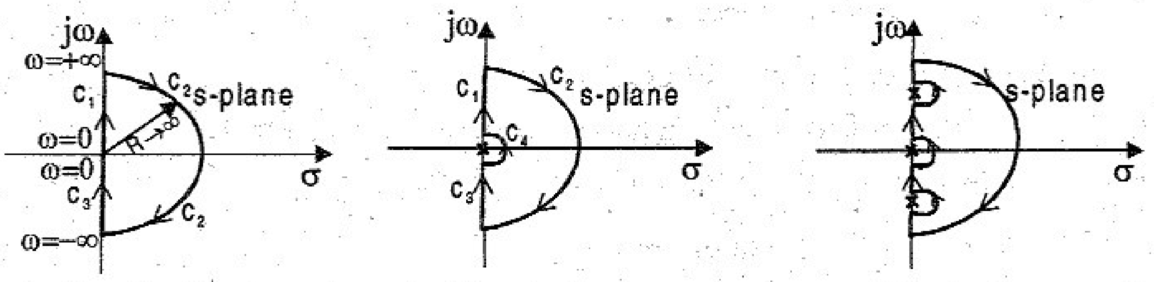


a. *Nyquist Contour when there is no pole on imaginary axis* **b.** *Nyquist Contour when there are poles at origin* **c.** *Nyquist Contour when there are poles on imaginary axis and at origin*

Fig 4.5 : *Nyquist Contour*

2. The Nyquist contour should be mapped in the G(s)H(s)-plane using the function G(s)H(s) to determine the encirclement $-1+j0$ point in the G(s)H(s)-plane. The Nyquist contour of fig 4.5b can be divided into four sections C_1, C_2, C_3 and C_4. The mapping of the four sections in the G(s)H(s)-plane can be carried sectionwise and then combined together to get entire G(s)H(s)-contour.

3. In section C_1 the value of ω varies from 0 to $+\infty$. The mapping of section C_1 is obtained by letting $s = j\omega$ in G(s)H(s) and varying ω from 0 to $+\infty$,

$$\text{i.e. } G(s)H(s)\Big|_{\substack{s=j\omega \\ \omega=0 \text{ to } \infty}} = G(j\omega)\,H(j\omega)\Big|_{\omega=0 \text{ to } \infty}$$

The locus of G(jω)H(jω) as ω is varied from 0 to $+\infty$ will be the G(s)H(s)-contour in G(s)H(s)-plane corresponding to section C_1 in s-plane. This locus is the polar plot of G(jω)H(jω). There are three ways of mapping this section of G(s)H(s)-contour, they are,

(i) Calculate the values of G(jω)H(jω) for various values of ω and sketch the actual locus of G(jω)H(jω).

(or)

(ii) Separate the real part and imaginary part of G(jω)H(jω). Equate the imaginary part to zero, to find the frequency at which the G(jω)H(jω) locus crosses real axis (to find phase crossover frequency). Substitute this frequency on real part and find the crossing point of the locus on real axis. Sketch the approximate locus of G(jω)H(jω) from the knowledge of type number and order of the system (or from the value of G(jω)H(jω) at $\omega = 0$ and $\omega = \infty$).

(iii) Separate the magnitude and phase of G(jω)H(jω). Equate the phase of G(jω)H(jω) to -180^0 and solve for ω. This value of ω is the phase crossover frequency and the magnitude at this frequency is the crossing point on real axis. Sketch the approximate root locus as mentioned in method (ii).

4. The section C_2 of Nyquist contour has a semicircle of infinite radius. Therefore, every point on section C_2 has infinite magnitude but the argument varies from $+\pi/2$ to $-\pi/2$. Hence the mapping of section C_2 from s-plane to G(s)H(s) plane can be obtained by letting $s = \underset{R\to\infty}{Lt}\; Re^{j\theta}$ in G(s)H(s) and varying θ from $+\pi/2$ to $-\pi/2$.

Consider the loop transfer function in time constant form and with y number of poles at origin, as shown below.

$$G(s)H(s) = \frac{K\,(1+sT_1)\,(1+sT_2)\,(1+sT_3)......}{s^y\,(1+sT_a)\,(1+sT_b)\,(1+sT_c)......}$$

Let G(s)H(s) has m zeros & n poles including poles at origin. For practical systems, n >m.

Since, $s \to Re^{j\theta}$ and $R \to \infty$, the term (1+sT) can be approximated to sT, [i.e., $(1+sT) \approx sT$].

$$\therefore G(s)H(s) \approx K\frac{sT_1 \times sT_2 \times sT_3.....}{s^y \times sT_a \times sT_b \times sT_c.....} = K_1\frac{s^m}{s^n} = \frac{K_1}{s^{n-m}}$$

On letting, $s = \underset{R\to\infty}{Lt}\; Re^{j\theta}$ we get,

$$G(s)H(s)\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K_1}{\underset{R\to\infty}{Lt}\;(Re^{j\theta})^{n-m}} = 0e^{-j\theta(n-m)}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{-j\frac{\pi}{2}(n-m)}$

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{+j\frac{\pi}{2}(n-m)}$

From the above two equations we can conclude that the section C_2 of Nyquist contour in s-plane is mapped as circles/circular arc around origin with radius tending to zero in the G(s)H(s)-plane.

5. In section C_3, the value of ω varies from $-\infty$ to 0. The mapping of section C_3 is obtained by letting $s = +j\omega$ in G(s)H(s) and varying ω from $-\infty$ to 0.

$$\text{i.e., } G(s)H(s)\Bigg|_{\substack{s=+j\omega \\ \omega=-\infty \text{ to } 0}} = G(j\omega)H(j\omega)\Bigg|_{\omega=-\infty \text{ to } 0.}$$

The locus of G(jω)H(jω) as ω is varied from $-\infty$ to 0 will be the G(s)H(s)-contour in G(s)H(s)-plane corresponding to section C_3 in s-plane. This locus is the inverse polar plot of G(jω)H(jω). The inverse polar plot is given by the mirror image of polar plot with respect to real axis.

6. The section C_4 of Nyquist contour has a semicircle of zero radius. Therefore every point on semicircle has zero magnitude but the argument varies from $-\pi/2$ to $+\pi/2$. Hence the mapping of section C_4 from s-plane to G(s)H(s)-plane can be obtained by letting $s = \underset{R\to 0}{Lt}\; Re^{-j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$.

Consider the loop transfer function in time constant form and with y number of poles at origin as shown below.

$$G(s)H(s) = \frac{K\,(1+sT_1)\,(1+sT_2)\,(1+sT_3)\;.....}{s^y\,(1+sT_a)\,(1+sT_b)\,(1+sT_c)\;.....}$$

Let G(s)H(s) has m zeros & n poles including poles at origin. For practical systems, n >m.

Since, $s \to R\,e^{j\theta}$ and $R \to 0$, the term 1+ sT can be approximated to 1, [i.e.$(1+sT) \approx 1$].

$$\therefore G(s)H(s) \approx K\frac{1}{s^y}$$

On letting, $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$ we get,

$$G(s)H(s)\Bigg|_{s=\underset{R\to 0}{Lt}\ Re^{j\theta}} = \frac{K}{\underset{R\to 0}{Lt}\ (Re^{j\theta})^y} = \infty e^{-j\theta y}$$

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{j\frac{\pi}{2}y}$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{-j\frac{\pi}{2}y}$

From the above two equations we can conclude that the section C_4 of Nyquist contour in s-plane is mapped as circles/circular arc in G(s)H(s)-plane with origin as centre and infinite radius.

<u>***Note :***</u>

1. *If there are no poles on the origin then the section C_4 of Nyquist contour will be absent.*
2. *If there are poles on imaginary axis as shown below then the Nyquist contour is divided into the following 8 sections and the mapping is performed sectionwise.*

Section C_1 : $s = j\omega\ ;\ \omega = 0^+$ to $+\omega_1^-$

Section C_2 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}\ ;\ \theta = -\frac{\pi}{2}$ *to* $+\frac{\pi}{2}$

Section C_3 : $s = j\omega\ ;\ \omega = +\omega_1^+$ *to* $+\infty$

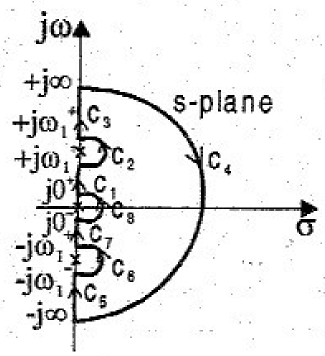
Section C_4 : $s = \underset{R\to \infty}{Lt}\ Re^{j\theta}\ ;\ \theta = +\frac{\pi}{2}$ *to* $-\frac{\pi}{2}$

Section C_5 : $s = j\omega\ ;\ \omega = -\infty$ *to* $-\omega_1^-$

Section C_6 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}\ ;\ \theta = -\frac{\pi}{2}$ *to* $+\frac{\pi}{2}$

Section C_7 : $s = j\omega\ ;\ \omega = -\omega_1^+$ *to* 0^-

Section C_8 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}\ ;\ \theta = -\frac{\pi}{2}$ *to* $+\frac{\pi}{2}$



EXAMPLE 4.13

Draw the Nyquist plot for the system whose open loop transfer function is, $G(s)H(s) = \frac{K}{s\,(s+2)\,(s+10)}$.

Determine the range of K for which closed loop system is stable.

SOLUTION

Given that, $G(s)H(s) = \frac{K}{s\,(s+2)\,(s+10)} = \frac{K}{s\times 2\left(\frac{s}{2}+1\right)\times 10\left(\frac{s}{10}+1\right)} = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$

The open loop transfer function has a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.13.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

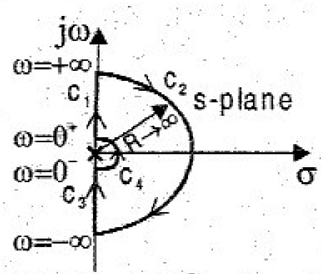


Fig 4.13.1 : *Nyquist Contour in s-plane*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{0.05K}{j\omega\,(1+j0.5\omega)\,(1+j0.1\omega)} = \frac{0.05K}{j\omega\,(1+j0.6\omega-0.05\omega^2)} = \frac{0.05K}{-0.6\omega^2 + j\omega\,(1-0.05\omega^2)}$$

When the locus of $G(j\omega)H(j\omega)$ crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad \omega_{pc}(1-0.05\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-0.05\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = \sqrt{\frac{1}{0.05}} = 4.472 \text{ rad/sec}$$

$$\text{At } \omega = \omega_{pc} = 4.472 \text{ rad/sec}, \qquad G(j\omega)H(j\omega) = \frac{0.05K}{-0.6\omega^2} = -\frac{0.05K}{0.6\times(4.472)^2} = -0.00417K$$

The open loop system is type-1 and third order system. Also it is a minimum phase system with all poles. Hence the polar plot of $G(j\omega)H(j\omega)$ starts at −90° axis at infinity, crosses real axis at −0.00417K and ends at origin in second quadrant. The section C_1 and its mapping are shown in fig 4.13.2. and 4.13.3.

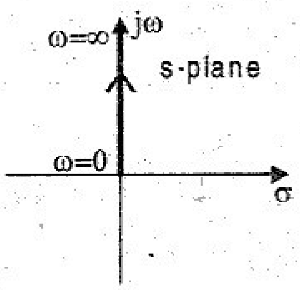


Fig 4.13.2 : Section C_1 in s-plane

⇒

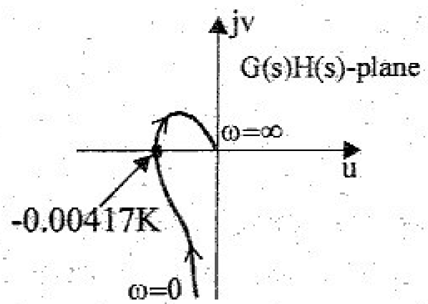


Fig 4.13.3 : *Mapping of section C_1 in G(s)H(s)-plane*

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt}\, Re^{j\theta}$ in G(s)H(s) and varying θ from +π/2 to −π/2. Since $s \to Re^{j\theta}$ and $R\to\infty$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)\,H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s\times 0.5s\times 0.1s} = \frac{K}{s^3}$$

Let, $s = \underset{R\to\infty}{Lt}\, Re^{j\theta}$.

$$\therefore G(s)H(s)\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{s^3}\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to\infty}{Lt}(Re^{j\theta})^3} = 0e^{-j3\theta}$$

$$\text{When } \theta = \frac{\pi}{2}, \qquad G(s)H(s) = 0e^{-j3\frac{\pi}{2}} \qquad .....(1)$$

$$\text{When } \theta = -\frac{\pi}{2}, \qquad G(s)H(s) = 0e^{+j3\frac{\pi}{2}} \qquad .....(2)$$

From the equations (1) and (2) we can say that section C_2 in s-plane (fig 4.13.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-3\pi/2$ to $+3\pi/2$ as shown in fig 4.13.5.

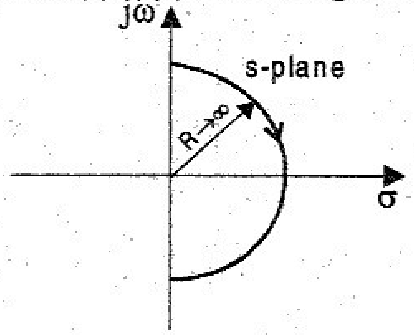


Fig 4.13.4 : Section C_2 in s-plane

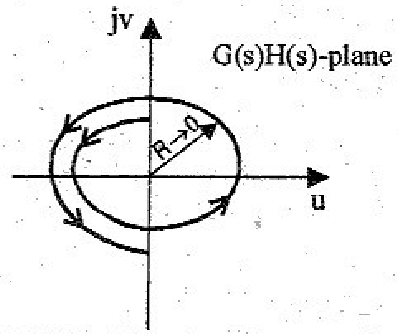


Fig 4.13.5 : *Mapping of section C_2 in G(s)H(s)-plane*

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of G(jω)H(jω) as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of G(jω)H(jω).

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.13.6 and fig 4.13.7.

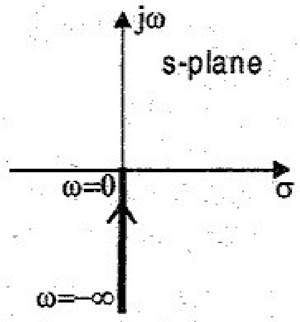


Fig 4.13.6 : Section C_3 in s-plane

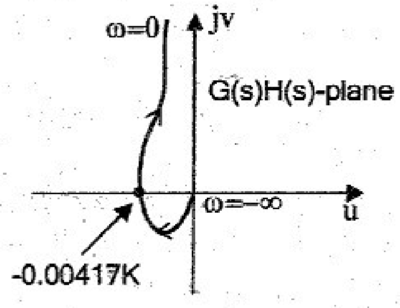


Fig 4.13.7 : *Mapping of section C_3 in G(s)H(s)-plane*

MAPPING OF SECTION C_4

The mapping of section C_4 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to 0$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s \times 1 \times 1} = \frac{0.05K}{s}$$

Let $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$.

$$\therefore\ G(s)H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{0.05K}{s}\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{0.05K}{\underset{R\to 0}{Lt}(Re^{j\theta})} = \infty e^{-j\theta}$$

When $\theta = -\dfrac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{+j\frac{\pi}{2}}$ .....(3)

When $\theta = \dfrac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{-j\frac{\pi}{2}}$ .....(4)

From the equations (3) and (4) we can say that section C_4 in s-plane (fig 4.13.8.) is mapped as a circular arc of infinite radius with argument (phase) varying from $+\pi/2$ to $-\pi/2$ as shown in fig 4.13.9.

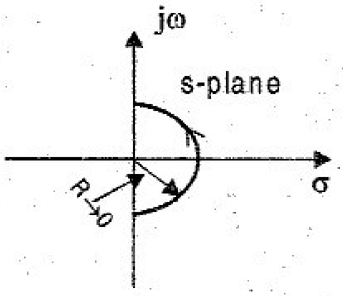


Fig 4.13.8 : Section C_4 in s-plane

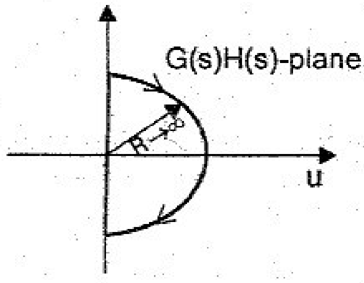


Fig 4.13.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.13.10.

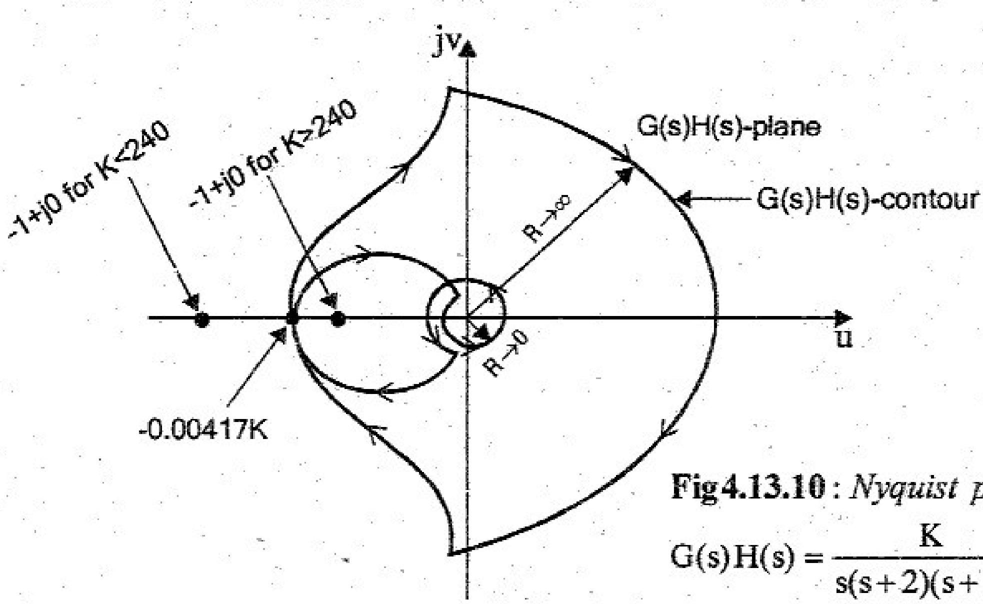


Fig 4.13.10 : *Nyquist plot of* $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$

STABILITY ANALYSIS

When, $-0.00417K = -1$, the contour passes through $(-1+j0)$ point and corresponding value of K is the limiting value of K for stability.

$$\therefore \text{Limiting value of } K = \frac{1}{0.00417} = 240$$

When K < 240

When K is less than 240, the contour crosses real axis at a point between 0 and $-1+j0$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is not encircled. Also the open loop transfer function has no poles on the right half of s-plane. Therefore the closed loop system is stable.

When K > 240

When K is greater than 240, the contour crosses real axis at a point between $-1+j0$ and $-\infty$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is encircled in clockwise direction two times. [Since there are two clockwise encirclement and no right half open loop poles, the closed loop system has two poles on right half of s-plane]. Therefore the closed loop system is unstable.

RESULT

The value of K for stability is $0 < K < 240$

EXAMPLE 4.14

Construct the Nyquist plot for a system whose open loop transfer function is given by $G(s)H(s) = \dfrac{K(1+s)^2}{s^3}$. Find the range of K for stability.

SOLUTION

Given that, $G(s)H(s) = \frac{K(1+s)^2}{s^3}$

The open loop transfer function has three poles at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.14.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

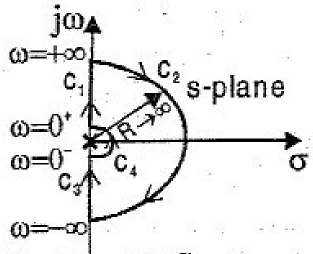


Fig 4.14.1 : *Nyquist Contour in s-plane*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{K(1+s)^2}{s^3}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K(1-\omega^2+2j\omega)}{-j\omega^3} = \frac{K(1-\omega^2)}{-j\omega^3} + \frac{K\,2j\omega}{-j\omega^3} = -\frac{2K}{\omega^2} + j\frac{K(1-\omega^2)}{\omega^3}$$

When the $G(j\omega)H(j\omega)$ locus crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad K(1-\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = 1\text{ rad/sec}$$

At $\omega = \omega_{pc} = 1$ rad/sec,

$$G(j\omega)H(j\omega) = -\frac{2K}{\omega^2} = -\frac{2K}{1^2} = -2K \qquad .....(1)$$

$$G(j\omega)\,H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega\,\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega}{\omega^3\angle 270^\circ} = \frac{K(1+\omega^2)}{\omega^3}\angle(2\tan^{-1}\omega - 270^\circ)$$

$$\text{As } \omega \to 0, \quad G(j\omega)H(j\omega) \to \infty\angle -270^\circ \qquad .....(2)$$

$$\text{As } \omega \to \infty, \quad G(j\omega)H(j\omega) \to 0\angle -90^\circ \qquad .....(3)$$

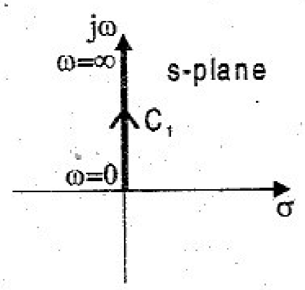


Fig 4.14.2 : *Section C_1 in s-plane*

⇒

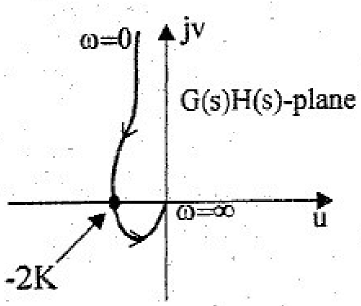


Fig 4.14.3 : *Mapping of section C_1 in G(s)H(s)-plane*

From equations (1), (2) and (3) we can say that the polar plot starts at −270° axis at infinity, crosses real axis at −2K and ends at origin in third quadrant. The section C_1 and its mapping are shown in fig 2 and 3.

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s) H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt} Re^{j\theta}$ in G(s)H(s) and varying θ from +π/2 to −π/2. Since $s \to Re^{j\theta}$ and $R \to \infty$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)H(s) = \frac{K(1+s)^2}{s^3} \approx \frac{Ks^2}{s^3} = \frac{K}{s}$$

Let $s = \underset{R\to\infty}{Lt} Re^{j\theta}$

$$\therefore G(s)H(s)\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{s}\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to\infty}{Lt} Re^{j\theta}} = 0e^{-j\theta}$$

When $\theta = \frac{\pi}{2}$, $G(s)H(s) = 0e^{-j\frac{\pi}{2}}$ .....(4)

When $\theta = -\frac{\pi}{2}$, $G(s)H(s) = 0e^{j\frac{\pi}{2}}$ .....(5)

From the equations (4) and (5) we can say that section C_2 in s-plane (fig 4.14.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-\pi/2$ to $+\pi/2$ as shown in fig 4.14.5.

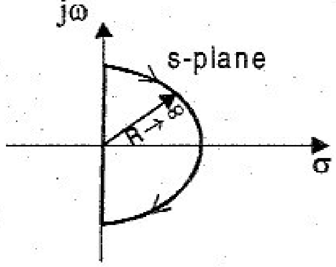


Fig 4.14.4 : *Section C_2 in s-plane*

⇒

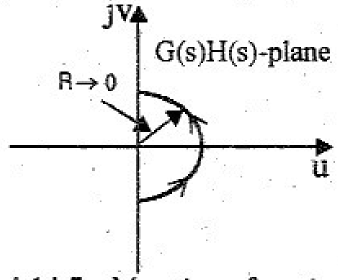


Fig 4.14.5 : *Mapping of section C_2 in G(s)H(s)-plane*

Mapping of section C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by locus of G(jω) H(jω) as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of G(jω) H(jω).

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.14.6 and fig 4.14.7.

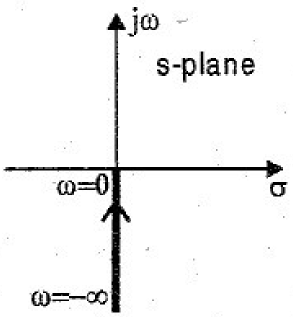


Fig 4.14.6 : *Section C_3 in s-plane*

⇒

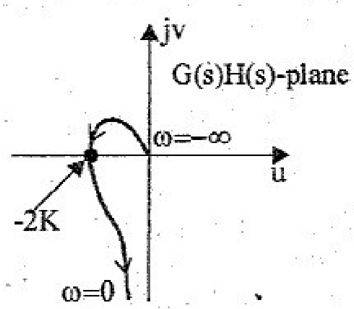


Fig 4.14.7 : *Mapping of section C_3 in G(s)H(s)-plane*

Mapping of section C_4

The mapping of section C_4 from s-plane to G(s) H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt} Re^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to Re^{j\theta}$ and $R \to 0$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{K(1+s)^2}{s^3} \approx \frac{K\times 1}{s^3} = \frac{K}{s^3}$$

Let $s = \underset{R\to 0}{Lt} Re^{j\theta}$

$$\therefore G(s)H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{+j\theta}} = \frac{K}{s^3}\Big|_{s=\underset{R\to 0}{Lt} Re^{+j\theta}} = \frac{K}{\underset{R\to 0}{Lt} (Re^{+j\theta})^3} = \infty e^{-j3\theta}$$

When $\theta = -\frac{\pi}{2}$, $\quad G(s)\,H(s) = \infty e^{+j3\frac{\pi}{2}}$ ......(6)

When $\theta = \frac{\pi}{2}$, $\quad G(s)\,H(s) = \infty e^{-j3\frac{\pi}{2}}$ ......(7)

From the equations (6) and (7) we can say that section C_4 in s-plane (fig 4.14.8.) is mapped as a circular arc of infinite radius with argument (phase) varying from $+3\pi/2$ to $-3\pi/2$ as shown in fig 4.14.9.

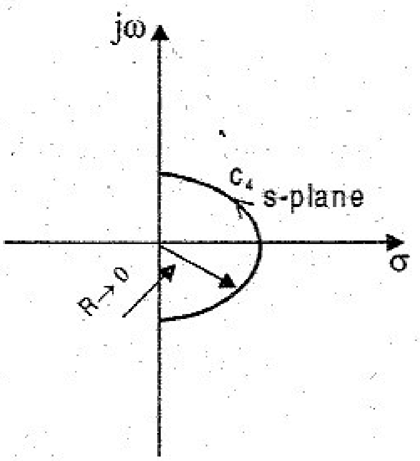


Fig 4.14.8 : *Section C_4 in s-plane*

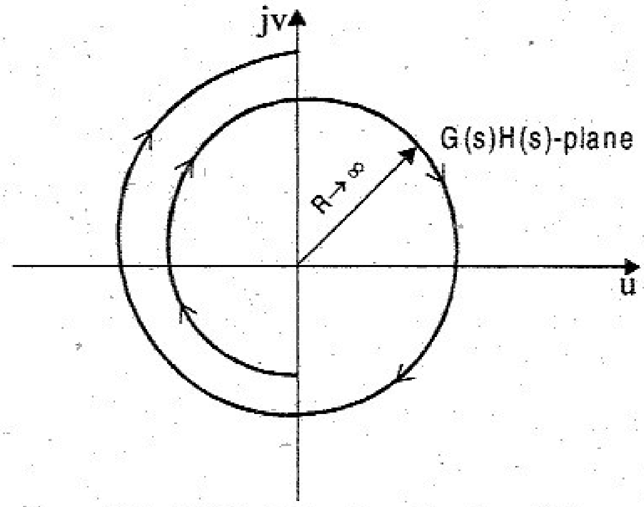


Fig 4.14.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.14.10.

STABILITY ANALYSIS

When, $-2K = -1$, the contour passes through $-1+j0$ point and corresponding value of K is the limiting value of K for stability.

$$\therefore \text{Limiting value of } K = \frac{1}{2} = 0.5$$

When K < 0.5

When K is less than 0.5, the contour crosses real axis at a point between 0 and $-1+j0$. On travelling through Nyquist plot along the indicated direction it is observed that the $-1+j0$ point is encircled in clockwise direction two times. Therefore the system is unstable. [Since there are two clockwise encirclement and no right half open loop poles, the closed loop system will have two poles on right half of s-plane]

When K > 0.5

When K is greater than 0.5, the contour crosses real axis at a point between $-1+j0$ and $-\infty$. On travelling through Nyquist plot along the indicated direction it is observed that (-1+j0) point is encircled in both clockwise and anticlockwise direction one time. Hence net encirclement is zero. Also the open loop system has no poles at the right half of s-plane. Therefore the closed loop system is stable.

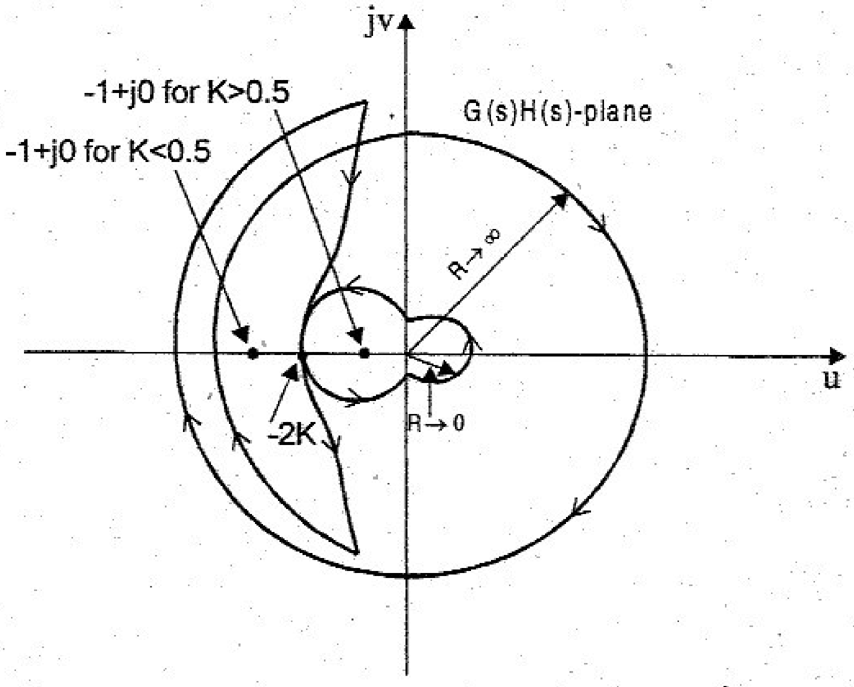


Fig 4.14.10 : *Nyquist plot of* $G(s)H(s) = \frac{K(1+s)^2}{s^3}$

RESULT

The system is stable when K > 0.5.

EXAMPLE 4.15

The open loop transfer function of a system is $G(s)H(s) = \frac{(1+4s)}{s^2(1+s)(1+2s)}$. Determine the stability of closed loop system. If the closed loop system is not stable then find the number of closed-loop poles lying on the right half of s-plane.

SOLUTION

Given that, $G(s)H(s) = \frac{(1+4s)}{s^2(1+s)(1+2s)}$

The open loop transfer function has two poles at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.15.1.

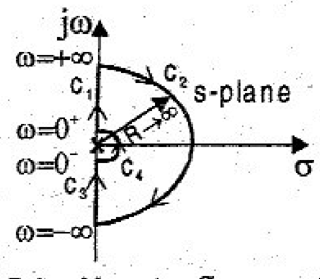


Fig 4.15.1 : *Nyquist Contour in s-plane*

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of G(jω)H(jω) as ω is varied from 0 to ∞. This locus is the polar plot of G(jω)H(jω).

$$G(s)H(s) = \frac{(1+4s)}{s^2(1+s)(1+2s)}$$

Let s = jω.

$$\therefore G(j\omega)H(j\omega) = \frac{(1+j4\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)} = \frac{\sqrt{1+16\omega^2}\ \angle\tan^{-1}4\omega}{\omega^2\angle 180^\circ\sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\ \sqrt{1+4\omega^2}\ \angle\tan^{-1}2\omega}$$

$$= \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}\ \angle(\tan^{-1}4\omega - 180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega)$$

$$\therefore |G(j\omega)H(j\omega)| = \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}4\omega - 180° - \tan^{-1}\omega - \tan^{-1}2\omega$$

When the G(jω)H(jω) locus crosses real axis, the phase will be –180° and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc},\quad \angle G(j\omega)H(j\omega) = -180°$$

$$\therefore \tan^{-1}4\omega_{pc} - 180° - \tan^{-1}\omega_{pc} - \tan^{-1}2\omega_{pc} = -180°$$

$$\tan^{-1}4\omega_{pc} = \tan^{-1}\omega_{pc} + \tan^{-1}2\omega_{pc}$$

On taking tan on both sides we get,

$$\tan\,[\tan^{-1}4\omega_{pc}] = \tan\,[\tan^{-1}\omega_{pc} + \tan^{-1}2\omega_{pc}]$$

$$4\omega_{pc} = \frac{\tan\tan^{-1}\omega_{pc} + \tan\tan^{-1}2\omega_{pc}}{1-\tan\tan^{-1}\omega_{pc}\times\tan\tan^{-1}2\omega_{pc}}$$

Note: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \times \tan B}$

$$4\omega_{pc} = \frac{\omega_{pc} + 2\omega_{pc}}{1-2\omega_{pc}^2} \quad\Rightarrow\quad 1-2\omega_{pc}^2 = \frac{3\omega_{pc}}{4\omega_{pc}} \quad\Rightarrow\quad -2\omega_{pc}^2 = \frac{3}{4} - 1$$

$$\therefore \omega_{pc} = \sqrt{\frac{-0.25}{-2}} = 0.354 \text{ rad / sec}$$

At $\omega = \omega_{pc} = 0.354$ rad / sec,

$$|G(j\omega)H(j\omega)| = \frac{\sqrt{1+16\omega_{pc}^2}}{\omega^2\sqrt{1+\omega_{pc}^2}\ \sqrt{1+4\omega_{pc}^2}} = \frac{\sqrt{1+16\times0.354^2}}{(0.354)^2\ \sqrt{1+0.354^2}\ \sqrt{1+4\times0.354^2}} = 10.64 \qquad .....(1)$$

Hence G(jω)H(jω) locus crosses the real axis at –10.64.

$$\text{At } \omega \to 0,\ G(j\omega)H(j\omega) \to \infty\angle -180° \qquad .....(2)$$

$$\text{At } \omega \to \infty,\ G(j\omega)H(j\omega) \to 0\angle -270° \qquad .....(3)$$

From equations (1), (2) and (3) we can say that the polar plot starts at –180° axis at infinity, travels in third quadrant and crosses real axis at –10.64 to enter second quadrant and then ends at origin in second quadrant. The section C_1 and its mapping are shown in fig 4.15.2. and 4.15.3.

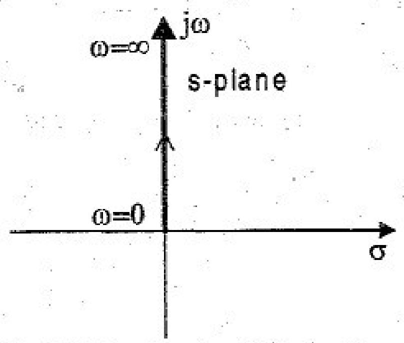


Fig 4.15.2 : *Section C_1 in s-plane*

⇒

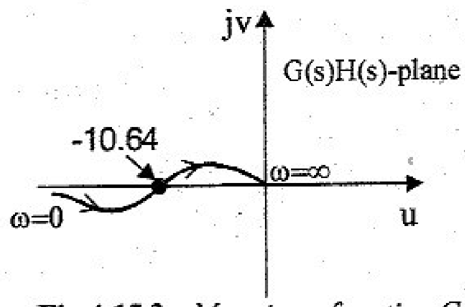


Fig 4.15.3 : *Mapping of section C_1 in G(s)H(s)-plane*

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from +π/2 to –π/2. Since $s \to R\,e^{j\theta}$ and $R \to \infty$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)H(s) = \frac{(1+4s)}{s^2(1+s)\,(1+2s)} \approx \frac{4s}{s^2 \times s \times 2s} = \frac{2}{s^3}$$

Let, $s = \underset{R\to\infty}{Lt}\ Re^{j\theta}$.

$$\therefore\ G(s)H(s)\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{2}{s^3}\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{2}{\underset{R\to\infty}{Lt}(Re^{j\theta})^3} = 0e^{-j3\theta}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{-j3\frac{\pi}{2}}$ .....(4)

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{j3\frac{\pi}{2}}$ .....(5)

From the equations (4) and (5) we can say that section C_2 in s-plane (fig 4.15.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-3\pi/2$ to $+3\pi/2$ as shown in fig 4.15.5.

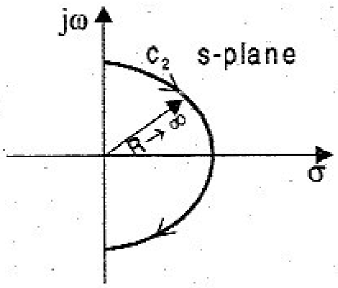


Fig 4.15.4 : *Section C_2 in s-plane*

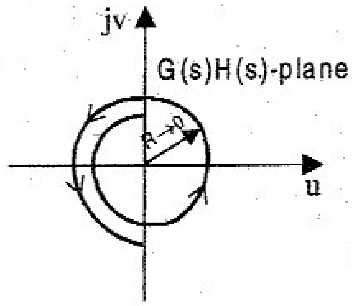


Fig 4.15.5 : *Mapping of section C_2 in G(s)H(s)-plane*

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of $G(j\omega)\,H(j\omega)$ as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of $G(j\omega)\,H(j\omega)$.

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.15.6 and fig 4.15.7.

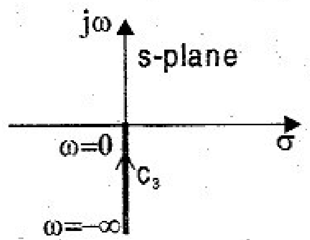


Fig 4.15.6 : *Section C_3 in s-plane*

⟹

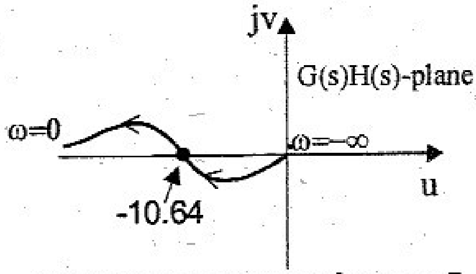


Fig 4.15.7 : *Mapping of section C_3 in G(s)H(s)-plane*

MAPPING OF SECTION C_4

The mapping of section C_4 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to 0$, the G(s)H(s) can be approximated as shown below [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{(1+4s)}{s^2(1+s)\,(1+2s)} \approx \frac{1}{s^2 \times 1 \times 1} = \frac{1}{s^2}$$

Let, $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$

$$\therefore\ G(s)H(s)\Bigg|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{1}{s^2}\Bigg|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{1}{\underset{R\to 0}{Lt}(Re^{j\theta})^2} = \infty e^{-j2\theta}$$

When $\theta = -\frac{\pi}{2}$, $G(s)H(s) = \infty e^{j\pi}$ .....(6)

When $\theta = \frac{\pi}{2}$, $G(s)H(s) = \infty e^{-j\pi}$ .....(7)

From the equations (6) and (7) we can say that section C_4 in s-plane (fig 4.15.8.) is mapped as a circle of infinite radius with argument (phase) varying from $+\pi$ to $-\pi$ as shown in fig 4.15.9.

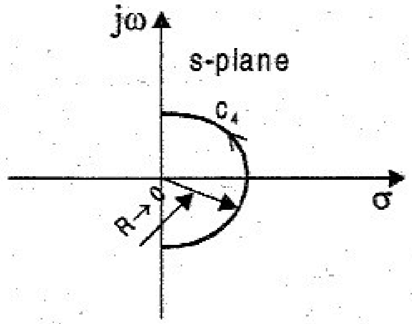


Fig 4.15.8 : *Section C_4 in s-plane*

⇒

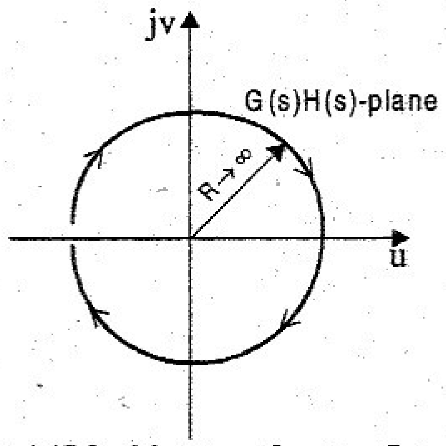


Fig 4.15.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.15.10.

STABILITY ANALYSIS

On travelling through Nyquist contour in G(s)H(s)-plane it is observed that (−1+j0) point is encircled in clockwise direction two times. Therefore the closed loop system is unstable.

Since the −1+j0 is encircled two times in clockwise and no right half open loop poles, two poles of closed loop system are lying on the right half s-plane.

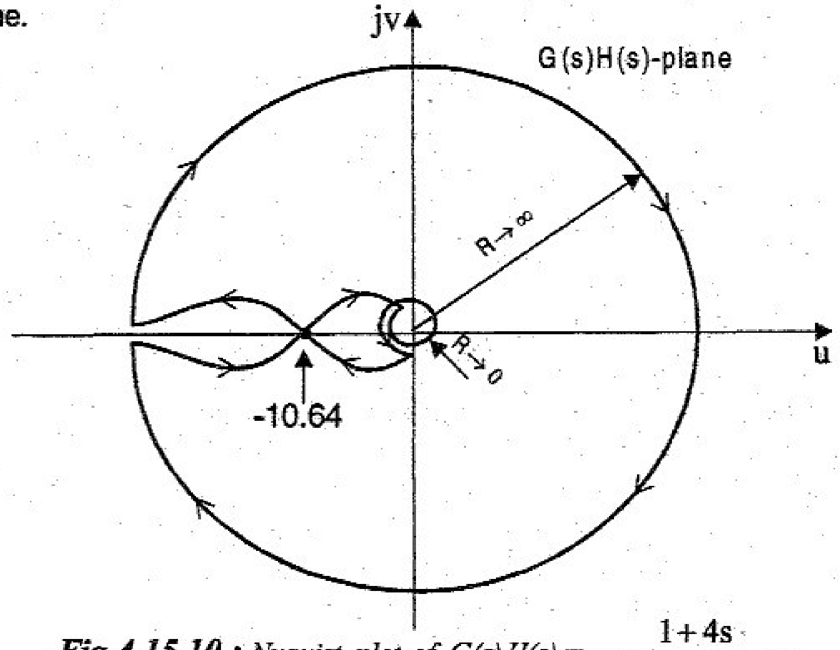


Fig 4.15.10 : Nyquist plot of $G(s)\,H(s) = \dfrac{1+4s}{s^2(1+s)(1+2s)}$

RESULT

(a) Closed loop system is unstable.

(b) Two poles of closed loop system are lying on the right half s-plane.

EXAMPLE 4.16

Sketch the Nyquist plot for a system with the open loop transfer function $G(s)H(s)=\dfrac{K(1+0.5s)\,(1+s)}{(1+10s)\,(s-1)}$. Determine the range of values of K for which the system is stable.

SOLUTION

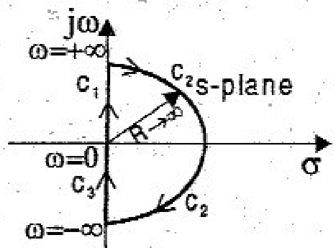


Fig 4.16.1 : *Nyquist Contour in s-plane*

Given that, $G(s)H(s)=\dfrac{K(1+0.5s)\,(1+s)}{(1+10s)\,(s-1)}$

The open loop transfer function does not have a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane as shown in fig 4.16.1.

The Nyquist contour has three sections C_1, C_2 and C_3. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to $+\infty$. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s)=\frac{K\,(1+0.5s)\,(1+s)}{(1+10s)\,(s-1)}$$

Let $s=j\omega$.

$$\therefore G(j\omega)H(j\omega)=\frac{K\,(1+j0.5\omega)\,(1+j\omega)}{(1+j10\omega)\,(-1+j\omega)}=\frac{K(1+j1.5\omega-0.5\omega^2)}{-1-j9\omega-10\omega^2}=\frac{K(1-0.5\omega^2)+j1.5\omega K}{-(1+10\omega^2)-j9\omega}$$

On multiplying the numerator and denominator by the complex conjugate of denominator we get,

$$G(j\omega)H(j\omega)=\frac{K(1-0.5\omega^2)+j1.5\omega K}{-(1+10\omega^2)-j9\omega}\times\frac{-(1+10\omega^2)+j9\omega}{-(1+10\omega^2)+j9\omega}$$

$$=\frac{-K(1-0.5\omega^2)\,(1+10\omega^2)-13.5\omega^2K+j\,[9\omega K(1-0.5\omega^2)-1.5\omega K(1+10\omega^2)]}{(1+10\omega^2)^2+(9\omega)^2}$$

When the $G(j\omega)H(j\omega)$ locus crosses real axis the imaginary term is zero and the corresponding frequency is the phase crossover frequency.

$$\therefore \text{At } \omega=\omega_{pc},\quad 9\omega_{pc}\,K(1-0.5\omega_{pc}^2)-1.5\omega_{pc}\,K(1+10\omega_{pc}^2)=0$$

$$\therefore 9\omega_{pc}\,K(1-0.5\omega_{pc}^2)=1.5\omega_{pc}\,K(1+10\omega_{pc}^2)\quad\Rightarrow\quad 1-0.5\omega_{pc}^2=\frac{1.5}{9}(1+10\omega_{pc}^2)$$

$$\therefore 1-0.5\omega_{pc}^2=0.167+1.67\omega_{pc}^2\quad\Rightarrow\quad 2.17\omega_{pc}^2=0.833\quad\Rightarrow\quad \omega_{pc}=\sqrt{\frac{0.833}{2.17}}=0.62\text{ rad/sec}$$

At $\omega=\omega_{pc}=0.62$ rad/sec

$$G(j\omega)H(j\omega) = \frac{-K(1-0.5\omega_{pc}^2)(1+10\omega_{pc}^2) - 13.5\omega_{pc}^2 K}{(1+10\omega_{pc}^2)^2 + (9\omega_{pc})^2}$$

$$= -K\left[\frac{(1-0.5\times0.62^2)(1+10\times0.62^2)+13.5\times0.62^2}{(1+10\times0.62^2)^2+(9\times0.62)^2}\right] = -K\left[\frac{3.913+5.189}{23.464+31.136}\right] = -0.1667K$$

Therefore, G(jω)H(jω) locus crosses real axis at a point -0.1667K.

The exact shape of G(jω)H(jω) locus is determined by calculating the magnitude and phase of G(jω)H(jω) for various values of ω.

$$G(j\omega)H(j\omega) = K\frac{(1+j0.5\omega)(1+j\omega)}{(1+j10\omega)(-1+j\omega)} = K\frac{\sqrt{1+(0.5\omega)^2}\angle\tan^{-1}0.5\omega\ \sqrt{1+\omega^2}\angle\tan^{-1}\omega}{\sqrt{1+(10\omega)^2}\angle\tan^{-1}10\omega\ \sqrt{1+\omega^2}\angle(180^\circ-\tan^{-1}\omega)}$$

$$= K\frac{\sqrt{1+0.25\omega^2}}{\sqrt{1+100\omega^2}}\angle(\tan^{-1}0.5\omega + 2\tan^{-1}\omega - \tan^{-1}10\omega - 180^\circ)$$

$$\therefore\ |G(j\omega)H(j\omega)| = K\frac{\sqrt{1+0.25\omega^2}}{\sqrt{1+100\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}0.5\omega + 2\tan^{-1}\omega - \tan^{-1}10\omega - 180^\circ$$

As $\omega \to 0$, $|G(j\omega)H(j\omega)| = K$

As $\omega \to 0$, $\angle G(j\omega)H(j\omega) = -180^\circ$

As $\omega \to \infty$, $$|G(j\omega)H(j\omega)| = \underset{\omega\to\infty}{Lt} K\frac{\sqrt{1+0.25\omega^2}}{\sqrt{1+100\omega^2}} = K\underset{\omega\to\infty}{Lt}\sqrt{\frac{\omega^2\left(\frac{1}{\omega^2}+0.25\right)}{\omega^2\left(\frac{1}{\omega^2}+100\right)}} = K\underset{\omega\to\infty}{Lt}\sqrt{\frac{\left(\frac{1}{\omega^2}+0.25\right)}{\left(\frac{1}{\omega^2}+100\right)}} = K\sqrt{\frac{0+0.25}{0+100}} = 0.05K$$

As $\omega \to \infty$, $\angle G(j\omega) = \tan^{-1}\infty + 2\tan^{-1}\infty - \tan^{-1}\infty - 180^\circ = 90^\circ + 180^\circ - 90^\circ - 180^\circ = 0$

ω rad/sec	0	0.1	0.5	1.5	2.0	5.0	∞
|G(jω)H(jω)|	K	0.707K	0.202K	0.083K	0.07K	0.054K	0.05K
∠G(jω)H(jω) deg	–180	–210	–191	–116	–95	–43	0

From the above analysis, the following conclusions are made,

1. The locus of G(jω)H(jω) starts at $K\angle-180^\circ$ when $\omega = 0$ and travels in second quadrant.
2. The locus crosses real axis at -0.1667K and enters third quadrant.
3. Then the locus crosses negative imaginary axis and enters fourth quadrant.
4. Finally the locus ends at $0.05K\angle0^0$ when $\omega = \infty$.

***Note:** The exact plot can also be sketched on polar graph sheet.*

The section C_1 in s-plane and its corresponding mapping in G(s)H(s) plane are shown in fig 4.16.2. and 4.16.3.

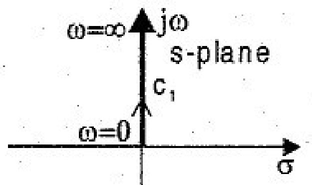


Fig 4.16.2 : *Section C_1 in s-plane*

⇒

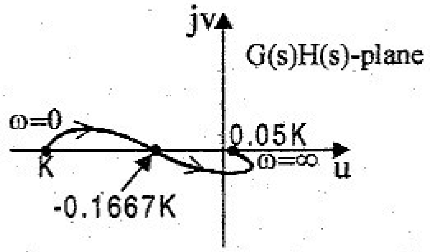


Fig 4.16.3 : *Mapping of section C_1 in G(s)H(s)-plane*

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt}\ Re^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to Re^{j\theta}$ and $R\to\infty$, G(s)H(s) can be approximated as shown below [i.e., $(1+sT)\approx sT$; Here $(s-1)\approx s$].

$$G(s)H(s) = \frac{K(1+0.5s)\,(1+s)}{(1+10s)\,(s-1)}$$

$$\approx \frac{K\,0.5s\times s}{10s\times s} = 0.05K$$

The approximate G(s)H(s) is independent of s and so the contour of section C_2 in s-plane is mapped as a point at 0.05K in G(s)H(s)-plane.

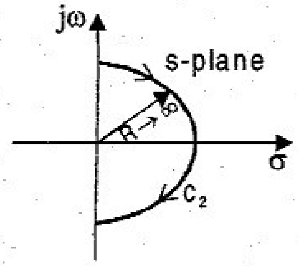


Fig 4.16.4 : *Section C_2 in s-plane*

⇒

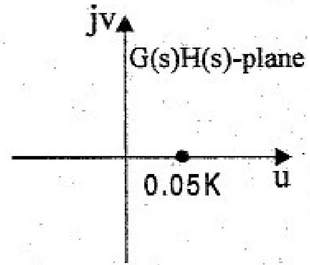


Fig 4.16.5 : *Mapping of section C_2 in G(s)H(s)-plane*

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of G(jω) H(jω) as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of G(jω) H(jω).

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.16.6 and fig 4.16.7.

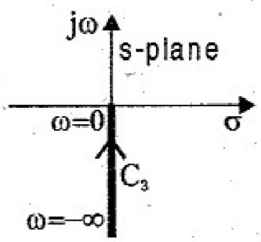


Fig 4.16.6 : *Section C_3 in s-plane*

⇒

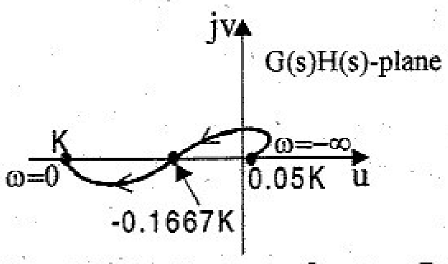


Fig 4.16.7 : *Mapping of section C_3 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.16.8.

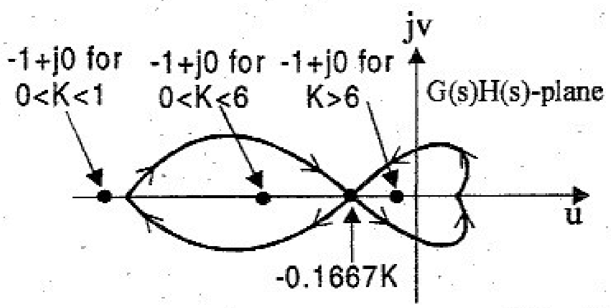


Fig 4.16.8 : Nyquist plot of $G(s)\,H(s) = \dfrac{K(1+0.5s)(1+s)}{(1+10s)(s-1)}$

STABILITY ANALYSIS

When $-0.1667K = -1$, the contour passes through $-1+j0$ point and this value of K is the limiting value for stability.

The limiting value of $K = \frac{1}{0.1667} = 6$

When 0<K<1

When 0<K<1, the $-1+j0$ point is not encircled, but there is one open loop right half pole and so system is unstable.

When 1<K<6

When 1<K<6, the locus crosses real axis between 0 and $-1+j0$. On travelling through the locus it is observed that the $-1+j0$ point is encircled clockwise and so the closed loop system is unstable.

When K>6

When K>6, the locus crosses real axis between $-1+j0$ and $-\infty$. On travelling through the locus it is observed that the $-1+j0$ point is encircled anticlockwise one time. Also the open loop system has one pole at the right half s-plane. Hence the system is stable.

RESULT

(a) The open loop system is unstable.

(b) For stability of the closed loop system, K>6.

EXAMPLE 4.17

Construct Nyquist plot for a feedback control system whose open loop transfer function is given by, $G(s)H(s) = \frac{5}{s(1-s)}$

Comment on the stability of open-loop and closed loop system.

SOLUTION

Given that, $G(s)H(s) = \frac{5}{s(1-s)}$

The open loop transfer function has a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half s-plane except the origin as shown in fig 4.17.1.

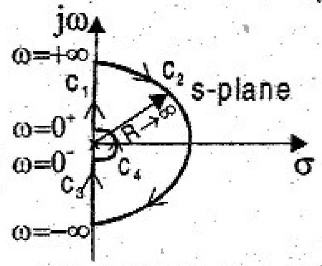


Fig 4.17.1 : *Nyquist Contour in s-plane*

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to $+\infty$. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{5}{s(1-s)}$$

Let $s = j\omega$. $\therefore G(j\omega)H(j\omega) = \frac{5}{j\omega(1-j\omega)}$

$$= \frac{5}{\omega\angle 90^\circ \sqrt{1+\omega^2}\angle -\tan^{-1}\omega}$$

$$= \frac{5}{\omega\sqrt{1+\omega^2}}\angle(-90+\tan^{-1}\omega)$$

$$\therefore |G(j\omega)H(j\omega)| = \frac{5}{\omega\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = -90^\circ + \tan^{-1}\omega$$

Note: (1–jω) represents a point in fourth quadrant

The exact shape of $G(j\omega)H(j\omega)$ locus is determined by calculating the magnitude and phase of $G(j\omega)H(j\omega)$ for various values of ω.

ω rad/sec	0	0.6	1.0	2.0	10.0	∞
$\lvert G(j\omega)H(j\omega)\rvert$	∞	7.15	3.53	1.12	0.05	0
$\angle G(j\omega)H(j\omega)$ deg	–90	–59	–45	–26	–5	0

From the above analysis, we can conclude that $G(j\omega)H(j\omega)$ locus starts at -90° axis at infinity for $\omega = 0$ and meets the origin along 0° axis when $\omega = \infty$.

The section C_1 in s-plane and its corresponding mapping in G(s)H(s)-plane are shown in fig 4.17.2 and 4.17.3.

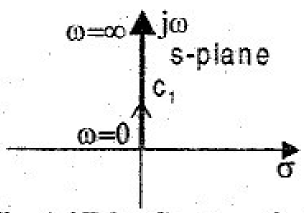


Fig 4.17.2 : Section C_1 in s-plane

⇒

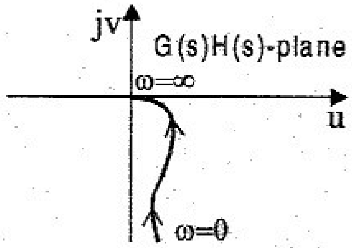


Fig 4.17.3 : Mapping of section C_1 in G(s)H(s)-plane

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to \infty$, the G(s)H(s) can be approximated as shown below, [i.e., $(1-s) \approx -s$)]

$$G(s)H(s) = \frac{5}{s(1-s)} \approx \frac{5}{s(-s)} = \frac{5}{s^2 e^{j\pi}}$$

Let, $s = \underset{R\to\infty}{Lt}\ R\,e^{j\theta}$.

Note : $-1 = e^{j\pi}$

$$\therefore G(s)H(s)\Big|_{s=\underset{R\to\infty}{Lt} R e^{j\theta}} = \frac{5}{\underset{R\to\infty}{Lt}(R\,e^{j\theta})^2 e^{j\pi}} = 0e^{-j(2\theta+\pi)}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{-j2\pi}$ .....(1)

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{j0}$ .....(2)

From the equations (1) and (2) we can say that section C_2 in s-plane (fig 4.17.4) is mapped as circular arc of zero radius around origin in G(s)H(s) plane with argument varying from -2π to $+0$ as shown in fig 4.17.5.

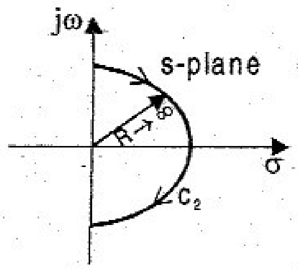


Fig 4.17.4 : Section C_2 in s-plane

⇒

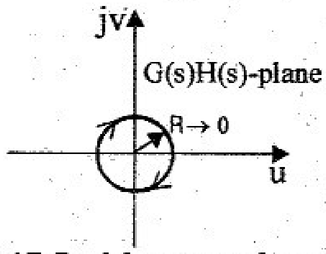


Fig 4.17.5 : *Mapping of section C_2 in G(s)H(s)-plane*

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of $G(j\omega)H(j\omega)$.

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.17.6 and fig 4.17.7.

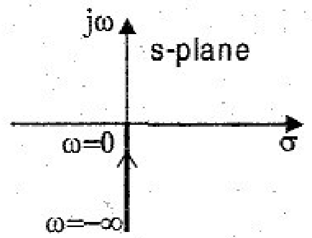


Fig 4.17.6 : Section C_3 in s-plane

$\Rightarrow$

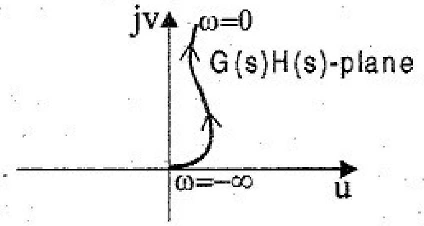


Fig 4.17.7 : Mapping of section C_3 in G(s)H(s)-plane

MAPPING OF SECTION C_4

The mapping of section C_4 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to 0$, G(s)H(s) can be approximated as shown below, [i.e., $(1-s) \approx 1$].

$$G(s)H(s) = \frac{5}{s\,(1-s)} \approx \frac{5}{s \times 1} = \frac{5}{s}$$

Let $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$.

$$\therefore\ G(s)H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{5}{\underset{R\to 0}{Lt}\ Re^{j\theta}} = \infty e^{-j\theta}$$

$$\text{When } \theta = -\frac{\pi}{2}, \quad G(s)H(s) = \infty e^{j\frac{\pi}{2}} \qquad .....(3)$$

$$\text{When } \theta = \frac{\pi}{2}, \quad G(s)H(s) = \infty e^{-j\frac{\pi}{2}} \qquad .....(4)$$

From the equations (3) and (4) we can say that section C_4 in s-plane (fig 4.17.8.) is mapped as a circular arc of infinite radius with argument varying from $\pi/2$ to $-\pi/2$ as shown in fig 4.17.9.

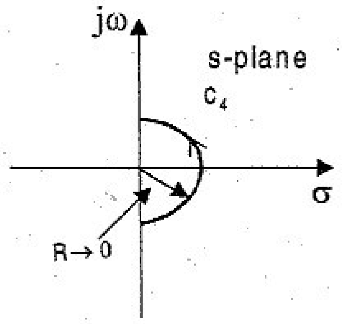


Fig 4.17.8 : Section C_4 in s-plane

$\Rightarrow$

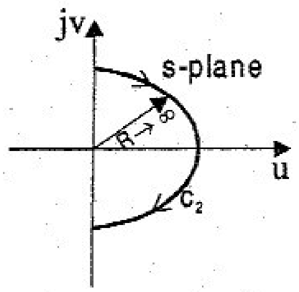


Fig 4.17.9 : Mapping of section C_4 in G(s)H(s)-plane

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.17.10.

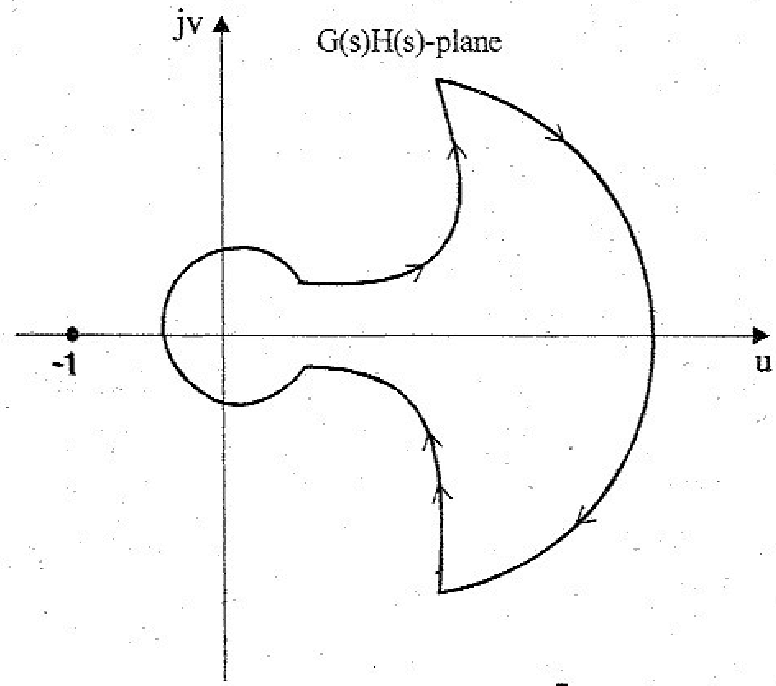


Fig 4.17.10 : Nyquist plot of $G(s)H(s) = \dfrac{5}{s(1-s)}$

STABILITY ANALYSIS

The Nyquist contour in G(s)H(s)-plane does not encircle the point $(-1+j0)$ but the open loop transfer function has one pole on the right half s-plane. Therefore the system is unstable.

RESULT

Both open loop and closed loop systems are unstable.

EXAMPLE 4.18

By Nyquist stability criterion determine the stability of closed loop system, whose open loop transfer function is given by,

$G(s)H(s) = \dfrac{(s+2)}{(s+1)\,(s-1)}$. Comment on the stability of open-loop and closed loop system.

SOLUTION

Given that, $G(s)H(s) = \dfrac{(s+2)}{(s+1)\,(s-1)}$

The open loop transfer function does not have a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane as shown in fig 4.18.1.

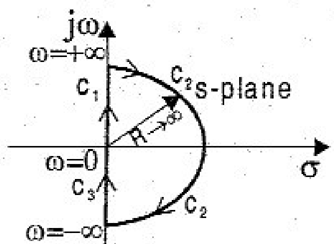


Fig 4.18.1 : *Nyquist Contour in s-plane*

The Nyquist contour has three sections C_1, C_2 and C_3. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to $+\infty$. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

> ***Note :** $(-1+j\omega)$ represents a point in second quadrant*

$$G(s)H(s) = \frac{s+2}{(s+1)\,(s-1)} = \frac{2(1+0.5s)}{(1+s)\,(-1+s)}$$

Let $s = j\omega$. $\quad \therefore G(j\omega)H(j\omega) = \dfrac{2(1+j0.5\omega)}{(1+j\omega)\,(-1+j\omega)} = \dfrac{2\sqrt{1+0.25\omega^2}\ \angle\tan^{-1}0.5\omega}{\sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\ \sqrt{1+\omega^2}\ \angle(180^\circ-\tan^{-1}\omega)}$