PROGRAM 3.8

Write a MATLAB program to draw the polar plot and calculate gain margin and phase margin for the open loop system governed by the following transfer function.

$$G(s)=1/(s(1+s)^2)$$

```
%program to draw nichols plot on nichols chart

clear all
clc

s=tf('s');
disp('The given transfer function is,');
Gs=20/(s^3+(7*s^2)+10*s)

[num_cof den_cof]=tfdata(Gs);  %determine numerator & denominator
                               %coefficients of G(s)
nichols(Gs);
axis([-180 0 -15 20]);         %specify the range of horizontal and
                               %vertical axis
ngrid
```

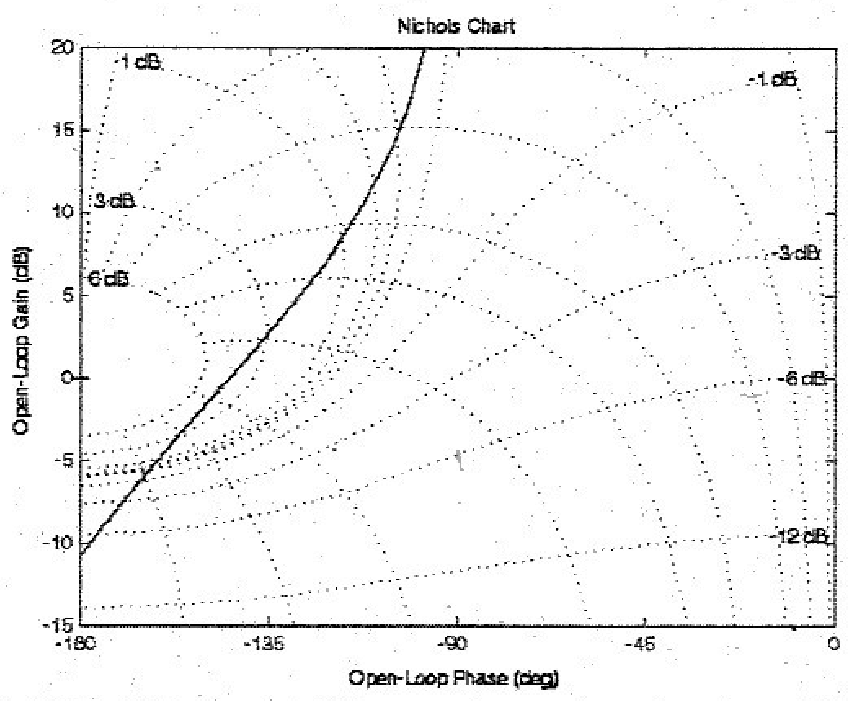

Fig P3.8 : Nichols plot of the open loop system given in problem 3.8.

OUTPUT

```
The given transfer function is,

Transfer function:
                    20
          ------------------
          s^3 + 7 s^2 + 10 s
```

3.13 SHORT QUESTIONS AND ANSWERS

Q3.1 ***What is frequency response ?***

The magnitude and phase function of sinusoidal transfer function of a system are real function of frequency ω, and so they are called frequency response.

Q3.2 ***What are advantages of frequency response analysis ?***

1. The absolute and relative stability of the closed loop system can be estimated from the knowledge of the open loop frequency response.
2. The practical testing of system can be easily carried with available sinusoidal signal generators and precise measurement equipments.
3. The transfer function of complicated functions can be determined experimentally by frequency response tests.
4. The design and parameter adjustment can be carried more easily.
5. The corrective measure for noise disturbance and parameter variation can be easily carried.
6. It can be extended to certain non-linear systems.

Q3.3 ***What are frequency domain specifications?***

The frequency domain specifications indicates the performance of the system in frequency domain, and they are,

1. Resonant peak, M_r
2. Resonant frequency, ω_r
3. Bandwidth, ω_b
4. Cut-off rate
5. Gain margin, K_g
6. Phase margin, γ

Q3.4 ***Define Resonant Peak?***

The maximum value of the magnitude of closed loop transfer function is called Resonant Peak.

Q3.5 ***What is Resonant frequency?***

The frequency at which the resonant peak occurs is called Resonant frequency. The resonant peak is the maximum value of the magnitude of closed loop transfer function.

Q3.6 ***Define Bandwidth?***

The Bandwidth is the range of frequencies for which the system gain is more than -3db.

Q3.7 ***What is cut-off rate?***

The slope of the log-magnitude curve near the cut-off frequency is called cut-off rate.

Q3.8 ***Define gain margin?***

The gain margin, K_g is defined as the value by which gain of the system has to be increased to drive system to be verge of instability. It is given by the reciprocal of the magnitude of open loop transfer function, at phase cross-over frequency, ω_{pc}.When expressed in decibels, it is given by, the negative of db magnitude of $G(j\omega)$ at phase cross-over frequency.

Gain margin, $K_g = \dfrac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$ and $K_g \text{ in db} = 20 \log \dfrac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = -20 \log |G(j\omega)|_{\omega=\omega_{pc}}$

Q3.9 ***Define phase margin?***

The phase margin, γ is that amount of additional phase lag at the gain cross-over frequency, ω_{gc} required to bring the system to the verge of instability. It is given by, $180° + \phi_{gc}$, where ϕ_{gc} is the phase of $G(j\omega)$ at the gain cross over frequency.

Phase margin, $\gamma = 180° + \phi_{gc}$; where, $\phi_{gc} = \angle G(j\omega)\Big|_{\omega=\omega_{gc}}$

Q3.10 ***What is phase and Gain cross-over frequency?***

The **gain cross over frequency** is the frequency at which the magnitude of the open loop transfer function is unity. The **phase cross over frequency** is the frequency at which the phase of the open loop transfer function is 180°.

Q3.11 ***Write the expression for resonant peak and resonant frequency.***

Resonant peak, $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$; Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

Q3.12 ***Write a short note on the correlation between the time and frequency response?***

Correlation exists between time and frequency response of first or second order systems. The frequency domain specifications can be expressed in terms of the time domain parameters ζ and ω_n. For a peak overshoot in time domain there is a corresponding resonant peak in frequency domain.

For higher order systems, there is no explicit correlation between time and frequency response. But if there is a pair of dominant complex conjugate poles, then the system can be approximated to second order system and the correlation between time and frequency response can be estimated.

Q3.13 ***The damping ratio and natural frequency of oscillation of a second order system is 0.5 and 8 rad/sec respectively. Calculate the resonant peak and resonant frequency?***

Resonant peak, $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = \dfrac{1}{2\times(0.5)\sqrt{1-(0.5)^2}} = 1.154$

Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\zeta^2} = 8\times\sqrt{1-2\times 0.5^2} = 5.657\text{ rad/sec}$

Q3.14 ***What is Bode plot?***

The bode plot is a frequency response plot of the transfer function of a system. It consists of two plots : *Magnitude plot and Phase plot.*

The magnitude plot is a graph between magnitude of a system transfer function in db and frequency, ω. The phase plot is a graph between the phase or argument of a system transfer function in degrees and the frequency, ω. Usually, both the plots are plotted on a common x-axis in which the frequencies are expressed in logarithmic scale.

Q3.15 ***What is approximate bode plot?***

In approximate bode plot, the magnitude plot of first and second order factors are approximated by two straight lines, which are asymptotes to exact plot. One straight line is at 0db, for the frequency range 0 to ω_c and the other straight line is drawn with a slope of $\pm 20n$ db/dec for frequency range ω_c to ∞. Here ω_c is the corner frequency.

Q3.16 ***Define corner frequency?***

The magnitude plot can be approximated by asymptotic straight lines. The frequencies corresponding to the meeting point of asymptotes are called corner frequency. The slope of the magnitude plot changes at every corner frequency.

Q3.17 ***What are the advantages of Bode Plot?***

1. The magnitudes are expressed in db, and so, a simple procedure is available to add magnitude of each term one by one.

2. The approximate bode plot can be quickly sketched, and the corrections can be made at corner frequencies to get the exact plot.
3. The frequency domain specifications can be easily determined.
4. The bode plot can be used to analyse both open loop and closed loop system.

Q3.18 ***What is the value of error in the approximate magnitude plot of a first order factor at the corner frequency?***

The error in the approximate magnitude plot of a first order factor at the corner frequency is ± 3m db, where m is multiplicity factor. Positive error for numerator factor and negative error for denominator factor.

Q3.19 ***What is the value of error in the approximate magnitude plot of a quadratic factor with $\zeta=1$ at the corner frequency?***

The error is ± 6db, for the quadratic factor with $\zeta=1$. Positive error for numerator factor and negative error for denominator factor.

Q3.20 ***Draw the bode plot of, $G(s) = \frac{K}{s^n}$.***

Let $s = j\omega$,

$$\therefore G(j\omega) = \frac{K}{(j\omega)^n}$$

The magnitude of $G(j\omega)$ is unity when $\omega = K^{1/n}$.

The magnitude plot is a straight line with slope of –20n db/dec and passing through $\omega = K^{1/n}$. The Phase plot is straight line parallel to x-axis at –90n°.

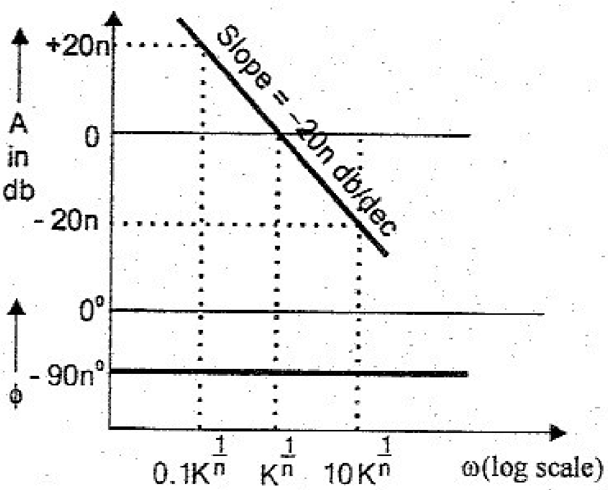

Fig Q3.20 : Bode plot of integral factor, $K/(j\omega)^n$.

Q3.21 ***Sketch the bode plot of $G(s) = 1/(1+sT)$.***

Let $s = j\omega$, $\therefore G(j\omega) = \frac{1}{1+j\omega T}$

The corner frequency, $\omega_c = \frac{1}{T}$

The magnitude plot is approximated by two straight lines : *one straight line at 0db in the frequency range 0 to ω_c and the other straight line with the slope of -20db/dec in the frequency range ω_c to ∞.* The phase of $G(j\omega)$ varies from 0 to -90° as ω is varied from 0 to ∞. Hence, the phase plot is a curve passing through -45° at the corner frequency.

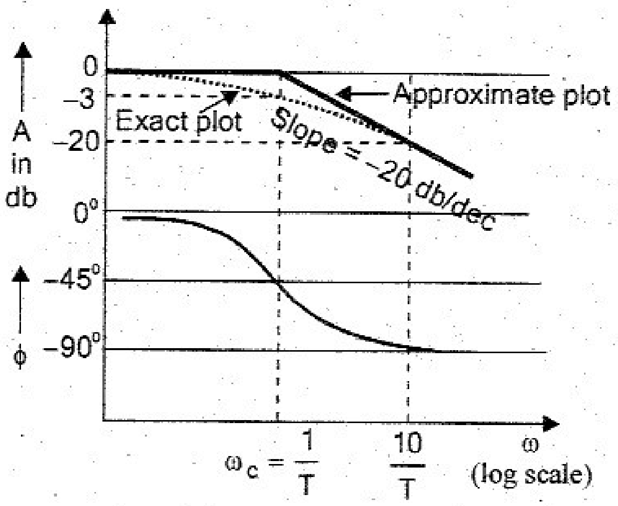

Fig Q3.21 : Bode plot of the factor $\frac{1}{1+j\omega T}$.

Q3.22 What is polar plot?

The polar plot of a sinusoidal transfer function G(jω) is a plot of the magnitude of G(jω) versus the phase angle/argument of G(jω) on polar or rectangular co-ordinates as ω is varied from zero to infinity.

Q3.23 What is minimum phase system?

The minimum phase systems are systems with minimum phase transfer functions. In minimum phase transfer functions, all poles and zeros will lie on the left half of s-plane.

Q3.24 What is All-Pass systems?

The all pass systems are systems with all pass transfer functions. In all pass transfer functions, the magnitude is unity at all frequencies and the transfer function will have anti-symmetric pole zero pattern (i.e., for every pole in the left half s-plane, there is a zero in the mirror image position with respect to imaginary axis).

Q3.25 What is non-minimum phase transfer function?

A transfer function which has one or more zeros in the right half s-plane is known as non-minimum phase transfer function.

Q3.26 In minimum phase system, how the start and end of polar plot are identified?

For minimum phase transfer functions, with only poles, the type number of the system determines the quadrant in which the polar plot starts, and the order of a system determines the quadrant in which the polar plot ends.

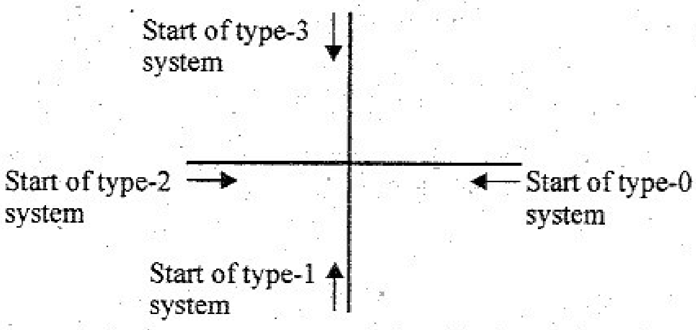

Fig Q3.26a : Start of polar plot of all pole minimum phase system.

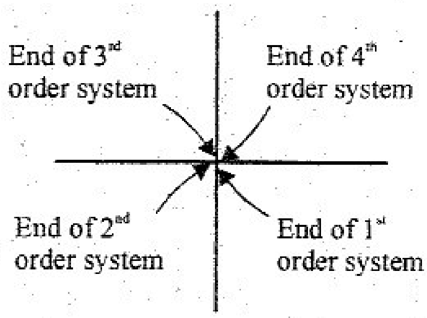

Fig Q3.26b : End of polar plot of all pole minimum phase system.

Q3.27 ***Draw the polar plot of G(s)= 1/(1+sT).***

Let $s = j\omega$, $\therefore G(j\omega) = \dfrac{1}{1+j\omega T} = \dfrac{1}{\sqrt{1+\omega^2T^2}\angle\tan^{-1}\omega T}$

$$= \frac{1}{\sqrt{1+\omega^2T^2}}\angle -\tan^{-1}\omega T$$

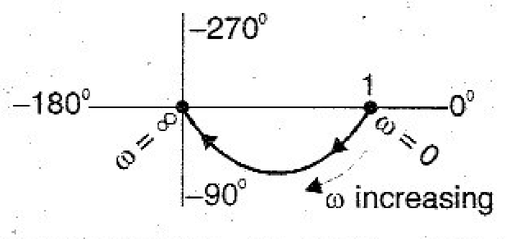

Fig Q3.27 : Polar plot of G(s) = 1/(1+sT).

As $\omega \to 0$, $G(j\omega) \to 1\angle 0°$

As $\omega \to \infty$, $G(j\omega) \to 0\angle -90°$

Q3.28 ***Sketch the polar plot of,*** $G(s) = \dfrac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$.

The given system is all pole minimum phase system. The type number of the system is 2 and the order is 5. Hence, the polar plot starts in second quadrant and ends in fourth quadrant.

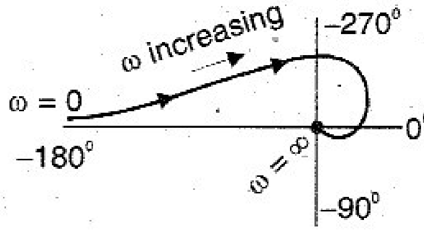

Fig Q3.28 : Polar plot of type-2, 5th order system.

Q3.29 ***What is Nichols plot?***

The Nichols plot is a frequency response plot of the open loop transfer function of a system. It is a graph between magnitude of G(jω) in db and the phase of G(jω) in degree, plotted on a ordinary graph sheet.

Q3.30 ***What are M and N circles?***

The magnitude, M of closed loop transfer function with unity feedback will be in the form of circle in complex plane for each constant value of M. The family of these circles are called M-circles.

Let N = tan α, where a is the phase of closed loop transfer function with unity feedback. For each constant value of N, a circle can be drawn in the complex plane. The family of these circles are called N-circles.

Q3.31 ***How closed loop frequency response is determined from open loop frequency response using M and N circles?***

The G(jω) locus or polar plot of open loop system is sketched on the standard M and N circles chart. The meeting point of M circle with G(jω) locus gives the magnitude of closed loop system. (the frequency being same as that of open loop system). The meeting point of G(jω) locus with N-circle gives the value of phase of closed loop system, (frequency being same as that of open loop system).

Q3.32 ***What is Nichols chart?***

The Nichols chart consists of M and N contours superimposed on ordinary graph. Along each M-contour the magnitude of closed loop system, M will be a constant. Along each N-contour, the phase α of closed loop system will be constant. The ordinary graph consists of magnitude in db, marked on the y-axis and the phase in degrees marked on x-axis. The Nichols chart is used to find the closed loop frequency response from the open loop frequency response.

Q3.33 ***How the closed loop frequency response is determined from the open loop frequency response using Nichols chart?***

The G(jω) locus or the Nichols plot is sketched on the standard Nichols chart. The meeting point of M-contour with G(jω) locus gives the magnitude of closed loop system and the meeting point with N-circle gives the argument/phase of the closed loop system.

Q3.34 ***What are the advantages of Nichols chart?***

1. It is used to find closed loop frequency response from open loop frequency response.
2. The frequency domain specifications can be determined from Nichols chart.
3. The gain of the system can be adjusted to satisfy the given specification.

3.14 EXERCISES

E3.1 *Sketch the bode plot of the following open loop transfer functions and from the plot determine the phase margin and gain margin.*

a) $G(s) = 100(1+0.1s)/s(1+0.2s)(1+0.5s)$

b) $G(s) = 50(1+0.1s)/(1+0.01s)(1+s)$

c) $G(s) = 30(1+0.1s)/s(1+0.01s)(1+s)$

d) $G(s) = s^2(s+10)/(s+5)^2(s+0.1)$

e) $G(s) = 40(1+s)/(1+5s)(s^2+2s+4)$

f) $G(s) = 10(1+s)e^{-0.1s}/s(1+0.2s)$

E3.2 *The open loop transfer function of a system is given by $G(s) = K/s(1+0.5s)(1+0.2s)$. Using bode plot find the value of K so that (i) The gain margin of the system is 6db and (ii) The phase margin of the system is 25°.*

E3.3 *Sketch the polar plot of the following transfer functions and from the plot, determine the phase margin and gain margin.*

a) $G(s) = 10(s+1)/(s+10)^2$

b) $G(s) = 200(s+2)/s(s^2+10s+100)$

c) $e^{-0.1s}/s(s+1)(s+5)$

d) $1/s(s+4)(s+8)$

E3.4 *The open loop transfer function of a system is given by $G(s) = K/s(s^2+s+4)$. Using polar plot, determine the value of K, so that phase margin is 50°. What is the corresponding value of gain margin?*

E3.5 *A unity feedback system has $G(s) = K/s(1+0.1s)$. Using Nichols chart find the value of K so that resonant peak, $M_r=1.4$. Find the corresponding value of ω_r.*

E3.6 *The open loop transfer function of unity feedback system is, $G(s) = K/(1+0.05s)(1+0.1s)(1+0.3s)$.*

Using Nichols chart find the value of K so that gain margin of the system is 10db. What is the corresponding value of phase margin.

E3.7 *Using Nichols chart determine the closed loop frequency response of the unity feedback system, whose open loop transfer function is, $G(s) = 200(s+1)/s(s+10)^2$.*

E3.8 *A unity feedback system has open loop transfer function $G(s) = 54/(1+0.1s)(s^2+8s+25)$.*

Using Nichols chart determine the closed loop frequency response. From the closed loop response determine, the resonant peak, resonant frequency and bandwidth.