Step 5 : To find angle of departure

Let us consider the complex pole p_3 shown in fig 4.25.2. Draw vectors from all other poles to the pole p_3 as shown in fig 4.25.2. Let angles of these vectors be θ_1, θ_2 and θ_3.

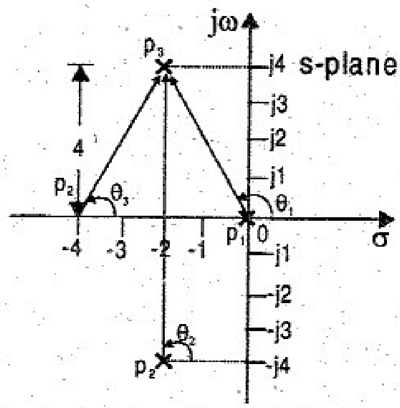

Fig 4.25.2

Here,

$$\theta_1 = 180° - \tan^{-1}\frac{4}{2} = 117°$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{4}{2} = 63°$$

$$\left.\begin{array}{l}\text{Angle of departure}\\ \text{from complex pole } p_3\end{array}\right\} = 180° - (\theta_1 + \theta_2 + \theta_3)$$

$$= 180° - (117° + 90° + 63°) = -90°$$

The angle of departure at complex pole p_4 is negative of the angle of departure at complex pole p_3.

∴ Angle of departure from complex pole p_4 = +90°

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point on imaginary axis

The characteristic equation is given by, $s^4 + 8s^3 + 36s^2 + 80s + K = 0$.

put $s = j\omega$,

$$(j\omega)^4 + 8(j\omega)^3 + 36(j\omega)^2 + 80(j\omega) + K = 0.$$

$$\omega^4 - j8\omega^3 - 36\omega^2 + j80\omega + K = 0.$$

On equating imaginary part to zero,

$$-j8\omega^3 + j80\omega = 0$$

$$-j8\omega^3 = -j80\omega$$

$$\omega^2 = 10$$

$$\omega = \pm\sqrt{10} = \pm 3.2$$

On equating real part to zero,

$$\omega^4 - 36\omega^2 + K = 0$$

$$K = -\omega^4 + 36\omega^2$$

Put $\omega^2 = 10$

$$\therefore K = -(10)^2 + (36 \times 10) = 260.$$

The crossing point of root locus is ±j3.2. The value of K at this crossing point is K = 260. (This is the limiting value of K for stability).

The complete root locus is sketched as shown in fig 4.25.3. The root locus has four branches. All the root locus branches goes to infinity along the asymptotic lines to meet the zeros at infinity.

EXAMPLE 4.26

Sketch root locus for the unity feedback system whose open loop transfer function is,

$$G(s)\,H(s) = \frac{K\,(s+1.5)}{s\,(s+1)\,(s+5)}.$$

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s(s+1)(s+5) = 0$ and the zeros are the roots of the equation, $(s+1.5) = 0$.

The poles are lying at, s = 0, –1, –5.

The zeros are lying at, s = –1.5 and infinity.

Let us denote poles by, p_1, p_2, p_3 and finite zero by z_1.

Here, $p_1 = 0$, $p_2 = -1$, $p_3 = -5$ and $z_1 = -1.5$.

The poles are marked by X(cross) and zeros by "o" (circle) as shown in fig 4.26.1.

Step 2 : To find root locus on real axis

The segment of real axis between s = 0 and s = –1 and the segment of real axis between s = –1.5 and s = –5 will be a part of root locus. Because if we choose a test point in this segment then to the right of this point we have odd number of real poles and zeros. The root locus on real axis are shown as bold lines in fig 4.26.1.

Step 3 : To find angles of asymptotes and centroid

Since there are three poles, the number of root locus branches are three. There is one finite zero, so one root locus branch will end at finite zero. The other two branches will meet the zeros at infinity. Hence the number of asymptotes required is two.

$$\text{Angles of asymptotes} = \frac{\pm 180^{\circ}\,(2q+1)}{n-m}\ ; \qquad q = 0, 1, 2, \ldots\ldots n-m.$$

Here, n = 3 and m = 1. $\therefore$ q = 0, 1, 2 .

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180^{\circ}}{2} = \pm 90^{\circ}$$

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-1-5-(-1.5)}{2} = -2.25$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.26.1.

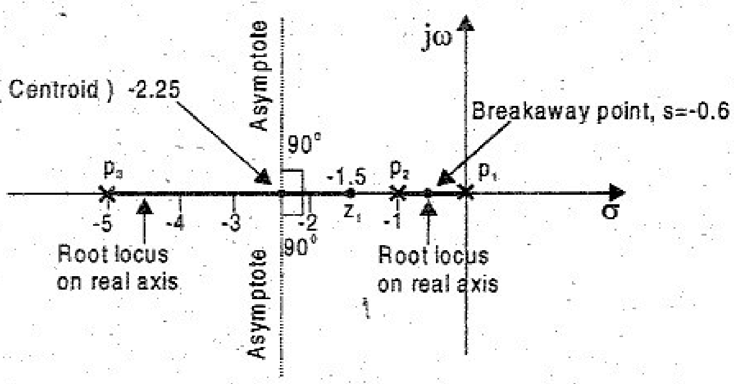

Fig 4.26.1 : Figure showing the asymptotes, root locus on real axis and location of poles, zeros, centroid and breakaway points.

Step 4 : To find the breakaway and breakin points

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\}\ \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K\,(s+1.5)}{s\,(s+1)\,(s+5)}}{1+\dfrac{K\,(s+1.5)}{s\,(s+1)\,(s+5)}} = \frac{K\,(s+1.5)}{s\,(s+1)\,(s+5)+K(s+1.5)}$$

The characteristic equation is, $s\,(s+1)\,(s+5) + K\,(s+1.5) = 0$

$$\therefore K = \frac{-s\,(s+1)\,(s+5)}{s+1.5} = \frac{-s\,(s^2+6s+5)}{s+1.5} = \frac{-(s^3+6s^2+5s)}{s+1.5}$$

On differentiating K with respect to s we get,

$$\frac{dK}{ds}=\frac{-(3s^2+12s+5)\,(s+1.5)-\left[-(s^3+6s^2+5s)\right](1)}{(s+1.5)^2}$$

$$=\frac{-3s^3-4.5s^2-12s^2-18s-5s-7.5+s^3+6s^2+5s}{(s+1.5)^2}$$

$$=\frac{-2s^3-10.5s^2-18s-7.5}{(s+1.5)^2}=\frac{-2(s^3+5.25s^2+9s+3.75)}{(s+1.5)^2}$$

For $\frac{dK}{ds}=0$, the numerator should be zero.

$$\therefore\ s^3+5.25s^2+9s+3.75=0$$

The third order polynomial will have one real root. The real root of the above polynomial can be determined by Lin's method. *(Refer Appendix II).*

To find the real root of, $s^3+5.25s^2+9s+3.75=0$, by Lin's method

The last two terms of the polynomial are chosen as I[st] trial divisor.

I[st] trial

I[st] Trial divisor $=9s+3.75=s+\frac{3.75}{9}=s+0.42$

$$\begin{array}{r|l} & s^2+4.83s+6.97 \\ \hline s+0.42 & s^3+5.25s^2+9s+3.75 \\ & s^3+0.42s^2 \\ \hline & 4.83s^2+9s \\ & 4.83s^2+2.03s \\ \hline \text{II}^{nd}\text{ trial divisor}\rightarrow & 6.97s+3.75 \\ & 6.97s+2.93 \\ \hline & 0.82 \end{array}$$

II[nd] trial

II[nd] Trial divisor $=6.97s+3.75=s+\frac{3.75}{6.97}=s+0.54$

$$\begin{array}{r|l} & s^2+4.71s+6.46 \\ \hline s+0.54 & s^3+5.25s^2+9s+3.75 \\ & s^3+0.54s^2 \\ \hline & 4.71s^2+9s \\ & 4.71s^2+2.54s \\ \hline \text{II}^{nd}\text{ trial divisor}\rightarrow & 6.46s+3.75 \\ & 6.46s+3.49 \\ \hline & 0.26 \end{array}$$

III[rd] trial

III[rd] Trial divisor $=6.46s+3.75=s+\frac{3.75}{6.46}=s+0.58$

$$\begin{array}{r|l} & s^2+4.67s+6.3 \\ \hline s+0.58 & s^3+5.25s^2+9s+3.75 \\ & s^3+0.58s^2 \\ \hline & 4.67s^2+9s \\ & 4.67s^2+2.7s \\ \hline \text{IV}^{th}\text{ trial divisor}\rightarrow & 6.3s+3.75 \\ & 6.3s+3.65 \\ \hline & 0.1 \end{array}$$

IV[th] trial

IV[th] Trial divisor $=6.3s+3.75=s+\frac{3.75}{6.3}=s+0.6$

$$\begin{array}{r|l} & s^2+4.65s+6.2 \\ \hline s+0.6 & s^3+5.25s^2+9s+3.75 \\ & s^3+0.6s^2 \\ \hline & 4.65s^2+9s \\ & 4.65s^2+2.8s \\ \hline & 6.2s+3.75 \\ & 6.2s+3.72 \\ \hline & 0.03 \end{array}$$

On neglecting the small value of 0.03, one of the root of the polynomial is, s = -0.6.

The polynomial, $s^3+5.25s^2+9s+3.75=0$, can be expressed,

$$s^3+5.25s^2+9s+3.75=(s+0.6)\,(s^2+4.65s+6.2)=0.$$

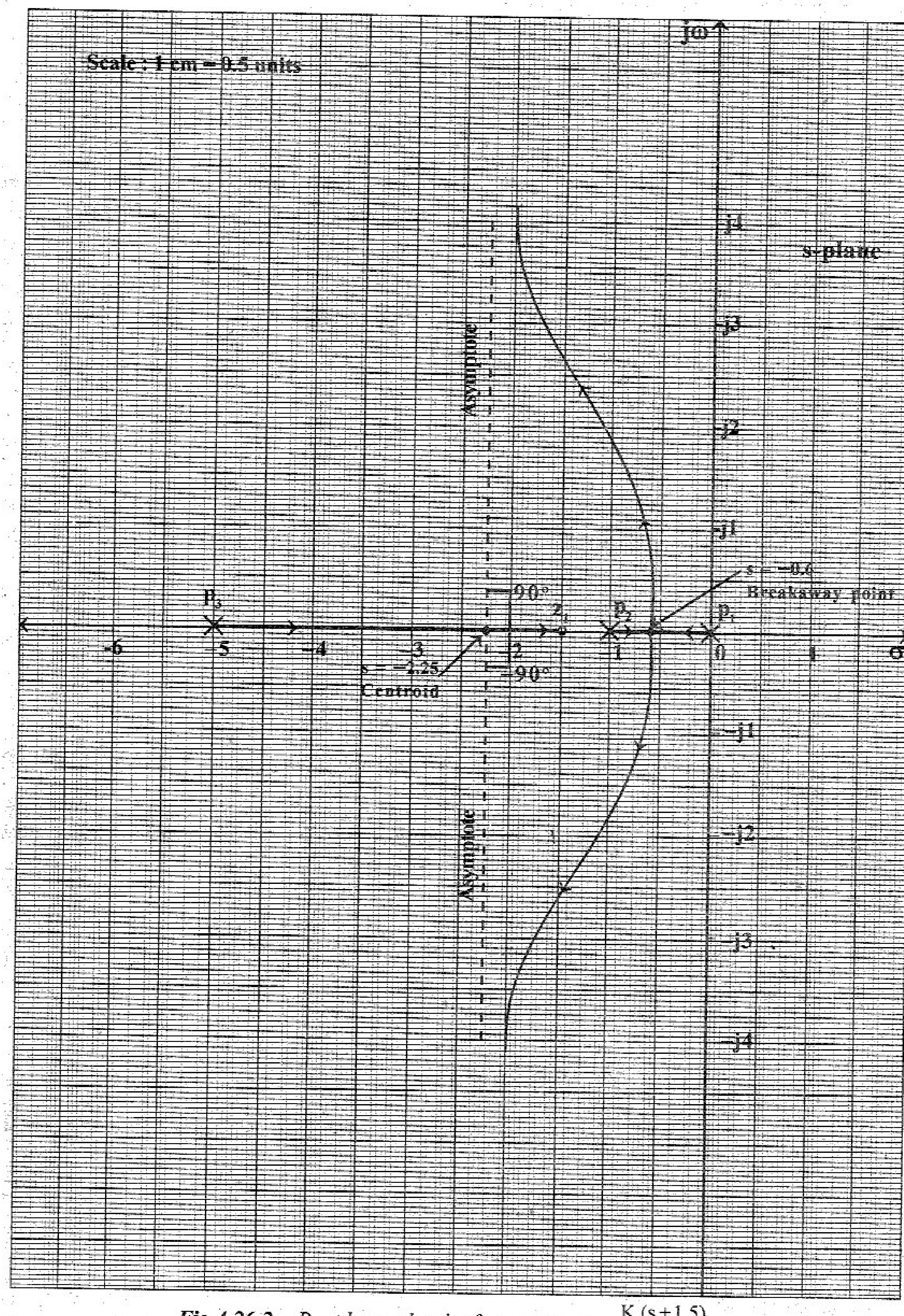

Fig 4.26.2. : Root locus sketch of, $1+G(s)=1+\dfrac{K(s+1.5)}{s(s+1)(s+5)}$.

The roots of the quadratic, $(s^2+4.65s+6.2)$, are,

$$s=\frac{-4.65\pm\sqrt{4.65^2-4\times 6.2}}{2}=-2.3\pm j0.89$$

Check for K : When $s=-0.6$, $K=\frac{-(s^3+6s^2+5s)}{s+1.5}=\frac{-\left[(-0.6)^3+6(-0.6)^2+5(-0.6)\right]}{-0.6+1.5}=1.17$

For $s=-0.6$, the value of K is positive and real and so it is actual breakaway point. It can be shown that for $s=-2.3\pm j0.86$ the value of K is not positive and real and so they cannot be breakaway points. The actual breakaway point is shown in fig 4.26.1.

Step 5 : To find angle of departure

Since there are no complex pole or zero we need not find angle of departure or arrival.

Step 6 : To find crossing point of imaginary axis.

The characteristic equation is,

$$s(s+1)(s+5)+K(s+1.5)=0 \Rightarrow s(s^2+6s+5)+Ks+1.5K=0 \Rightarrow s^3+6s^2+5s+Ks+1.5K=0$$

Put $s=j\omega$

$$(j\omega)^3+6(j\omega)^2+5(j\omega)+K(j\omega)+1.5K=0 \Rightarrow -j\omega^3-6\omega^2+j5\omega+jK\omega+1.5K=0$$

On equating imaginary part to zero, we get,

$$-j\omega^3+j5\omega+jK\omega=0$$
$$-j\omega^3=-j5\omega-jK\omega$$
$$\omega^2=5+K$$

On equating real part to zero we get,

$$-6\omega^2+1.5K=0$$

put $\omega^2=5+K$

$$-6(5+K)+1.5K=0$$
$$-30-4.5K=0$$
$$-4.5K=30 \Rightarrow K=-\frac{30}{4.5}=-6.67$$

Since the value of K is negative, there is no crossing point on imaginary axis, or for any positive values of K, and so the root locus will not cross imaginary axis.

The complete root locus sketch is shown in figure 4.26.2. The root locus has three branches. One branch starts at $s=-5$ and ends at finite zero at $s=-1.5$. The other two root locus starts at $s=0$ and $s=-1$ and breakaway from real axis at $s=-0.6$, then travel parallel to asymptotes to meet the zeros at infinity.

EXAMPLE 4.27

Sketch root locus for the unity feedback system whose open loop transfer function is,

$$G(s)=\frac{K(s^2+6s+25)}{s(s+1)(s+2)}.$$

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation $s(s+1)(s+2)=0$ and the zeros are the roots of the equation $(s^2+6s+25)=0$.

The roots of quadratic are, $s=\frac{-6\pm\sqrt{6^2-4\times 25}}{2}=-3\pm j4$

The poles are lying at, $s=0,-1,-2$

The zeros are lying at, $s=-3+j4, -3-j4$.

Let us denote poles by p_1, p_2, p_3 and zeros by z_1, z_2.

Here, $p_1 = 0$, $p_2 = -1$, $p_3 = -2$, $z_1 = -3 + j4$, $z_2 = -3 - j4$.

The poles are marked by X (cross) and zeros by "o" (circle) as shown in fig 4.27.1.

Step 2 : To find root locus on real axis

The segment of real axis between s = 0 and s = -1 and the entire negative real axis from s = -2 will be part of root locus. Because if we choose a test point in this segment then to the right of this point we have odd number of real poles and zeros. The root locus on real axis are shown as a bold line in fig 4.27.1.

Step 3 : To find angles of asymptotes and centroid

Since there are three poles the number of root locus branches are three. There are two finite zeros, so two root locus branch will end at finite zeros. The third root locus will meet the zero at infinity by travelling through negative real axis. Here the number of asymptote is one and the angle of asymptote is ±180°.

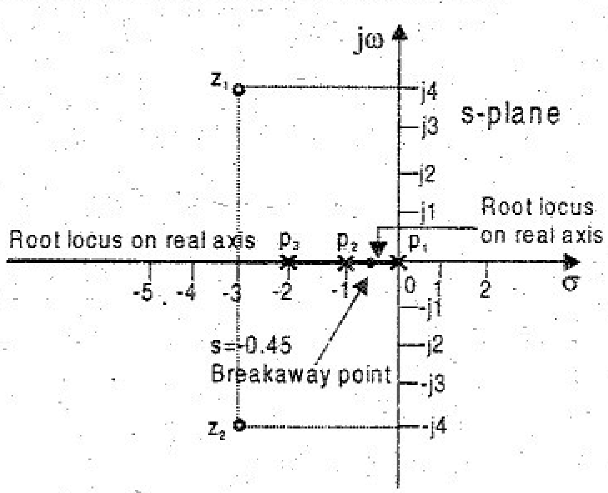

Fig 4.27.1 : *Figure showing the root locus on real axis location of poles, zeros and breakaway points.*

Step 4 : To find the breakaway and breakin points

The closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K(s^2+6s+25)}{s(s+1)(s+2)}}{1+\dfrac{K(s^2+6s+25)}{s(s+1)(s+2)}} = \frac{K(s^2+6s+25)}{s(s+1)(s+2)+K(s^2+6s+25)}$$

The characteristic equation is, $s(s+1)(s+2) + K(s^2+6s+25) = 0$.

$$\therefore K = \frac{-s(s+1)(s+2)}{s^2+6s+25} = \frac{-s(s^2+3s+2)}{s^2+6s+25} = \frac{-s^3-3s^2-2s}{s^2+6s+25}$$

On differentiating K with respect to s we get,

$$\frac{dK}{ds} = \frac{(-3s^2-6s-2)(s^2+6s+25)-(-s^3-3s^2-2s)(2s+6)}{(s^2+6s+25)^2}$$

$$= \frac{-3s^4-18s^3-75s^2-6s^3-36s^2-150s-2s^2-12s-50+2s^4+6s^3+6s^3+18s^2+4s^2+12s}{(s^2+6s+25)^2} = \frac{-(s^4+12s^3+91s^2+150s+50)}{(s^2+6s+25)^2}$$

For $\frac{dK}{ds} = 0$, the numerator should be zero.

$$\therefore s^4+12s^3+91s^2+150s+50 = 0$$

The fourth order polynomial can be split into two quadratic equations. The two quadratic factors can be obtained b Lin's method. *(Refer Appendix-II).*

To find quadratic factors by Lin's Method.

The first trial divisor be the last three terms

Ist trial

I^{st} Trial divisor $= 91s^2 + 150s + 50$

$$= s^2 + \frac{150}{91}s + \frac{50}{91} = s^2 + 1.65s + 0.55$$

$$\begin{array}{r|l} & s^2 + 10.35s + 73.37 \\ \hline s^2 + 1.65s + 0.55 & s^4 + 12s^3 + 91s^2 + 150s + 50 \\ & s^4 + 1.65s^3 + 0.55s^2 \\ \hline & 10.35s^3 + 90.45s^2 + 150s \\ & 10.35s^3 + 17.08s^2 + 5.7s \\ \hline \text{II}^{nd} \text{ trial divisor} \rightarrow & 73.37s^2 + 144.3s + 50 \\ & 73.37s^2 + 121.1s + 40.35 \\ \hline & 23.2s + 9.65 \\ \hline \end{array}$$

IInd trial

II^{nd} Trial divisor $= 73.37s^2 + 144.3s + 50$

$$= s^2 + \frac{144.3}{73.37}s + \frac{50}{73.37} = s^2 + 2s + 0.7$$

$$\begin{array}{r|l} & s^2 + 10s + 70.3 \\ \hline s^2 + 2s + 0.7 & s^4 + 12s^3 + 91s^2 + 150s + 50 \\ & s^4 + 2s^3 + 0.7s^2 \\ \hline & 10s^3 + 90.3s^2 + 150s \\ & 10s^3 + 20s^2 + 7s \\ \hline & 70.3s^2 + 143s + 50 \\ & 70.3s^2 + 140.6s + 49.2 \\ \hline & 2.4s + 0.8 \\ \hline \end{array}$$

On neglecting the small remainder we can write,

$$s^4 + 12s^3 + 91s^2 + 150s + 50 \approx (s^2 + 2s + 0.7)(s^2 + 10s + 70.3)$$

The roots of the quadratic, $s^2 + 2s + 0.7 = 0$, are,

$$s = \frac{-2 \pm \sqrt{2^2 - 4 \times 0.7}}{2} = -0.45, \; -155$$

The roots of the quadratic, $s^2 + 10s + 70.3 = 0$, are,

$$s = \frac{-10 \pm \sqrt{10^2 - 4 \times 70.3}}{2} = -5 \pm j6.73$$

Here, $s = -1.55$ is not a point on root locus, hence it cannot be a breakaway point.

Check the other three values for actual breakaway point.

$$\text{When } s = -0.45, \; K = \frac{-s^3 - 3s^2 - 2s}{s^2 + 6s + 25} = \frac{-(-0.45)^3 - 3(-0.45)^2 - 2(-0.45)}{(-0.45)^2 + 6(-0.45) + 25} = 0.017$$

For $s = -0.45$, the value of K is positive and real and so it is actual **breakaway point**. It can be shown that for $s = -5 \pm j6.73$ the value of K is not positive and real and so they cannot be breakaway points. The actual breakaway point is shown in fig 4.27.1.

Step 5 : To find angle of arrival

Let us consider the complex zero z_1 shown in fig 4.27.2. Draw vectors from all other poles and zero to the zero z_1 as shown in fig 4.27.2. Let the angles of these vectors be θ_1, θ_2, θ_3 and θ_4.

Here, $\theta_1 = 180° - \tan^{-1}\frac{4}{3} = 126.9°$

$\theta_2 = 180° - \tan^{-1}\frac{4}{2} = 116.6°$

$\theta_3 = 180° - \tan^{-1}\frac{4}{1} = 104°.$

$\theta_4 = 90°$

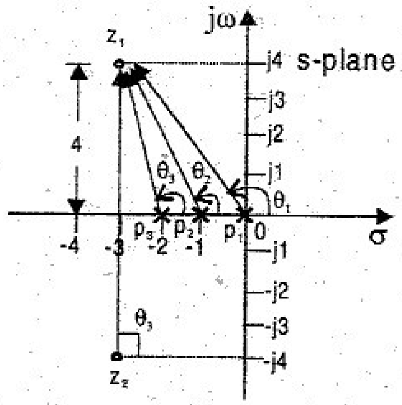

Fig 4.27.2

$$\left.\begin{array}{l}\text{Angle of arrival at} \\ \text{complex zero } z_1\end{array}\right\} = 180° - (\theta_4) + (\theta_1 + \theta_2 + \theta_3)$$

$$= 180° - 90° + 126.9° + 116.6° + 104°$$

$$= 437.5° = 77.5°$$

Angle of arrival at complex zero z_2 is negative of the angle of arrival at complex zero z_1.

$$\therefore \left.\begin{array}{l}\text{Angle of arrival at} \\ \text{complex zero } z_2\end{array}\right\} = -77.5°$$

Mark the angles of arrival at complex zeros using protractor.

Step 6 : To find the crossing point on imaginary axis

The characteristic equation is,

$$s(s+1)(s+2) + K(s^2+6s+25) = 0$$

$$s(s^2+3s+2) + Ks^2 + 6Ks + 25K = 0$$

$$s^3 + 3s^2 + 2s + Ks^2 + 6Ks + 25K = 0$$

$$s^3 + (3+K)s^2 + (2+6K)s + 25K = 0$$

Put $s = j\omega$.

$$(j\omega)^3 + (3+K)(j\omega)^2 + (2+6K)(j\omega) + 25K = 0 \Rightarrow -j\omega^3 - (3+K)\omega^2 + j(2+6K)\omega + 25K = 0$$

On equating imaginary part to zero

$$-j\omega^3 + j(2+6K)\omega = 0$$

$$-j\omega^3 = -j(2+6K)\omega$$

$$\omega^2 = (2+6K)$$

On equating real part to zero

$$-(3+K)\omega^2 + 25K = 0$$

Put $\omega^2 = 2 + 6K$

$$-(3+K)(2+6K) + 25K = 0$$

$$-(6+18K+2K+6K^2) + 25K = 0$$

$$-6K^2 + 5K - 6 = 0$$

$$K = \frac{-5 \pm \sqrt{5^2 - 4\times(-6)(-6)}}{2\times(-6)} = 0.4 \pm j0.9$$

Since the value of K is not real and positive, there is no crossing point on imaginary axis, or for any positive values of K the root locus will not cross imaginary axis.

Step 7 : To find points on root locus

Choose test points a, b, c, d on the s-plane and adjust the test points to satisfy angle criterion. The test points are shown in fig 4.27.3. On the upper half of s-plane the root locus is sketched through the test points a, b, c and d. The root locus on the lower half of s-plane is the mirror image of the root locus on the upper half of s-plane.

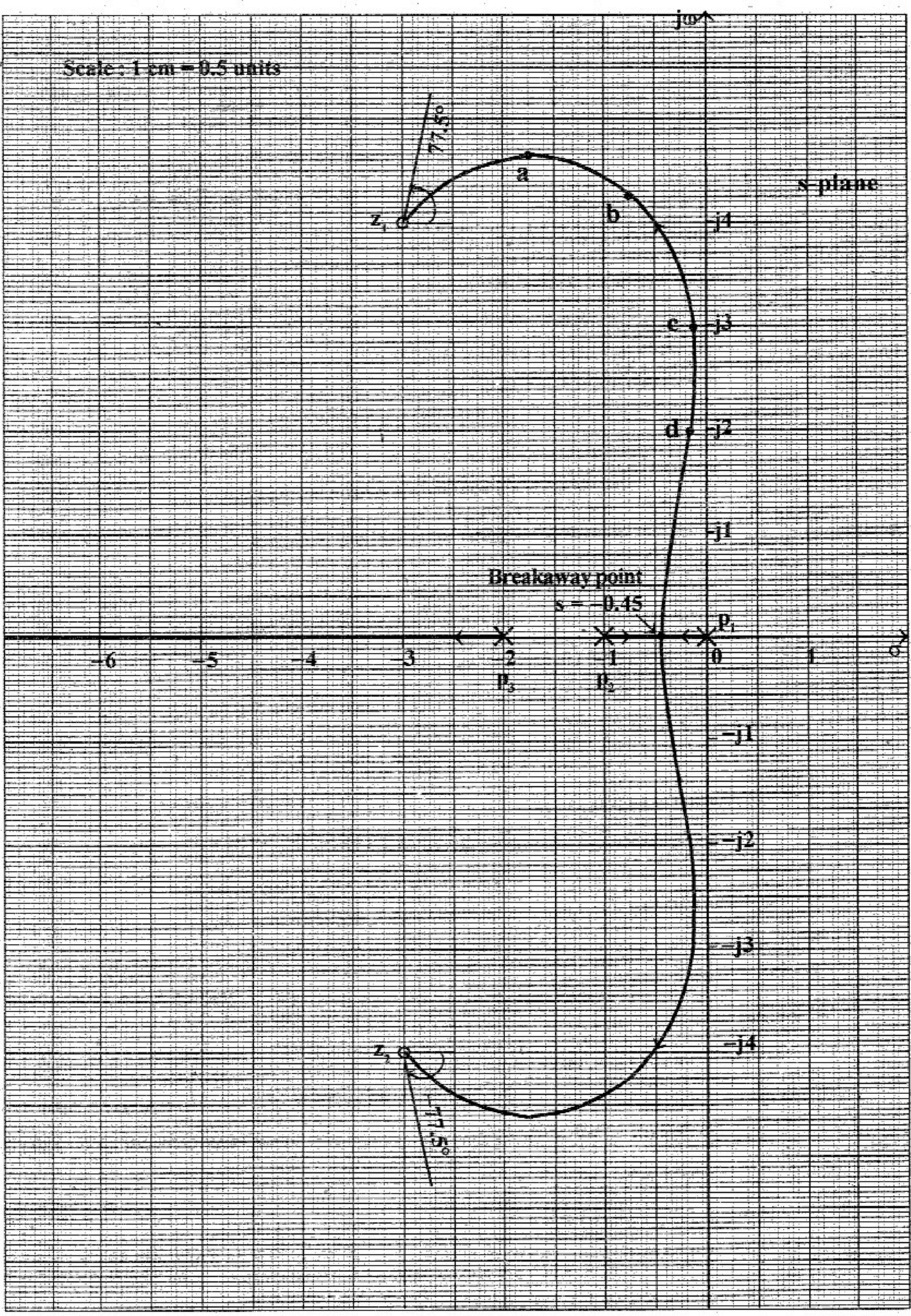

Fig 4.27.3. : Root locus sketch of, $1+G(s)=1+\frac{K(s^2+6s+25)}{s\,(s+1)\,(s+2)}$.

The complete root locus sketch is shown in fig 4.27.3. The root locus has three branches. One branch starts at s = -2 and goes to infinity along negative real axis. The other two root locus branches starts at s = 0 and s = -1 and breaks from real axis at s = -0.45, then meets the complex zeros.

EXAMPLE 4.28

Sketch the root locus for the unity feedback system whose open loop transfer function is,

$$G(s) = \frac{K}{s(s^2+6s+10)}.$$

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s(s^2+6s+10)=0$.

The roots of the quadratic are, $s = \frac{-6 \pm \sqrt{6^2 - 4 \times 10}}{2} = -3 \pm j1$

The poles are lying at, s= 0, –3+j1 and –3 –j1

Let us denote the poles as p_1, p_2, and p_3.

Here, $p_1 = 0$, $p_2 = -3+j1$, and $p_3 = -3-j1$.

The poles are marked by X(cross) as shown in fig 4.28.1

Step 2 : To find the root locus on real axis

There is only one pole on real axis at the origin. Hence if we choose any test point on the negative real axis then to the right of that point the total number of real poles and zeros is one, which is an odd number. Hence the entire negative real axis will be part of root locus. The root locus on real axis is shown are three.

> *Note : For the given transfer function one root locus branch will start at the pole at the origin and meet the zero at infinity through the negative real axis.*

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. There is no infinite zero. Hence all the three root locus branches ends at zeros at infinity. The number of asymptotes required are three

$$\text{Angles of asymptotes} = \frac{\pm 180^o\ (2q+1)}{n-m}; \qquad q = 0,\ 1,\ 2,\ ....n-m.$$

Here, n = 3 and m = 0. $\quad \therefore q = 0, 1, 2, 3, ...$

When q = 0, $\quad \text{Angles} = \pm\frac{180^\circ}{3} = \pm 60^\circ$

When q = 1, $\quad \text{Angles} = \pm\frac{180^o \times 3}{3} = \pm 180^\circ$

$$\text{Centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{n-m} = \frac{-3+j1-3-j1}{3} = -2$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.28.1.

Step 4: To find the breakaway and breakin poir.ts

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s^2+6s+10)}}{1+\frac{K}{s(s^2+6s+10)}} = \frac{K}{s(s^2+6s+10)+K}$

The characteristic equation is, $s(s^2+6s+10)+K=0$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds} = -3s^2 - 12s - 10$$

Put $\frac{dK}{ds} = 0$

$$-3s^2 - 12s - 10 = 0 \quad \Rightarrow \quad 3s^2 + 12s + 10 = 0$$

$$\therefore s = \frac{-12 \pm \sqrt{12^2 - 4 \times 3 \times 10}}{2 \times 3} = -1.18 \text{ or } -2.82$$

Check for K : When, $s = -1.18$, $K = -s^3 - 6s^2 - 10s = -(-1.18)^3 - 6(-1.18)^2 - 10(-1.18) = 5.09$

When, $s = -2.82$, $K = -s^3 - 6s^2 - 10s = -(-2.82)^3 - 6(-2.82)^2 - 10(-2.82) = 2.91$

- Since the values of K for s = –1.18 and –2.82 are positive and real, both the points are actual breakaway or breakin points. It can be proved that s = –2.82 is a breakin point and s = –1.18 is a breakaway point. The breakin and breakaway points are shown in fig 4.28.1.

[Also the value of K for s = –2.82 is less than the value of K for s = –1.18, therefore when root locus travel from s = –2.82 to –1.18, the value of K increases]

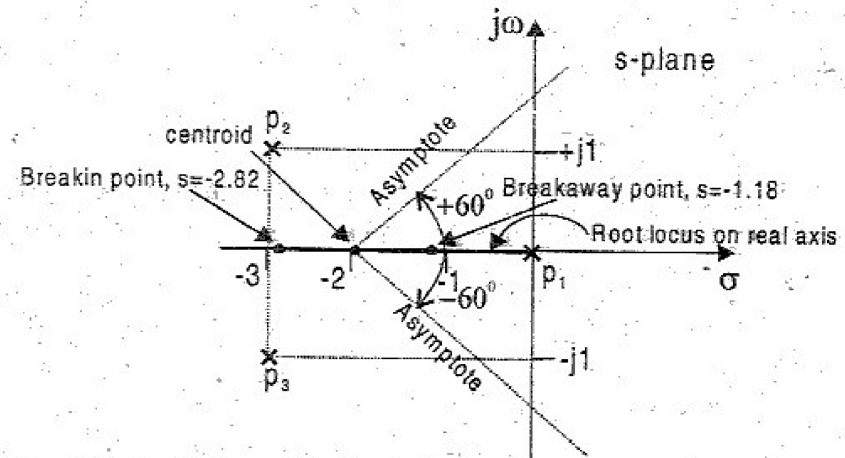

Fig 4.28.1 : Figure showing the asymptotes, root locus on real axis and location of poles, zeros, centroid, breakin and breakaway points.

Step 5: To find the angle of departure

Consider the complex pole p_2 shown in fig 4.28.2. Draw vectors from all other poles to the pole p_2 as shown in fig 4.28.2. Let the angle of these vectors be θ_1 and θ_2.

Here, $\theta_1 = 180° - \tan^{-1}(1/3) = 161.6°$

$\theta_2 = 90°$

$$\left.\begin{array}{l}\text{Angle of departure from} \\ \text{the complex pole } p_2\end{array}\right\} = 180° - (\theta_1 + \theta_2)$$

$$= 180° - (161.6° + 90°) = -71.6° \approx -72°$$

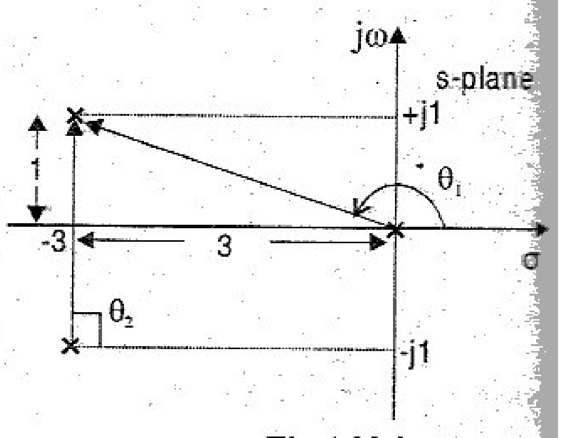

Fig 4.28.2

The angle of departure at complex pole p_3 is negative of the angle of departure at complex pole p_2.

$\therefore$ Angle of departure at pole p_3 = +72

Mark the angles of departure at complex poles using protractor.

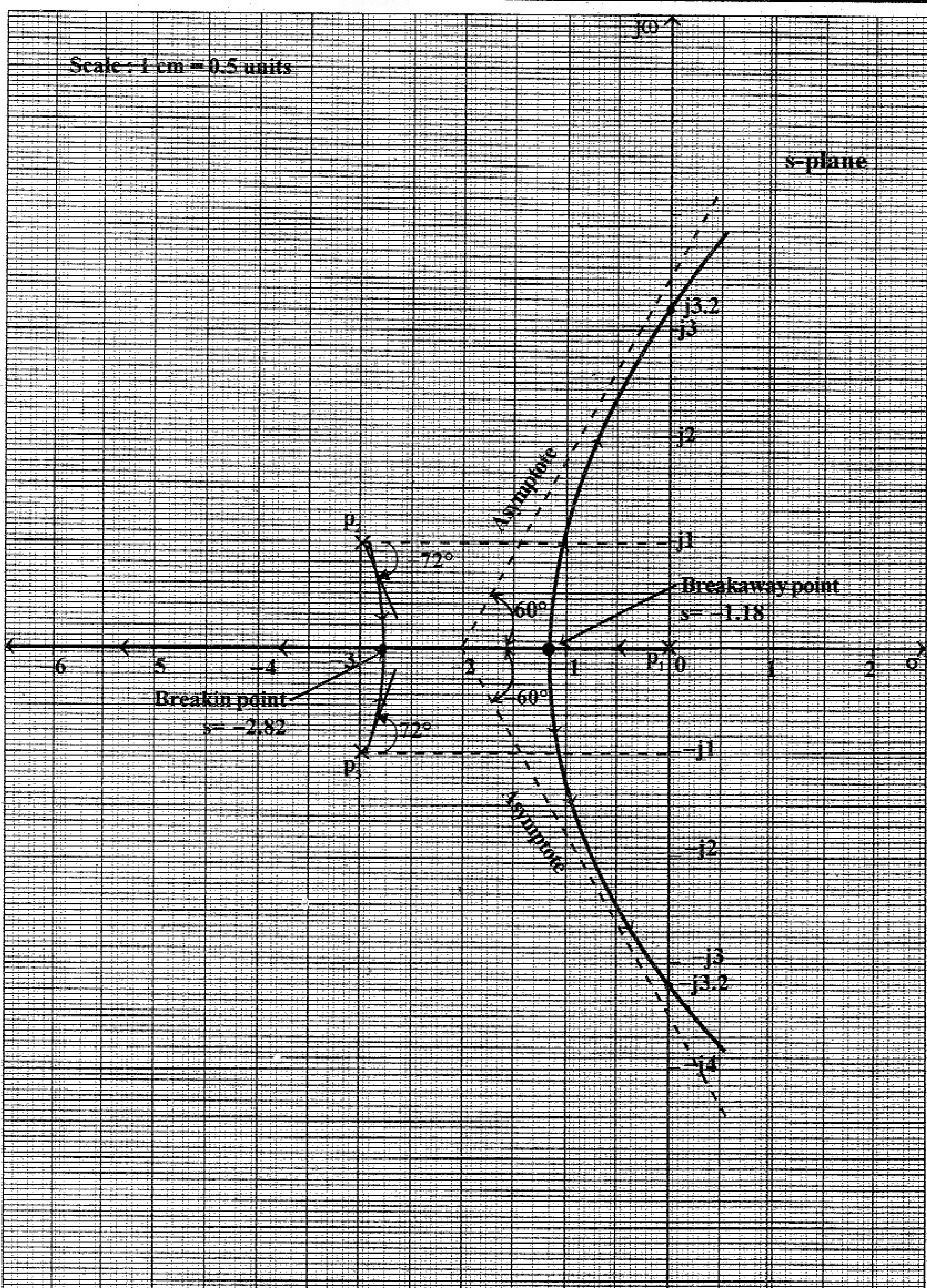

Fig 4.28.3. : Root locus sketch of, $1+G(s)=1+\dfrac{K}{s\,(s^2+6s+10)}$.

Step 6 : To find the crossing point on imaginary axis

The characteristic equation is given by, $s(s^2+6s+10)+K=0 \Rightarrow s^3+6s^2+10s+K=0$

Put $s=j\omega$.

$$(j\omega)^3+6(j\omega)^2+10(j\omega)+K=0 \Rightarrow -j\omega^3-6\omega^2+j10\omega+K=0$$

On equating imaginary part to zero we get,

$$-\omega^3+10\omega=0$$
$$\omega^3=10\omega$$
$$\omega^2=10$$
$$\omega=\pm\sqrt{10}=\pm3.16\approx\pm3.2$$

On equating real part to zero we get,

$$-6\omega^2+K=0$$
$$K=6\omega^2$$
$$=6\times10=60$$

The root locus crosses imaginary axis at ±j3.2 and the gain K corresponding to this point is 60. This is the limiting value of K for the stability of the system.

The complete root locus sketch is shown in fig 4.28.3. The root locus has three branches. One branch starts at s = 0 and goes to infinity along negative real axis. The other two root locus branches starts at s = −3 ± j1 and enter the real axis at s = −2.82 and then breakaway from real axis at s = −1.18. Finally they travel parallel to asymptotes to meet the zeros at infinity.

4.9 NYQUIST AND ROOT LOCUS PLOTS USING MATLAB

In general, the open loop transfer function of a system is denoted as G(s).

Let, G(s) be a rational function of "s", as shown below.

$$G(s)=\frac{b_0s^M+b_1s^{M-1}+b_2s^{M-2}+\dots\dots\dots+b_{M-1}s+b_M}{a_0s^N+a_1s^{N-1}+a_2s^{N-2}+\dots\dots\dots+a_{N-1}s+a_N}$$

For drawing Nyquist and root locus plots, the transfer function G(s) is declared as a function of s using the following commands.

```
s=tf('s');
Gs=(b0*s^M+b1*s^(M-1)+...+bM)/(a0*s^N+a1*s^(N-1)+...+aN);
```

The coefficients of numerator and denominator polynomials of the transfer function are determined using the following command.

```
[num_cof  den_cof]=tfdata(Gs);
```

The horizontal and vertical axes range for the Nyquist and root locus plots can be specified using the axis command as shown below.

```
axis([x_start   x_end   y_start   y_end]);
```

NYQUIST PLOT

The Nyquist plot can be plotted using any one of the following commands.

```
nyquist(Gs);
nyquist(Gs,'k');
nyquist(num_cof, den_cof);
```

ROOT LOCUS PLOT

The root locus plot can be plotted using any one of the following commands.

```
rlocus(Gs);
rlocus(Gs,'k');
rlocus(num_cof, den_cof);
```

PROGRAM 4.1

Write a MATLAB program to draw the Nyquist plot of the system governed by the following open loop transfer function.

G(s) = 240/s(s+2)(s+10).

```
%program to plot Nyquist plot

clear all
clc
s=tf('s');
disp('The given transfer function is');
Gs=240/(s*(s+2)*(s+10))

nyquist(Gs,'k');
axis([-4 0.5 -2 2]); grid;
```

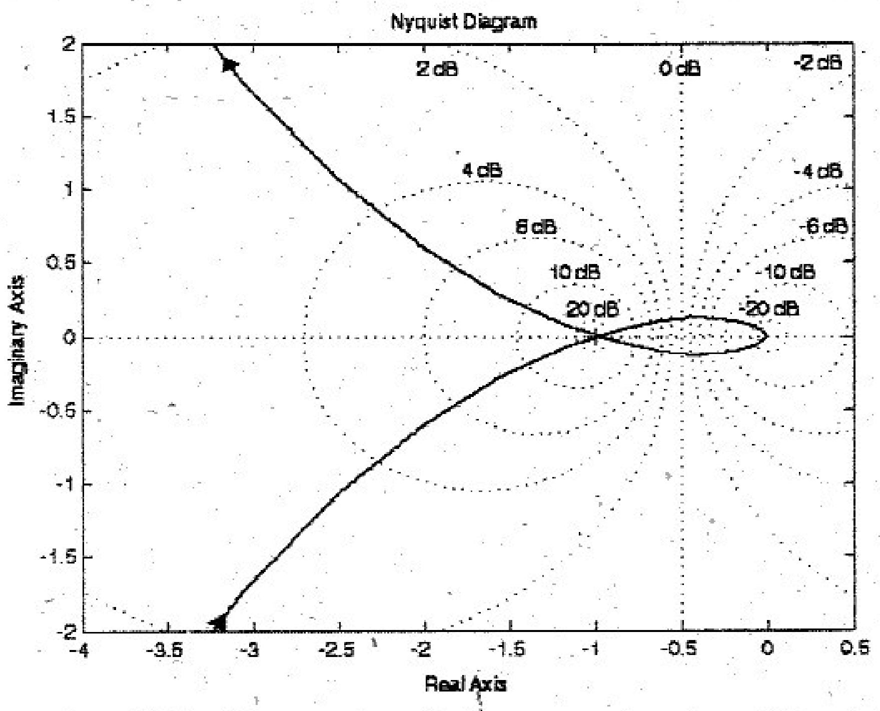

Fig P4.1 : Nyquist plot of the system given in problem 4.1.

OUTPUT

The given transfer function is,

Transfer function:

```
        240
--------------------
s^3 + 12 s^2 + 20 s
```

The Nyquist plot of program 4.1 is shown in fig P4.1.

PROGRAM 4.2

Write a MATLAB program to draw the Nyquist plot of the system governed by the following open loop transfer function.

G(s) = (1+0.5s)(1+s)/(1+10s)(s-1)

```
%program to plot Nyquist plot

clear all
clc
```

```
s=tf('s');
disp('The given transfer function is,')
Gs=((1+0.5*s)*(1+s))/((1+10*s)*(s-1))

nyquist(Gs,'k');
axis([-1.2 0.2 -1 1]);
grid;
```

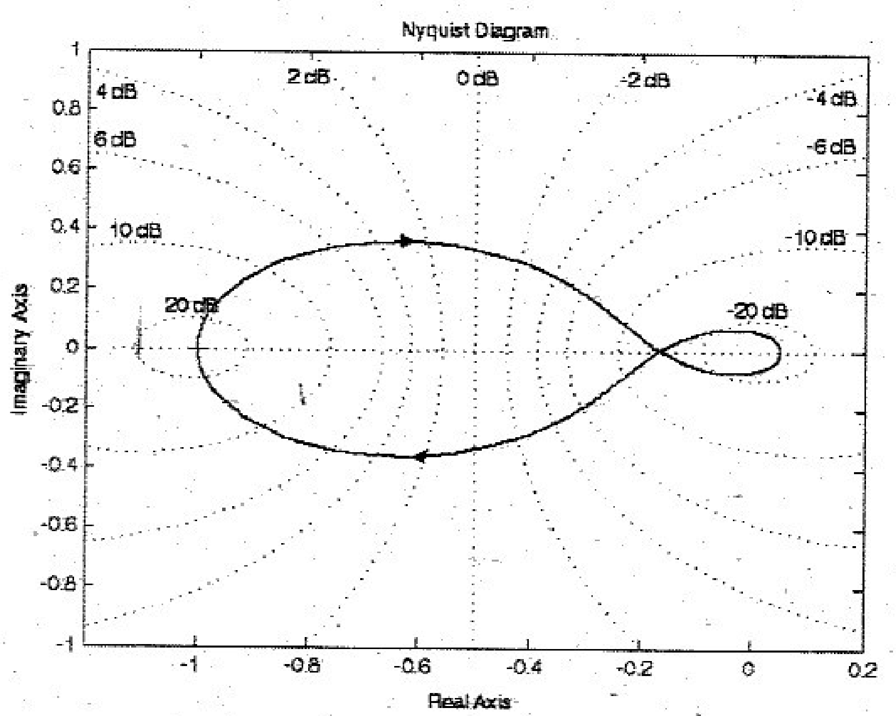

Fig P4.2 : Nyquist plot of the system given in problem 4.2.

OUTPUT

The given transfer function is,

Transfer function:

```
0.5 s^2 + 1.5 s + 1
--------------------
10 s^2 - 9 s - 1
```

The Nyquist plot of program 4.2 is shown in fig P4.2.

PROGRAM 4.3

Write a MATLAB program to draw the Nyquist plot of the system governed by the following open loop transfer function.

G(s) = (s+2)/(s+1)(s-1)

```
%program to plot Nyquist plot

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=(s+2)/((s+1)*(s-1))

nyquist(Gs,'k');
axis([-2.5 0.2 -1 1]);
grid;
```