CS - UNIT - V

3] Soln:

Given, $G(s) = \frac{K}{s(0.1s+1)(0.2s+1)}$

~~To~~ satisfy,
phase margin $\geq 30°$

~~K_v~~ K_v is missing. Let us assume $K_v = 1$ and solve it.

4] Soln:

Given, $G(s) = \frac{K}{s(s+1)(s+4)}$

and

$K_v > 5$ & phase margin, $\gamma > 43°$.

To find:- the design of lag compensator.

Step-1: To find K

By defn of K_v,

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$K_v = \lim_{s \to 0} \cancel{s} \frac{K}{\cancel{s}(s+1)(s+4)} \cdot 1 \quad [\because H(s) = 1, \text{ it is unity feedback}]$$

$$5 = \frac{K}{1 \times 4}$$

$$\therefore K = 20$$

Step-2: To draw the bode plot.

$$G(s) = \frac{20}{s(1+s)(4+s)} = \frac{\cancel{20}\,5}{s(1+s)\cancel{4}(1+s/4)}$$

$$\therefore G(s) = \frac{5}{s(1+s)(1+0.25s)}$$

Sub, $s = j\omega$, $\therefore G(j\omega) = \frac{5}{(j\omega)(1+j\omega)(1+j0.25\omega)}$

MAGNITUDE PLOT :

$\omega_{c1} = 1$ rad/sec

$\omega_{c2} = \frac{1}{0.25} = 4$ rad/sec.

TABLE-1.

Term.	Corner freq rad/sec.	Slope db/dec	Change in slope db/sec.
$5/j\omega$	—	-20	—
$\frac{1}{1+j\omega}$	$\omega_{c1} = 1$ rad/sec.	-20	$-20+(-20) = -40$
$\frac{1}{1+j0.25\omega}$	$\omega_{c2} = 4$ rad/sec.	-20	$-40+(-20) = -60$

Now, choosing the freq, $\omega_l < \omega_{c1}$ & $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec & $\omega_h = 10$ rad/sec.

Let gain, $A = |G(j\omega)|$ in db

At $\omega = \omega_l$, $A = 20\log\left|\frac{5}{j\omega}\right| = 20\log(5/0.5) = 20$ db.

" $\omega = \omega_{c1}$, $A = 20\log\left(\frac{5}{1}\right) = 14$ db

" $\omega = \omega_{c2}$, $A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(at\ \omega = \omega_{c1})}$

$= -40 \times \log(4/1) + 14$ db.

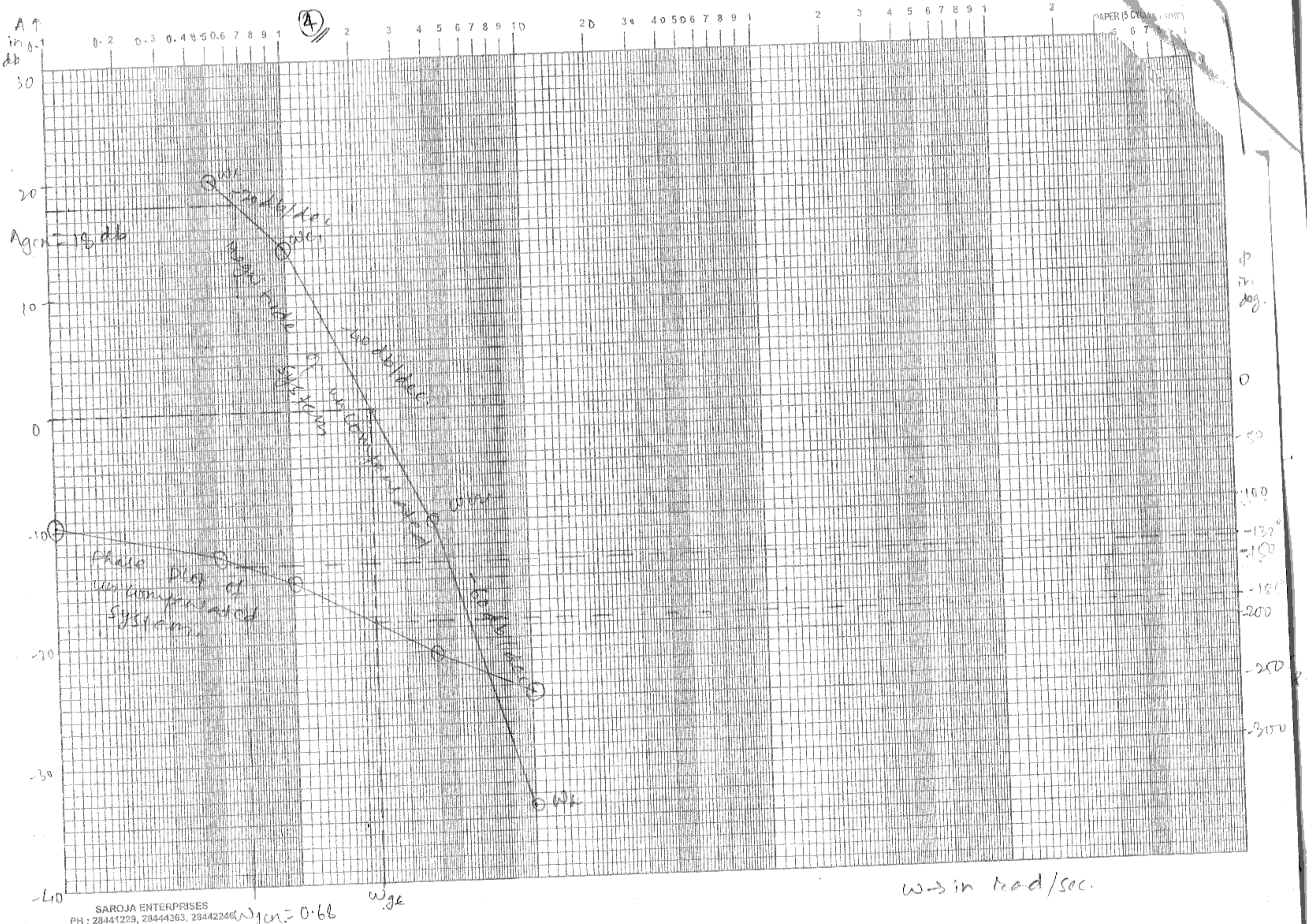
$= -24 + 14$

$= -10$ db

" $\omega = \omega_h$, $A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(at\ \omega = \omega_{c2})}$

$= -60 \times \log(10/4) + (-10)$

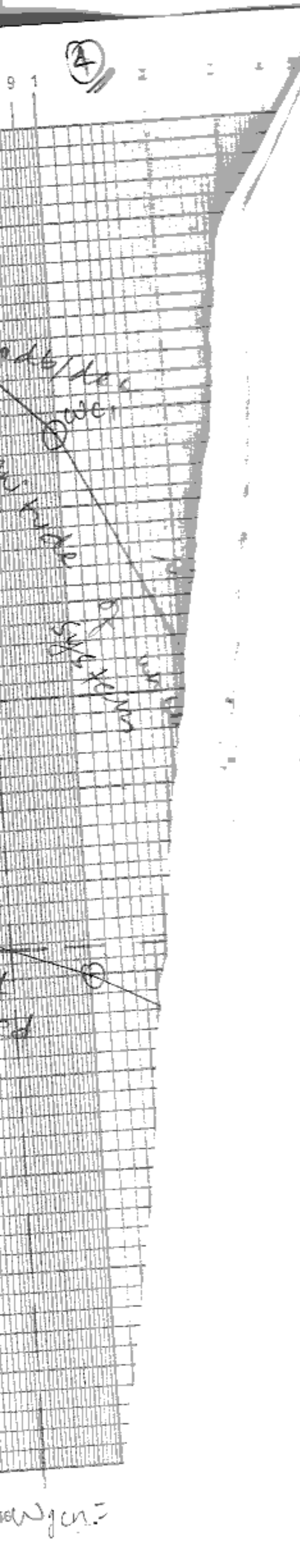
$= -24 - 10 = -34$ db.



4
A in db
φ in deg.
Agcn = 18 db
-20 db/dec
-40 db/dec
-60 db/dec
Magnitude plot of uncompensated system
Phase plot of uncompensated system
ωgc
ωgcn = 0.68
ω → in rad/sec.

Phase plot : $\phi = -90° - \tan^{-1}\omega - \tan^{-1} 0.25\omega$

Table - 2



ω rad/sec	0.1	0.5	1	4	10
φ in deg	$-97°$ ~~-90°~~	-124 ~~-90.5~~	$-150°$ ~~-91~~	-211 ~~-92°~~	$-243°$ ~~-93°~~

Step-3 : Determination of phase margin of uncompensated system.

Let, ϕ_{gc} = phase of $G(j\omega)$ at gain cross over frequency (ω_{gc}).

and γ = phase margin of uncompensated system.

from the bode plot we are getting,

$\phi_{gc} = -185°$

Now, $\gamma = 180° + \phi_{gc} = 180° - 185° = -5°$

The system requires a phase margin of 43°, but the available phase margin is $-5°$ & so lag compensation should be employed to improve the phase margin.

Step-4 : Choose the suitable ϕ value for the phase margin of compensated system.

The desired phase margin, $\gamma_d = 43°$

∴ phase margin of compensated system, $\gamma_n = \gamma_d + \epsilon$

∴ $\gamma_n = 43° + 5°$ [∵ initial choice of $\epsilon = 5$]

$\gamma_n = 48°$

$\gamma_n = 180° + \phi_{gc}$

$48 - 180° = +\phi_{gc}$

$\phi_{gc} =$

5. To determine the new gain crossover frequency.

Let, ω_{gcn} = new gain cross over freq and ϕ_{gcn} = phase of $G(j\omega)$ at ω_{gcn}.

Now,
$$\gamma_n = 180^\circ + \phi_{gcn}$$
$$48^\circ = 180^\circ + \phi_{gcn}$$
$$\phi_{gcn} = 48^\circ - 180^\circ$$
$$\phi_{gcn} = -132^\circ$$

From the bode plot,

$\therefore$ New gain cross over frequency, $\omega_{gcn} = 0.68$ rad/sec

Step-6: To determine the parameter, β.

we have, $A_{gcn} = 18$ db.

$\therefore$ $|G(j\omega)|$ in db at $(\omega = \omega_{gcn}) = A_{gcn} = 20$ db.

Also, $A_{gcn} = 20 \log \beta$.
$$18 = 20 \log \beta$$
$$\therefore \beta = 10^{18/20} = 7.9 \simeq 8$$
$$\therefore \boxed{\beta = 8}$$

Step-7: To determine the transfer funch of lag compensator.

The zero of the compensator is placed at a frequency one-tenth of ω_{gcn}.

$\therefore$ zero of the lag compensator, $z_c = \frac{1}{T} = \frac{\omega_{gcn}}{8}$
$$T = \frac{8}{\omega_{gcn}} = \frac{8}{0.68} = \cancel{14.7 \simeq 15}$$
$$\therefore \boxed{T = 13.33}$$

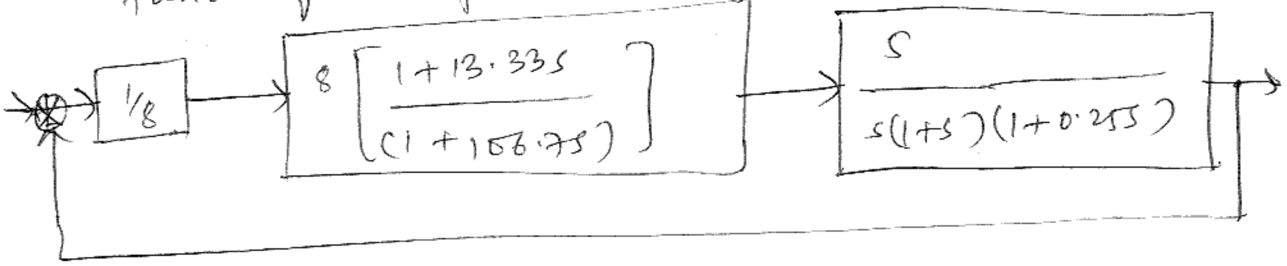
pole of lag compensator, $P_c = \frac{1}{\beta T} = \frac{1}{8\times 13.33} = \frac{1}{106.7}$.

$\therefore \boxed{P_c = 0.00937}$.

Transfer func of lag compensator $\Big\}$ $(G_c) = \dfrac{s + 1/T}{s + 1/\beta T}$

$$= \beta\left[\frac{1+sT}{1+s\beta T}\right] = 8\left[\frac{1+13.33s}{1+106.7s}\right]$$

Step-8: To determine the open-loop transfer func of compensated system.



Open loop transfer function of compensated system $\Big\}$ $G_o(s) = \frac{1}{8}\times\frac{8(1+13.33s)}{(1+106.7s)}\times\frac{5}{s(1+s)(1+0.25s)}$

$$= \frac{5(1+13.33s)}{s(1+s)(1+106.7s)(1+0.25s)} \quad \text{— ⊛}$$

Step-9: To determine the actual phase margin of compensated system.

On sub $s = j\omega$ in ⊛ then

$$G_o(s) = \frac{5(1+j13.33\omega)}{j\omega\,(1+j106.7\omega)\,(1+j0.25\omega)}$$

Let ϕ_0 = phase of $G_0(j\omega)$

ϕ_{gco} = phase of $G_0(j\omega)$ at $\omega = \omega_{gcn}$

$$\phi_0 = \tan^{-1} 13.33\omega - 90^\circ - \tan^{-1} 106.7\omega - \tan^{-1} 0.25\omega$$

At, $\omega \to \omega_{gcn}$, $\phi_0 \to \phi_{gco}$ then,

$$\phi_{gco} = \tan^{-1} 13.33\omega_{gcn} - 90^\circ - \tan^{-1} 106.7\omega_{gcn} - \tan^{-1} 0.25\omega_{gcn}$$

$$= -105^\circ$$

$\therefore$ Actual phase, $\gamma_0 = 180^\circ + \phi_{gco} = 180^\circ - 105^\circ$

$\boxed{\gamma_0 = 75^\circ}$ $> 43^\circ$ which is satisfied & acceptable.

RESULT :

Transfer funch of lag compensator } $G_c(s) = \dfrac{8(1+13.33s)}{(1+106.75s)}$

$$= \frac{s+0.075}{s+0.0094}$$

open loop transfer functions of compensated system } $G_0(s) = \dfrac{8(1+13.33s)}{s(1+106.75s)(1+0.25s)(1+s)}$