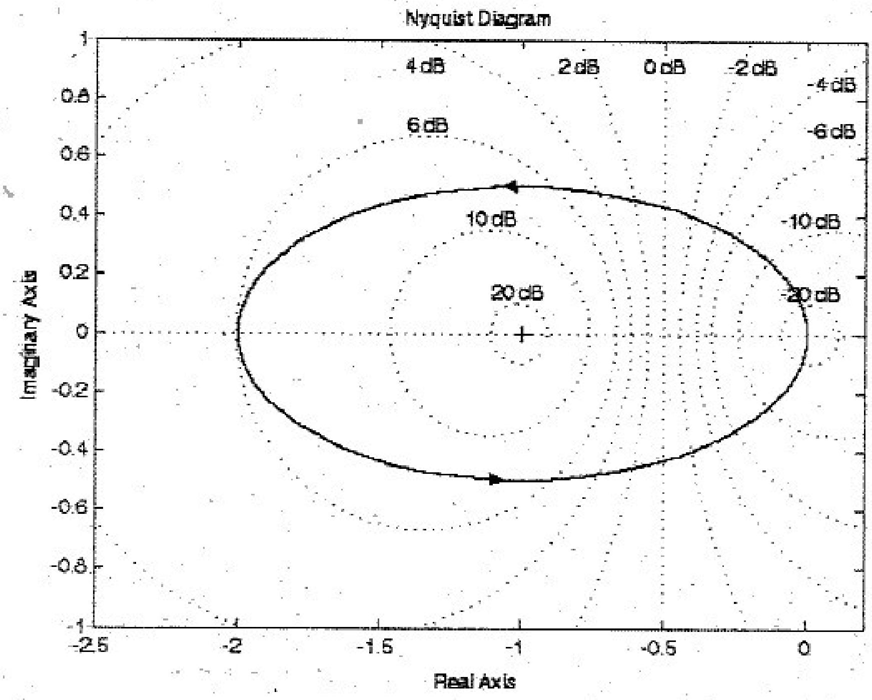

Fig P4.3 : Nyquist plot of the system given in problem 4.3.

OUTPUT

```
The given transfer function is,
Transfer function:
                         s + 2
                         ------
                         s^2 - 1

The Nyquist plot of program 4.3 is shown in fig P4.3.
```

PROGRAM 4.4

Write a MATLAB program to draw the root locus plot of the unity feedback system governed by the following open loop transfer function.

$G(s) = 1/s(s^2+4s+13)$

```
%program to plot root locus

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=1/(s*(s^2+4*s+13))
rlocus(Gs,'k');
axis([-3 2 -6 6]);                  %specify x and y axis limits
sgrid([0.5,0.707],[90.5,1,2]);   %specify the s-grid lines to draw
```

OUTPUT

```
The given open loop transfer function G(S) is,
Transfer function:
                              1
                     ----------------
                     s^3 + 4 s^2 + 13 s

The root locus plot of program 4.4 is shown in fig P4.4.
```

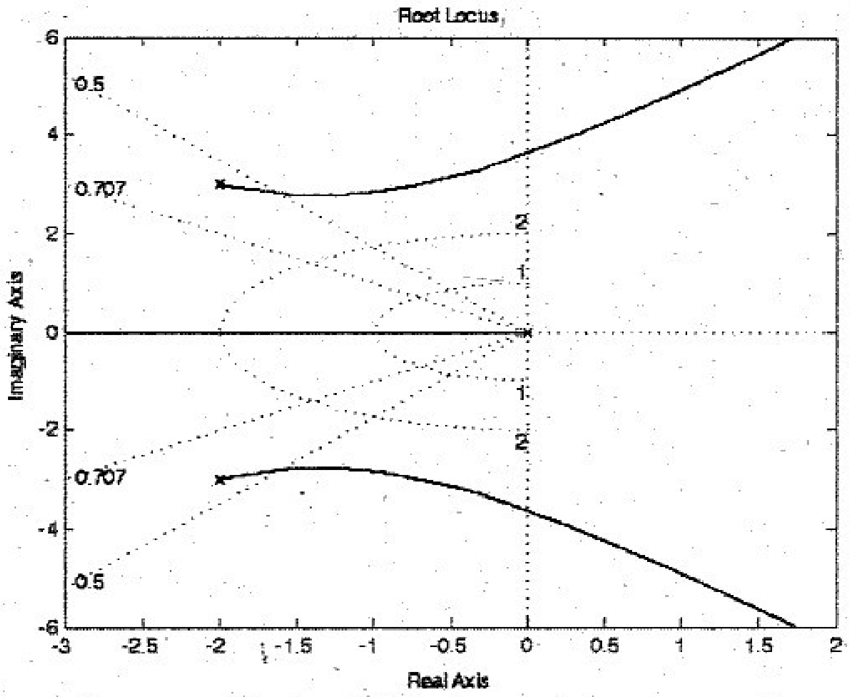

Fig P4.4 : Root locus plot of the system given in problem 4.4.

PROGRAM 4.5

Write a MATLAB program to draw the root locus plot of the unity feedback system governed by the following open loop transfer function.

$$G(s) = 1/s(s+4)(s^2+4s+20)$$

```
%program to plot root locus
clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=1/(s*(s+4)*(s^2+4*s+20))
rlocus(Gs,'k'); axis([-8 4 -6 6]); sgrid;
```

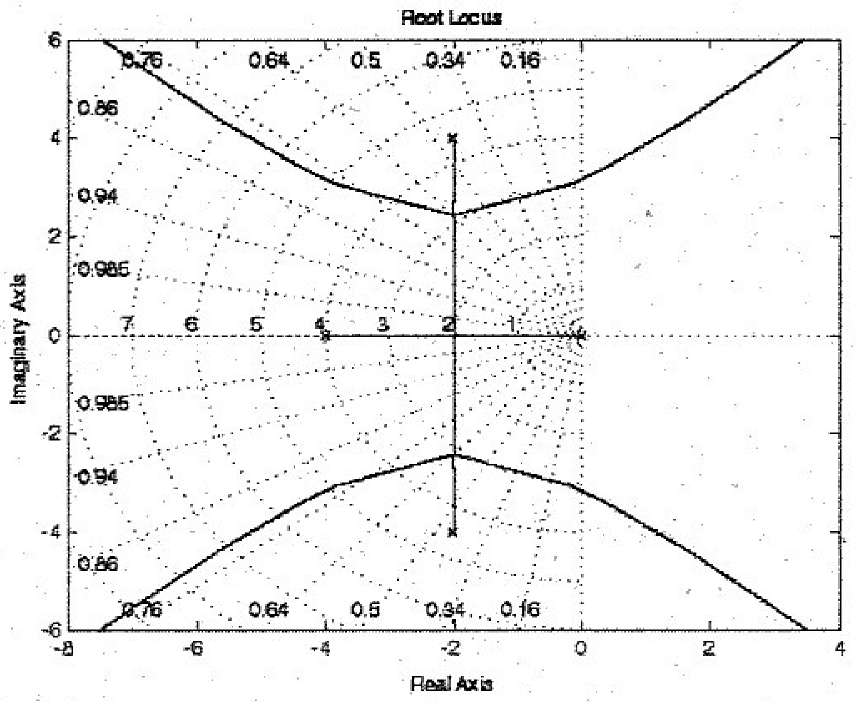

Fig P4.5 : Root locus plot of the system given in problem 4.5.

OUTPUT

```
The given open loop transfer function G(S) is,
Transfer function:
                        1
            ----------------------------
            s^4 + 8 s^3 + 36 s^2 + 80 s

The root locus plot of program 4.5 is shown in fig P4.5.
```

PROGRAM 4.6

Write a MATLAB program to draw the root locus plot of the unity feedback system governed by the following open loop transfer function.

$$G(s) = (s^2+6s+25)/s(s+1)(s+2)$$

```
%program to plot root locus

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=(s^2+6*s+25)/(s*(s+1)*(s+2))

rlocus(Gs,'k');
axis([-6 2 -6 6]); sgrid;
```

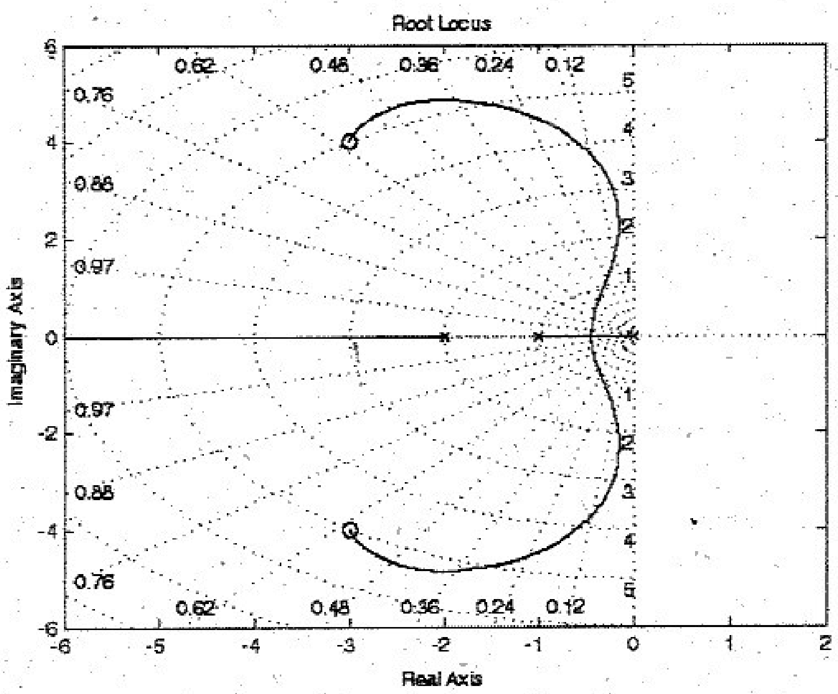

Fig P4.6 : Root locus plot of the system given in problem 4.6.

OUTPUT

```
The given open loop transfer function G(S) is,
Transfer function:
                 s^2 + 6 s + 25
               ------------------
               s^3 + 3 s^2 + 2 s

The root locus plot of program 4.6 is shown in fig P4.6.
```

4.10 SHORT QUESTIONS AND ANSWERS

Q4.1 ***Define BIBO stability.***

A linear relaxed system is said to have BIBO stability if every bounded (finite) input results in a bounded (finite) output.

Q4.2 ***What is impulse response?***

The impulse response of a system is the response of a system for impulse input and it is given by inverse Laplace transform of the system transfer function.

Q4.3 ***What is the requirement for BIBO stability?***

The requirement for BIBO stability is that, $\int_0^{\infty} m(t)\,dt < \infty$,

where m(t) is impulse response of the system.

Q4.4 ***What is characteristic equation?***

The denominator polynomial of C(s)/R(s) is the characteristic equation of the system.

Q4.5 ***How the roots of characteristic equation are related to stability?***

If the roots of characteristic equation has positive real part then the impulse response of the system is not bounded (the impulse response will be infinite as $t \to \infty$). Hence the system will be unstable. If the roots have negative real part then the impulse response is bounded (the impulse response becomes 0 as $t \to \infty$). Hence the system will be stable.

Q4.6 ***What is the necessary condition for stability?***

The necessary condition for stability is that all the coefficients of the characteristic polynomial must be positive.

Q4.7 ***What is the relation between stability and coefficient of characteristic polynomial?***

If the coefficients of characteristic polynomial are negative or zero, then some of roots lie on right half of s-plane. Hence the system is unstable. If the coefficients of characteristic polynomial are positive and if no coefficient is zero then there is a possibility of the system to be stable provided all the roots are lying on left half of s-plane.

Q4.8 ***What will be the nature of impulse response when the roots of characteristic equation are lying on imaginary axis?***

If the roots of characteristic equation lies on imaginary axis the nature of impulse response is oscillatory.

Q4.9 ***What will be the nature of impulse response if the roots of characteristic equation are lying on right half of s-plane?***

When the roots are lying on the real axis on the right half of s-plane, then the response is exponentially increasing. When the roots are complex conjugate and lying on the right half of s-plane, then the response is oscillatory with exponentially increasing amplitude.

Q4.10 ***What is the principle of argument?***

The principle of argument states that let F(s) be an analytic function and if an arbitrary closed contour in the clockwise direction is chosen in the s-plane so that F(s) is analytic at every point of the contour. Then the corresponding F(s)-plane contour mapped in the F(s)-plane will encircle the origin, N times in the anticlockwise direction, where N is the difference between number of poles ,P and zeros Z of F(s) that are enclosed by the chosen closed contour in the s-plane. (i.e., N = P -Z).

Q4.11 ***What is the necessary and sufficient condition for stability?***

The necessary and sufficient condition for stability is that all of the elements in the first column of the routh array should be positive.

Q4.12 ***What is routh stability criterion?***

Routh criterion states that the necessary and sufficient condition for stability is that all of the elements in the first column of the routh array be positive. If this condition is not met, the system is unstable and the number of sign changes in the elements of the first column of routh array corresponds to the number of roots of characteristic equation in the right half of the s-plane.

Q4.13 ***What is auxiliary polynomial?***

In the construction of routh array a row of all zero indicates the existence of an even polynomial as a factor of the given characteristic equation. In an even polynomial the exponents of s are even integers or zero only. This even polynomial factor is called auxiliary polynomial. The coefficients of auxiliary polynomial are given by the elements of the row just above the row of all zeros.

Q4.14 ***What is quadrantal symmetry?***

The symmetry of roots with respect to both real and imaginary axis is called quadrantal symmetry.

Q4.15 ***In routh array what conclusion you can make when there is a row of all zeros?***

All zero row in routh array indicates the existence of an even polynomial as a factor of the given characteristic equation. The even polynomial may have roots on imaginary axis.

Q4.16 ***What is limitedly stable system ?***

For a bounded input signal, if the output has constant amplitude oscillations then the system may be stable or unstable under some limited constraints. Such a system is called limitedly stable.

Q4.17 ***What is Nyquist stability criterion?***

If G(s)H(s)-contour in the G(s)H(s)-plane corresponding to Nyquist contour in s-plane encircles the point −1+j0 in the anti-clockwise direction as many times as the number of right half s-plane poles of G(s)H(s). Then the closed loop system is stable.

Q4.18 ***What is root locus?***

The path taken by a root of characteristic equation when open loop gain K is varied from 0 to ∞ is called root locus.

Q4.19 ***What is magnitude criterion?***

The magnitude condition states that $s=s_a$ will be a point on root locus if for that value of s magnitude of G(s)H(s) is equal to 1, (i.e. $|G(s)H(s)| = 1$).

$$\text{Let, } G(s)\,H(s) = \frac{K\,(s+z_1)\,(s+z_2)\,(s+z_3)\,.....}{(s+p_1)\,(s+p_2)\,(s+p_3)\,.....}$$

∴ For $s = s_a$ be a point in root locus,

$$|G(s)\,H(s)| = \frac{K\,|s_a+z_1|\,|s_a+z_2|\,|s_a+z_3|\,.....}{|s_a+p_1|\,|s_a+p_2|\,|s_a+p_3|\,.....} = 1$$

$$\left[\text{or } |G(s)\,H(s)| = K = \frac{\text{Product of length of vectors from open loop zeros to the point } s_a}{\text{Product of length of vectors from open loop poles to the point } s_a} = 1\right]$$

Q4.20 ***What is angle criterion?***

The angle criterion states that $s = s_a$ will be a point on root locus if for that value of s the argument or phase of G(s)H(s) is equal to an odd multiple of 180°, [i.e., $\angle G(s)H(s) = \pm 180°\,(2q+1)$].

$$\text{Let, } G(s)H(s) = K\frac{(s+z_1)\,(s+z_2)\,(s+z_3)\,.....}{(s+p_1)\,(s+p_2)\,(s+p_3)\,.....}$$

∴ For $s = s_a$ be a point on root locus,

$$\angle G(s)\,H(s) = \angle(s_a+z_1) + \angle(s_a+z_2) + \angle(s_a+z_3)\,..... - \angle(s_a+p_1) - \angle(s_a+p_2)\,.... = \pm 180°(2q+1)$$

$$\left[\text{or}\left(\begin{array}{c}\text{Sum of anges}\\ \text{of vectors from zeros}\\ \text{to the point } s=s_a\end{array}\right)-\left(\begin{array}{c}\text{Sum of angles}\\ \text{of vectors from poles}\\ \text{to the point } s=s_a\end{array}\right)=\pm180°(2q+1)\right]$$

Q4.21. ***How will you find the gain K at a point on root locus?***

The gain K at a point $s = s_a$ on root locus is given by,

$$K=\frac{\text{Product of length of vector from open loop poles to the point } s_a}{\text{Product of length of vector from open loop zeros to the point } s_a}$$

Q4.22 ***How will you find root locus on real axis?***

To find the root locus on real axis, choose a test point on real axis. If the total number of poles and zeros on the real axis to the right of this test point is odd number, then the test point lies on the root locus. If it is even then the test point does not lie on the root locus.

Q4.23 ***What are asymptotes? How will you find the angle of asymptotes?***

Asymptotes are straight lines which are parallel to root locus going to infinity and meet the root locus at infinity.

$$\text{Angles of asymptotes} = \frac{\pm180°\ (2q+1)}{n-m}\ ;\ q=0,\ 1,\ 2,\ldots\ldots\ (n-m)$$

Q4.24 ***What is centroid? How the centroid is calculated?***

The meeting point of asymptotes with real axis is called centroid. The centroid is given by,

$$\text{Centroid}=\frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

Q4.25 ***What are breakaway and breakin point? How to determine them?***

At breakaway point the root locus breaks from the real axis to enter into the complex plane. At breakin point the root locus enters the real axis from the complex plane.

To find the breakaway or breakin points, form an equation for K from the characteristic equation, and differentiate the equation of K with respect to s. Then find the roots of equation $dK/ds = 0$. The roots of $dK/ds = 0$ are breakaway or breakin points, provided for this value of root, the gain K should be positive and real.

Q4.26 ***How to find the crossing points of root locus in imaginary axis.***

Method (i) : By Routh hurwitz criterion.

Method (ii) : By letting $s = j\omega$ in the characteristic equation and separate the real and imaginary parts. These two equations are equated to zero. Solve the two equations for ω and K. The value of ω gives the point where the root locus crosses imaginary axis and the value of K is the gain corresponding to the crossing point.

Q4.27 ***What is dominant pole?***

The dominant pole is a pair of complex conjugate pole which decides transient response of the system. In higher order systems the dominant poles are very close to origin and all other poles of the system are widely separated and so they have less effect on transient response of the system.

Q4.28 ***How will you fix dominant pole on root locus and find the gain K corresponding to the dominant pole?***

The dominant poles are given by roots of a quadratic factor, $s^2+2\zeta\omega_n s+\omega_n^2=0$.

$$\therefore s=\frac{-2\zeta\omega_n\pm\sqrt{4\zeta^2\omega_n^2-4\omega_n^2}}{2}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$

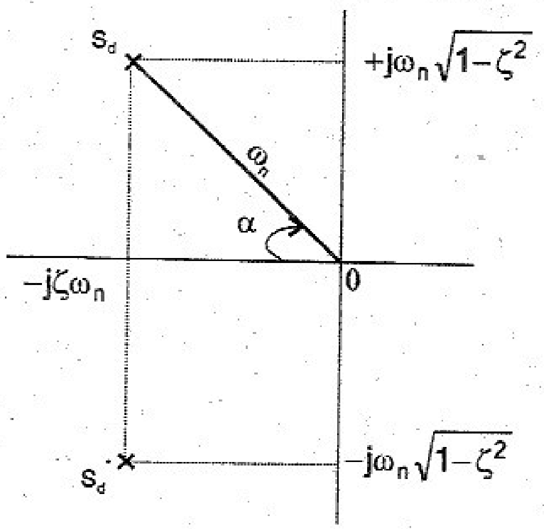

Fig Q4.28

The dominant pole can be plotted on the s-plane as shown in fig Q4.28.

In the right angle triangle OAP,

$$\cos\alpha = \frac{\zeta\omega_n}{\omega_n} = \zeta \qquad \therefore \alpha = \cos^{-1}\zeta$$

To fix a dominant pole on root locus draw a line at an angle of $\cos^{-1}\zeta$ with respect to negative real axis. The meeting point of this line with root locus will give the location of dominant pole. The value of K corresponding to dominant pole can be obtained from magnitude condition.

$$K = \frac{\text{Product of length of vectors from open loop poles to dominant pole}}{\text{Product of length of vectors from open loop zeros to dominant pole}}$$

Q4.29 ***For the system represented by the following characteristic equation say whether the necessary condition for stability is satisfied or not.***

(i) $s^4 + 3s^3 + 4s^2 + 5s + 10 = 0$

(ii) $s^6 - 2s^5 + s^3 + s^2 + s + 6 = 0$

(iii) $s^3 - 8s^2 + 7s + 6 = 0$

(iv) $s^5 + 4s^4 - 5s^3 - 4s^2 + 2s + 1 = 0$

In equation (i) All the coefficients are positive and so the necessary condition for stability is satisfied.

In equation (ii), (iii) and (iv) some of the coefficients are negative and some of the coefficients are missing. Hence the necessary condition for stability is not satisfied.

Q4.30 ***Check the stability of the system whose Nyquist plot are shown in figQ 4.30 a & b, if there is no open loop poles on right half of s-plane.***

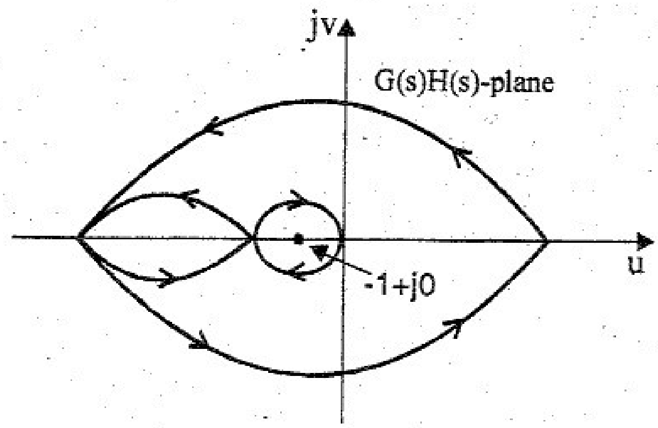

Fig Q4.30 a

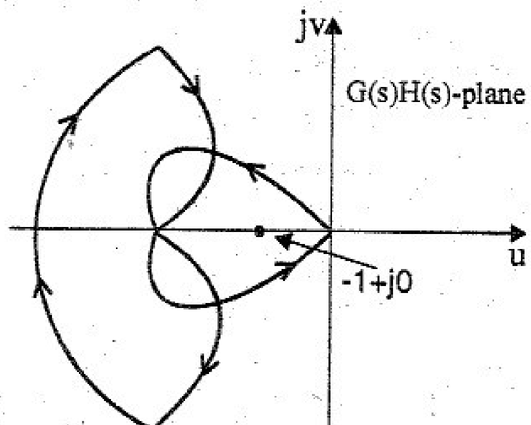

Fig Q4.30 b

In contour shown in fig Q 4.30a the $-1 + j0$ point is encircled once in clockwise direction and once in anticlockwise direction. Hence net encirclement is zero. Since no poles are lying on right half of s-plane and net encirclement of $-1+j0$ is zero, and so the system is stable.

In contour shown in fig Q4.30b the $-1+j0$ point is encircled once in anticlockwise direction but there is no pole on right half hence and hence the system is unstable.

4.11 EXERCISES

E.4.1 *Using routh criterion determine the locations of the roots of the following characteristic equations and comment on the stability of the systems.*

a) $2s^5 + 2s^4 + 5s^3 + 5s^2 + 3s + 5 = 0$

b) $3s^4 + 10s^3 + 5s^2 + 5s + 3 = 0$

c) $2s^6 + 4s^5 + s^4 - 32s^3 + 51s^2 + 3s + 15 = 0$

d) $s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63 = 0$

e) $s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$

f) $s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$

E.4.2 *The characteristic equations for certain feedback control systems are given below. In each case, determine the range of values of K, for which the system is stable.*

a) $s^4 + 3s^3 + 3s^2 + s + K = 0$

b) $s^5 + s^4 + Ks^3 + s^2 + s + 1 = 0$

c) $s^5 + s^4 + s^2 + s + K = 0$

d) $s^4 + s^3 + 3Ks^2 + (K+2)s + 4 = 0$

e) $s^4 + s^3 + 3(K+1)s^2 + (7K+5)s + (4K+7) = 0$

E.4.3 *Open-loop transfer functions of certain unity feedback systems are given below. In each case determine the location of closed loop poles in the s-plane, using routh criterion. Comment on the stability of closed loop system.*

a) $G(s) = \dfrac{200\,(1+s)}{s\,(1+0.1s)\,(1+0.2s)\,(1+0.5s)}$

b) $G(s) = \dfrac{10}{(s+2)\,(s+4)\,(s^2+6s+25)}$

c) $G(s) = \dfrac{(s+1)}{s(s-1)\,(s^2+4s+16)}$

d) $G(s) = \dfrac{2.5}{s(s+5)\,(0.1s+1)}$

E.4.4 *Open-loop transfer functions of certain unity feedback systems are given below. In each case determine the range of values of K for which the system is stable.*

a) $G(s) = \dfrac{K(s+13)}{s(s+3)(s+7)}$

b) $G(s) = \dfrac{K(s+1)}{s(s-1)(s+6)}$

c) $G(s) = \dfrac{K(s+2)}{s(s+5)(s^2+2s+5)}$

d) $G(s) = \dfrac{K(s-1)}{s(s+2)}$

E.4.5 *The open-loop transfer function of a unity feedback control system is given by $G(s)=K(s+2)/s(s-2)(s^2+5s+16)$. Determine the value of K which will cause sustained oscillations in the closed-loop system and what is the corresponding oscillation frequencies?*

E.4.6 *The open-loop transfer functions of certain unity feedback system are given below. In each case, sketch the Nyquist plot and determine the stability of the system.*

a) $G(s) = \dfrac{K(s+3)}{s(s-1)}$

b) $G(s) = \dfrac{-1}{2s(1-20s)}$

c) $G(s) = \dfrac{K(1+2s)}{s(1+s)(1+s+s^2)}$

d) $G(s) = \dfrac{K(s+5)(s+40)}{s^3(s+200)(s+1000)}$

e) $G(s) = \dfrac{K}{(s+1)(s+1.5)(s+2)}$

f) $G(s) = \dfrac{K}{s(s+ \quad ^2+2s+2)}$

E.4.7 *Determine the phase margin and gain margin of the system with following transfer functions.*

a) $G(s) = \dfrac{2}{(s+1)^2}$ b) $\dfrac{20}{s(s+1)(s^2+2s+2)}$

E.4.8 *The open-loop transfer function of a unity feedback system is given by, G(s)= K/s(1+0.5s) (1+s).*

a) Determine the value of K so that the gain margin of the system is 6 db.

b) Determine the value of K so that the phase margin of the system is 30°.

E.4.9 *The open-loop transfer functions of certain unity feedback systems are given below. Sketch the root locus of each system.*

a) $G(s) = \dfrac{K(s+2)}{(s+3)^2(s^2+2s+17)}$

b) $G(s) = \dfrac{K}{s(s+3)(s^2+2s+2)}$

c) $G(s) = \dfrac{K(s^2+2s+10)}{s(s+2)(s+4)}$

d) $G(s) = \dfrac{K(s+1)}{s^2(s+12)}$

e) $G(s) = \dfrac{K(s+4)}{s(s+0.5)(s+2)}$

f) $G(s) = \dfrac{K}{s(s^2+8s+20)}$

g) $G(s) = \dfrac{K}{s(s+2)(s^2+2s+2)}$

h) $G(s) = \dfrac{K(s^2+1)}{s(s+2)}$

E.4.10 *A unity feedback system has an open-loop transfer function, $G(s) = K/s(s^2+8s+32)$. Sketch the root locus and determine the dominant closed loop poles with ζ=0.5. Determine the value of K at this point.*

E.4.11 *Draw the root locus plot for a unity feedback system having forward path transfer function,*

$G(s) = K/s\,(s+1)\,(s+5)$.

a) Determine the value of K which gives continuous oscillations and the frequency of oscillation.

b) Determine the value of K corresponding to a dominant closed loop pole with damping ratio, ζ= 0.7.