks

Ans. : Refer section 5.4.1.3.

Q.7 *Discuss briefly about minimum shift keying for a CPFSK signal.*

April/May-2005; May/June-2006, 8 Marks

Ans. : Refer section 5.9.

Q.8 *Explain BPSK signal transmission and coherent BPSK reception with suitable diagrams. Derive an expression for the probability of symbol error for the scheme.*

Nov./Dec.-2005, 16 Marks; Nov./Dec.-2010, 8 Marks

Ans. : Refer section 5.2.3.1 for BPSK transmission, section 5.2.3.2 for BPSK reception and section 5.13.2 for symbol error probability of BPSK.

Q.9 *With necessary equations and signal space diagram, explain briefly about FSK system.*

May/June-2006, 8 Marks

Ans. : Refer section 5.7.

Q.10 *Obtain probability of error in terms of E_b / N_0 for QPSK.* **May/June-2006, 8 Marks**

Ans. : Refer section 5.13.5.

Q.11 *Describe the binary PSK scheme in detail with a neat diagram.*

Nov./Dec.-2006, 8 Marks

Ans. : Refer sections 5.2.1 and 5.2.3.

Q.12 *With necessary equations and signal space diagram, obtain the error probability for MSK systems.* **Nov./Dec.-2006, 12 Marks**

Ans. : Signal space diagram of MSK : Refer section 5.9.1.

Error probability for MSK : By union bound approximation,

$$P(e / s_1) \leq \sum_{k=2}^{M} \frac{1}{2} erfc \sqrt{\frac{d_{k1}^2}{4N_0}}$$

From signal space diagram of MSK, observe that, there are two equidistant points from any signal point and the distance is $d = 2\sqrt{E_b}$. Hence above equation can be written as,

$$P(e / s_1) = \frac{1}{2} erfc \sqrt{\frac{d_{21}^2}{4N_0}} + \frac{1}{2} erfc \sqrt{\frac{d_{41}}{4N_0}} \quad \text{... (1)}$$

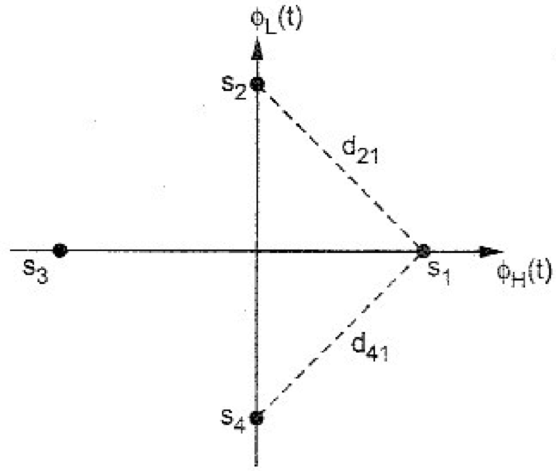

Fig. 1 Signal space diagram of MSK

Here d_{21} is distance between signal points s_1 and s_2. Similarly d_{41} is distance between signal points s_4 and s_1. Here s_2 and s_4 are the two signal points nearest to s_1 as shown in Fig. 1.

From this Fig. 1 observe that

$d_{21} = d_{41} = 2\sqrt{E_b}$.

Hence equation (1) can be expressed as,

$$P(e / s_1) = 2 \times \frac{1}{2} erfc \sqrt{\frac{4E_b}{4N_0}} = erfc \sqrt{\frac{E_b}{N_0}}$$

This is true for all the symbols, hence error probability for MSK is,

$$P_e = erfc \sqrt{\frac{E_b}{N_0}}$$

Q.13 *Explain the principle, operation, signals, constellation diagram, transmitter and receiver of a MSK system.* **May/June-2007, Nov./Dec.-2009, 16 Marks**

Ans. : Refer section 5.9.

Q.14 *Draw the block diagram of MSK transmitter and receiver and discuss in detail with required waveforms.* **Nov./Dec.-2007, 10 Marks**

Ans. : MSK transmitter : Refer section 5.9.4.1.

MSK receiver : Refer section 5.9.4.2.

MSK waveforms : Refer section 5.9 and Fig. 5.9.1.

Ans. : Signal space diagram of MSK : Refer section 5.9.1.

Error probability for MSK : By union bound approximation,

$$P(e / s_1) \le \sum_{k=2}^{M} \frac{1}{2} erfc \sqrt{\frac{d_{k1}^2}{4N_0}}$$

From signal space diagram of MSK, observe that, there are two equidistant points from any signal point and the distance is $d = 2\sqrt{E_b}$. Hence above equation can be written as,

$$P(e / s_1) = \frac{1}{2} erfc \sqrt{\frac{d_{21}^2}{4N_0}} + \frac{1}{2} erfc \sqrt{\frac{d_{41}}{4N_0}} \quad \dots (1)$$

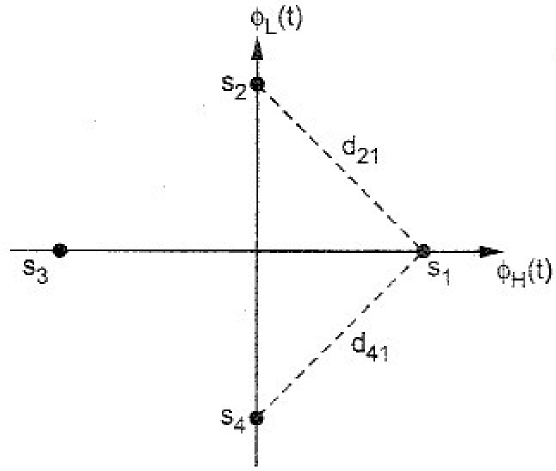

Fig. 1 Signal space diagram of MSK

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From this Fig. 1 observe that

$d_{21} = d_{41} = 2\sqrt{E_b}$.

Hence equation (1) can be expressed as,

$$P(e / s_1) = 2 \times \frac{1}{2} erfc \sqrt{\frac{4E_b}{4N_0}} = erfc \sqrt{\frac{E_b}{N_0}}$$

This is true for all the symbols, hence error probability for MSK is,

$$P_e = erfc \sqrt{\frac{E_b}{N_0}}$$

Q.13 *Explain the principle, operation, signals, constellation diagram, transmitter and receiver of a MSK system.* **May/June-2007, Nov./Dec.-2009, 16 Marks**

Ans. : Refer section 5.9.

Q.14 *Draw the block diagram of MSK transmitter and receiver and discuss in detail with required waveforms.* **Nov./Dec.-2007, 10 Marks**

Ans. : MSK transmitter : Refer section 5.9.4.1.

MSK receiver : Refer section 5.9.4.2.

MSK waveforms : Refer section 5.9 and Fig. 5.9.1.

Q.15 *What are the advantages and disadvantages of MSK as compared to QPSK system ?*

Nov./Dec.-2007, 6 Marks

Ans. : Refer section 5.9.5.

Q.16 *Explain carrier synchronization in QPSK signal.* **Nov./Dec.-2007, 6 Marks**

Ans. : Refer section 5.4.1.4.

Q.17 *Explain the detection of binary FSK signal with block diagram.*

Nov./Dec.-2007, 7 Marks

Ans. : Refer section 5.7.3.

Q.18 *Explain binary PSK signal with geometrical representation.*

Nov./Dec.-2007, 3 Marks

Ans. : BPSK signal : Refer section 5.2.1.

Geometric representation : Refer section 5.2.5.

Q.19 *Give the block diagram for the generation and detection of BFSK signal and give a brief explanations of the same.* **April/May-2008, 8 Marks**

Ans. : BFSK generation : Refer section 5.7.1.

BFSK detection : Refer section 5.7.3.

Q.20 *What are the special properties of MSK scheme ?* **April/May-2008, 4 Marks**

Ans. : Special properties of MSK :

i) MSK signal have continuous phase in all the cases.

ii) There are no amplitude variations in MSK signal.

iii) The difference between the two transmitted frequencies (f_H and f_L) is minimum and at the same time they are orthogonal.

iv) The sidelobes of MSK are very small and hence interchannel interference is minimum.

v) MSK is the special case of CPFSK.

Q.21 *Compare the performance of BPSK with that of BFSK.* **April/May-2008, 4 Marks**

Ans. : Refer section 5.14.4.

Q.22 *Discuss the generation and detection of QPSK with suitable block diagrams.*

April/May-2008, 10 Marks

Ans. : Refer section 5.4.1.

Q.23 *Write a note on differential phase shift keying.* **April/May-2008, 6 Marks**

Ans. : Refer section 5.3.

Q.24 *Draw the block diagrams of MSK transmitter and receiver and explain the functions of each block. Draw the constellation diagram. Derive probability of error.*

Nov./Dec.-2008, 16 Marks

Ans. : Refer section 5.9.

Q.25 *Explain the generation, detection, signal space diagram, bit error probability and power spectra of QPSK.* **May/June-2009, 16 Marks; Nov./Dec.-2010, 8 Marks**

Ans. : Refer section 5.4.

Q.26 *Compare the Digital modulation techniques in terms of bit error rate and bandwidth efficiency.* **May/June-2009, 6 Marks**

Ans. : Bandwidth efficiency : It is the ratio of transmission data rate to minimum channel bandwidth. Thus,

$$\text{BW efficiency} = \frac{\text{Transmission data rate } (r_b)}{\text{Minimum channel bandwidth } (B_T)}$$

Here r_b is in bits/sec and

B_T is in Hz.

Thus bandwidth efficiency is given in bits/sec/Hz. Following table lists the bit error rate and bandwidth efficiencies of various binary modulation techniques.

Sr. No.	Binary modulation	Bit error rate	BW efficiency
1.	PSK	$\frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$	1
2.	FSK	$\frac{1}{2} erfc \sqrt{\frac{0.6E}{N_0}}$	1
3.	DPSK	$\frac{1}{2} e^{-E_b/N_0}$	1
4.	QPSK	$erfc \sqrt{\frac{E_b}{2N_0}}$	2

Table 1 Comparison of binary modulation techniques for bandwidth efficiency and bit error rate

Q.27 *Discuss π/4 shifted QPSK and offset QPSK scheme in detail.*

Nov./Dec.-2009, 16 Marks

Ans. : Refer sections 5.4.1.1 and 5.4.1.2.

Q.28 *Derive the expression for probability of error of BPSK and QAM system.*

May/June-2010, 6 Marks

Ans. : Refer section 5.13.2.

Q.29 *Explain the working of DPSK scheme. With help of suitable circuit.*

May/June-2010, 6 Marks

Ans. : Refer section 5.3.

Q.30 *Distinguish coherent and non-coherent detection.* **Nov./Dec.-2010, 4 Marks**

Ans. : Refer section 5.1.2.

Q.31 *Explain non-coherent detection methods of binary frequency shift keying scheme.*

Nov./Dec.-2010, 12 Marks

Ans. : Refer section 5.7.4.

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A Trigonometric Relations

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad \text{(A-1)}$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad \text{(A-2)}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{e^{jx} - e^{-jx}}{j(e^{jx} + e^{-jx})} \quad \text{(A-3)}$$

$$e^{\pm jx} = \cos(x) \pm j\sin(x) \text{ (Euler's theorem)} \quad \text{(A-4)}$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) \quad \text{(A-5)}$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \quad \text{(A-6)}$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x) \quad \text{(A-7)}$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x) \quad \text{(A-8)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \text{(A-9)}$$

$$\sin(2x) = 2\sin(x)\cos(x) \quad \text{(A-10)}$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y) \quad \text{(A-11)}$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y) \quad \text{(A-12)}$$

$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y) \quad \text{(A-13)}$$

$$2\cos^2(x) = 1 + \cos(2x) \quad \text{(A-14)}$$

$$2\sin^2(x) = 1 - \cos(2x) \quad \text{(A-15)}$$

$$4\cos^3(x) = 3\cos(x) + \cos(3x) \quad \text{(A-16)}$$

$$4\sin^3(x) = 3\sin(x) - \sin(3x) \quad \text{(A-17)}$$

$$8\cos^4(x) = 3 + 4\cos(2x) + \cos(4x) \quad \text{(A-18)}$$

$$8\sin^4(x) = 3 - 4\cos(2x) + \cos(4x) \quad \text{(A-19)}$$

$$A\cos(x) - B\sin(x) = R\cos(x + \theta) \quad \text{(A-20a)}$$

where

$$R = \sqrt{A^2 + B^2} \quad \text{(A-20b)}$$

$$\theta = \tan^{-1}(B/A) \quad \text{(A-20c)}$$

$$A = R\cos(\theta) \quad \text{(A-20d)}$$

$$B = R\sin(\theta) \quad \text{(A-20e)}$$

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