The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.22.1.

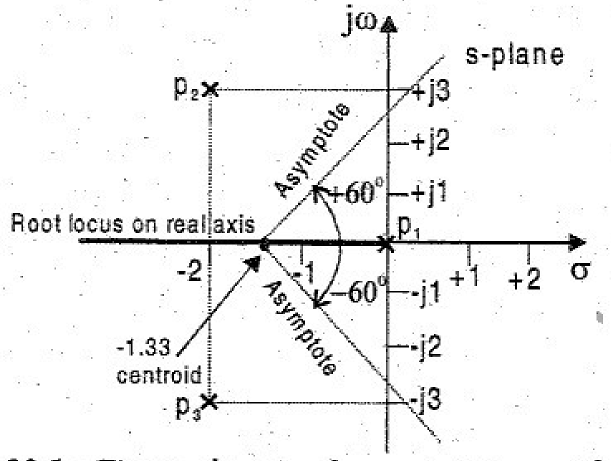

Fig 4.22.1 : Figure showing the asymptote, root locus on real axis and location of poles and centroid

Step 4 : To find the breakaway and breakin points

$$\left.\begin{matrix}\text{The closed loop}\\\text{transfer function}\end{matrix}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K}{s(s^2+4s+13)}}{1+\dfrac{K}{s(s^2+4s+13)}} = \frac{K}{s(s^2+4s+13)+K}$$

The characteristic equation is, $s(s^2+4s+13)+K=0$

$$\therefore \quad s^3+4s^2+13s+K=0 \quad \Rightarrow \quad K=-s^3-4s^2-13s$$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds} = -(3s^2+8s+13)$$

Put $\dfrac{dK}{ds}=0$

$$\therefore \quad -(3s^2+8s+13)=0 \quad \Rightarrow \quad (3s^2+8s+13)=0$$

$$\therefore s = \frac{-8 \pm \sqrt{8^2-4\times13\times3}}{2\times3} = -1.33 \pm j1.6$$

Check for K : When, $s=-1.33+j1.6$, the value of K is given by,

$$K=-(s^3+4s^2+13s) = -[(-1.33+j1.6)^3+4(-1.33+j1.6)^2+13(-1.33+j1.6)]$$

$$\neq \text{positive and real.}$$

Also it can be shown that when $s=-1.33-j1.6$ the value of K is not equal to real and positive.

Since the values of K for, $s=-1.33\pm j1.6$, are not real and positive, these points are not an actual breakaway or breakin points. The root locus has neither breakaway nor breakin point.

Step 5 : To find the angle of departure

Let us consider the complex pole p_2 shown in fig 4.22.2. Draw vectors from all other poles to the pole p_2 as shown in fig 4.22.2. Let the angles of these vectors be θ_1 and θ_2.

Here, $\theta_1 = 180° - \tan^{-1}(3/2) = 123.7°$; $\theta_2 = 90°$

Angle of departure from the complex pole $p_2 = 180° - (\theta_1+\theta_2)$

$$= 180° - (123.7° + 90°)$$

$$= -33.7°$$

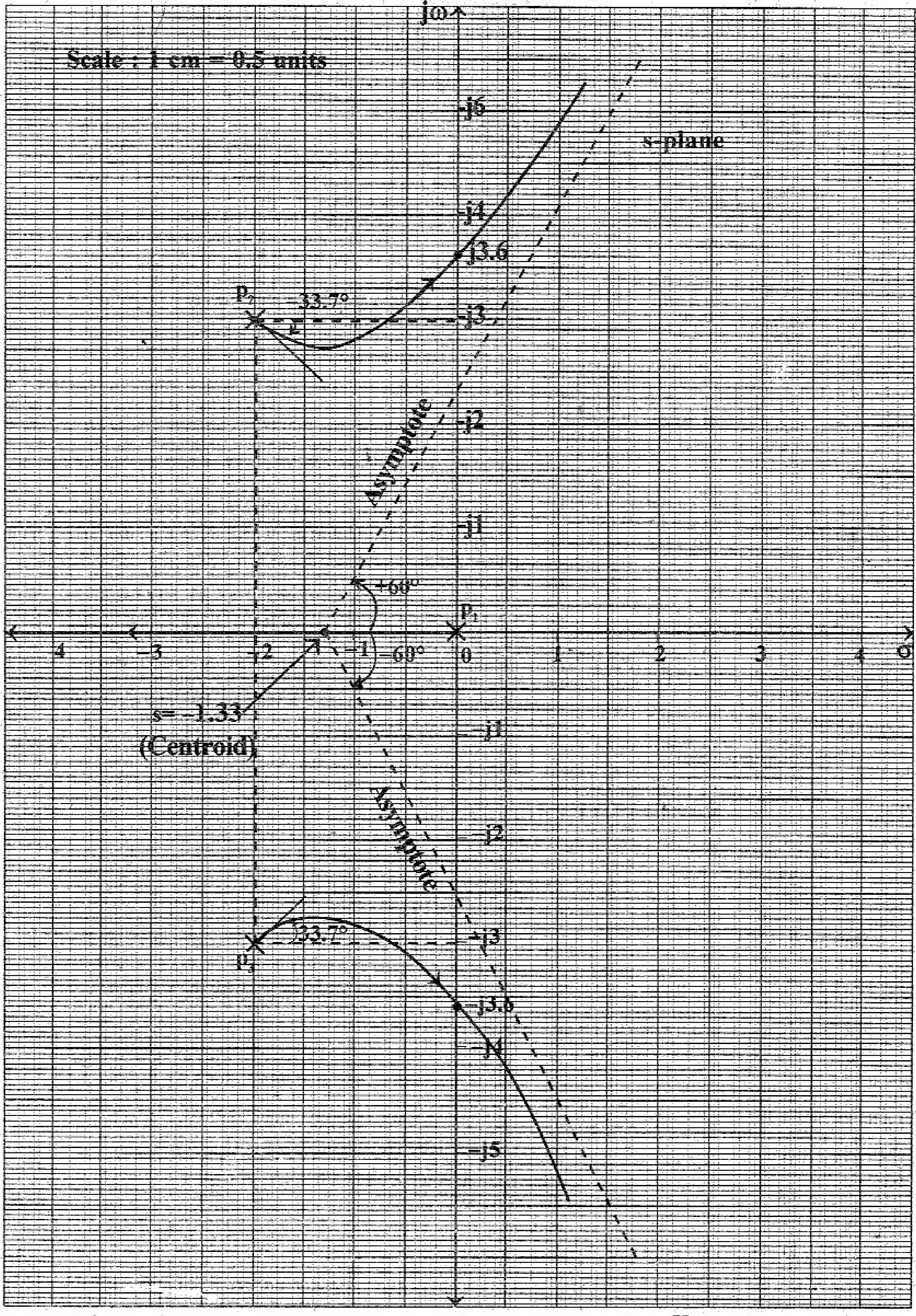

Fig 4.22.3. : Root locus sketch of, $1+G(s)=1+\dfrac{K}{s\,(s^2+4s+13)}$.

The angle of departure at complex pole p_3 is negative of the angle of departure at complex pole A.

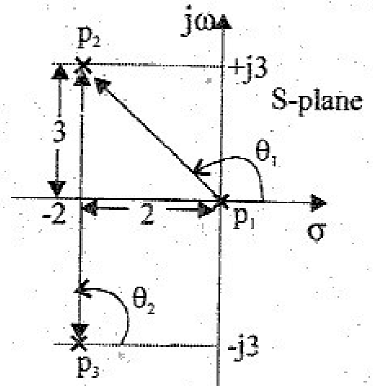

Fig 4.22.2

∴ Angle of departure at pole $p_3 = +33.7°$

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point on imaginary axis

The characteristic equation is given by,

$$s^3 + 4s^2 + 13s + K = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0 \quad \Rightarrow -j\omega^3 - 4\omega^2 + 13j\omega + K = 0$$

On equating imaginary part to zero, we get,

$$-\omega^3 + 13\omega = 0$$

$$-\omega^3 = -13\omega$$

$$\omega^2 = 13 \Rightarrow \quad \omega = \pm\sqrt{13} = \pm 3.6$$

On equating real part to zero, we get,

$$-4\omega^2 + K = 0$$

$$K = 4\omega^2$$

$$= 4 \times 13 = 52$$

The crossing point of root locus is $\pm$ j3.6. The value of K at this crossing point is K = 52. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig 4.22.3. The root locus has three branches one branch starts at the pole at origin and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at complex poles (along the angle of departure), crosses the imaginary axis at ± j3.6 and travel parallel to asymptotes to meet the zeros at infinity.

EXAMPLE 4.23

Sketch the root locus of the system whose open loop transfer function is, $G(s) = \frac{K}{s\,(s+2)\,(s+4)}$. Find the value of K so that the damping ratio of the closed loop system is 0.5.

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, s(s+2) (s+4) = 0.

∴ The poles are lying at, s = 0, –2, – 4.

Let us denote the poles as p_1, p_2, and p_3.

Here, $p_1 = 0$, $p_2 = -2$, $p_3 = -4$.

The poles are marked by X(cross) as shown in fig 4.23.1.

Step 2 : To find the root locus on real axis

There are three poles on the real axis.

Choose a test point on real axis between s = 0 and s = –2. To the right of this point the total number of real poles and zeros is one, which is an odd number. Hence the real axis between s = 0 and s = –2 will be a part of root locus.

Choose a test point on real axis between s = –2 and s = –4. To the right of this point, the total number of real poles and zeros is two which is an even number. Hence the real axis between s = –2 and s = – 4 will not be a part of root locus.

Choose a test point on real axis to the left of s = – 4. To the right of this point, the total number of real poles and zeros is three, which is an odd number. Hence the entire negative real axis from s = – 4 to – ∞ will be a part of root locus.

The root locus on real axis are shown as bold lines in fig 4.23.1.

Step 3 : To find asymptotes and centroid

Since there are three poles the number of root locus branches are three. There is no finite zero. Hence all the three root locus branches ends at zeros at infinity. The number of asymptotes required are three.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ\ (2q+1)}{n-m} \quad ; \quad q = 0, 1, 2, ...n-m.$$

Here, n = 3 and m = 0. $\therefore q = 0, 1, 2, 3$.

When $q = 0$, $\text{Angles} = \pm\frac{180^\circ}{3} = \pm 60^\circ$

When $q = 1$, $\text{Angles} = \pm\frac{180^\circ \times 3}{3} = \pm 180^\circ$

> ***Note :** It is enough if you calculate the required number of angles. Here it is given by first three values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-2-4-0}{3} = -2$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.23.1.

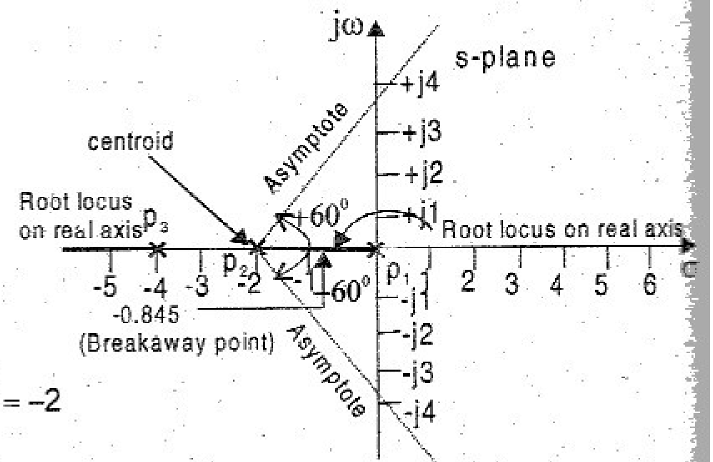

Fig 4.23.1 : Figure showing the asymptotes root locus on real axis and location of poles centroid ,and breakaway points.

Step 4 : To find the breakaway and breakin points

$$\left.\begin{matrix}\text{The closed loop}\\ \text{transfer function}\end{matrix}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s\,(s+2)\,(s+4)}}{1+\frac{K}{s\,(s+2)\,(s+4)}} = \frac{K}{s\,(s+2)\,(s+4)\ +\ K}$$

The characteristic equation is given by,

$$s\,(s+2)\,(s+4) + K = 0 \quad \Rightarrow \quad s\,(s^2+6s+8) + K = 0 \quad \Rightarrow \quad s^3 + 6s^2 + 8s + K = 0$$

$$\therefore K = -s^3 - 6s^2 - 8s$$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds} = -(3s^2 + 12s + 8)$$

Put $\frac{dK}{ds} = 0$

$$\therefore -(3s^2 + 12s + 8) = 0 \quad \Rightarrow \quad (3s^2 + 12s + 8) = 0$$

$$s = \frac{-12 \pm \sqrt{12^2 - 4 \times 3 \times 8}}{2 \times 3} = -0.845 \quad \text{or} \quad -3.154$$

Check for K : When s = –0.845, the value of K is given by,

$$K = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)] = 3.08$$

Since K, is positive and real for, s = –0.845, this point is actual breakaway point.

When s = –3.154, the value of K is given by,

$$K = -\ [(-3.154)^3 + 6(-3.154)^2 + 8(-3.154)] = -3.08$$

Since K, is negative for, s = –3.154, this is not a actual breakaway point.

The breakaway point is marked on the negative real axis as shown in fig 4.23.1.

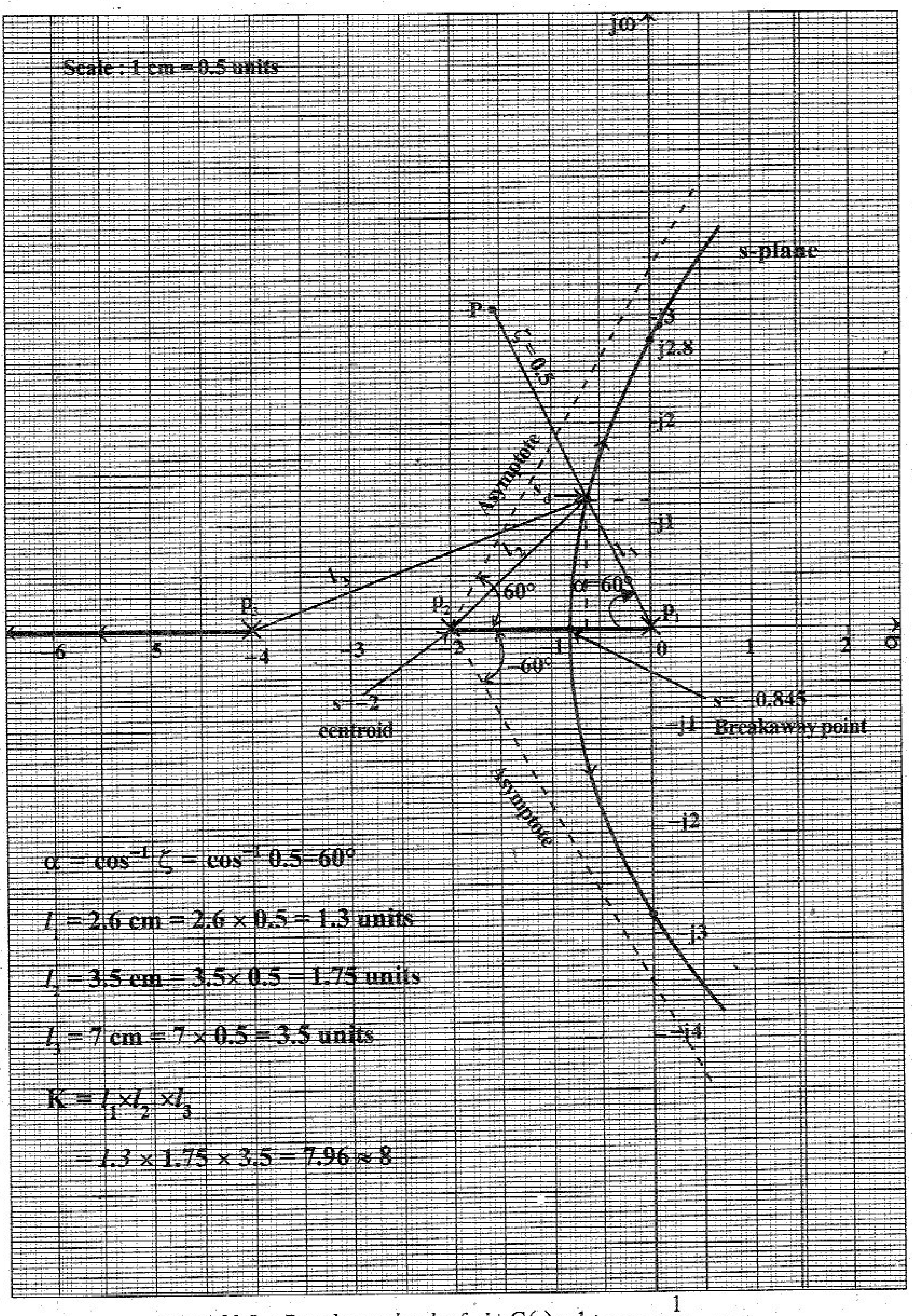

Fig 4.23.2. : Root locus sketch of, $1 + G(s) = 1 + \dfrac{1}{s\,(s+2)\,(s+4)}$.

Step 5 : To find angle of departure

Since there are no complex pole or zero, we need not find angle of departure or arrival.

Step 6 : To find the crossing point of imaginary axis

The characteristic equation is given by,

$$s^3 + 6s^2 + 8s + K = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

Equating imaginary part to zero

$$-j\omega^3 + j8\omega = 0$$

$$-j\omega^3 = -j8\omega$$

$$\omega^2 = 8 \quad \Rightarrow \quad \omega = \pm\sqrt{8} = \pm 2.8$$

Equating real part to zero

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2 = 6 \times 8 = 48$$

The crossing point of root locus is $\pm j2.8$. The value of K corresponding to this point is K = 48. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig 4.23.2. The root locus has three branches. One branch starts at the pole at s = –4 and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at s = 0 and s = –2 and travel through negative real axis, breakaway from real axis at s = –0.845, then crosses imaginary axis at s = ± j2.8 and travel parallel to asymptotes to meet the zeros at infinity

To find the value of K corresponding to ζ = 0.5

Given that $\zeta = 0.5$

Let $\alpha = \cos^{-1}\zeta = \cos^{-1} 0.5 = 60°$

Draw a line OP, such that the angle between line OP and negative real axis is 60° (α = 60°) as shown in fig 4.23.2. The meeting point of the line OP and root locus gives the dominant pole, s_d.

Let K_{sd} be value of K corresponding to the point $s = s_d$

$$K_{sd} = \frac{\text{Product of length of vector from all poles to the point, } s = s_d}{\text{Product of length of vector from all zeros to the point, } s = s_d}$$

$$= \frac{l_1 \times l_2 \times l_3}{1} = 1.3 \times 1.75 \times 3.5 = 7.96 \approx 8$$

Note : The length of vectors are measured to scale.

EXAMPLE 4.24

The open loop transfer function of a unity feedback system is given by, $G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$. Sketch the root locus of the system.

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equations, $(s^2 + 4s + 11) = 0$.

The roots of the quadratic are, $s = \frac{-4 \pm \sqrt{4^2 - 4 \times 11}}{2} = -2 \pm j2.64$

$\therefore$ The poles are lying at, s = 0, –2 + j2.64, –2 –j2.64

The zeros are lying at, s = –9 and infinity.

Let us denote the poles as p_1, p_2, p_3 finite zero by z_1.

Here, $p_1 = 0$, $p_2 = -2+j2.64$, $p_3 = -2-j2.64$ and $z_1 = -9$.

The poles are marked by X(cross) and zeros by "o" (circle) as shown in fig 4.24.1.

Step 2 : To find the root locus on real axis.

One pole and one zero lie on real axis.

Choose a test point to the left of s = 0, then to the right of this point, the total number of poles and zeros is one which is an odd number. Hence the portion of real axis from s = 0 to s = –9 will be a part of root locus.

If we choose a test point to the left of s = –9 then to the right of this point, the total number of poles and zeros is two, which is an even number. Hence the real axis from s = –9 to –∞ will not be a part of root locus.

The root locus on real axis is shown as a bold line in fig 4.24.1.

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. One root locus branch starts at the pole at origin and travel along negative real axis to meet the zero at s = –9. The other two root locus branches meet the zeros at infinity. The number of asymptotes required are two.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ\ (2q+1)}{n-m}; \qquad q = 0,\ 1,\ 2,\ ....n-m.$$

Here, n = 3 and m = 0. $\therefore$ q = 0, 1, 2, 3 .

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180^\circ}{3} = \pm 60^\circ$$

$$\text{When } q = 1, \quad \text{Angles} = \pm\frac{180^\circ \times 3}{2} = \pm 270^\circ = \mp 90^\circ$$

$$\text{When } q = 2, \quad \text{Angles} = \pm\frac{180^\circ \times 5}{2} = \pm 450^\circ = \pm 90^\circ$$

> ***Note :** It is enough if you calculate the required number of angles. Here it is given by first two values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-2+j2.64-2-j2.64-(-9)}{2} = 2.5$$

The centroid is marked and from the centroid, the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown 4.24.1.

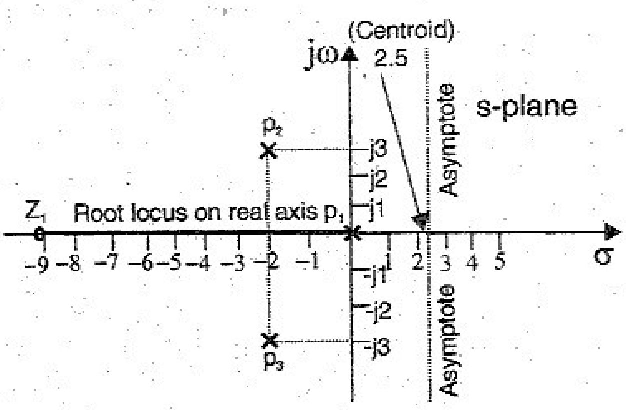

Fig 4.24.1 : Figure showing the asymptotes, root locus on real axis and location of poles, zero and centroid

Step 4 : To find the breakaway and breakin points

From the location of poles and zero and from the knowledge of typical sketches of root locus, it can be concluded that there is no possibility of breakaway or breakin points.

Step 5 : To find the angle of departure

Let us consider the complex pole p_2 as shown in fig 4.24.2. Draw vectors from all other poles and zero to the pole p_2 as shown in fig 4.24.2. Let the angles of these vectors be θ_1, θ_2 and θ_3.

$$\text{Here, } \theta_1 = 180° - \tan^{-1}\frac{2.64}{2} = 127.1°$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{2.64}{7} = 20.7°$$

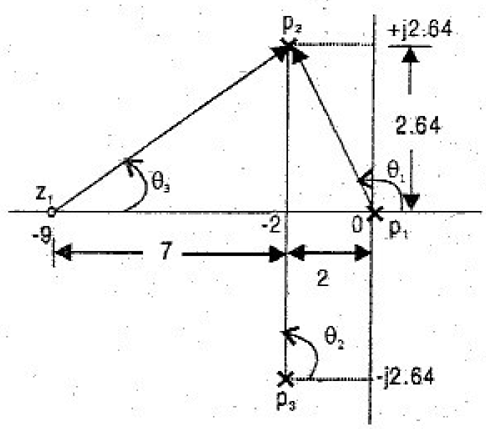

Fig 4.24.2

$$\left.\begin{array}{l}\text{Angle of departure from}\\ \text{the complex pole } p_2\end{array}\right\} = 180° - (\theta_1 + \theta_2) + \theta_3$$

$$= 180° - (127.1° + 90°) + 20.7° = -16.4°$$

The angle of departure at the complex pole p_3 is negative of the angle of departure at complex pole p_3.

$$\therefore \text{Angle of departure at pole } p_3 = -(-16.4) = +16.4°$$

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point of imaginary axis

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K(s+9)}{s\,(s^2+4s+11)}}{1+\dfrac{K\,(s+9)}{s\,(s^2+4s+11)}} = \frac{K(s+9)}{s\,(s^2+4s+11)+K(s+9)}$$

The characteristic equation is the denominator polynomial of C(s)/R(s).

∴ The characteristic equation is,

$$s\,(s^2+4s+11)+K\,(s+9)=0 \;\Rightarrow\; (s^3+4s^2+11s)+Ks+9K=0$$

put $s = j\omega$

$$(j\omega)^3+4(j\omega)^2+11(j\omega)+K(j\omega)+9K=0 \quad\Rightarrow\quad -j\omega^3-4\omega^2+j11\omega+jK\omega+9K=0$$

On equating imaginary part to zero,

$$-j\omega^3+j11\omega+jK\omega=0 \Rightarrow -j\omega^3=-j11\omega-jK\omega$$

$$\therefore \omega^2=11+K$$

$$\text{Put } K=8.8, \quad \therefore \omega^2=11+8.8=19.8$$

$$\omega=\pm\sqrt{19.8}=\pm 4.4$$

On equating real part to zero,

$$-4\omega^2+9K=0 \Rightarrow 9K=4\omega^2$$

$$\text{Put, } \omega^2=11+K \quad \therefore 9K=4(11+K)=44+4K$$

$$\therefore 9K-4K=44$$

$$\therefore 5K=44 \Rightarrow K=\frac{44}{5}=8.8$$

The crossing point of root locus is $\pm j4.4$. The value of K at this crossing point is K = 8.8 (This is the limiting value of K for the stability of the system).

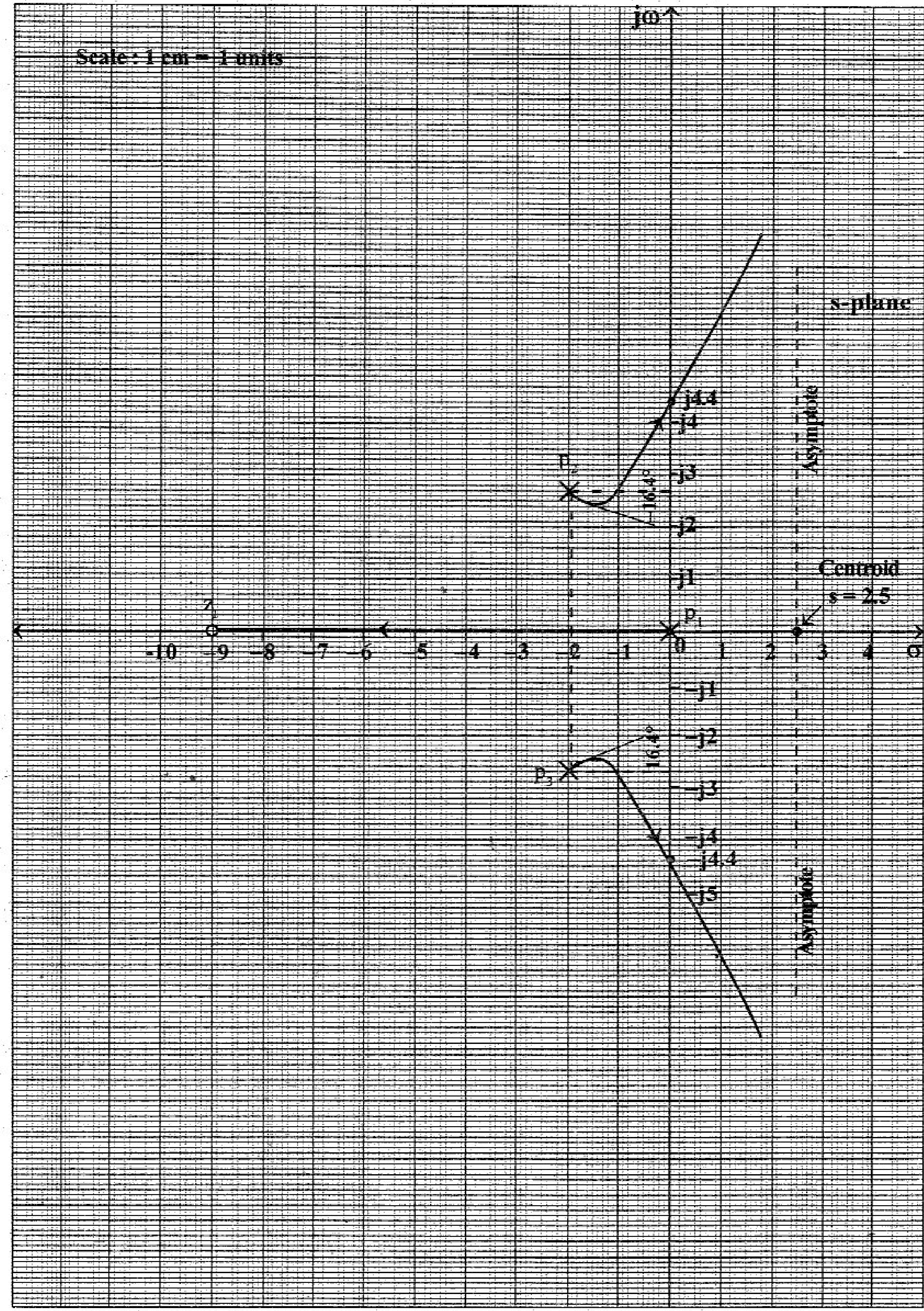

Fig 4.24.3. : Root locus sketch for, $1+G(s)=1+\dfrac{K\,(s+9)}{s\,(s^2+4s+11)}$.

The complete root locus sketch is shown in fig 4.24.3. The root locus has three branches. One branch starts at pole at origin and travel through negative real axis to meet the zero at s = –9.

The other two root locus branches starts at complex poles (along the angle of departure) crosses the imaginary axis at ±j4.4 and travel parallel to asymptotes to meet the zeros at infinity.

EXAMPLE 4.25

Sketch the root locus for the unity feedback system whose open loop transfer function is,

$$G(s)\,H(s) = \frac{K}{s\,(s+4)\,(s^2+4s+20)}.$$

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s\,(s+4)\,(s^2+4s+20) = 0$.

$$\text{The roots of the quadratic are, } s = \frac{-4 \pm \sqrt{4^2 - 4\times1\times20}}{2} = -2 \pm j4$$

$\therefore$ The poles are lying at, $s = 0, -4, -2+j4$ and $-2-j4$.

The zeros are lying at infinity.

Let us denote the poles as p_1, p_2, p_3 and p_4.

Here, $p_1 = 0$, $p_2 = -4$, $p_3 = -2+j4$, and $p_4 = -2-j4$.

The poles are marked by X (cross) as shown in fig 4.25.1.

Step 2 : To find root locus on real axis

There are two poles on the real axis. Choose a test point on real axis between s = 0 and s = –4. To the right of this point, the total number of real poles is one which is an odd number. Hence the real axis between s = 0 and s = –4 will be a part of root locus. Choose a test point to the left of s = –4, now to the right of this test point the total number of poles and zeros is two which is even number. Hence the real axis from s = –4 to s = –∞ will not be a part of root locus. The root locus on real axis is shown as a bold line in fig 4.25.1.

Step 3 : To find angles of asymptotes and centroid

Since there are four poles, the number of root locus branches are four. There is no finite zero. Hence all the four root locus branches ends at zeros at infinity. Hence the number of asymptotes required is four.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ\,(2q+1)}{n-m}; \qquad q = 0, 1, 2, \ldots\ldots n-m.$$

Here, n = 3 and m = 0. $\quad\therefore q = 0, 1, 2, 3$.

> *Note : It is enough if you calculate the required number of angles. Here it is given by first four values of angles. The remaining will be repetiticns of the previous values.*

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180^\circ}{4} = \pm 45^\circ$$

$$\text{When } q = 1, \quad \text{Angles} = \pm\frac{180^\circ \times 3}{4} = \pm 135^\circ$$

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0-4-2+j4-2-j4-0}{4-0} = \frac{-8}{4} = -2.$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 4.25.1.

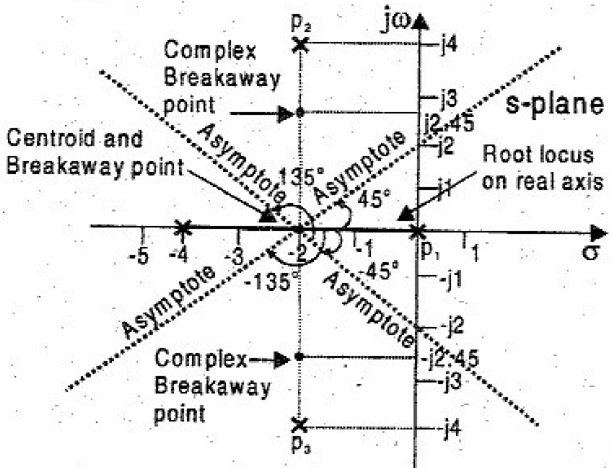

Fig 4.25.1 : Figure showing the asymptotes, root locus on real axis and location of poles, centroid and breakaway points.

Step 4 : To find the breakaway and breakin point

The closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K}{s\,(s+4)\,(s^2+4s+20)}}{1+\dfrac{K}{s\,(s+4)\,(s^2+4s+20)}} = \frac{K}{s\,(s+4)\,(s^2+4s+20)+K}$$

The characteristic equation is, $s\,(s+4)\,(s^2+4s+20)+K=0$.

$$\therefore K = -s\,(s+4)\,(s^2+4s+20) = -(s^2+4s)\,(s^2+4s+20)$$

$$\therefore K = -(s^4+8s^3+36s^2+80s)$$

On differentiating the equation of K with respect to s we get, $\dfrac{dK}{ds} = -(4s^3+24s^2+72s+80)$.

To find the real root of $\dfrac{dK}{ds} = 0$ by Lin's method.

The first trial divisor is chosen as the last two terms of the polynomial

Ist trial

Trial divisor $= 18s+20 = s+\dfrac{20}{18} = s+1.11$

$$\begin{array}{r|l} & s^2+4.89s+12.57 \\ s+1.11 & s^3+6s^2+18s+20 \\ & s^3+1.11s^2 \\ \hline & \quad 4.89s^2+18s \\ & \quad 4.89s^2+5.43s \\ \hline \end{array}$$

Next trial divisor → $12.57s+20$

$$\begin{array}{r} 12.57s+13.95 \\ \hline 6.05 \\ \hline \end{array}$$

IInd trial

Trial divisor $= 12.57s+20 = s+\dfrac{20}{12.57} = s+1.59$

$$\begin{array}{r|l} & s^2+4.41s+11 \\ s+1.59 & s^3+6s^2+18s+20 \\ & s^3+1.59s^2 \\ \hline & \quad 4.41s^2+18s \\ & \quad 4.41s^2+7s \\ \hline \end{array}$$

Next trial divisor → $11s+20$

$$\begin{array}{r} 11s+17.49 \\ \hline 2.51 \\ \hline \end{array}$$

III^rd trial

Trial divisor = 11s + 20

$$= s + \frac{20}{11} = s + 1.82$$

$$\begin{array}{r|l} & s^2 + 4.18s + 10.4 \\ \hline s + 1.82 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 1.82s^2 \\ \hline & \quad 4.18s^2 + 18s + 20 \\ & \quad 4.18s^2 + 7.6s \\ \hline & \qquad 10.4s + 20 \\ & \qquad 10.4s + 18.9 \\ \hline & \qquad\qquad 1.1 \end{array}$$

Since the remainder converge for every trial, let us approximate the root to s = –2. On dividing the polynomial by s + 2, we found that (s+2) is a divisor of the polynomial.

$$\begin{array}{r|l} & s^2 + 4s + 10 \\ \hline s + 2 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 2s^2 \\ \hline & \quad 4s^2 + 18s \\ & \quad 4s^2 + 8s \\ \hline & \qquad 10s + 20 \\ & \qquad 10s + 20 \\ \hline & \qquad\quad 0 \end{array}$$

Put, $\frac{dK}{ds} = 0$. $\therefore (-4s^3 + 24s^2 + 72s + 80) = 0 \quad \Rightarrow \quad 4s^3 + 24s^2 + 72s + 80 = 0.$

On dividing by 4 we get, $s^3 + 6s^2 + 18s + 20 = 0.$

The equation $s^3 + 6s^2 + 18s + 20 = 0$ will have atleast one real root. By trial and error, the real root is found to be s = –2. *(Refer Appendix II for Lin's method.)*

The polynomial, $(s^3 + 6s^2 + 18s + 20) = 0$, can be expressed as,

$$s^3 + 6s^2 + 18s + 20 = (s + 2)(s^2 + 4s + 10) = 0$$

The root of the quadratic, $s^2 + 4s + 10 = 0$, are given by,

$$s = \frac{-4 \pm \sqrt{4^2 - 4 \times 10}}{2} = -2 \pm j2.45$$

Check for K : When, s = –2, $K = -(s^4 + 8s^3 + 36s^2 + 80s) = -\left[(-2)^4 + 8 \times (-2)^3 + 36 \times (-2)^2 + 80 \times (-2)\right]$

$$= -[-64] = 64$$

When, $s = -2 \pm j2.45 = 3.16\angle \pm 129°$

$$K = -(s^4 + 8s^3 + 36s^2 + 80s)$$

$$= -(3.16\angle \pm 129°)^4 + 8 \times (3.16\angle \pm 129°)^3 + 36 \times (3.16\angle \pm 129°)^2 + 80 \times 3.16\angle \pm 129°$$

$$= -[99.7\angle \pm 156° + 252.4\angle \pm 27° + 359.5\angle \pm 258° + 252.8\angle \pm 129°]$$

For positive values of angles,

$$K = -[-91 + j40 + 225 + j115 - 75 - j351 - 159 + j196] = -[-100] = 100$$

For negative values of angles,

$$K = -[-91 - j40 + 225 + j115 - 75 + j351 - 159 - j196] = -[-100] = 100$$

For all the roots of the equation dK/ds = 0, the value of K is positive and real. Hence all the three roots are actual breakaway points. The breakaway points are shown in fig 4.25.1.

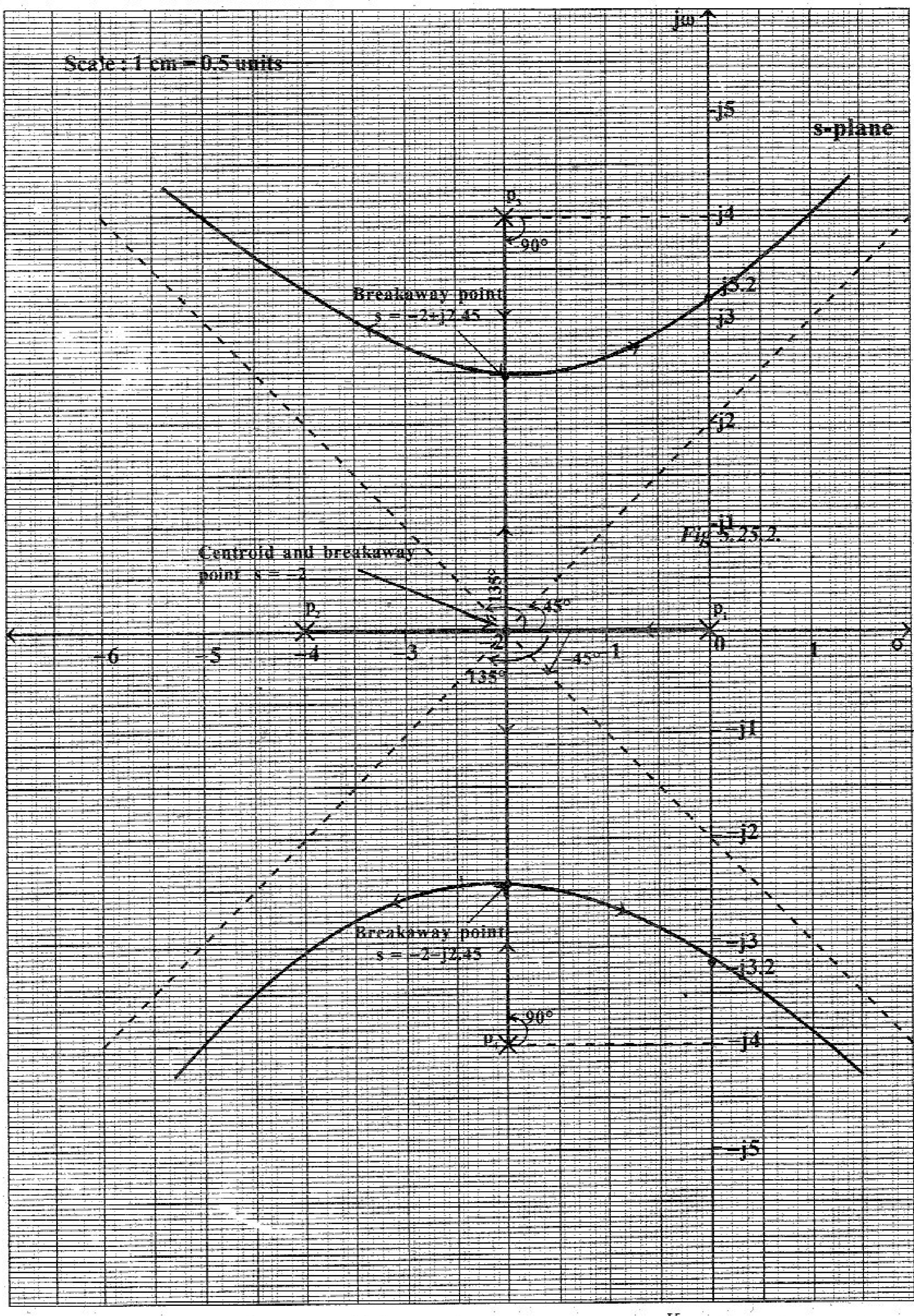

Fig 4.25.3. : Root locus sketch of, $1 + G(s) = 1 + \dfrac{K}{s\,(s+4)\,(s^2+4s+20)}$.