20/6/4

Communication

Analog — Digital.

Continious Sgl. — Pulse

SAMPLING THEOREM :-

A Band limitted signal of finite Energy has no frequency Component higher than W Hertz., is Completly described by specifying the value of the signal at instant of Time $\frac{1}{2W}$ seconds.

W → max^m frequency. $T_s = \frac{1}{2W}$

$f_s = 2W.$

Sampling Theory can also be stated as band limited signal of finite energy which has more frequency Component than W Hertz, may be Completly recovered from the knowledge of its samples taken at the rate of 2W samples/sec.

A Continous time signal can be completly represented in its sample & recovered back if the sampling frequency is $f_s \geq 2W$.

Here,

f_s is the sampling frequency, &

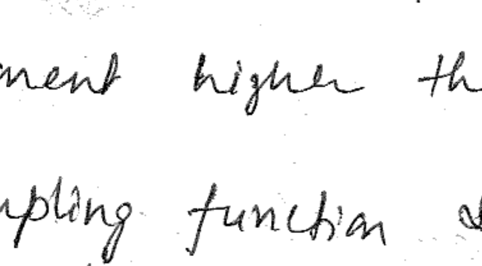
W is the max^m frequency.

Proof ~~Sampling theorem~~

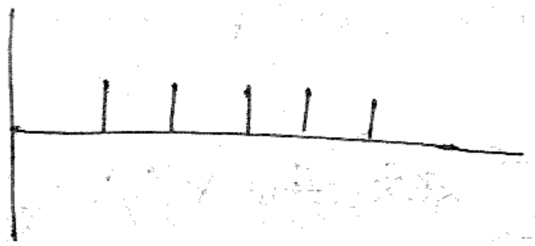
Let $x(t)$ be a continious time signal which doesn't contain any frequency component higher than W hertz.

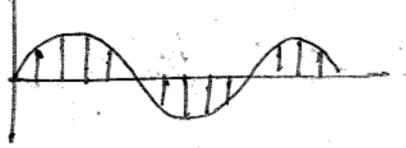
A sampling function samples the signal ~~at~~ regularly at the rate of f_s samples / sec. i.e

$$T_s = \frac{1}{f_s} \Rightarrow T_s = \text{Sampling period}$$



sampling i/p





The Pulse train of Pulse is rep as,

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \quad \text{— } ①$$

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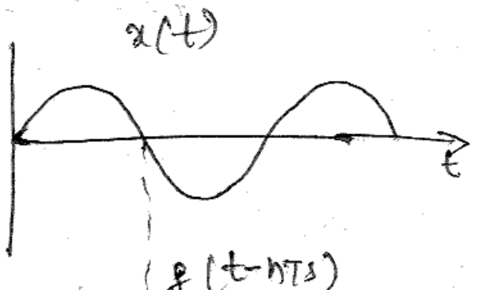
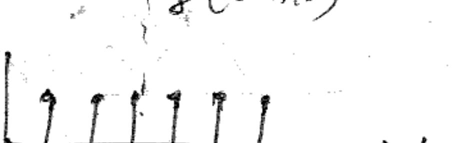
The impulse train pulse is :-

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \quad \text{— } ①$$

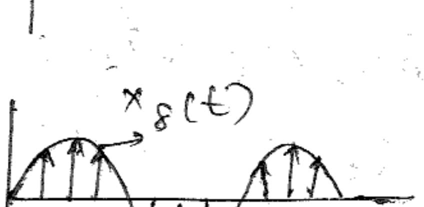


The waveform (3) represented as

$$x_\delta(t) = \delta(t-nT_s)\, x(nT_s)$$



The fourier transform of the pulse train pulse is:-



$$X(f) = f_s \sum_{n=-\infty}^{\infty} \delta(t-nf_s)$$

$$\left(\text{where, } f_s = \frac{1}{T_s}\right)$$

Taking fourier transform of waveform (3),

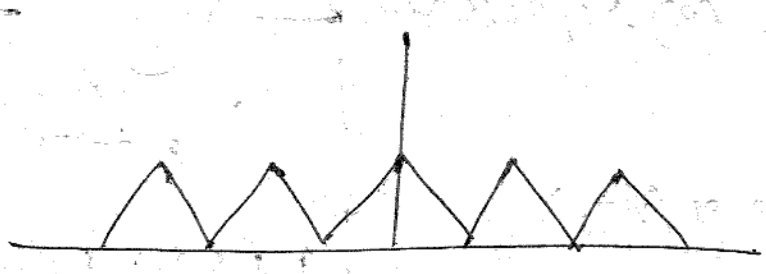
$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s) \quad -(2)$$

This Eq shows that the process of uniformly sampling a Continuous time signal results in a periodic Spectrum with periods equal to the sampling rate

The fourier of signal $x(t)$ results in

$X(f-nf_s) = X(f)$, where

$$f = 0, \pm f_s, \pm 2f_s.$$



Thus the same spectrum $X(f)$ appear as,

$f=0,\ f=\pm f_s,\ f=\pm 2f_s.$

Eq (2) Can be written as,

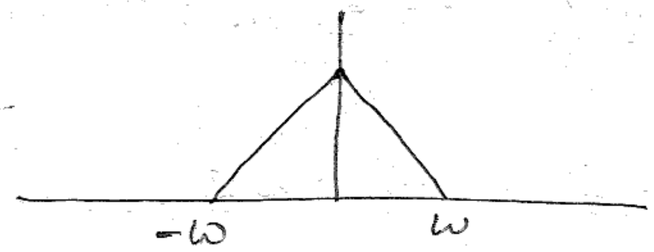
$$X_\delta(f) = f_s X(f) + f_s X(f \pm f_s)$$
$$+ f_s X(f \pm 2f_s) + \dots \quad -(3)$$

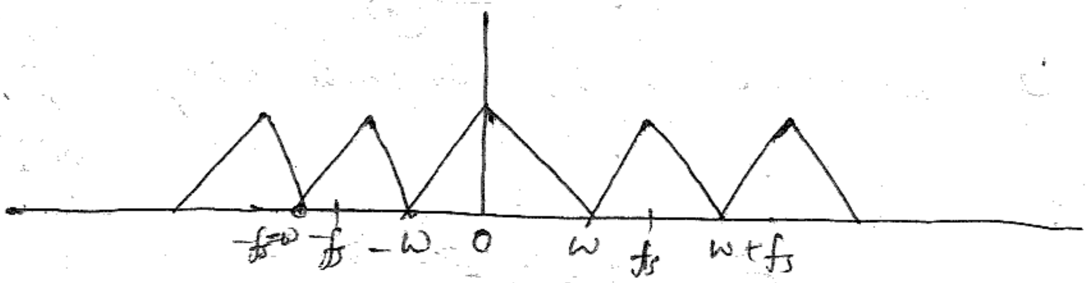
Eqⁿ (1) can also be written as,

$$X_\delta(f) = f_s X(f) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} f_s X(f-nf_s) \quad \text{--- (3)}$$

w.k.t

Sampling frequency, $f_s = 2W$ $\quad T_s = \frac{1}{2W}$.





⇒ The fig represent spectrum of sampling frequency.

By the defⁿ of Fourier transform of C.T s/g.

$$FT[X(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

The fourier transform of D.T signal is

$$F[X_\delta(t)] = \sum_{-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}$$

from Eq^n (3), we can write,

$$f_s X(f) = X_\delta(f) - \sum_{\substack{n=-\infty \\ n=0}}^{\infty} f_s X(f_s - nf_s).$$

$$X(f) = \frac{1}{f_s} X_\delta(f) - \sum_{n=-\infty}^{\infty} X(f_s - nf_s)$$

Substitute,

$f_s = 2w$

$$X(f) = \frac{1}{2w} X_\delta(f) - \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} X(f - nf_s)$$

$$X(f) = \frac{1}{2w} X_\delta(f) \quad \text{for } -w \leq f \leq w \qquad (4).$$

Eq^n (4) can be written as, with a help of

$$X(f) = \frac{1}{2w} \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}.$$

$T_s = \frac{1}{f_s}$, $\quad$ in $T_s = \frac{1}{2w}$

$$X(f) = \frac{1}{2w} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2w}\right) e^{-j2\pi \frac{fn}{2w}}$$

$$X(f) = \frac{1}{2w} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2w}\right) e^{-\frac{j\pi f n}{w}}.$$