5.1.5 Passband Transmission Model

Fig. 5.1.2 shows the model of passband data transmission system

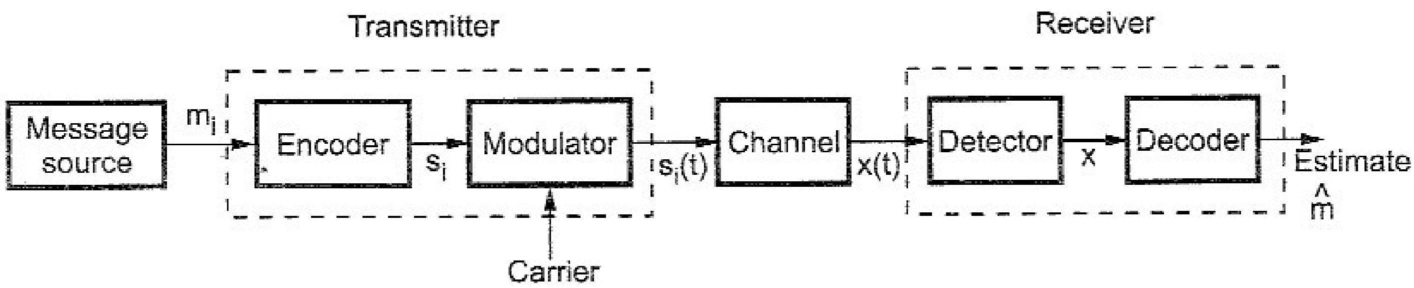

Fig. 5.1.2 Model of passband data transmission system

1. **Message source** : It emits the symbol at the rate of T seconds.
2. **Encoder** : It is signal transmission encoder. It produces the vector s_i made up of 'N' real elements. The vector s_i is unique for each set of 'M' symbols.
3. **Modulator** : It constructs the modulated carrier signal $s_i(t)$ of duration 'T' seconds for every symbol m_i. The signal $s_i(t)$ is energy signal.
4. **Channel** : The modulated signal $s_i(t)$ is transmitted over the communication channel.
   - The channel is assumed to be linear and of enough bandwidth to accommodate the signal $s_i(t)$.
   - The channel noise is white Gaussian of zero mean and psd of $\frac{N_0}{2}$.
5. **Detector** : It demodulates the received signal and obtains an estimate of the signal vector.
6. **Decoder** : The decoder obtains the estimate of symbol back from the signal vector. Here note that the detector and decoder combinely perform the reception of the transmitted signal. The effect of channel noise is minimized and correct estimate of symbol $\hat{m}$ is obtained.

5.2 Binary Phase Shift Keying (BPSK)

5.2.1 Principle of BPSK

- In binary phase shift keying (BPSK), binary symbol '1' and '0' modulate the phase of the carrier. Let the carrier be,

$$s(t) = A\cos(2\pi f_0 t) \quad \text{... (5.2.1)}$$

'A' represents peak value of sinusoidal carrier. In the standard 1Ω load register, the power dissipated will be,

$$P = \frac{1}{2}A^2$$

$\therefore \qquad A = \sqrt{2P}$ ... (5.2.2)

- When the symbol is changed, then the phase of the carrier is changed by 180 degrees (π radians).
- Consider for example,

Symbol '1' $\Rightarrow s_1(t) = \sqrt{2P}\cos(2\pi f_0 t)$ ... (5.2.3)

if next symbol is '0' then,

Symbol '0' $\Rightarrow s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$ ... (5.2.4)

Since $\cos(\theta + \pi) = -\cos\theta$, we can write above equation as,

$$s_2(t) = -\sqrt{2P}\cos(2\pi f_0 t) \qquad \text{... (5.2.5)}$$

With the above equation we can define BPSK signal combinely as,

$$\boxed{s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)} \qquad \text{... (5.2.6)}$$

Here $b(t) = +1$ when binary '1' is to be transmitted

$= -1$ when binary '0' is to be transmitted

5.2.2 Graphical Representation of BPSK Signal

Fig. 5.2.1 shows binary signal and its equivalent signal $b(t)$.

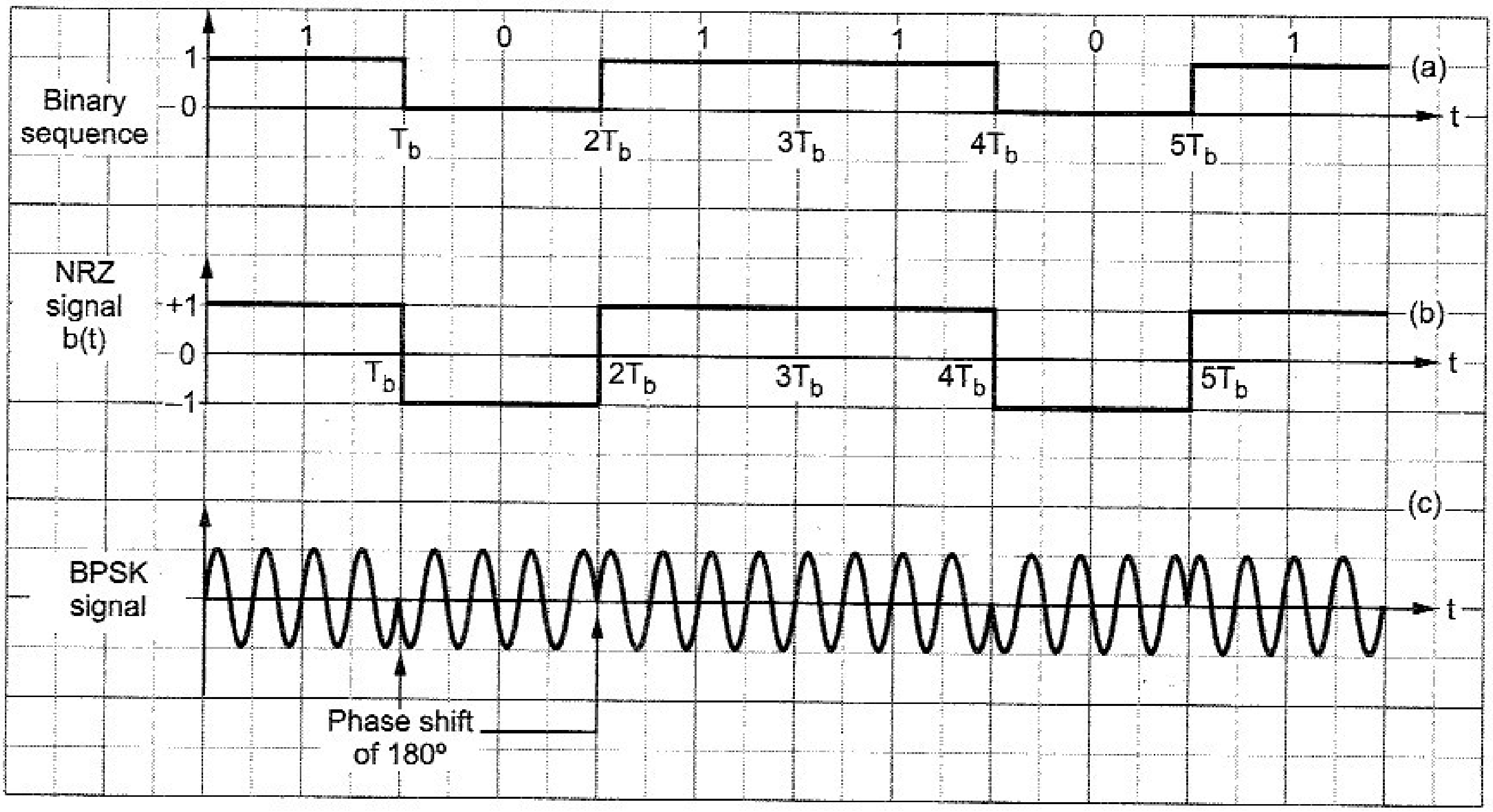

Fig. 5.2.1 (a) Binary sequence (b) Its equivalent bipolar signal $b(t)$ (c) BPSK signal

As can be seen from Fig. 5.2.1 (b), the signal $b(t)$ is NRZ bipolar signal. This signal directly modulates carrier $\cos(2\pi f_0 t)$.

5.2.3 Generation and Reception of BPSK Signal

Nov./Dec.- 2005

5.2.3.1 Generator of BPSK Signal

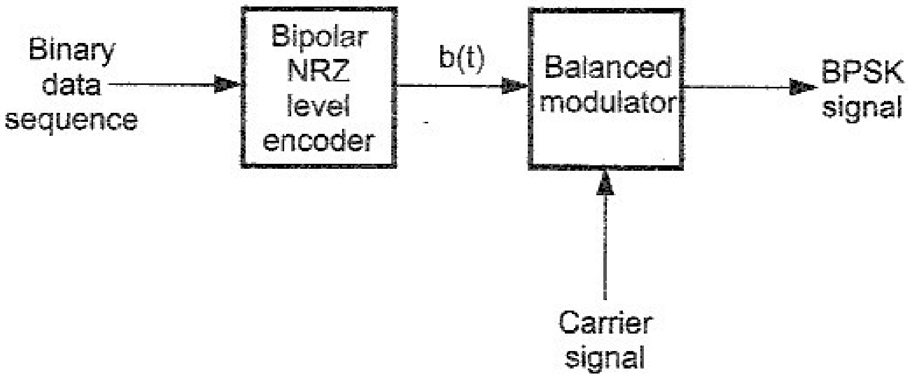

Fig. 5.2.2 BPSK generation scheme

- The BPSK signal can be generated by applying carrier signal to the balanced modulator.
- The baseband signal $b(t)$ is applied as a modulating signal to the balanced modulator. Fig. 5.2.2 shows the block diagram of BPSK signal generator.
- The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

5.2.3.2 Reception of BPSK Signal

Fig. 5.2.3 shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmitted BPSK signal is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

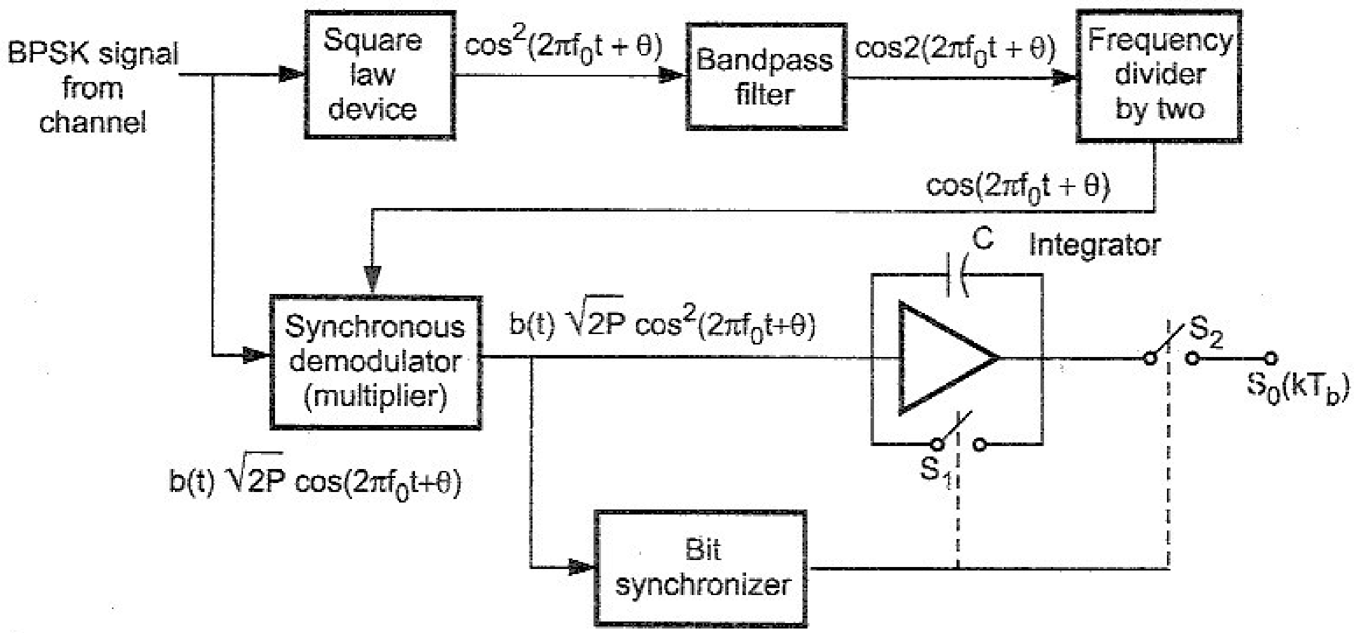

Fig. 5.2.3 Reception BPSK scheme

Operation of the receiver

1) **Phase shift in received signal** : This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in the transmitted signal. Let the phase shift be θ. Therefore the signal at the input of the receiver is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta) \quad \text{... (5.2.7)}$$

2) **Square law device** : Now from this received signal, a carrier is separated since this is coherent detection. As shown in the figure, the received signal is passed through a square law device. At the output of the square law device the signal will be,

$$\cos^2(2\pi f_0 t+\theta)$$

Note here that we have neglected the amplitude, because we are only interested in the carrier of the signal.

We know that,

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\therefore \quad \cos^2(2\pi f_0 t+\theta) = \frac{1+\cos 2(2\pi f_0 t+\theta)}{2}$$

or

$$= \frac{1}{2}+\frac{1}{2}\cos 2(2\pi f_0 t+\theta)$$

Here $\frac{1}{2}$ represents a DC level.

3) **Bandpass filter** : This signal is then passed through a bandpass filter whose passband is centered around $2f_0$. Bandpass filter removes the DC level of $\frac{1}{2}$ and at its output we get,

$$\cos 2(2\pi f_0 t+\theta)$$

This signal has frequency of $2f_0$.

4) **Frequency divider** : The above signal is passed through a frequency divider by two. Therefore at the output of frequency divider we get a carrier signal whose frequency is f_0 i.e. $\cos(2\pi f_0 t+\theta)$.

5) **Synchronous demodulator** : The synchronous (coherent) demodulator multiplies the input signal and the recovered carrier. Therefore at the output of multiplier we get,

$$b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)\times\cos(2\pi f_0 t+\theta) = b(t)\sqrt{2P}\cos^2(2\pi f_0 t+\theta)$$

$$= b(t)\sqrt{2P}\times\frac{1}{2}[1+\cos 2(2\pi f_0 t+\theta)]$$

$$= b(t)\sqrt{\frac{P}{2}}[1+\cos 2(2\pi f_0 t+\theta)] \quad \text{... (5.2.8)}$$

6) **Bit synchronizer and integrator :** The above signal is then applied to the bit synchronizer and integrator. The integrator integrates the signal over one bit period. The bit synchronizer takes care of starting and ending times of a bit.

- At the end of bit duration T_b, the bit synchronizer closes switch S_2 temperorily. This connects the output of an integrator to the decision device. It is equivalent to sampling the output of integrator.
- The synchronizer then opens switch S_2 and switch S_1 is closed temperorily. This resets the integrator voltage to zero. The integrator then integrates next bit.
- Let us assume that one bit period 'T_b' contains integral number of cycles of the carrier. That is the phase change occurs in the carrier only at zero crossing. This is shown in Fig. 5.2.1 (c). Thus BPSK waveform has full cycles of sinusoidal carrier.

To show that output of integrator depends upon transmitted bit

- In the k^{th} bit interval we can write output signal as,

$$s_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} [1+\cos 2(2\pi f_0 t+\theta)]\, dt$$

from equation 5.2.8

The above equation gives the output of an interval for k^{th} bit. Therefore integration is performed from $(k-1)T_b$ to kT_b. Here T_b is the one bit period.

- We can write the above equation as,

$$s_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\left[\int_{(k-1)T_b}^{kT_b} 1\, dt + \int_{(k-1)T_b}^{kT_b} \cos 2(2\pi f_0 t+\theta)\, dt\right]$$

Here $\int_{(k-1)T_b}^{kT_b} \cos 2(2\pi f_0 t+\theta)\, dt = 0$, because average value of sinusoidal waveform is zero if integration is performed over full cycles. Therefore we can write above equation as,

$$s_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} 1\, dt$$

$$= b(kT_b)\sqrt{\frac{P}{2}}\,[t]_{(k-1)\,T_b}^{k\,T_b}$$

$$= b(kT_b)\sqrt{\frac{P}{2}}\left\{kT_b - (k-1)\,T_b\right\}$$

$$\therefore \qquad \boxed{s_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\,T_b} \qquad \text{... (5.2.9)}$$

This equation shows that the output of the receiver depends on input i.e.

$$s_o(kT_b) \;\alpha\; b(kT_b)$$

Depending upon the value of $b(kT_b)$, the output $s_0(kT_b)$ is generated in the receiver.

This signal is then given to a decision device (not shown in Fig. 5.2.3), which decides whether transmitted symbol was zero or one.

5.2.4 Spectral Characteristics of BPSK Signals

Step 1 : Fourier transform of basic NRZ pulse.

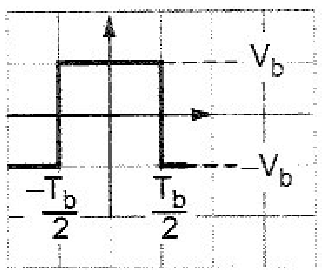

Fig. 5.2.4 NRZ pulse

We know that the waveform $b(t)$ is NRZ bipolar waveform. In this waveform there are rectangular pulses of amplitude $\pm V_b$. If we say that each pulse is $\pm\frac{T_b}{2}$ around its center as shown in Fig. 5.2.4. then it becomes easy to find fourier transform of such pulse. The fourier transform of this type of pulse is given as,

$$X(f) = V_b\,T_b\,\frac{\sin(\pi f T_b)}{(\pi f T_b)} \quad \text{By standard relations} \qquad \text{... (5.2.10)}$$

Step 2 : PSD of NRZ pulse.

For large number of such positive and negative pulses the power spectral density $S(f)$ is given as

$$S(f) = \frac{\overline{|X(f)|^2}}{T_s} \qquad \text{... (5.2.11)}$$

Here $\overline{X(f)}$ denotes average value of $X(f)$ due to all the pulses in $b(t)$. And T_s is symbol duration. Putting value of $X(f)$ from equation 5.2.10 in equation 5.2.11 we get,

$$S(f) = \frac{V_b^2 T_b^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

Step 3 : PSD of baseband signal *b*(*t*)

For BPSK since only one bit is transmitted at a time, symbol and bit durations are same i.e. $T_b = T_s$. Then above equation becomes,

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \quad \text{... (5.2.12)}$$

The above equation gives the power spectral density of baseband signal $b(t)$.

Step 4 : PSD of BPSK signal.

The BPSK signal is generated by modulating a carrier by the baseband signal $b(t)$. Because of modulation of the carrier of frequency f_0, the spectral components are translated from f to $f_0 + f$ and $f_0 - f$. The magnitude of those components is divided by half.

Therefore from equation 5.2.12 we can write the power spectral density of BPSK signal as,

$$S_{BPSK}(f) = V_b^2 T_b \left\{\frac{1}{2}\left[\frac{\sin\pi(f_0 - f)T_b}{\pi(f_0 - f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f_0 + f)T_b}{\pi(f_0 + f)T_b}\right]^2\right\}$$

The above equation is composed of two half magnitude spectral components of same frequency 'f' above and below f_0. Let us say that the value of $\pm V_b = \pm\sqrt{P}$. That is the NRZ signal is having amplitudes of $+\sqrt{P}$ and $-\sqrt{P}$. Then above equation becomes,

$$S_{BPSK}(f) = \frac{P T_b}{2}\left\{\left[\frac{\sin\pi(f - f_0)T_b}{\pi(f - f_0)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f_0 + f)T_b}{\pi(f_0 + f)T_b}\right]^2\right\} \quad \text{... (5.2.13)}$$

The above equation gives power spectral density of BPSK signal for modulating signal $b(t)$ having amplitudes of $\pm\sqrt{P}$. We know that modulated signal is given by equation 5.2.3 and equation 5.2.5 as,

$$s(t) = \pm\sqrt{2P}\cos(2\pi f_0 t) \qquad \text{since } A = \sqrt{2P}$$

If $b(t) = \pm\sqrt{P}$, then the carrier becomes,

$$\phi(t) = \sqrt{2}\cos(2\pi f_0 t) \quad \text{... (5.2.14)}$$

Plot of PSD

- Equation 5.2.12 gives power spectral density of the NRZ waveform. For one rectangular pulse, the shape of $S(f)$ will be a sinc pulse as given by equation 5.2.12. Fig. 5.2.5 shows the plot of magnitude of $S(f)$.

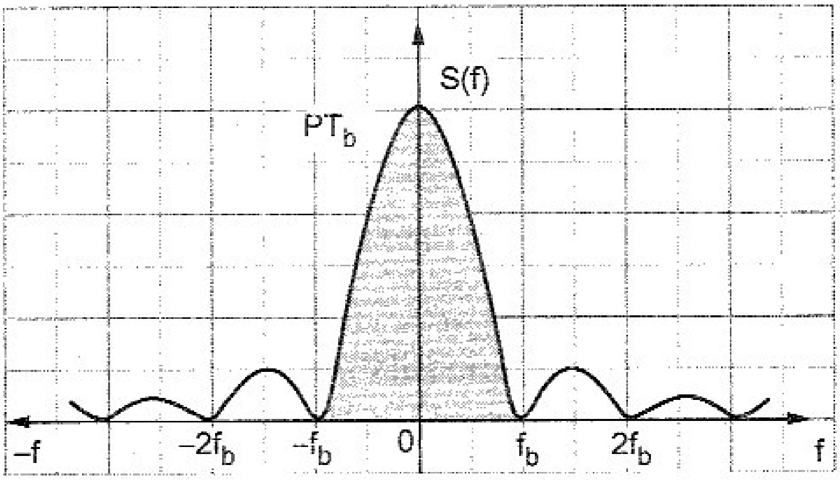

Fig. 5.2.5 Plot of power spectral density of NRZ baseband signal

Above figure shows that the main lobe ranges from $-f_b$ to $+f_b$. Here $f_b = \frac{1}{T_b}$.

Since we have taken $\pm V_b = \pm\sqrt{P}$ in equation 5.2.12, the peak value of the main lobe is $P T_b$.

- Now let us consider the power spectral density of BPSK signal given by equation 5.2.15. Fig. 5.2.6 shows the plot of this equation. The figure thus clearly shows that there are two lobes ; one at f_0 and other at $-f_0$. The same spectrum of Fig. 5.2.5 is placed at $+f_0$ and $-f_0$. But the amplitudes of main lobes are $\frac{P T_b}{2}$ in Fig. 5.2.6.

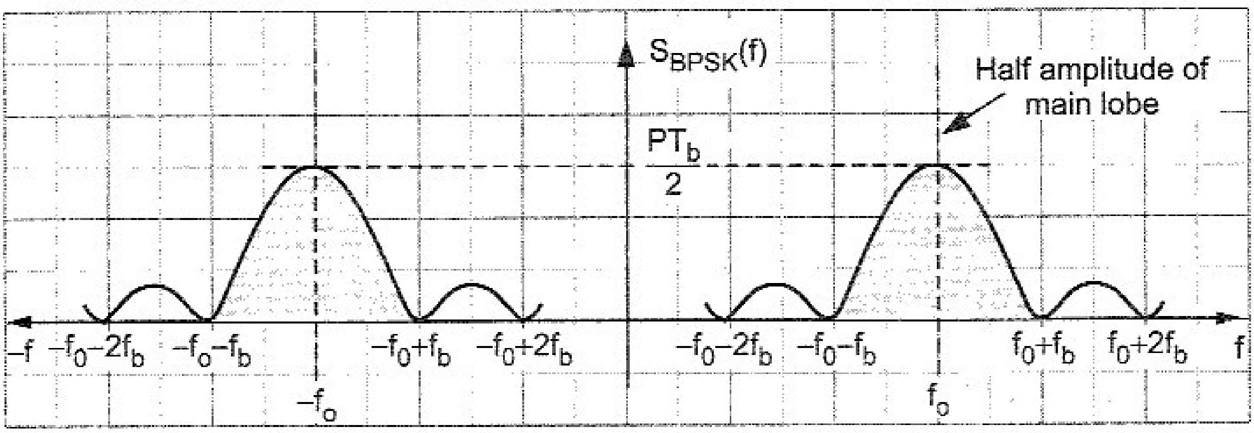

Fig. 5.2.6 Plot of power spectral density of BPSK signal

Thus they are reduced to half. The spectrums of $S(f)$ as well as $S_{BPSK}(f)$ extends over all the frequencies.

Interchannel Interference and ISI :

- Let's assume that BPSK signals are multiplexed with the help of different carrier frequencies for different baseband signals. Then at any frequency, the spectral components due to all the multiplexed channels will be present. This is because $S(f)$ as well as $S_{BPSK}(f)$ of every channel extends over all the frequency range.
- Therefore a BPSK receiver tuned to a particular carrier frequency will also receive frequency components due to other channels. This will make interference with the required channel signals and error probability will increase. This result is called *Interchannel Interference.*
- To avoid interchannel interference, the BPSK signal is passed through a filter.This filter attenuates the side lobes and passes only main lobe. Since side lobes are attenuated to high level, the interference is reduced. Because of this filtering the phase distortion takes place in the bipolar NRZ signal, i.e. $b(t)$. Therefore the individual bits (symbols) mix with adjacent bits (symbols) in the same channel. This effect is called *intersymbol interference* or ISI.
- The effect of ISI can be reduced to some extent by using equalizers at the receiver. Those equalizers have the reverse effect to that filter's adverse effects. Normally equalizers are also filter structures.

5.2.5 Geometrical Representation of BPSK Signals

We know that BPSK signal carries the information about two symbols. Those are symbol '1' and symbol '0'. We can represent BPSK signal geometrically to show those two symbols.

(i) From equation 5.2.6 we know that BPSK signal is given as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t) \qquad \text{... (5.2.15)}$$

(ii) Let's rearrange the above equation as,

$$s(t) = b(t)\sqrt{P T_b} \cdot \sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t) \qquad \text{... (5.2.16)}$$

(iii) Let $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)$ represents an orthonormal carrier signal. Equation 5.2.14 also gives equation for carrier. It is slightly different than $\phi_1(t)$ defined here. Then we can write equation 5.2.16 as,

$$s(t) = b(t)\sqrt{P T_b}\,\phi_1(t) \qquad \text{... (5.2.17)}$$

(iv) The bit energy E_b is defined in terms of power 'P' and bit duration T_b as,

$$E_b = P T_b \quad \text{... (5.2.18)}$$

∴ Equation 5.2.17 becomes,

$$s(t) = \pm \sqrt{E_b}\, \phi_1(t) \quad \text{... (5.2.19)}$$

Here $b(t)$ is simply ± 1.

(v) Thus on the single axis of $\phi_1(t)$ there will be two points. One point will be located at $+\sqrt{E_b}$ and other point will be located at $-\sqrt{E_b}$. This is shown in Fig: 5.2.7.

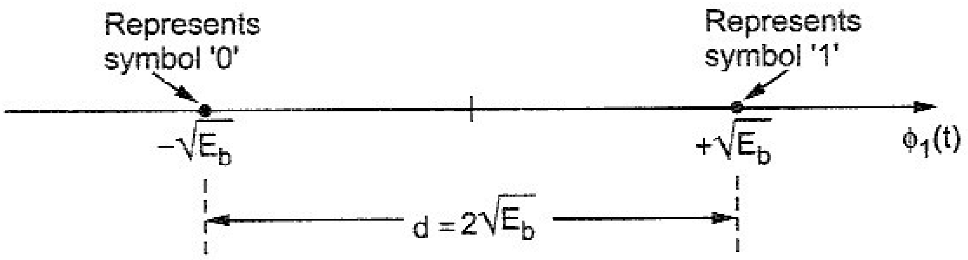

Fig. 5.2.7 Geometrical representation of BPSK signal

At the receiver the point at $+\sqrt{E_b}$ on $\phi_1(t)$ represents symbol '1' and point at $-\sqrt{E_b}$ represents symbol '0'. The separation between these two points represent the isolation in symbols '1' and '0' in BPSK signal. This separation is normally called distance 'd'. From Fig.5.2.7 it is clear that the distance between the two points is,

$$d = +\sqrt{E_b} - (-\sqrt{E_b})$$

$$\therefore \quad d = 2\sqrt{E_b} \quad \text{... (5.2.20)}$$

As this distance 'd' increases, the isolation between the symbols in BPSK signal is more. Therefore probability of error reduces.

5.2.6 Bandwidth of BPSK Signal

The spectrum of the BPSK signal is centered around the carrier frequency f_0.

If $f_b = \frac{1}{T_b}$, then for BPSK the maximum frequency in the baseband signal will be f_b see Fig. 5.2.6. In this figure the main lobe is centered around carrier frequency f_0 and extends from $f_0 - f_b$ to $f_0 + f_b$. Therefore bandwidth of BPSK signal is,

$$BW = \text{Highest frequency} - \text{Lowest frequency in the main lobe}$$

$$= f_0 + f_b - (f_0 - f_b)$$

$$\therefore \quad \boxed{BW = 2f_b} \quad \text{... (5.2.21)}$$

Thus the minimum bandwidth of BPSK signal is equal to twice of the highest frequency contained in baseband signal.

5.2.7 Drawbacks of BPSK : Ambiguity in Output Signal

Fig. 5.2.3 shows the block diagram of BPSK receiver. To regenerate the carrier in the receiver, we start by squaring $b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$. If the received signal is $-b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$ then the squared signal remains same as before. Therefore the recovered carrier is unchanged even if the input signal has changed its sign. Therefore it is not possible to determine whether the received signal is equal to $b(t)$ or $-b(t)$. This result in ambiguity in the output signal.

This problem can be removed if we use differential phase shift keying. But Differential Phase Shift Keying (DPSK) also has some other problems. DPSK is given in detail in the next section. Other problems of BPSK are ISI and Interchannel interference. These problems are reduced to some extent by use of filters.

Example 5.2.1 : *Determine the minimum bandwidth for a BPSK modulator with a carrier frequency of 40 MHz and an input bit rate of 500 kbps.*

Solution : The input bit rate indicates highest frequency of the baseband signal.

Hence,

$$f_b = 500 \text{ kbps} = 500 \text{ kHz.}$$

From equation 5.2.21, the bandwidth of the BPSK system is given as,

$$\text{BW} = 2f_b = 2 \times 500 \text{ kHz} = 1 \text{ MHz}$$

Review Questions

1. *Explain BPSK system with the help of transmitter and receiver, and state its advantages/disadvantages over other system.*
2. *Derive an expression for spectrum of BPSK system and hence calculate the bandwidth required.*

In the Fig. 5.3.1 observe that the transmitted signal is given as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_o t)$$

$$= \pm\sqrt{2P}\cos(2\pi f_o t)$$

i.e.

$$s(t) = \begin{cases} \sqrt{2P}\cos(2\pi f_o t) & \text{when} \quad b(t)=1 \\ -\sqrt{2P}\cos(2\pi f_o t) & \text{when} \quad b(t)=0 \end{cases}$$

The above equations can also be written as,

$$s(t) = \begin{cases} \sqrt{2P}\cos(2\pi f_o t + 0) & \text{when} \quad b(t)=1 \\ -\sqrt{2P}\cos(2\pi f_o t + \pi) & \text{when} \quad b(t)=0 \end{cases}$$

The transmitted phase sequence is shown in Fig. 5.3.4 as per the above equation.

Review Questions

1. *With the help of block diagram, waveforms and expressions explain the operation of DPSK transmitter and receiver.*
2. *What are the advantages and disadvantages of DPSK ? What is the bandwidth requirement of DPSK ?*

5.4 Quadrature Phase Shift Keying

May/June-2006

Principle

- In communication systems we know that there are two main resources, i.e. transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or signalling rate f_b. In digital bandpass transmission, a carrier is used for transmission. This carrier is transmitted over a channel.
- If two or more bits are combined in some symbols, then the signalling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the transmission channel bandwidth. Thus because of grouping of bits in symbols, the transmission channel bandwidth is reduced.
- In quadrature phase shift keying, two successive bits in the data sequence are grouped together. This reduces the bits rate of signalling rate (i.e. f_b) and hence reduces the bandwidth of the channel.
- In BPSK we know that when symbol changes the level, the phase of the carrier is changed by 180°. Since there were only two symbols in BPSK, the phase shift occurs in two levels only.
- In QPSK two successive bits are combined. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the

phase of the carrier is changed by 45° (π / 4 radians). Table 5.4.1 shows these symbols and their phase shifts.

Sr.No.	Input successive bits		Symbol	Phase shift in carrier
$i = 1$	1(1V)	0 (−1V)	S_1	$\pi / 4$
$i = 2$	0 (−1V)	0 (−1V)	S_2	$3\pi / 4$
$i = 3$	0 (−1V)	1(1V)	S_3	$5\pi / 4$
$i = 4$	1(1V)	1(1V)	S_4	$7\pi / 4$

Table 5.4.1 Symbol and corresponding phase shifts in QPSK

Thus as shown in above table, there are 4 symbols and the phase is shifted by π / 4 for each symbol.

5.4.1 QPSK Transmitter and Receiver

5.4.1.1 Offset QPSK (OQPSK) or Staggered QPSK Transmitter

Operation and waveforms April/May-2004

Step 1 : Input Sequence Converted to NRZ type :

Fig. 5.4.1 shows the block diagram of OQPSK transmitter. The input binary sequence is first converted to a bipolar NRZ type of signal. This signal is called $b(t)$. It represents binary '1' by $+1V$ and binary '0' by $-1V$. This signal is shown in Fig.5.4.2(a).

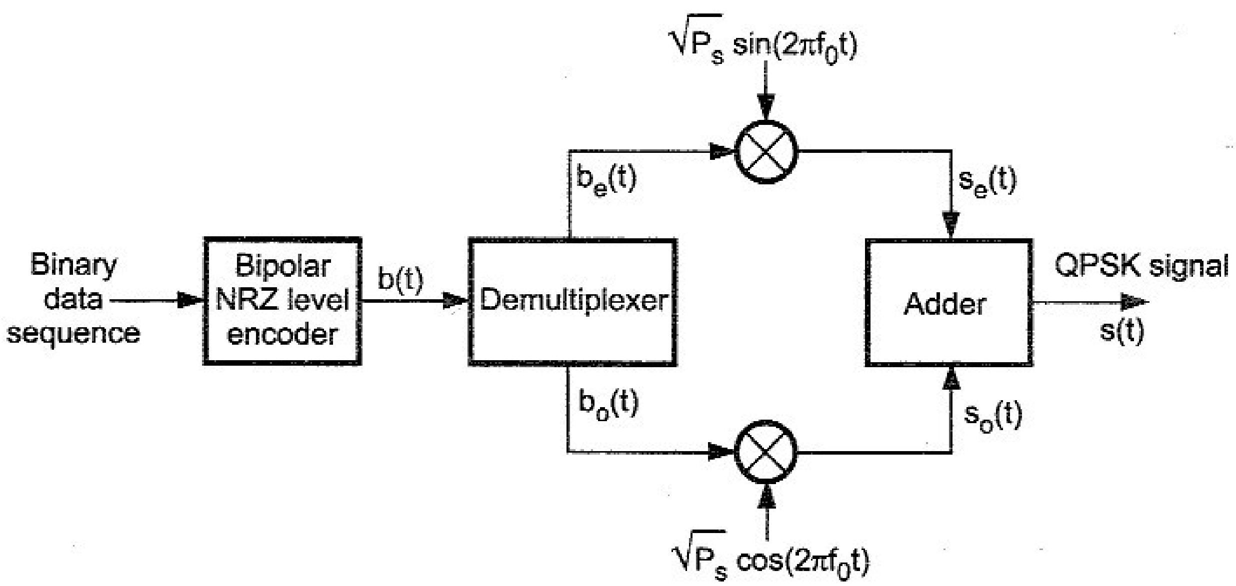

Fig. 5.4.1 An offset QPSK transmitter

Step 2 : Demultiplexing into odd and even numbered sequences

The demultiplexer divides $b(t)$ into two separate bit streams of the odd numbered and even numbered bits. $b_e(t)$ represents even numbered sequence and $b_o(t)$ represents odd numbered sequence. The symbol duration of both of these odd and even numbered sequences is $2T_b$. Thus every symbol contains two bits. Fig. 5.4.2 (b) and (c) shows the waveforms of $b_e(t)$ and $b_o(t)$.

Observe that the first even bit occurs after the first odd bit. Therefore even numbered bit sequence $b_e(t)$ starts with the delay of one bit period due to first odd bit. Thus first symbol of $b_e(t)$ is delayed by one bit period 'T_b' with respect to first symbol of $b_o(t)$. This delay of T_b is called offset. Hence the name offset QPSK is given. This shows that the change in levels of $b_e(t)$ and $b_o(t)$ cannot occur at the same time because of offset or staggering.

Step 3 : Modulation of quadrature carriers

The bit stream $b_e(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_0 t)$ and $b_o(t)$ modulates $\sqrt{P_s}\sin(2\pi f_0 t)$. These modulators are balanced modulator. The two carriers $\sqrt{P_s}\cos(2\pi f_0 t)$ and $\sqrt{P_s}\sin(2\pi f_0 t)$ are shown in Fig. 5.4.2 (d) and (e). These carriers are also called quadrature carriers. The two modulated signals are,

$$s_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t) \quad \text{... (5.4.1)}$$

and

$$s_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) \quad \text{... (5.4.2)}$$

Thus $s_e(t)$ and $s_o(t)$ are basically BPSK signals and they are similar to equation 5.2.3 and equation 5.2.5. The only difference is that $T = 2T_b$ here. The value of $b_e(t)$ and $b_o(t)$ will be $+1V$ or $-1V$. Fig. 5.4.2 (f) and (g) shows the waveforms of $s_e(t)$ and $s_o(t)$.

Step 4 : Addition of modulated carriers

The adder of Fig. 5.4.1 adds these two signals $b_e(t)$ and $b_o(t)$. The output of the adder is OQPSK signal and it is given as,

$$\begin{aligned} s(t) &= s_o(t) + s_e(t) \\ &= b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t) \end{aligned} \quad \text{... (5.4.3)}$$

Step 5 : QPSK signal and phase shift

Fig. 5.4.2 (h) shows the QPSK signal represented by above equation. In QPSK signal of Fig. 5.4.2 (h), if there is any phase change, it occurs at minimum duration of T_b. This is because the two signals $s_e(t)$ and $s_o(t)$ have an offset of 'T_b'. Because of this offset, the phase shift in QPSK signal is $\frac{\pi}{2}$. It is clear from the waveforms of Fig. 5.4.2 that $b_e(t)$ and $b_o(t)$ cannot change at the same time because of offset between them. Fig. 5.4.3 shows the phasor diagram of QPSK signal of equation 5.4.2.

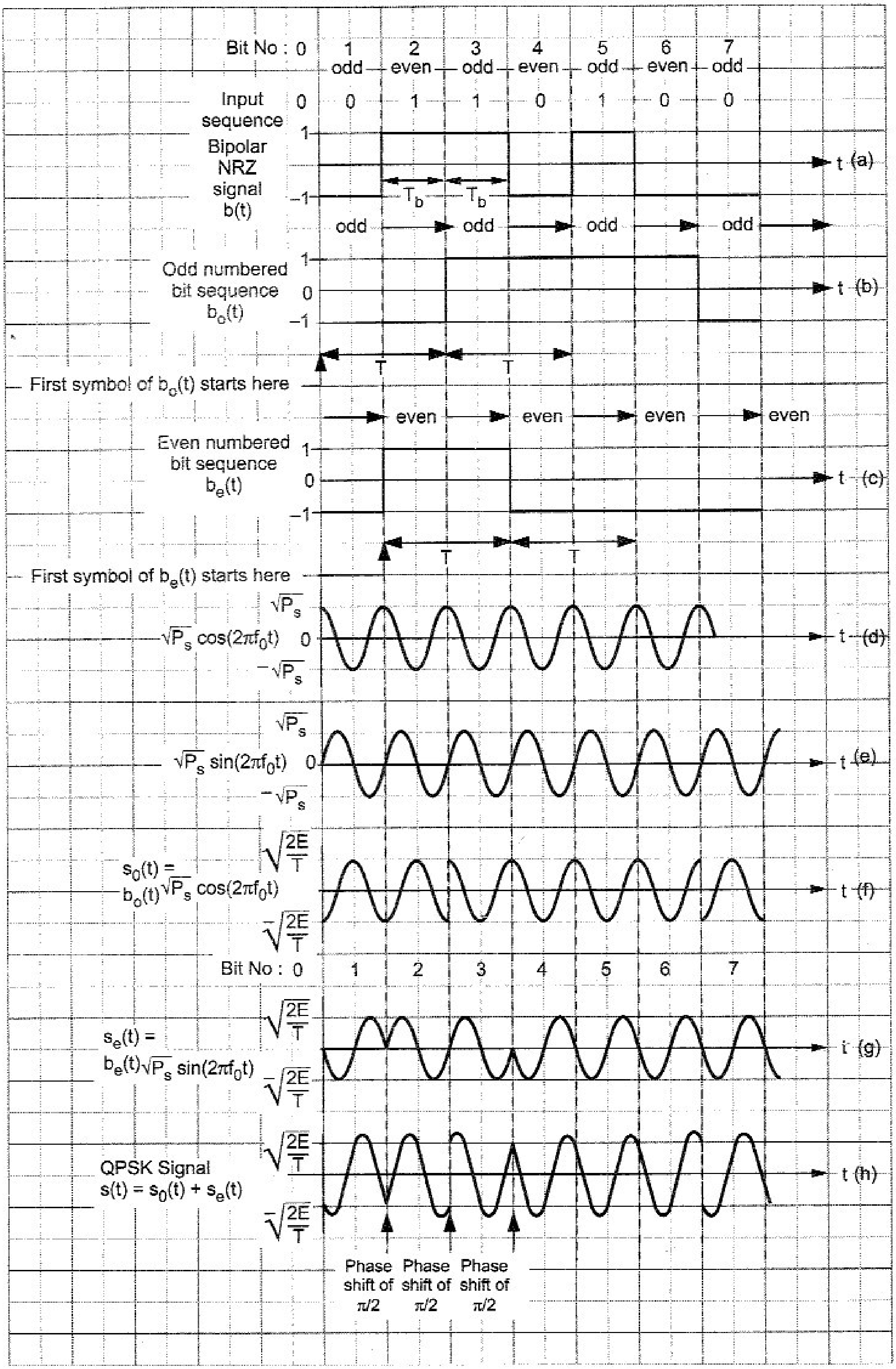

Fig. 5.4.2 QPSK waveforms (a) Input sequence and its NRZ waveform (b) Odd numbered bit sequence and its NRZ waveform (c) Even numbered bit sequence and its NRZ waveform (d) Basis function $\phi_1(t)$ (e) Basis function $\phi_2(t)$ (f) Binary PSK waveform for odd numbered channel (g) Binary PSK waveform for even numbered channel (h) Final QPSK waveform representing equation

Since $b_o(t)$ and $b_e(t)$ cannot change at the same time, the phase change in QPSK signal will be maximum $\pi / 2$. This is clear from Fig. 5.4.3.

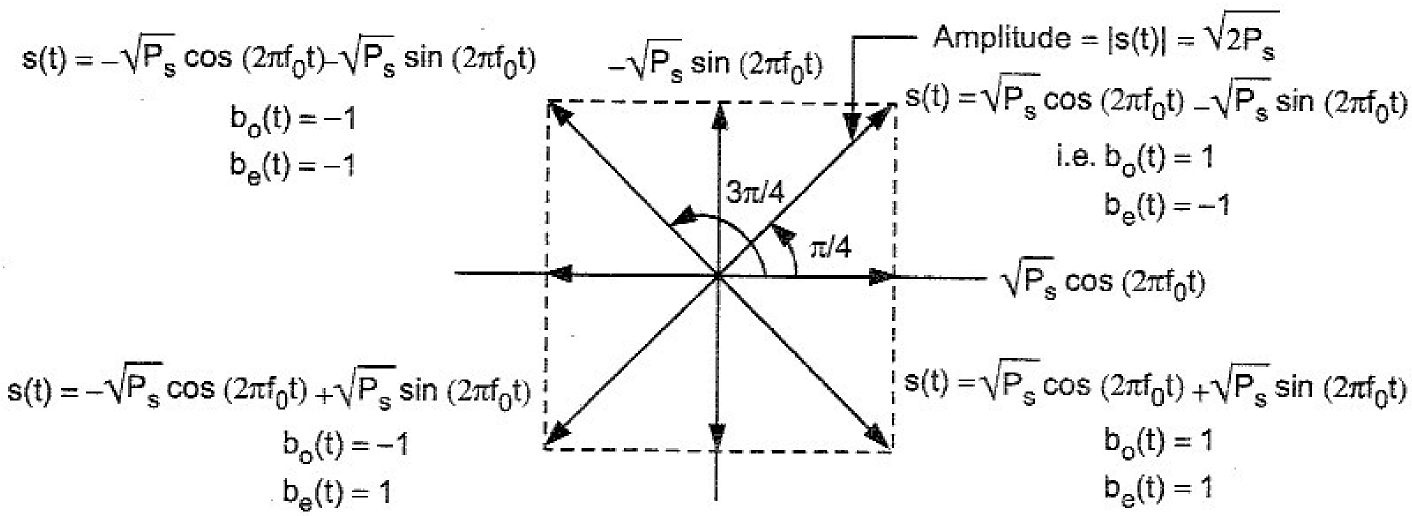

Fig. 5.4.3 Phasor diagram of QPSK signal

5.4.1.2 Non-Offset QPSK

- We known that there is an offset of 'T_b' between $b_e(t)$ and $b_o(t)$. If we delay $b_e(t)$ by 'T_b' then there will be no offset. Then the sequences $b_e(t)$ and $b_o(t)$ will change at the same time. This change will occur after minimum of '$2T_b$'.
- As a result, the signals $s_o(t)$ and $s_e(t)$ will have phase shifts at the same time. The individual phase shifts of $s_o(t)$ and $s_e(t)$ are 180°. Because of this the amplitude variations in the waveform will occur at the same time in $s_o(t)$ and $s_e(t)$. Therefore these variations will be more pronounced in non offset QPSK than OQPSK.
- Filters are used to suppress side bands in QPSK. Since phase changes by 180° in non offset QPSK, amplitude changes are more. Hence filtering affects the amplitude of non-offset QPSK. In OQPSK, the phase changes by 90°, hence amplitude changes during filtering are less.
- Since amplitude variations are more in non-offset QPSK, the signal is affected if communication takes place through repeators. These repeators highly affect the amplitude and phase of the QPSK signal.

5.4.1.3 The QPSK Receiver

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Fig. 5.4.4 shows the QPSK receiver. This is synchronous reception. Therefore coherent carrier is to be recovered from the received signal $s(t)$.

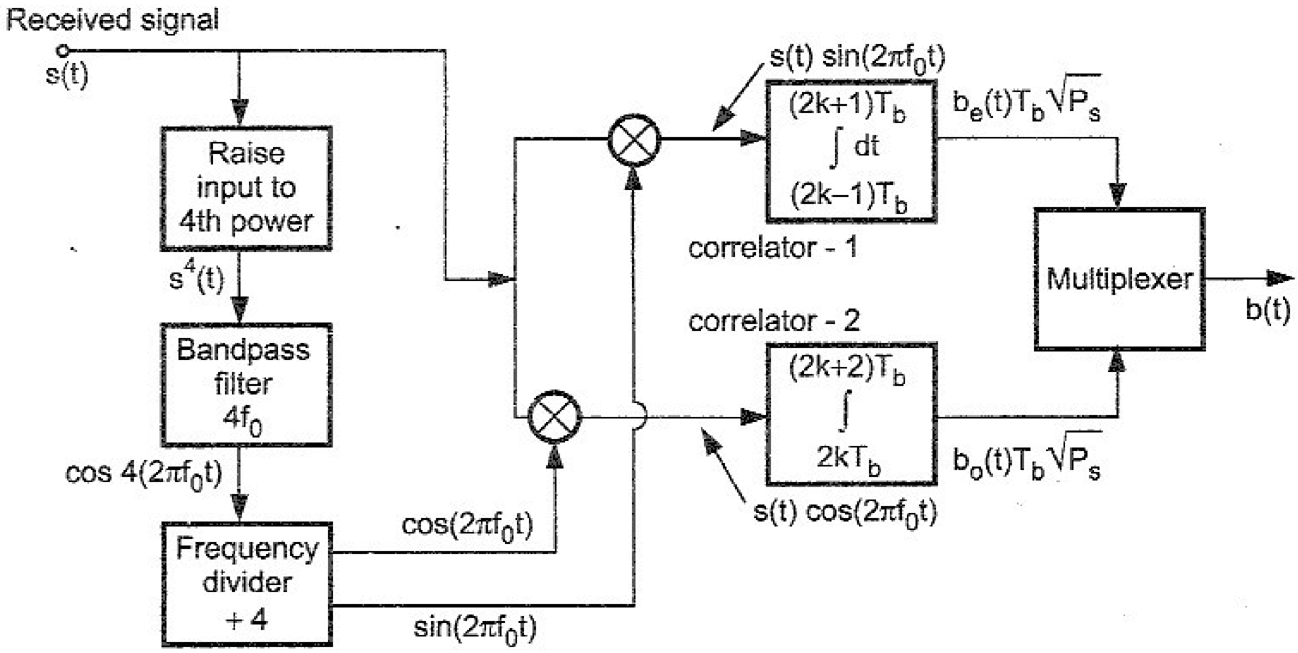

Fig. 5.4.4 QPSK receiver

Operation

Step 1 : Isolation of carrier

The received signal $s(t)$ is first raised to its 4^{th} power, i.e. $s^4(t)$. Then it is passed through a bandpass filter centered around $4f_0$. The output of the bandpass filter is a coherent carrier of frequency $4f_0$. This is divided by 4 and it gives two coherent quadrature carriers $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 (t)$.

Step 2 : Synchronous detection

These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.

Step 3 : Integration over two bits interval

The incoming signal is applied to both the multipliers. The integrator integrates the product signal over two bit interval (i.e. $T_s = 2T_b$).

Step 4 : Sampling and multiplexing odd and even bit sequences

At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b. Hence the output of multiplexer is the signal $b(t)$. That is, the odd and even sequences are combined by multiplexer.

To show that output of integrator depends upon respective bit sequence.

- Let's consider the product signal at the output of upper multiplier.

$$s(t)\sin(2\pi f_0 t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)\sin(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin^2(2\pi f_0 t) \quad \ldots (5.4.4)$$

- This signal is integrated by the upper integrator in Fig. 5.4.4.

$$\therefore \int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt$$

$$+ b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin^2(2\pi f_0 t)\,dt$$

Since $\frac{1}{2}\sin(2x) = \sin x\cdot\cos x$

and $\sin^2(x) = \frac{1}{2}[1-\cos(2x)]$

- Using the above two trigonometric identities in the above equation,

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = \frac{b_o(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin 4\pi f_0 t\,dt + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} 1\cdot dt$$

$$-\frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos 4\pi f_0 t\,dt$$

- In the above equation, the first and third integration terms involves integration of sinusoidal carriers over two bit period. They have full (integral number of) cycles over two bit period and hence integration will be zero.

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = \frac{b_e(t)\sqrt{P_s}}{2}[t]_{(2k-1)T_b}^{(2k+1)T_b}$$

$$= \frac{b_e(t)\sqrt{P_s}}{2}\times 2T_b$$

$$= b_e(t)\sqrt{P_s}\,T_b \quad \ldots (5.4.5)$$

- Thus the upper integrator responds to even sequence only. Similarly we can obtain the output of lower integrator as $b_o(t)\sqrt{P_s}\,T_b$.

Eventhough bit synchronizer is not shown in Fig. 5.4.4, it is assumed to be present with the integrator to locate starting and ending times of integration. The multiplexer is also operated by bit synchronizer. The amplitudes of signals marked in Fig. 5.4.4 are arbitrary. They can change depending upon the gains of integrator.

Ambiguity in the output :

In Fig. 5.4.4 observe that even if the received signal is negative, the recovered carrier remains unaffected because of the 4^{th} power conversion of the signal. Therefore it will not be possible to determine whether the transmitted signals were positive or negative [i.e. $+b_e(t)$ or $-b_e(t)$ and $+b_o(t)$ or $-b_o(t)$]. This is phase ambiguity in output similar to BPSK. This problem can be recovered by employing differential encoding and decoding of $b(t)$.

5.4.1.4 Carrier Synchronization in QPSK

Both the carriers are to be synchronized properly in coherent detection in QPSK. Fig. 5.4.5 shows the PLL system for carrier synchronization in QPSK.

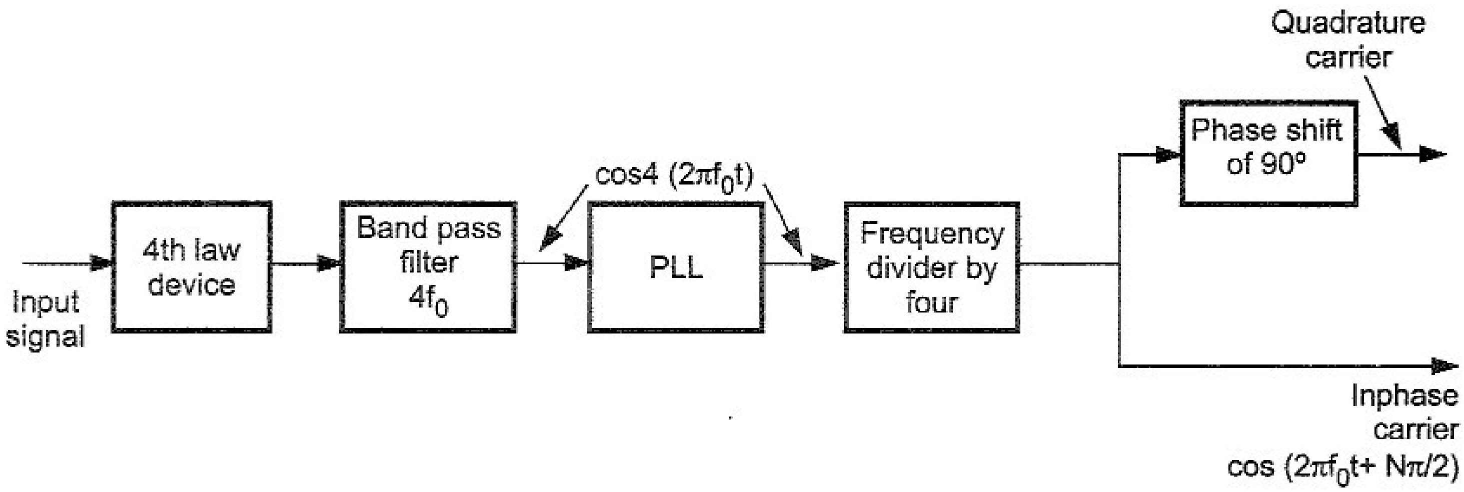

Fig. 5.4.5 PLL system for carrier synchronization

The fourth power of the input signal contains discrete frequency component at $4f_0$. We know that,

$$\cos 4(2\pi f_0 t) = \cos(8\pi f_0 t + 2\pi N)$$

Here 'N' is the number of cycles over the bit period. It is always integer value. When the frequency division by four takes place, the RHS of above equation becomes $\cos\left(2\pi f_0 t + \frac{N\pi}{2}\right)$. This shows that the output has a fixed phase error of $\frac{N\pi}{2}$. Differential encoding may be used to nullify the phase error events. The PLL remains locked with the phase of '$4f_0$' and then output of PLL is divided by 4. This gives a coherent carrier. A 90° phase shift is added to this carrier to generate a quadrature carrier.

5.4.2 Signal Space Representation of QPSK Signals

(1) Fig. 5.4.3 shows the phasor diagram of QPSK signal. Depending upon the combination of two successive bits, the phase shift occurs in carrier (see table 5.4.1). That is the QPSK signal of equation 5.4.3 can be written as,

$$s(t) = \sqrt{2P_s} \cos\left[2\pi f_0 t + (2m+1)\frac{\pi}{4}\right] \quad m = 0, 1, 2, 3 \quad \text{... (5.4.6)}$$

Here, the above equation takes four values and they represent the phasors of Fig.5.4.3.

(2) The above equation can be expanded as,

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \cos\left[(2m+1)\frac{\pi}{4}\right] - \sqrt{2P_s} \sin(2\pi f_0 t) \sin\left[(2m+1)\frac{\pi}{4}\right]$$

(3) Let's rearrange the above equation as,

$$s(t) = \left\{\sqrt{P_s T_s} \cos\left[(2m+1)\frac{\pi}{4}\right]\right\} \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$$

$$- \left\{\sqrt{P_s T_s} \sin\left[(2m+1)\frac{\pi}{4}\right]\right\} \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t) \quad \text{... (5.4.7)}$$

(4) Let $\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$ ... (5.4.8)

and $\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$ ... (5.4.9)

The above two signals are called orthogonal signals and they are used as carriers in QPSK modulator.

(5) Let $b_o(t) = \sqrt{2} \cos\left[(2m+1)\frac{\pi}{4}\right]$ ... (5.4.10)

and $b_e(t) = -\sqrt{2} \sin\left[(2m+1)\frac{\pi}{4}\right]$ ... (5.4.11)

(6) With the use of equation 5.4.8 to equation 5.4.9 we can write equation 5.4.7 as,

$$s(t) = \sqrt{P_s T_s} \cdot \frac{1}{\sqrt{2}} b_o(t)\, \phi_1(t) + \sqrt{P_s T_s} \cdot \frac{1}{\sqrt{2}} b_e(t)\, \phi_2(t)$$

$$= \sqrt{P_s \cdot \frac{T_s}{2}}\, b_o(t)\, \phi_1(t) + \sqrt{P_s \frac{T_s}{2}}\, b_e(t)\, \phi_2(t)$$

(7) T_s = symbol duration and $T_s = 2T_b$