or $$T_b = \frac{T_s}{2} \quad \text{... (5.4.12)}$$

Then the above equation becomes,

$$s(t) = \sqrt{P_s T_b}\, b_o(t)\, \phi_1(t) + \sqrt{P_s T_b}\, b_e(t)\, \phi_2(t) \quad \text{... (5.4.13)}$$

Since bit energy $E_b = P_s T_b$ ... (5.4.14)

$$\boxed{s(t) = \sqrt{E_b}\, b_o(t)\, \phi_1(t) + \sqrt{E_b}\, b_e(t)\, \phi_2(t)} \quad \text{... (5.4.15)}$$

Comments

- The above equation gives signal space representation of QPSK signal. The two orthogonal signals, $\phi_1(t)$ and $\phi_2(t)$ form the two axes of the signal space. Fig. 5.4.6 shows the signal space representation.

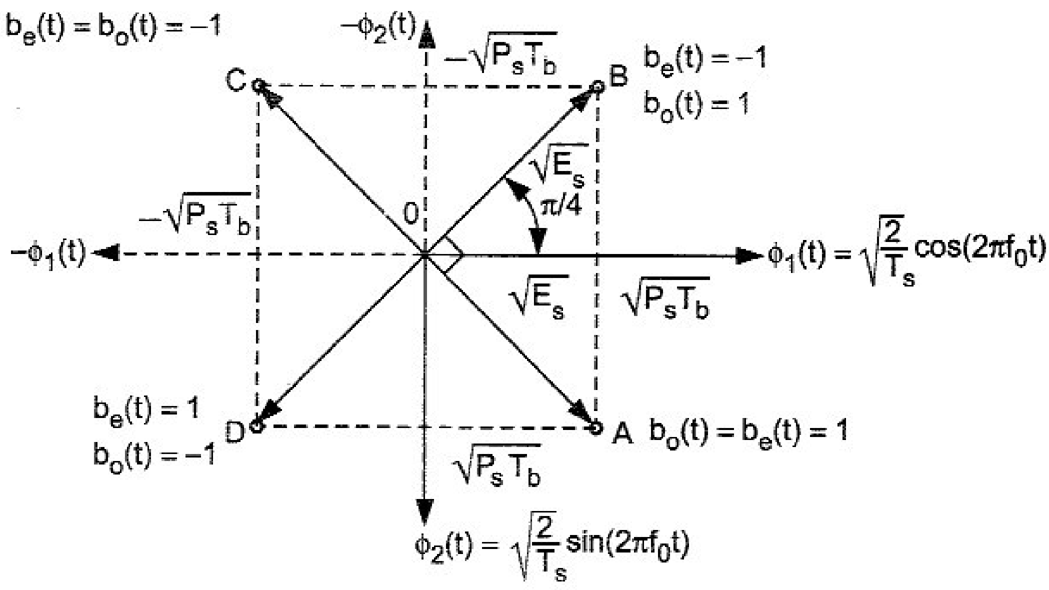

Fig. 5.4.6 Signal space representation of QPSK signals

- The possible 4 signal points are shown by small circles on $\phi_1\ \phi_2$ plane. From each signal point, we obtain two bits. For example from point 'A', we obtain two bits as (1, 1) and from 'B' we obtain bits, as (–1, 1).
- The distance of any signal point from origin '0' given as,

$$\begin{aligned} \text{Distance 'OB'} &= \sqrt{P_s T_b + P_s T_b} \\ &= \sqrt{2P_s T_b} \\ &= \sqrt{P_s T_s} \qquad (\because 2T_b = T_s) \quad \text{... (5.4.16)} \\ &= \sqrt{E_s} \qquad (\because P_s T_s = E_s) \quad \text{... (5.4.17)} \end{aligned}$$

Thus the length of each signal point from origin is $\sqrt{E_s}$.

- We know that $b_e(t)$ and $b_o(t)$ represent two successive bits. There is an offset of 'T_b' between $b_e(t)$ and $b_o(t)$. Therefore $b_e(t)$ and $b_o(t)$ both cannot change their levels simultaneously. Therefore either $b_e(t)$ or $b_o(t)$ can change at a time.
- Let's say that $b_e(t) = b_o(t) = 1$ representing signal point 'A' in Fig. 5.4.6. In the next bit interval if $b_o(t) = -1$, then signal point will be 'D'. Otherwise if $b_e(t)$ changes its level (i.e. $b_e(t) = -1$), then next signal point will be 'B'. Thus from signal point 'A', then next signal points will be either 'D' or 'B'.

Distance between signal points :

Normally the ability to determine a bit without error is measured by the distance between two nearest possible signal points in the signal space. Such points differed in a single bit. For example signal points 'A' and 'B' are two nearest points since they differ by a single bit $b_e(t)$. As 'A' and 'B' becomes closer to each other, the possibility of error increases. Hence this distance should be as large as possible. This distance is denoted by 'd'. In Fig. 5.4.6, the distance between signal points 'A' and 'B' is given as,

$$d^2 = (\sqrt{E_s})^2 + (\sqrt{E_s})^2$$

$$\therefore \quad d = \sqrt{2E_s} \qquad \text{... (5.4.18)}$$

or

$$d = 2\sqrt{P_s T_b} = 2\sqrt{E_b} \qquad \text{... (5.4.19)}$$

Compare this distance with the distance of BPSK signals given by equation 5.2.20. This shows that the distance for QPSK is the same as that for BPSK. Since this distance represents noise immunity of the system, it shows that noise immunities of BPSK and QPSK are same.

5.4.3 Spectral Characteristics of QPSK Signal

Step 1 : PSD of NRZ waveform

The input sequence $b(t)$ is of bit duration T_b. It is NRZ bipolar waveform. In section 5.2.4 we have obtained the power spectral density of such waveform as,

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \qquad \text{from equation 5.2.12}$$

and $V_b = \sqrt{P_s}$, then above equation becomes,

$$S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \qquad \text{... (5.4.20)}$$

The above equation gives power spectral density of signal $b(t)$.

Step 2 : PSDs of even and odd numberd sequence

This signal is divided into $b_e(t)$ and $b_o(t)$ each of bit period $2T_b$. If we consider that symbols 1 and 0 are equally likely, then we can write power spectral densities of $b_e(t)$ and $b_o(t)$ as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \quad \text{... (5.4.21)}$$

and

$$S_o(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \quad \text{... (5.4.22)}$$

In the above two equations we have just replaced T_b by T_s and T_s is the period of bit in $b_e(t)$ and $b_o(t)$.

Step 3 : PSD of QPSK signal

Since inphase and quadrature components $[b_e(t) \text{ and } b_o(t)]$ are statistically independent, the baseband power spectral density of QPSK signal equals the sum of the individual power spectral densities of $b_e(t)$ and $b_o(t)$ i.e.,

$$S_B(f) = S_e(f) + S_o(f)$$

$$= 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \quad \text{... (5.4.23)}$$

This equation gives baseband power spectral density of QPSK signal. Upon modulation of carrier of frequency f_0, the spectral density given by above equation is shifted at $\pm f_0$. Thus plots of power spectral density of QPSK will be similar to that BPSK given in Fig. 5.2.5 and Fig. 5.2.6.

5.4.4 Bandwidth of QPSK Signal

We have seen that the bandwidth of BPSK signal is equal of $2f_b$. Here $T_b = \frac{1}{f_b}$ is the one bit period. In QPSK the two waveforms $b_e(t)$ and $b_o(t)$ from the baseband signals. One bit period for both of these signals is equal to $2T_b$. Therefore bandwidth of QPSK signal is,

$$BW = 2 \times \frac{1}{2T_b} \quad \text{or} \quad BW = f_b \quad \text{... (5.4.24)}$$

Thus the bandwidth of QPSK signal is half of the bandwidth of BPSK signal. Earlier we have seen that noise immunity of QPSK and BPSK is same. This shows that inspite of the reduction in bandwidth in QPSK, the noise immunity remains same as compared to BPSK. BW of QPSK can also be obtained by plotting equation 5.4.20 as shown in Fig. 5.4.7 below.

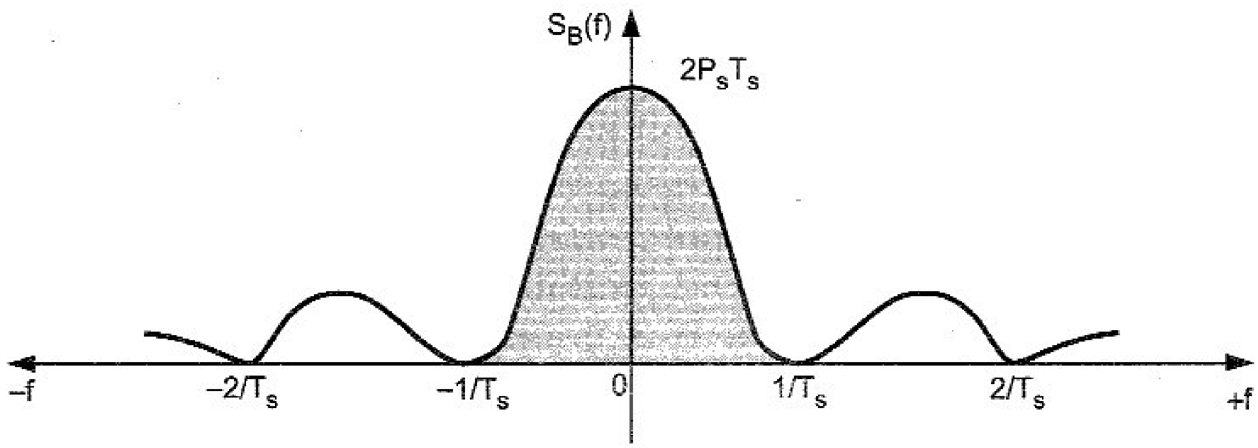

Fig. 5.4.7 Plot of power spectral density of QPSK signal

$$BW = \text{Highest frequency} - \text{Lowest frequency in main lobe}$$

$$= \frac{1}{T_s} - \left(-\frac{1}{T_s}\right) \text{ since carrier frequency } f_0 \text{ cancels out}$$

$$= \frac{2}{T_s}$$

We know that $T_s = 2T_b$

$$\therefore \quad BW = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$$

which is same as we obtained in equation 5.4.24.

5.4.5 Advantages of QPSK

QPSK has some definite advantages and disadvantages as compared to BPSK and DPSK.

Advantages :

1) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.
2) Because of reduced bandwidth, the information transmission rate of QPSK is higher.
3) Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

Example 5.4.1 : *Write the waveforms for a binary sequence 101100 modulated under QPSK.*

Solution : Let us assume the offset QPSK or OQPSK system. The waveforms are drawn similar to those in Fig. 5.4.2. Fig. 5.4.8 shows the waveforms for given sequence. The input sequence is written at the top alongwith bit numbers Fig.(a) shows NRZ waveform for odd numbered bits. Fig.(b) shows the NRZ waveform for even numbered bits. Observe that the signal is assumed 'O' at the beginning for even numbered bit sequence i.e. $b_e(t)$. The two quadrature carriers are shown in Fig. (c) and (d). Fig. (e) shows the binary PSK signal generated due to $b_o(t)$, i.e. $s_o(t)$. Fig. (f) shows the binary PSK signal generated due to $b_e(t)$, i.e. $s_e(t)$. The signals $s_o(t)$ and $s_e(t)$ are added to get the final QPSK waveform s(t). It is shown in Fig. 5.4.8(g). Observe that the phase shifts of $\frac{\pi}{2}$ occur in this waveform. The individual binary PSK waveforms are staggered due to offset QPSK. (See Fig. 5.4.8 on next page)

Example 5.4.2 : *In a QPSK system, the bit rate of NRZ stream is 10 Mbps and carrier frequency is 1 GHz. Find the symbol rate of transmission and bandwidth requirement of the channel. Sketch the power spectral density of the QPSK signal.*

Solution : (i) To obtain symbol rate and bandwidth

Bit rate $f_b = 10$ Mbps

Bandwidth of the QPSK system is equal to bit rate.

Hence $BW = f_b = 10$ MHz

Symbol duration and bit duration are related as,

$$T_s = 2T_b$$

$$\therefore \quad \text{Symbol rate} = \frac{1}{T_s} = \frac{1}{2T_b} = \frac{f_b}{2}$$

$$= 5 \text{ MHz}$$

(ii) To obtain power spectral density :

Carrier frequency $f_o = 1$ GHz

psd of NRZ signal is given as,

$$S_B(f) = 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

Upon modulation of the carrier of frequency f_o, the spectral density given by above equation is shifted at $\pm f_o$.

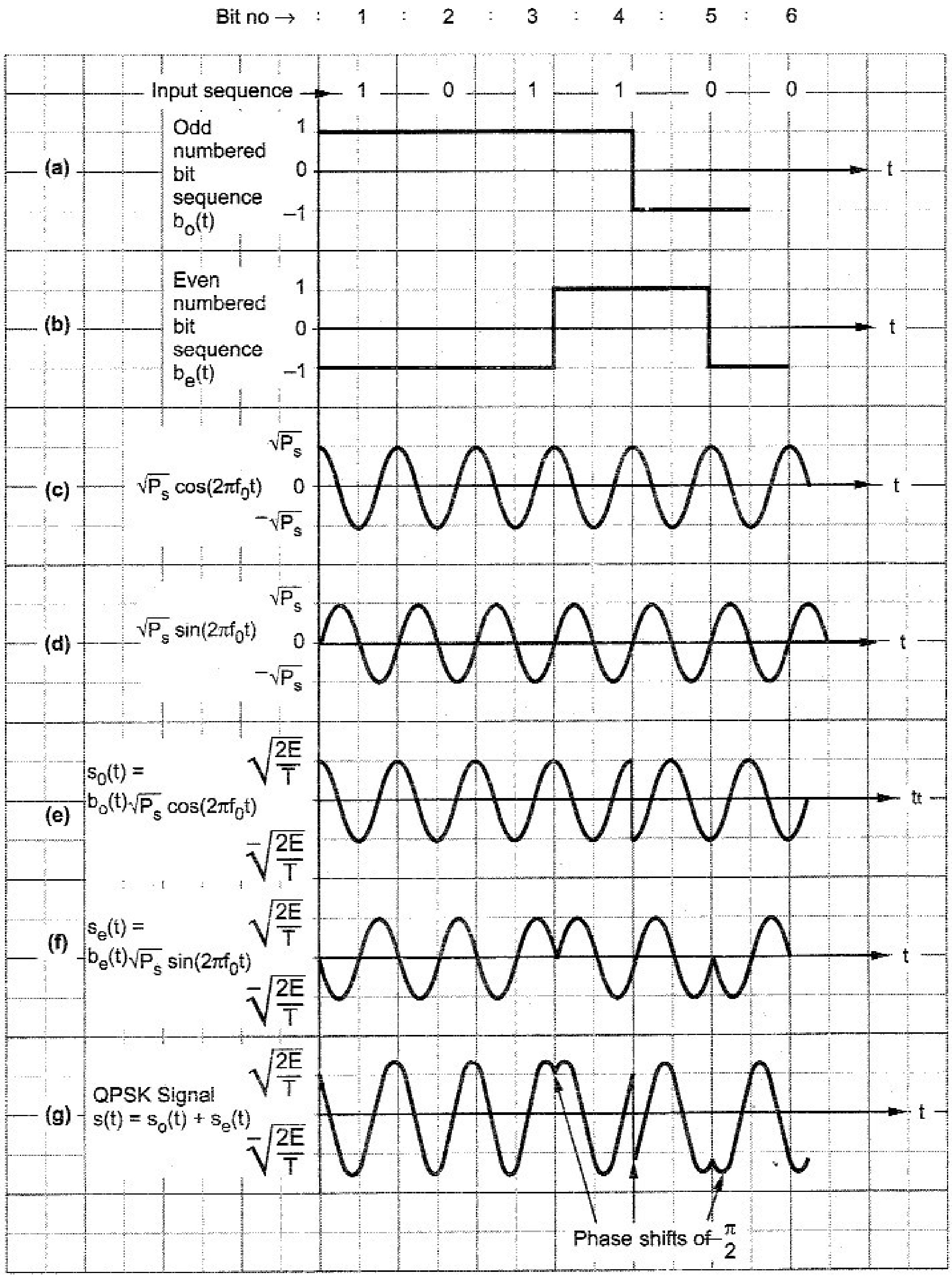

Fig. 5.4.8 QPSK waveform

Fig. 5.4.9 shows the psd plot.

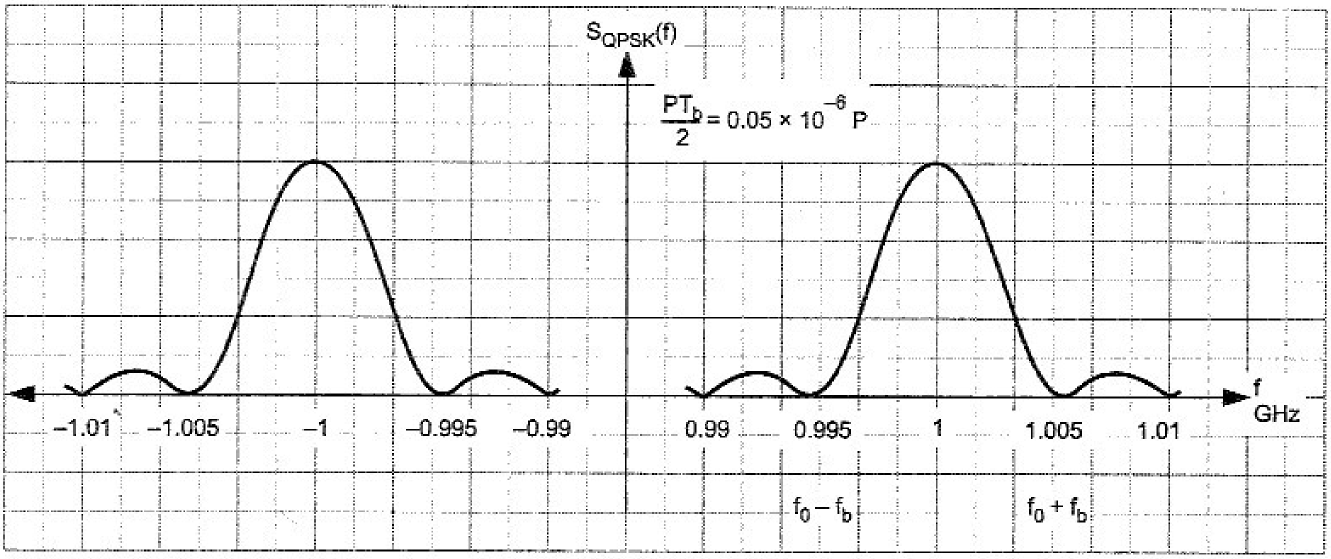

Fig. 5.4.9 Spectral density plot of QPSK

In above figure observe that the two lobes are placed at ± 1 GHz. Width of the main lobe is (1 + 0.005) GHz to (1 − 0.005) GHz. This is equal to bandwidth requirement of the system.

That is, $BW = 1.005 - 0.995 = 0.01\ GHz = 10\ MHz$

Example 5.4.3 : *For the input binary sequence*

$\{b_k\} = \{1, -1, 1, -1, -1, -1, 1, 1\}$. *Find the transmitted phase sequence and sketch the transmitted waveform for QPSK*

Solution : The given sequence $\{b_k\}$ is in NRZ form. Fig. 5.4.10 shows the waveforms of QPSK. (See Fig. 5.4.10 on next page)

Fig. (a) and (b) shows the NRZ waveforms of even numbered and odd numbered samples. Fig. (c) and (d) shows the quadrature carriers. Fig. (e) and (f) shows the PSK modulated quadrature carriers. Fig. (g) shows the QPSK waveform. The transmitted phase is shown at the bottom.

Review Questions

1. *With the help of block diagram and relevent expressions/waveforms explain QPSK transmitter and receiver.*
2. *Compare psd and bandwidth requirements of QPSK with that of BPSK.*
3. *Represent QPSK signals in the signal space and find distance between them. What is the significance of this distance ?*

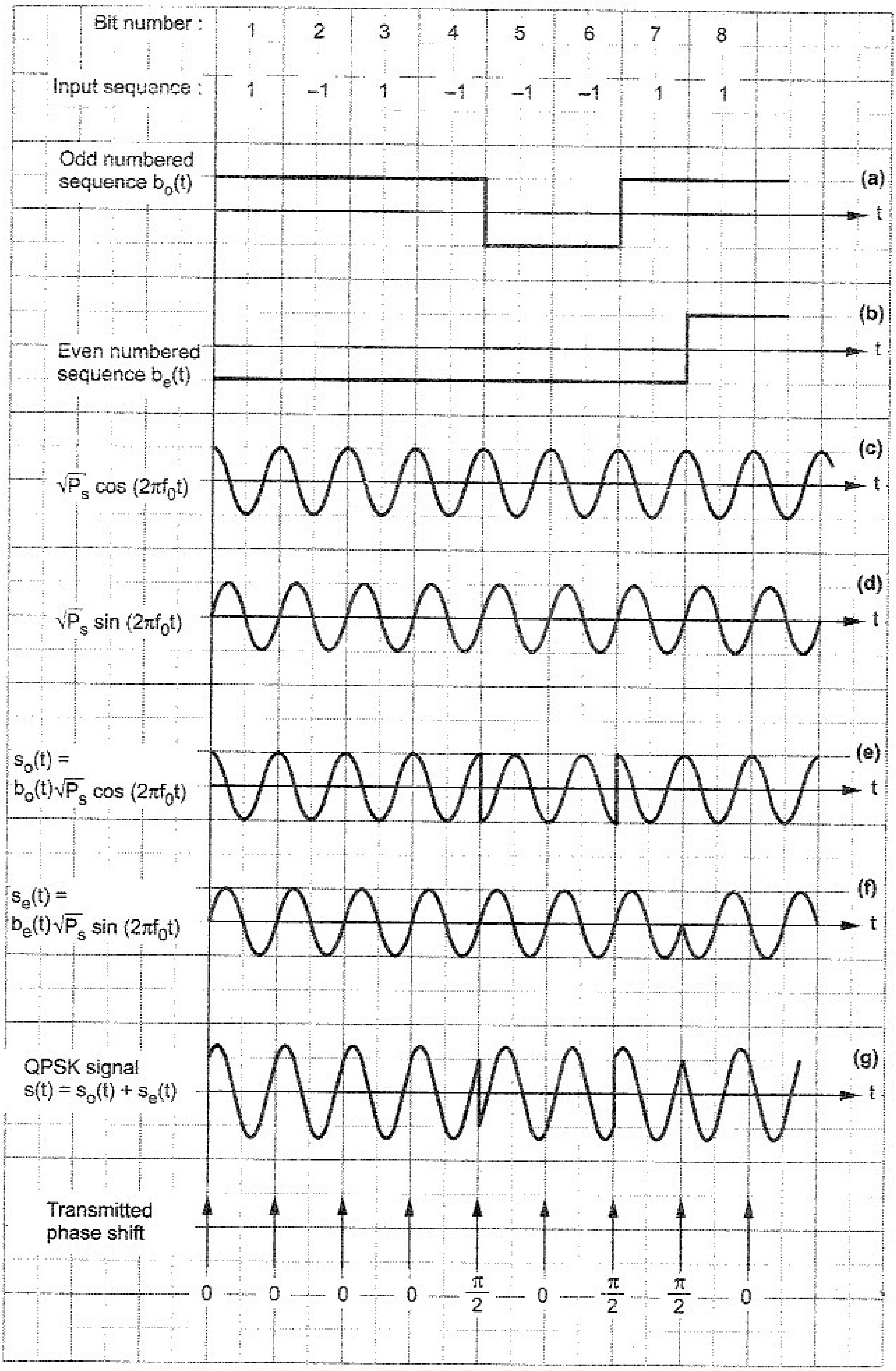

Fig. 5.4.10 Waveforms of QPSK

5.5 M-ary PSK

BPSK transmits one bit at a time and it has only two symbols. Hence whenever the symbol is changed, the phase shift is,

$$\text{Phase shift in QPSK} = \frac{2\pi}{\text{number of symbols}} = \frac{2\pi}{2} = \pi \text{ or } 180^\circ$$

Review Questions

1. *Explain the differences between QASK and QPSK systems giving corresponding expressions and signal space representations.*
2. *Explain QASK system with its transmitter, receiver and signal space representation.*
3. *What is the bandwidth of the transmitter in terms of input bit duration i.e., input signal bandwidth? Explain the mechanism by which the bandwidth reduction is made possible in QASK system ?*

5.7 Binary Frequency Shift Keying (BFSK)

May/June - 2006

In binary frequency shift keying, the frequency of the carrier is shifted according to the binary symbol. The phase of the carrier is unaffected. That is we have two different frequency signals according to binary symbols. Let there be a frequency shift by Ω. Then we can write following equations.

If $b(t) = '1'$; $s_H(t) = \sqrt{2P_s}\cos(2\pi f_0 + \Omega)t$ (5.7.1)

If $b(t) = '0'$; $s_L(t) = \sqrt{2P_s}\cos(2\pi f_0 - \Omega)t$ (5.7.2)

Thus there is increase or decrease in frequency by Ω. Let us use the following conversion table to combine above two FSK equations.

b (*t*) Input	*d*(*t*)	$P_H(t)$	$P_L(t)$
1	+ 1V	+1 V	0 V
0	− 1 V	0 V	+1 V

Table 5.7.1 Conversion table for BPSK representation

We can write equation 5.7.1 and equation 5.7.2 combinely as,

$$s(t) = \sqrt{2P_s}\cos\left[(2\pi f_0 + d(t)\Omega)t\right] \quad \text{... (5.7.3)}$$

Thus when symbol '1' is to be transmitted, the carrier frequency will be $f_0 + \left(\frac{\Omega}{2\pi}\right)$.

If symbol '0' is to be transmitted, the carrier frequency will be $f_0 - \left(\frac{\Omega}{2\pi}\right)$. i.e.,

$$f_H = f_0 + \frac{\Omega}{2\pi} \quad \text{for symbol '1'} \quad \text{... (5.7.4)}$$

$$f_L = f_0 - \frac{\Omega}{2\pi} \quad \text{for symbol '0'} \quad \text{... (5.7.5)}$$

$P_H(t)$ = High i/p

$P_L(t)$ = Low i/p

↓ in

5.7.1 BFSK Transmitter

From the table 5.7.1, we know that $P_H(t)$ is same as $b(t)$. And $P_L(t)$ is inverted version of $b(t)$. The block diagram of BFSK transmitter is shown in Fig. 5.7.1.

We know that input sequence $b(t)$ is same as $P_H(t)$. An inverter is added after $b(t)$ to get $P_L(t)$. $P_H(t)$ and $P_L(t)$ are unipolar signals. The level shifter converts the '+1' level to $\sqrt{P_s T_b}$. Zero level is unaffected. Thus the output of the level shifters will be either $\sqrt{P_s T_b}$ (if '+1') or zero (if input is zero). Further there are product modulators after level shifter. The two carrier signals $\phi_1(t)$ and $\phi_2(t)$ are used. $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other. In one bit period of input signal (i.e. T_b), $\phi_1(t)$ or $\phi_2(t)$ have integral number of cycles.

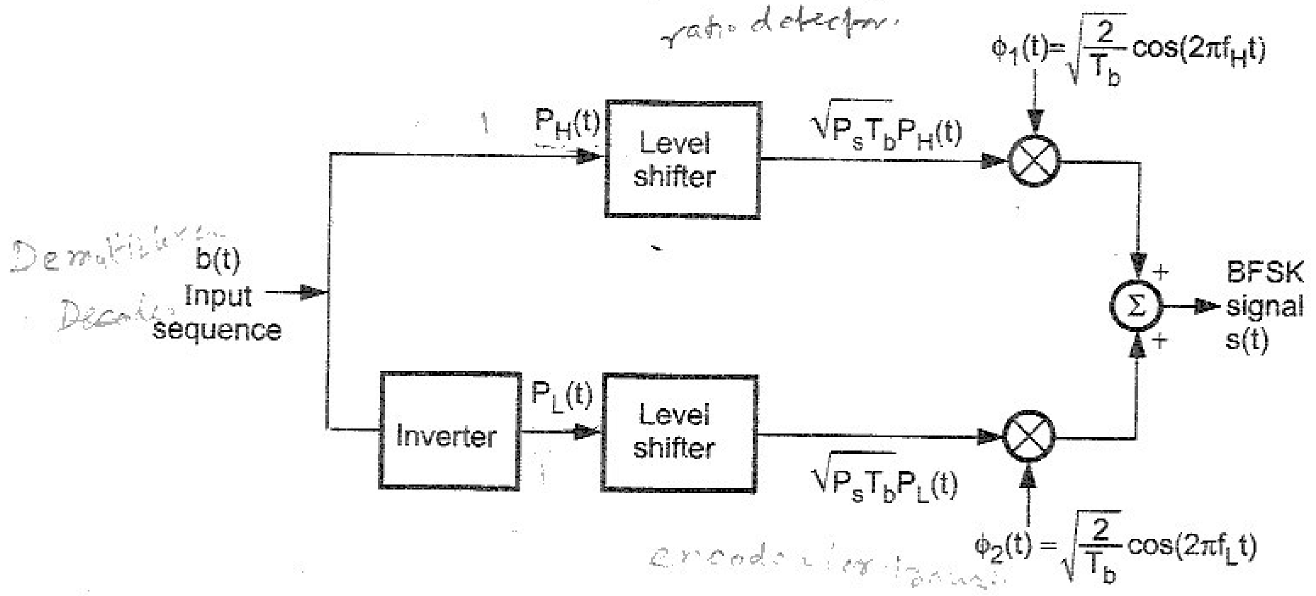

Fig. 5.7.1 Block diagram of BFSK transmitter

Therefore the modulated signal has continuous phase. Such BFSK signal is shown in Fig.5.7.2. The adder then adds the two signals.

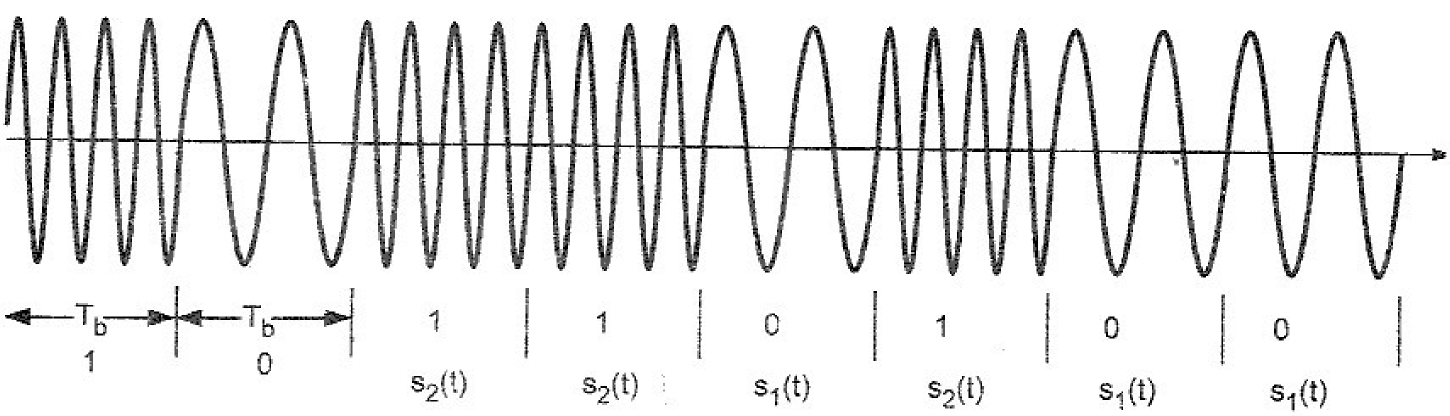

Fig. 5.7.2 BFSK signal

Here note that outputs from both the multipliers are not possible at a time. This is because $P_H(t)$ and $P_L(t)$ are complementary to each other. Therefore if $P_H(t) = 1$, then output will be only due to upper modulator and lower modulator output will be zero (since $P_L(t) = 0$).

5.7.2 Spectral Characteristics and Bandwidth of BFSK

In Fig. 5.7.1 we can write BFSK signal $s(t)$ as,

$$s(t) = \sqrt{2P_s}\, P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t) \cos(2\pi f_L t) \qquad \ldots (5.7.6)$$

This is the BFSK signal equation. Let's compare this equation with BPSK equation of equation 5.2.6 i.e.,

$$s_{BPSK}(t) = b(t)\sqrt{2P} \cos(2\pi f_0 t) \qquad \ldots (5.7.7)$$

We observe that the above equation is similar to BFSK equation. In BPSK equation, $b(t)$ is a bipolar signal but in BFSK the similar coefficients $P_H(t)$ or $P_L(t)$ are unipolar. Therefore let us convert those coefficients in bipolar form as follows

$$P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t) \qquad \ldots (5.7.8)$$

and

$$P_L(t) = \frac{1}{2} + \frac{1}{2} P_L'(t) \qquad \ldots (5.7.9)$$

Here $P_H'(t)$ and $P_L'(t)$ will be bipolar (i.e. +1 or −1). Putting those values in equation 5.7.6 we get,

$$s(t) = \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2} P_H'(t)\right] \cos(2\pi f_H t) + \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2} P_L'(t)\right] \cos(2\pi f_L t)$$

$$= \sqrt{\frac{P_s}{2}} \cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}} \cos(2\pi f_L t) + \sqrt{\frac{P_s}{2}}\, P_H'(t) \cos(2\pi f_H t)$$

$$+ \sqrt{\frac{P_s}{2}}\, P_L'(t) \cos(2\pi f_L t) \qquad \ldots (5.7.10)$$

In the above equation, the first term represent the single frequency impulse at f_H. The second term represent the pulse at f_L. Those are constant amplitude pulses. The last two terms are similar to BPSK equation of equation 5.7.7. Here $P_H'(t)$ and $P_L'(t)$ are equivalent to $b(t)$. Therefore those last two terms in equation 5.7.10 produce the spectrum which are similar to that of BPSK. One spectrum is located at f_H and other at f_L. Therefore we can write the power spectral density of BFSK as,

$$S(f) = \sqrt{\frac{P_s}{2}} \left\{ \delta(f - f_H) + \delta(f + f_L) + \frac{P_s T_b}{2} \left\{ \frac{\sin(\pi f_H T_b)}{\pi f_H T_b} \right\}^2 + \frac{P_s T_b}{2} \left\{ \frac{\sin(\pi f_L T_b)}{\pi f_L T_b} \right\}^2 \right\} \quad \dots (5.7.11)$$

Fig. 5.7.3 shows the plot of power spectral density of BFSK signal given by above equation.

f_H and f_L are selected. Such that,

$$f_H - f_L = 2f_b \quad \dots (5.7.12)$$

With such selection, it is clear from the spectrums in the above figure that, the two

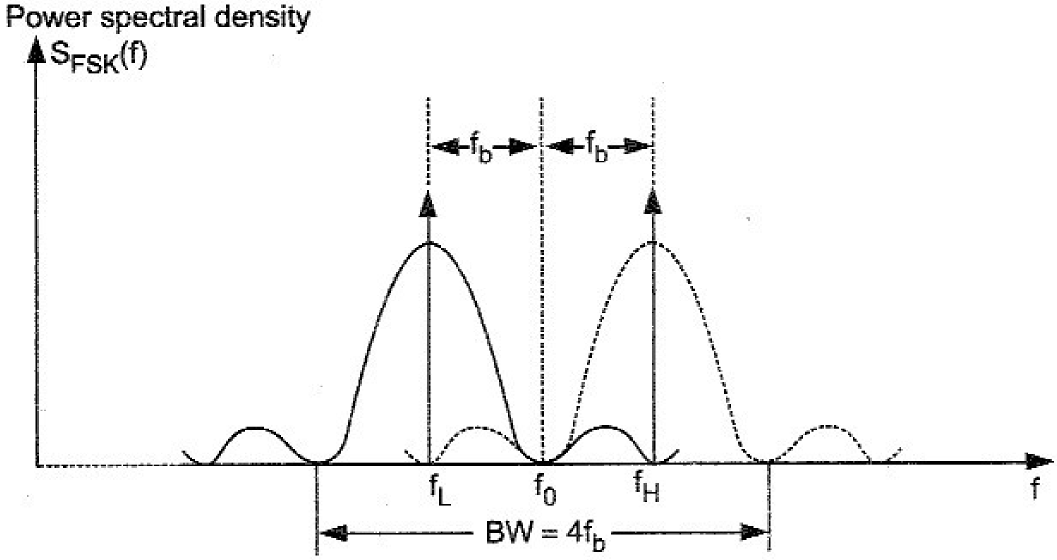

Fig. 5.7.3 Power spectral density of BFSK signal

frequencies f_H and f_L can be identified properly. The interference between the spectrums is not much with the above assumption.

Bandwidth of BFSK Signal :

From Fig. 5.7.3 it is clear that the width of one lobe is $2f_b$. The two main lobes due to f_H and f_L are place such that the total width due to both main lobes is $4f_b$. i.e.,

$$\text{Bandwidth of BFSK} = 2f_b + 2f_b$$

or

$$BW = 4f_b \quad \dots (5.7.13)$$

If we compare this bandwidth with that of BPSK given by equation 5.2.21, we observe that,

$$\text{BW (BFSK)} = 2 \times \text{BW (BPSK)}$$

5.7.3 Coherent BFSK Receiver

Fig. 5.7.4 shows the block diagram of coherent BFSK receiver. There are two correlators for two frequencies of FSK signal. These correlators are supplied with locally generated carriers $\phi_1(t)$ and $\phi_2(t)$. If the transmitted frequency is f_H, then output $s_1(t)$ will be higher than $s_2(t)$. Hence y(t) will be greater than zero.

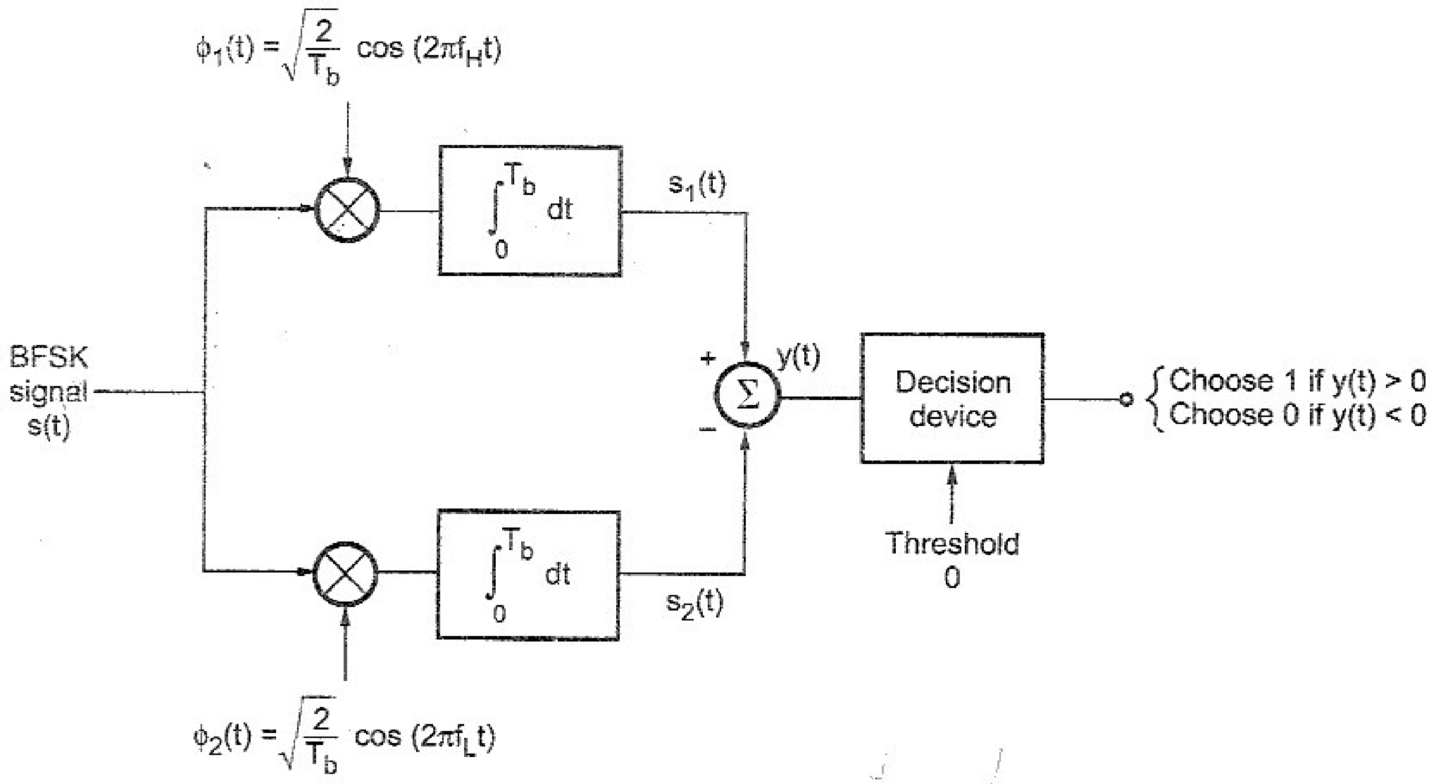

Fig. 5.7.4 Coherent BFSK reciever

The decision device then decides in favour of binary '1'. If $s_2(t) > s_1(t)$, then $y(t) < 0$ and decision device decides in favour of 0. The coherent carriers are generated using similar methods discussed earlier.

5.7.4 Noncoherent BFSK Receiver

Fig. 5.7.5 shows the block diagram of BFSK receiver. The receiver consists of two bandpass filters ; one with centre frequency f_H and other with centre frequency f_L. Since $f_H - f_L = 2f_b$, the outputs of filters do not overlap. The bandpass filters pass their corresponding main lobes without much distortion.

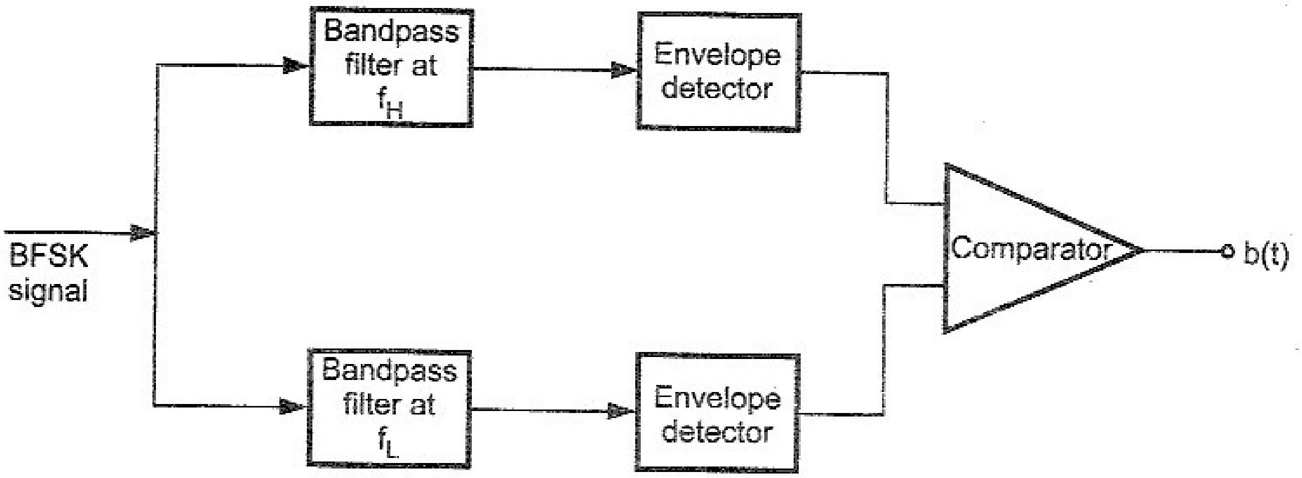

Fig. 5.7.5 Block diagram of BFSK receiver

The outputs of filters are applied to envelop detectors. The outputs of detectors are compared by the comparator. If unipolar comparator is used, then the output of comparator is the bit sequence $b(t)$.

5.7.5 Geometrical Representation of Orthogonal BFSK or Signal Space Representation of Orthogonal BFSK

Orthogonal carriers are used for M-ary PSK and QASK. The different signal points are represented geometrically in $\phi_1\ \phi_2$ plane. For geometrical representation of BFSK signals such orthogonal carriers are required. From Fig. 5.7.1, we know that, two carriers $\phi_1(t)$ and $\phi_2(t)$ of two different frequencies f_H and f_L are used for modulation. To make $\phi_1(t)$ and $\phi_2(t)$ orthogonal, the frequencies f_H and f_L should be some integer multiple of base band frequency 'f_b'.

i.e. $$f_H = m f_b \qquad \ldots (5.7.14)$$

and $$f_L = n f_b \qquad \ldots (5.7.15)$$

Here $f_b = \frac{1}{T_b}$, then the carriers will be

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi\, m f_b\, t) \qquad \ldots (5.7.16)$$

and $$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi\, m f_b\, t) \qquad \ldots (5.7.17)$$

The carriers $\phi_1(t)$ and $\phi_2(t)$ are orthogonal over the period T_b. We can write equation 5.7.1 and equation 5.7.2 as,

$$s_H(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_H t)$$

and $$s_L(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_L t)$$

Here $$f_H = f_0 + \frac{\Omega}{2\pi} \text{ and } f_L = f_0 - \frac{\Omega}{2\pi}$$

Using the relations of equation 5.7.14 to equation 5.7.17 we can write above equations as,

$$s_H(t) = \sqrt{P_s T_b}\cdot\phi_1(t) \qquad \text{... (5.7.18)}$$

and $$s_L(t) = \sqrt{P_s T_b}\cdot\phi_2(t) \qquad \text{... (5.7.19)}$$

Based on the above two equations we can draw the signal space diagram as shown in Fig. 5.7.6.

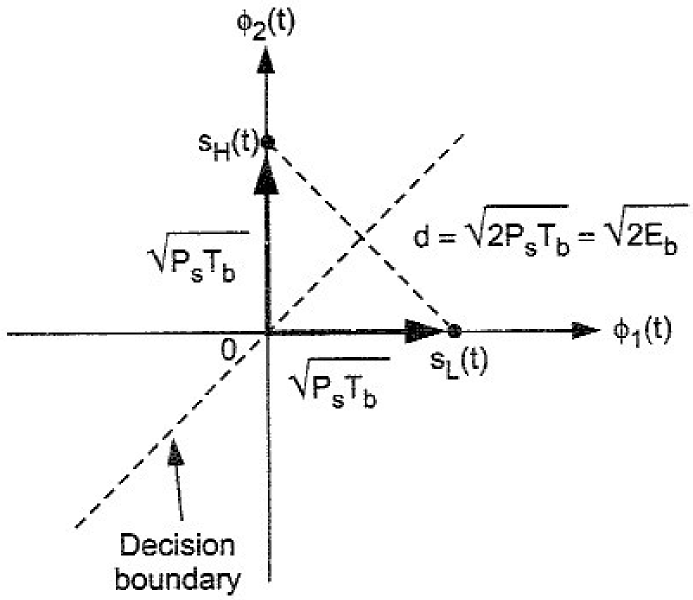

Fig. 5.7.6 Signal space representation of orthogonal BFSK

Distance between signal points :

There are two signal points in the signal space. The distance between these two points can be obtained as,

$$d^2 = (\sqrt{P_s T_b})^2 + (\sqrt{P_s T_b})^2 = 2P_s T_b$$

$$\therefore \quad d = \sqrt{2P_s T_b} \qquad \text{... (5.7.20)}$$

Since $P_s T_b = E_b$, we can write above relation as,

$$d = \sqrt{2E_b} \qquad \text{... (5.7.21)}$$

As compared to the distance of BPSK, we observe that this distance is smaller.

5.7.6 Geometrical Representation of Non Orthogonal BFSK Signals

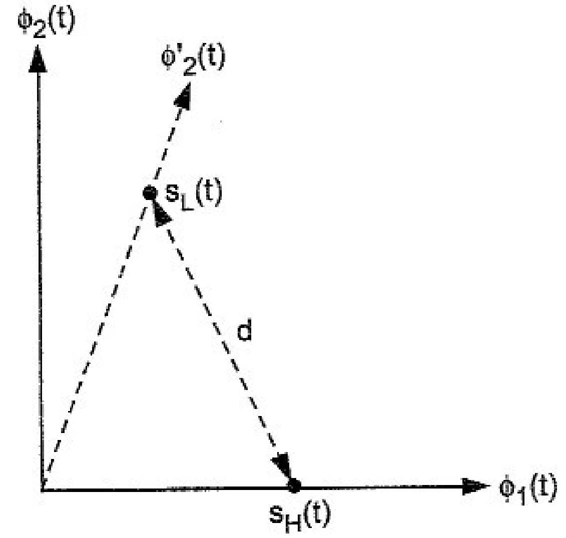

Fig. 5.7.7 Geometrical representation of non-orthogonal BFSK signals

Whenever the carriers $\phi_1(t)$ and $\phi_2(t)$ are non orthogonal, then the signal point $S_H(t)$ or $S_L(t)$ will not lie exactly on the axes $\phi_1(t)$ and $\phi_2(t)$. Such a representation is shown in Fig. 5.7.7.

The distance 'd' for non orthogonal signal shown in Fig. 5.7.6 is given approximately as,

$$d^2 = 2E_b\left[1 - \frac{\sin 2\pi(f_H - f_L)T_b}{2\pi(f_H - f_L)T_b}\right] \quad \text{... (5.7.22)}$$

5.7.7 Advantages and Disadvantages of BFSK

Even though the generation of BFSK is easier it has many disadvantages compared to BPSK. First thing is that its bandwidth is greater than $4f_b$, which is almost double the bandwidth of BPSK. The distance between the signal points is less in BFSK. Hence the error rate of BFSK is more compared to BPSK.

From equation 5.7.3 we can write,

$$s(t) = \sqrt{2P_s}\cos\{d(t)\Omega t\}\cos(2\pi f_0 t) - \sqrt{2P_s}\sin\{d(t)\Omega t\}\sin(2\pi f_0 t)$$

Since $\quad d(t) = \pm 1$

$\therefore \quad \cos\{\pm\Omega t\} = \cos(\Omega t)$

and $\quad \sin\{\pm\Omega t\} = \pm\sin(\Omega t) = d(t)\sin(\Omega t)$

By standard trigonometric relations

$$s(t) = \sqrt{2P_s}\cos(\Omega t)\cos(2\pi f_0 t) - \sqrt{2P_s}\,d(t)\sin(\Omega t)\sin(2\pi f_0 t) \quad \text{... (5.7.23)}$$

In the above relation the first term, $\sqrt{2P_s}\cos(\Omega t)\cos(2\pi f_0 t)$ carries no information. The second term, $\sqrt{2P_s}\,d(t)\sin(\Omega t)\sin(2\pi f_0 t)$ carries the information signal $d(t)$. Thus only half of the transmitted energy carries the information signal.

Review Questions

1. *Draw the block diagrams and explain the operation of BFSK transmitter and receiver.*
2. *Compare BFSK and BPSK.*

5.8 M-ary FSK

In the last section we studied BFSK for two symbols. This principle can be extended further to 'N' successive bits. These 'N' bits from $2^N = M$ different symbols. Every symbol uses separate frequency for transmission. Such system is called M-ary FSK system. The principle of transmission and reception of M-ary FSK is different than BFSK.

5.8.1 Transmitter and Receiver of FSK

5.8.1.1 Transmitter

Fig. 5.8.1 shows the M-ary FSK transmitter. The 'N' successive bits are presented in parallel to digital to analog converter. These 'N' bits forms a symbol at the output of digital to analog converter. There will be total $2^N = M$ possible symbols. The symbol is presented every $T_s = NT_b$ period. The output of digital to analog converter is given to a frequency modulator. Thus depending upon the value of symbol, the frequency modulator generates the output frequency. For every symbol, the frequency modulator produces different frequency output. This particular frequency signal remains at the output for one symbol duration. Thus for 'M' symbols, there are 'M' frequency signals at the output of modulator. Thus the transmitted frequencies are $f_0, f_1, f_2, ...f_{M-1}$ depending upon the input symbol to the modulator.

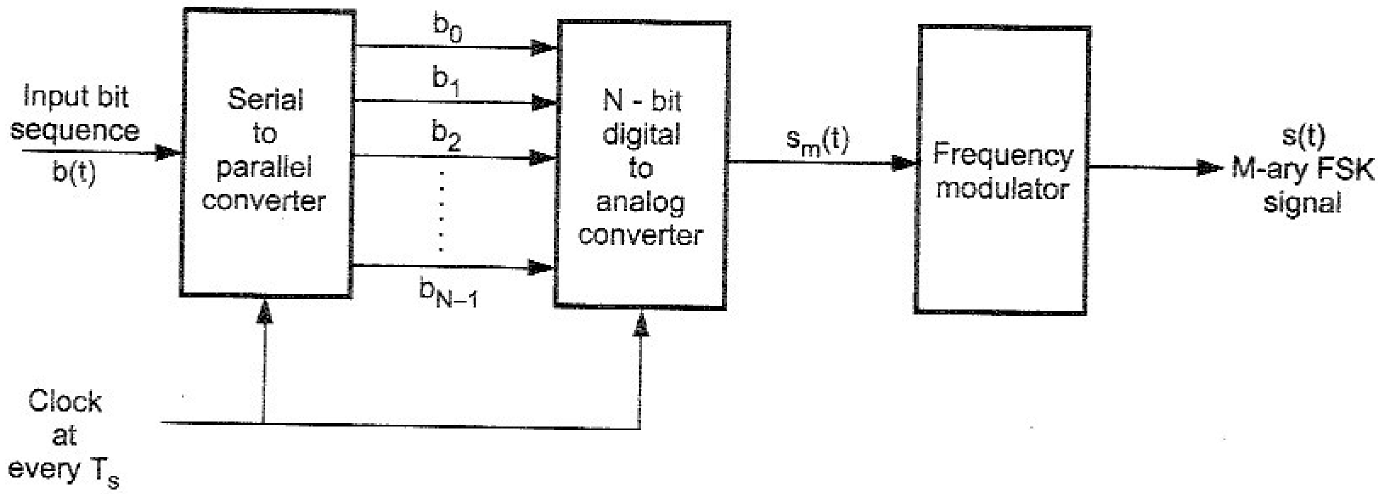

Fig. 5.8.1 M-ary FSK transmitter

5.8.1.2 Receiver

Fig. 5.8.2 shows block diagram of M-ary FSK receiver. It is the extension of BFSK receiver of Fig. 5.8.1. The M-ary FSM signal is given to the set of 'M' bandpass filters. The center frequencies of those filters are $f_0, f_1, f_2, ...f_{M-1}$. These filters pass their particular frequency and alternate others. The envelope detectors outputs are applied to a decision device. The decision device produces its output depending upon the highest input. Depending upon the particular symbol, only one envelope detector will have higher output. The outputs of other detectors will be very low. The output of the decision device is given to 'N' bit analog to digital converter. The analog to digital converter output is the 'N' bit symbol in parallel. These bits are then converted to serial bit stream by parallel to serial converter. In some cases the bits appear in parallel. Then there is no need to use serial to parallel and parallel to serial converters.

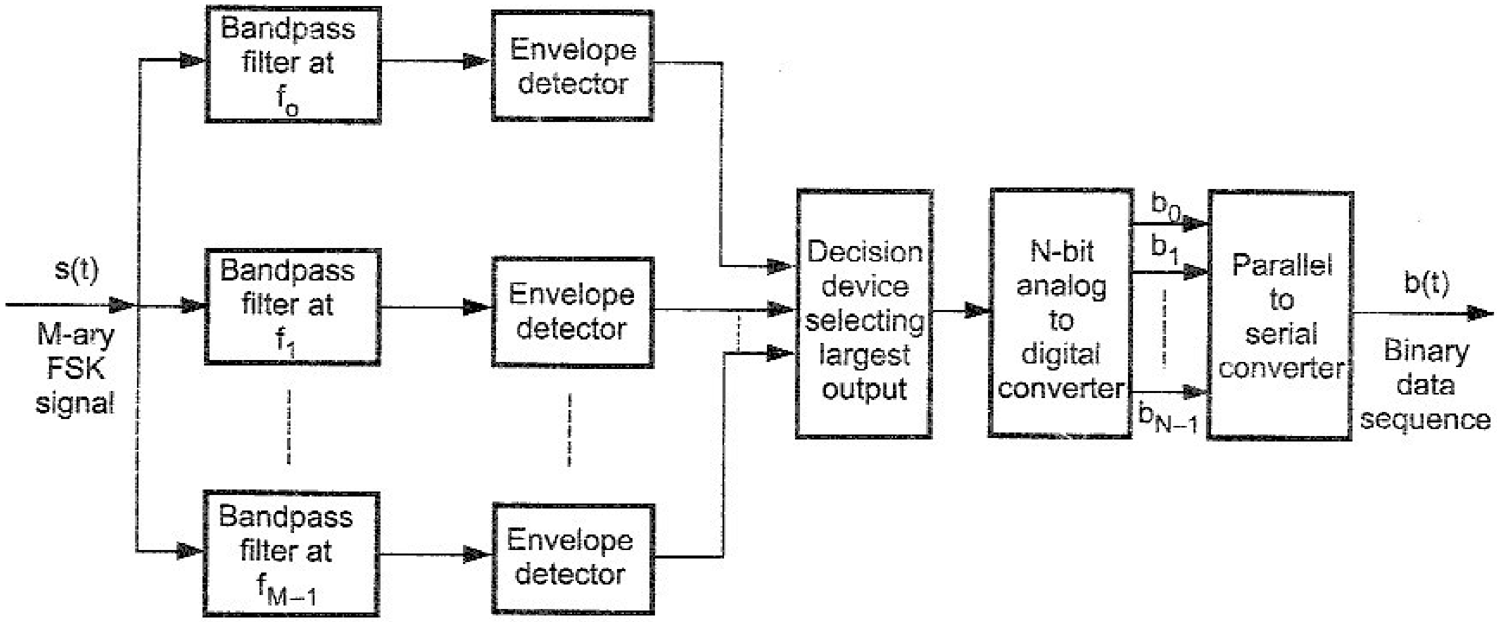

Fig. 5.8.2 Block diagram of M-ary FSK system

5.8.2 Power Spectral Density and Bandwidth of M-ary FSK

We know that for M symbol $f_0, f_1, f_2 \cdots f_{m-1}$ frequencies are used for transmission. The probability of error is minimized by selecting those frequencies such that transmitted signals are mutually orthogonal. If those frequencies are selected as successive even harmonics of symbol frequency f_s, then transmitted signals will be orthogonal.

Let's say that the lowest carrier frequency f_0 is the k^{th} harmonic of symbol frequency i.e.,

$$f_0 = kf_s \quad \text{... (5.8.1)}$$

Then the other frequencies will be,

$$f_1 = (k+2) f_s, f_2 = (k+4) f_s \text{ ... etc} \quad \text{... (5.8.2)}$$

Thus every carrier frequency is separated by $2f_s$ from its nearest carriers. Fig.5.7.2 shows the power spectral density of BFSK (for two symbol FSK). In this plot the two symbol frequencies f_L and f_H are separated by $2f_s$ (Here $f_s = f_b$ for BFSK). The same principle of BFSK is extended to M-ary FSK. That is M-carriers are added with separation of $2f_s$ between the carrriers (Note here that f_s is symbol frequency and not f_b). Therefore power spectral density for M-ary FSK will be simply extension of BFSK. Fig. 5.8.3 shows the power spectral density of M-ary FSK.

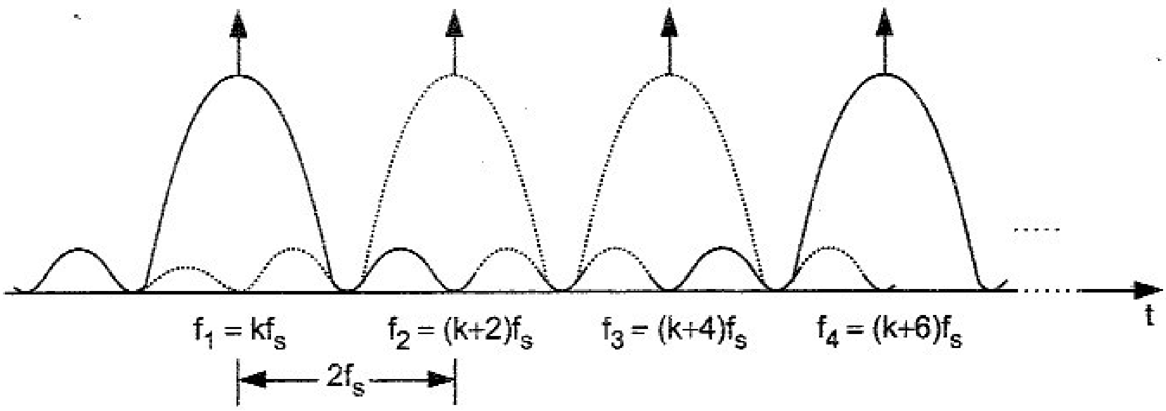

Fig. 5.8.3 Power spectral density M-ary FSK

Observe in the above figure that, the separation between the two nearest main lobes is $2f_s$.

Bandwidth of M-ary FSK :

From Fig. 5.8.3 it is clear that the width of one main lobe is $2f_s$. If there are M-symbols, then power spectral density spectrum will have M lobes. Therefore bandwidth of the system for M-symbols will be

$$BW = M \times (2f_s)$$

$$= 2Mf_s \quad \dots (5.8.3)$$

We know that $2^N = M$ and $f_s = \frac{f_b}{N}$ we can write the above equations,

$$BW = 2 \cdot 2^N \cdot \frac{f_b}{N} \quad \dots (5.8.4)$$

$$= \frac{2^{N+1} f_b}{N} \quad \dots (5.8.5)$$

On comparison of above equation with M-ary PSK bandwidth of equation 5.5.12, it is clear that M-ary FSK needs comparatively large bandwidth.

5.8.3 Geometrical Representation of M-ary FSK or Signal Space Representation

We know that in M-ary FSK, mutually orthogonal signals are used for transmission. Equation 5.7.18 and 5.7.19 give the two mutually orthogonal signals for BFSK. Similarly we can write the equation for M-ary FSK i.e.

$$\left.\begin{aligned} s_0(t) &= \sqrt{P_s T_s}\,\phi_0(t) \\ s_1(t) &= \sqrt{P_s T_s}\,\phi_1(t) \\ s_2(t) &= \sqrt{P_s T_s}\,\phi_2(t) \\ &\vdots \\ &\vdots \\ s_M(t) &= \sqrt{P_s T_s}\,\phi_{M-1}(t) \end{aligned}\right\} \quad \ldots (5.8.6)$$

Here $s_0(t), s_1(t), s_2(t) \ldots s_{M-1}(t)$ are mutually orthogonal signals for 'M' symbols. The orthogonal carriers $\phi_0(t), \phi_1(t), \phi_2(t) \ldots \phi_{M-1}(t)$ etc. can be represented as follows (i.e. extension of equation 5.5.16 and equation 5.5.17).

$$\left.\begin{aligned} \phi_0(t) &= \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t) \\ \phi_2(t) &= \sqrt{\frac{2}{T_s}}\cos(2\pi f_1 t) \\ &\vdots \\ \phi_{M-1}(t) &= \sqrt{\frac{2}{T_s}}\cos(2\pi f_{M-1} t) \end{aligned}\right\} \quad \ldots (5.8.7)$$

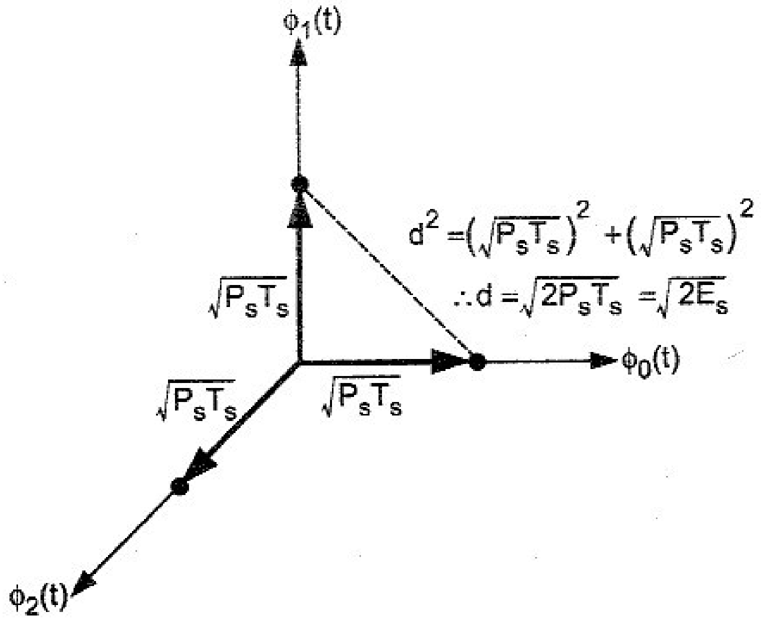

Fig. 5.8.4 Signal space (Geometrical) representation of M-ary FSK for M = 3

In the signal space $\phi_1(t), \phi_2(t), \phi_3(t)$ ..... etc. form mutually perpendicular axes. For simplicity of understanding we will consider $M = 3$. Then the three carriers $\phi_0(t), \phi_1(t)$ and $\phi_2(t)$ will form three axes. Then the signals $s_0(t), s_1(t)$ and $s_2(t)$ will be represented by vectors of length $\sqrt{P_s T_s}$ along those axes. It is shown in Fig. 5.8.4.

Distance between signal points :

From the above figure it is clear that the distance between the signal points is,

$$d^2 = (\sqrt{P_s T_s})^2 + (\sqrt{P_s T_s})^2$$

$$= 2P_s T_s$$

$$\therefore \quad d = \sqrt{2P_s T_s} \quad \text{... (5.8.8)}$$

$$= \sqrt{2E_s} \quad \text{... (5.8.9)}$$

This equation gives the minimum distance between any two signal points. This relation holds for 'M' signal points since all axes are perpendicular to each other.

Since $E_s = NE_b$, we can write above equation as,.

$$d = \sqrt{2NE_b} \quad \text{... (5.8.10)}$$

The distance of M-ary FSK given by above equation is greater as compared to the distance of M-ary PSK and QASK.

Review Questions

1. *With relevant expressions and block diagrams explain the operation of M-ary FSK transmitter and receiver.*
2. *Determine the bandwidth required for M-ary FSK system. Draw the geometrical representation of M-ary FSK signals and findout distance between the signal points. What is the bandwidth of this system ?*
3. *Compare the performance of M-ary PSK and FSK.*

5.9 Minimum Shift Keying (MSK)

Nov./Dec.-2004, April/May-2005, May/June-2006

We studied QPSK technique. In QPSK the phase changes by 90° (OQPSK) or 180° (QPSK). This creates abrupt amplitude variations in the waveform. Therefore bandwidth requirement of QPSK is more. Filters of other methods can overcome those problems, but they have other side effects. For example, filters alters the amplitude of the waveform.

MSK overcomes those problems. In MSK the output waveform is continuous in phase hence there are no abrupt changes in amplitude. The sidelobes of MSK are very small hence bandpass filtering is not required to avoid interchannel interference. Fig. 5.9.1 shows the waveform of MSK. The binary bit sequence is shown at the top.

Review Questions

1. *Explain MSK with the help of relevent expressions and waveforms. Show that MSK is basically continuous phase FSK (CPFSK).*
2. *Explain the transmitter/receiver arrangement for MSK.*
3. *Compare MSK and QPSK. What is the bandwidth requirement of MSK ?*

5.10 Amplitude Shift Keying or ON-OFF Keying

Amplitude shift keying (ASK) or ON-OFF keying (OOK) is the simplest digital modulation technique. In this method, there is only one unit energy carrier and it is switched on or off depending upon the input binary sequence. The ASK waveform can be represented as,

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \qquad (T_0 \text{ transmit '1'}) \qquad \ldots (5.10.1)$$

To transmit symbol '0', the signal $s(t) = 0$. That is no signal is transmitted. $s(t)$ contains some complete cycles of carrier frequency 'f'. Thus,

symbol '1' ⇒ pulse is transmitted, symbol '0' ⇒ no pulse is transmitted.

Thus the ASK waveform looks like an ON-OFF of the signal. Hence it is also called ON-OFF keying (OOK). Fig. 5.10.1 below shows the ASK waveform.

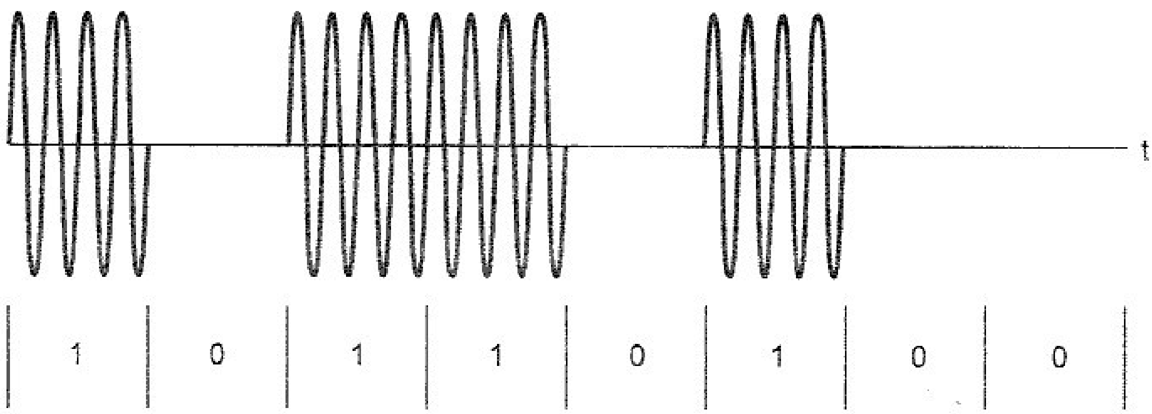

Fig. 5.10.1 ASK waveform

5.10.1 Signal Space Diagram of ASK

The ASK waveform of equation 5.10.1 for symbol '1' can be represented as,

$$s(t) = \sqrt{P_s T_b} \cdot \sqrt{2 / T_b} \cos(2\pi f_0 t)$$

$$= \sqrt{P_s T_b}\, \phi_1(t) \qquad \ldots (5.10.2)$$

Thus there is only one carrier function $\phi_1(t)$. The signal space diagram will have two points on $\phi_1(t)$. One will be at zero and other will be at $\sqrt{P_s T_b}$. Fig. 5.10.2 shows this.

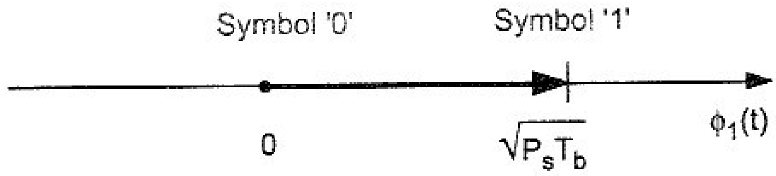

Fig. 5.10.2 Signal space diagram of ASK

Therefore the distance between the two signal points will be,

$$d = \sqrt{P_s T_b} = \sqrt{E_b} \quad \dots (5.10.3)$$

5.10.2 Generator and Detector of ASK

5.10.2.1 ASK Generator

Fig. 5.10.3 shows the ASK generator. The input binary sequence is applied to the product modulator. The product modulator amplitude modulates the sinusoidal carrier. It passes the carrier when input bit is '1'. It blocks the carrier (i.e. zero output) when intput bit is '0'. The wavefrom of ASK is as shown in Fig. 5.10.1.

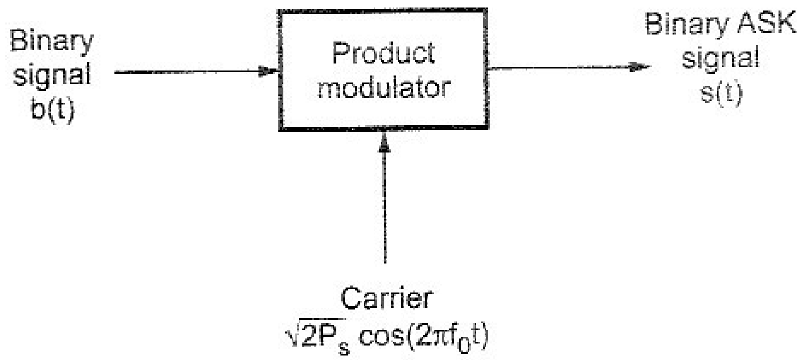

Fig. 5.10.3 Block diagram of ASK generator

5.10.2.2 ASK Detector

Fig. 5.10.4 shows the block diagram of coherent ASK detector. The ASK signal is applied to the correlator consisting of multiplier and integrator. The locally generated coherent carrier is applied to the multiplier. The output of multiplier is integrated over one bit period. The decision device takes the decision at the end of every bit period. It compares the output of integrator with the threshold. Decision is taken in favour of '1' when threshold is exceeded. Decision is taken as '0' if threshold is not exceeded.