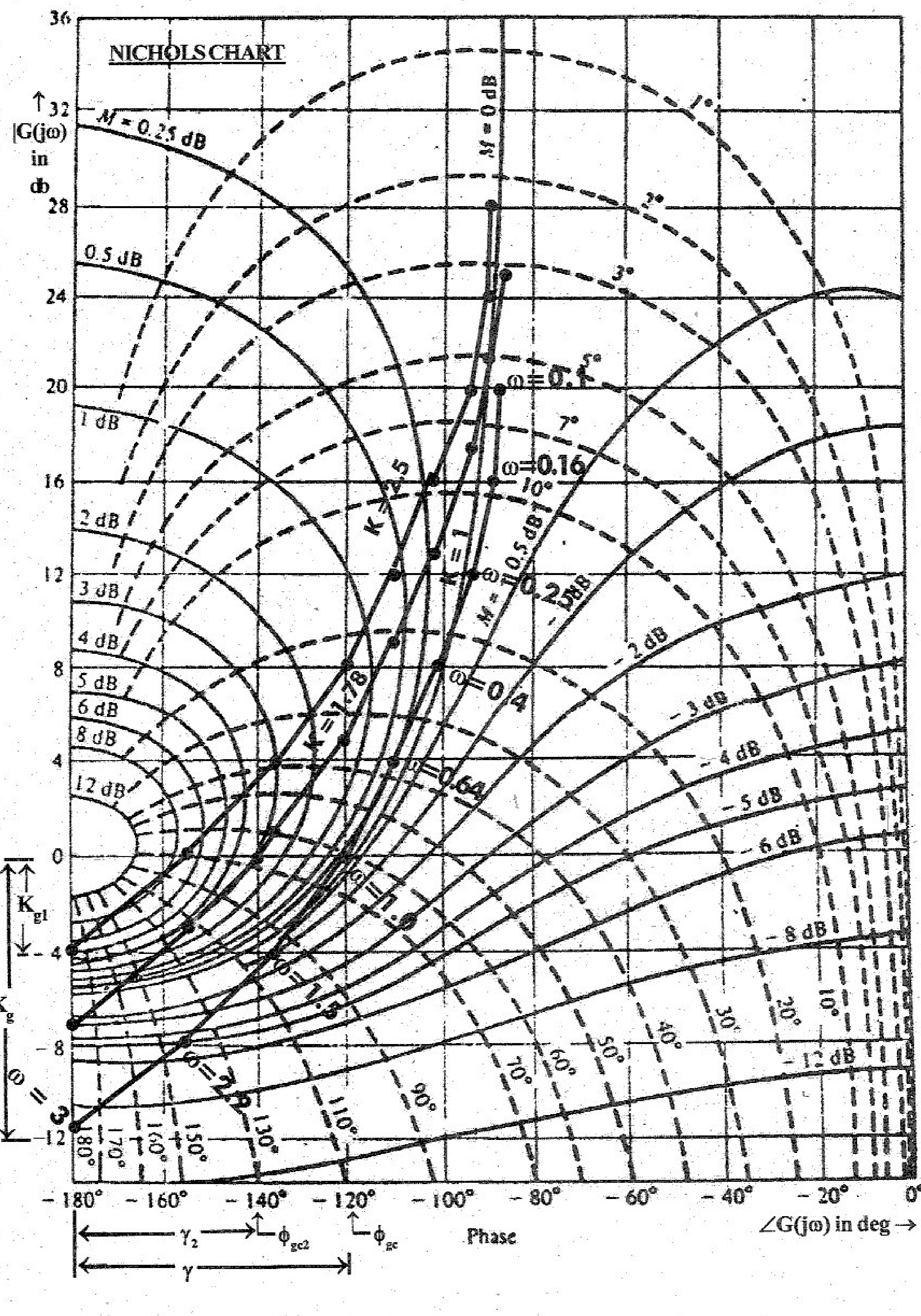

Fig 3.14.2 : Nichols plot of $G(j\omega) = Ke^{-j0.2\omega}/j\omega(1 + j0.25\omega)\,(1 + j\,0.1\omega)$

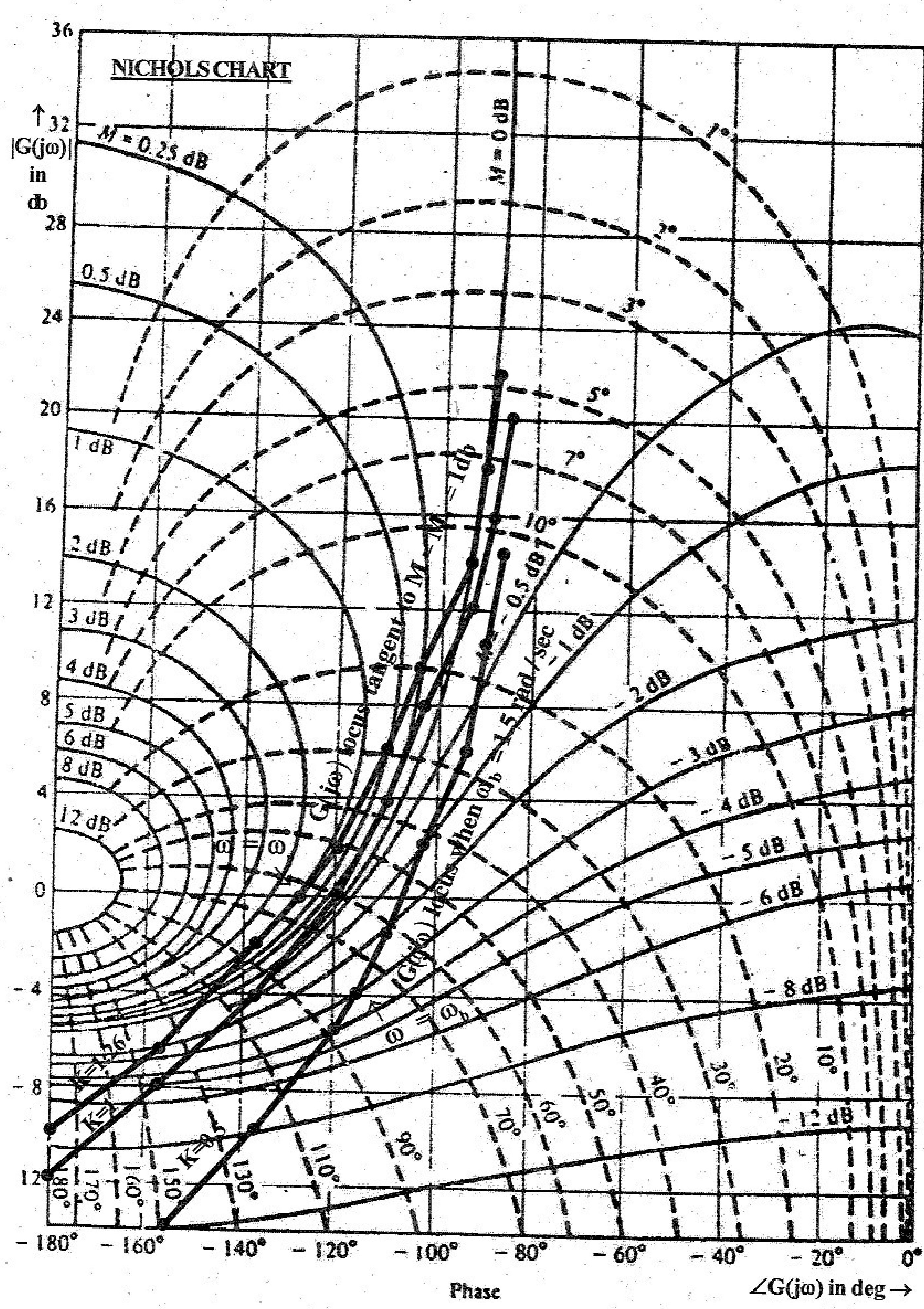

Fig 3.14.3 : Nichols plot of $G(j\omega) = Ke^{-j0.2\omega}/j\omega(1 + j0.25\omega)\,(1 + j\,0.1\omega)$

EXAMPLE 3.15

A unity feedback system has open loop transfer function, $G(s) = \frac{20}{s(s+2)(s+5)}$. Using Nichols chart, determine the closed loop frequency response and estimate M_r, ω_r and ω_b.

SOLUTION

Given that,

$$G(s) = \frac{20}{s(s+2)(s+5)}$$

The transfer function G(s) is converted to time constant or bode form.

$$G(s) = \frac{20}{s \times 2\left(\frac{s}{2}+1\right) \times 5\left(\frac{s}{5}+1\right)} = \frac{20/(2 \times 5)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)} = \frac{2}{s\,(1+0.5s)\,(1+0.2s)}$$

Put, $s = j\omega$, in G(s) to get $G(j\omega)$.

$$\therefore G(j\omega) = \frac{2}{j\omega(1+j0.5\omega)(1+j0.2\omega)}$$

$$= \frac{2}{\omega\angle 90^\circ \sqrt{1+0.25\omega^2}\angle \tan^{-1}0.5\omega \sqrt{1+0.04\omega^2}\angle \tan^{-1}0.2\omega}$$

$$= \frac{2}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+0.04\omega^2}}\angle(-90^\circ - \tan^{-1}0.5\omega - \tan^{-1}0.2\omega)$$

$$\therefore |G(j\omega)| = \frac{2}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+0.04\omega^2}}$$

$$|G(j\omega)|_{\text{in db}} = 20\log\left[\frac{2}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+0.04\omega^2}}\right]$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}0.5\omega - \tan^{-1}0.2\omega$$

The magnitude of $G(j\omega)$ in db and phase of $G(j\omega)$ for various frequencies are calculated and listed in table-1. The choice of frequencies are chosen such that the magnitude plot extends in the range of 40 db to –14 db and the phase plot extends in the range of 0° to –180°.

Using table-1, the actual bode plot of $G(j\omega)$ is plotted on semilog graph sheet, as shown in fig 3.15.1.

TABLE-1: Calculated values of $|G(j\omega)|$ and $\angle G(j\omega)$

ω, rad/sec	0.2	0.5	1.0	2.0	3.0	4.0
|G(jω)|, db	20	12	5	–4	–10	–15
∠G(jω), deg	–98	–110	–128	–157	–177	–192

The magnitude and phase plot of bode plot of $G(j\omega)$ are shown in fig 3.15.1 From the bode plot, the phase and frequency for various values of magnitudes are noted and tabulated in table-2. (The choice of magnitudes are 20, 16, 12,..., i.e., in steps of 4 db, which is convenient for Nichols plot on Nichols chart).

Using the values listed in table-2, the locus of $G(j\omega)$ is sketched on the Nichols chart as shown in fig 3.15.2.

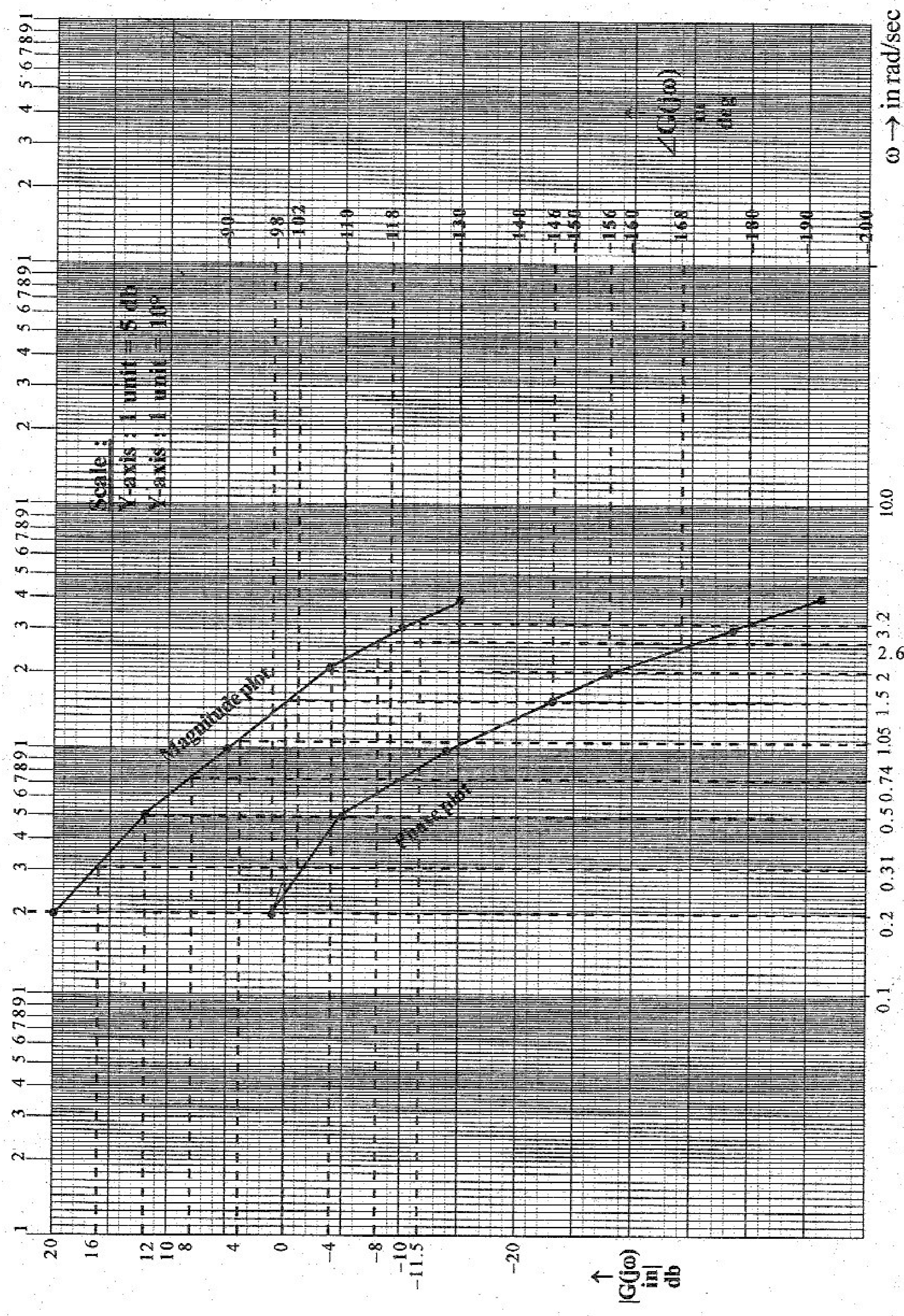

Fig 3.15.1 : *Bode plot of* $G(j\omega) = 2/[j\omega\,(1 + j0.5\,\omega)\,(1 + j0.2\,\omega)]$

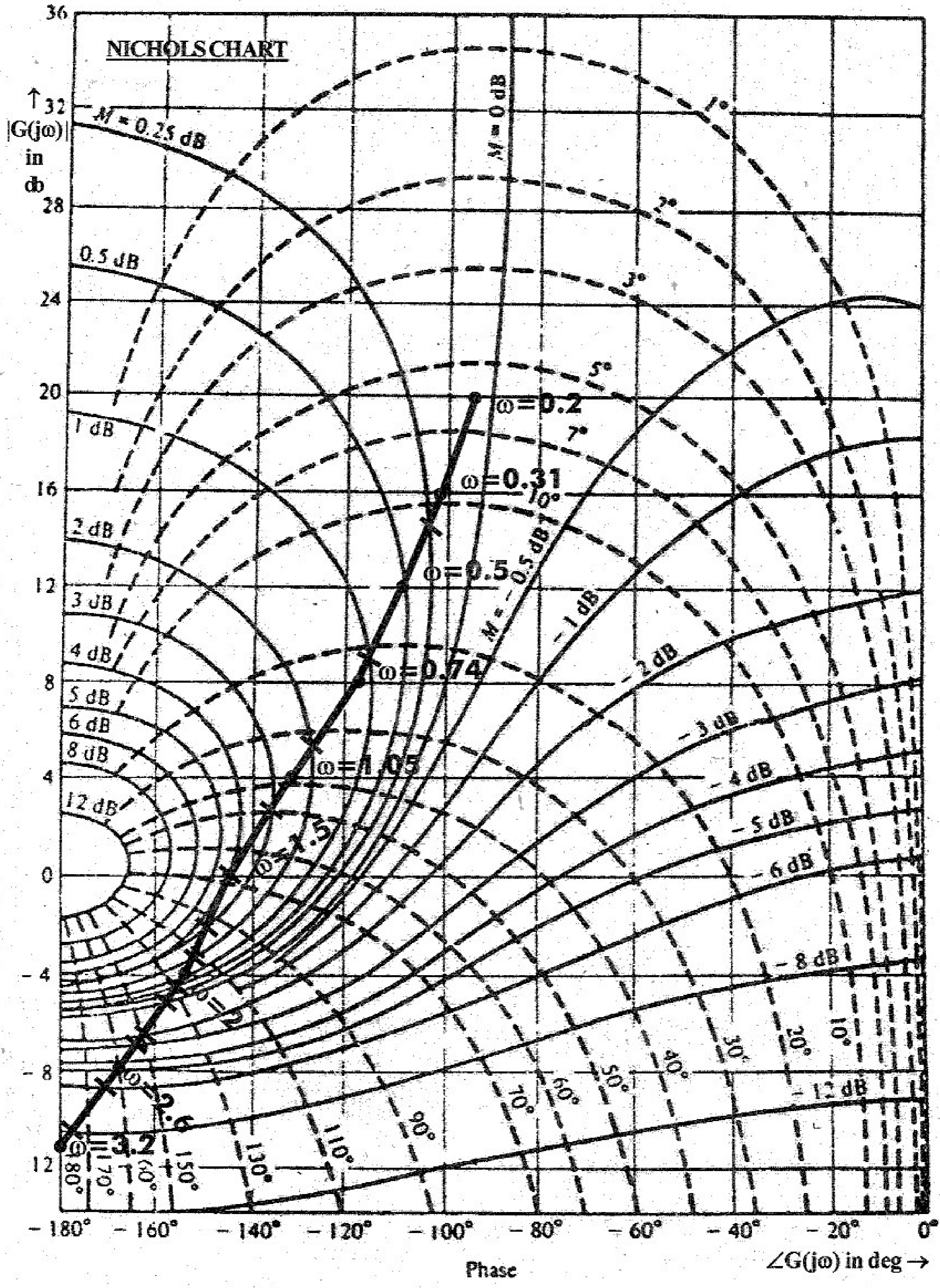

Fig 3.15.2 : Nichols plot of $G(j\omega) = 2/[j\omega\,(1 + j0.5\omega)\,(1 + j\,0.2\omega)]$.

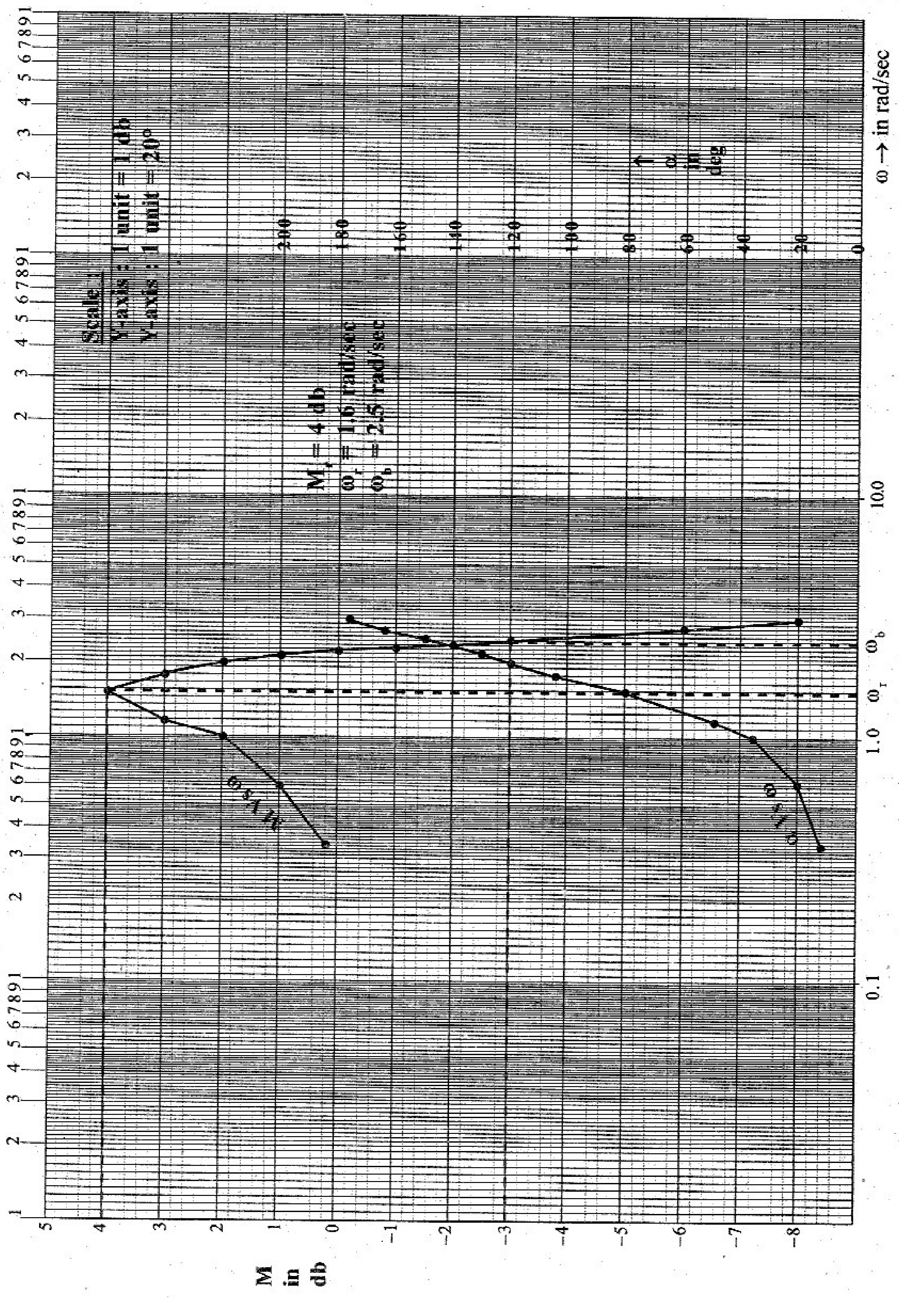

Fig 3.15.3 : *Closed loop frequency response of* $G(j\omega) = 2/[j\omega\,(1 + j0.5\omega)\,(1 + j0.2\omega)]$

TABLE-2: Values of |G(jω)| and ∠G(jω) Obtained from Bode Plot

ω rad/sec	0.2	0.31	0.5	0.74	1.05	1.5	2.0	2.6	3.2
|G(jω)| db	20	16	12	8	4	0	–4	–8	–11.5
∠G(jω) deg	–98	–102	–110	–118	–130	–146	–156	–168	–180

The locus of G(jω) drawn on the Nichols chart cuts the M-contour and N-contour at various points. The meeting points of G(jω) locus and various M-contours are noted.

The phase α corresponding to the meeting point are noted from N-contours passing through the meeting point. (If a meeting point lies between two N-contours, then choose an approximate value of α).

The frequency corresponding to the meeting point are noted from bode plot by transferring the |G(jω)| corresponding to meeting point to bode plot. The values of ω, M and α are listed in table-3.

The values of M and α are the magnitude and phase of closed loop frequency response of G(jω) with unity feedback.

TABLE-3: Values of M and α from Nichols Chart

ω rad/sec	0.36	0.62	1.0	1.2	1.6	1.8	2.0	2.1	2.3	2.4	2.5	2.8	3.0
M db	0.25	1	2	3	4	3	2	1	0	–2	–3	–6	–8
α deg	12	21	35	50	78	102	120	130	140	151	155	165	175

Using the values listed in table-3, the closed loop frequency response plots are sketched as shown in fig 3.15.3. The closed loop frequency response consists of two plots and they are magnitude plot, M Vs ω and phase plot, α Vs ω.

From the closed loop frequency response the values of M_r, ω_r and ω_b are noted.

Resonant peak, M_r = +4 db

Resonant Frequency ω_r = 1.6 rad/sec

Bandwidth, ω_b = 2.5 rad/sec

3.12 FREQUENCY RESPONSE ANALYSIS USING MATLAB

In general, the open loop or closed loop transfer function of a system is denoted as T(s).

Let, T(s) be a rational function of "s", as shown below.

$$T(s) = \frac{b_0 s^M + b_1 s^{M-1} + b_2 s^{M-2} + ... + b_{M-1} s + b_M}{a_0 s^N + a_1 s^{N-1} + a_2 s^{N-2} + ... + a_{N-1} s + a_N}$$

For frequency response analysis, the transfer function T(s) is declared as a function of s using the following commands.

```
s=tf('s');
Ts=(b0*s^M+b1*s^(M-1)+...+bM)/(a0*s^N+a1*s^(N-1)+...+aN);
```

The coefficients of numerator and denominator polynomials of the transfer function are determined using the following command.

```
[num_cof den_cof]=tfdata(Ts);
```

The gain margin, phase margin, gain crossover frequency and phase crossover frequency can be determined using the following command.

```
[GM PM wgc wpc] = margin(Ts);
```

BODE PLOT

In order to draw Bode plot the frequecy range can be specified using following commands.

```
w = logspace(ds, de, n);
where, ds represents Start decade as 10^ds
       de represents end decade as 10^de
       n represents number of points to be calculated between 10^ds & 10^de
```

Method 1 :

The Bode plot can be plotted using any one of the following command.

```
bode(Ts,'k');
bode(Ts, w);
bode(num_cof, den_cof);
bode(num_cof, den_cof,w);
```

Method 2 :

The Bode plot can also be plotted using semilog plot command as shown below.

```
[Mag Phase w] = bode(Ts,w);
MagdB = 20*log10(Mag);
subplot(2,1,1);semilogx(w,MagdB,'k');
subplot(2,1,2);semilogx(w,Phase,'k');
```

In this method the magnitude and phase can be scaled by drawing two lines at two specified upper and lower values. For drawing these lines, one dimensional arrays consisting of same values has to be created by multiplying the specified value with one. The length of the array should be same as number of frequency points for which the magnitude and phase are computed. (Refer program 3.3).

POLAR PLOT

The polar plot can be plotted using the following commands.

```
w = w_start : w_step : w_end ;
[re, im, w] = nyquist(num_cof,den_cof,w);
z = re + i*im; r = abs(z); theta = angle(z);
polar(theta,r,'w')
```

NICHOLS PLOT

The Nichols plot of open loop transfer function, G(s) can be plotted using the following commands.

```
[num_cof den_cof ] = tfdata(Gs);
nichols(Gs);
axis([ph_start, ph_end, mag_start, mag_end]);
```

PROGRAM 3.1

Write a MATLAB program to draw the Bode plot for the open loop system governed by the following transfer function.

$$G(s)=s^2/(1+0.2s)(1+0.02s)$$

```
%program to plot Bode plot

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=(s^2)/((1+0.2*s)*(1+0.02*s))

w=logspace(-1,2,200);          %specify the frequency range
bode(Gs,w)
grid
```

OUTPUT

The given transfer function is,

Transfer function:

```
          s^2
----------------------
0.004 s^2 + 0.22 s + 1
```

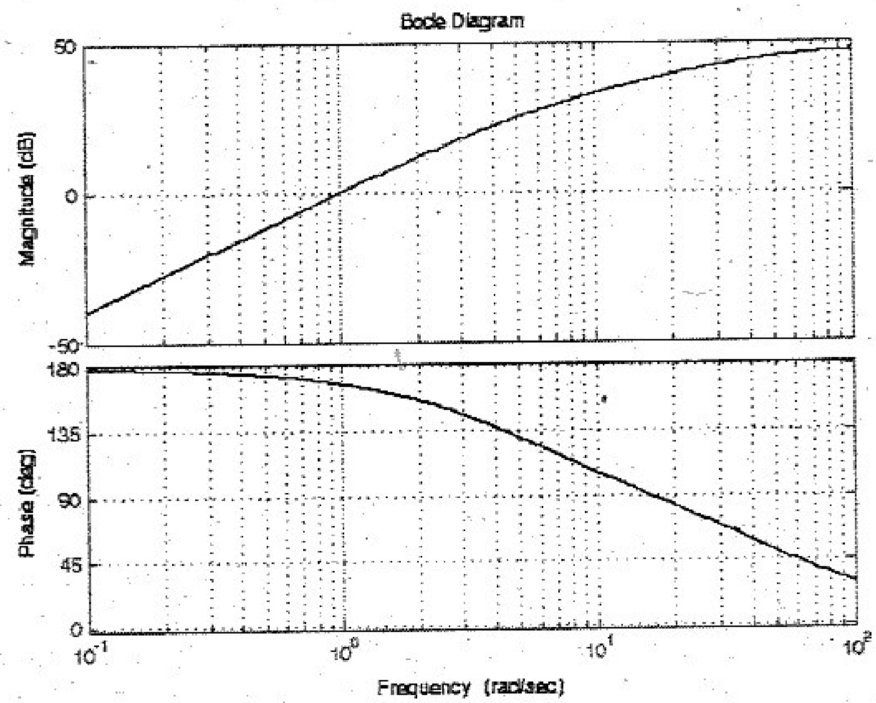

***Fig P3.1**: Bode plot of the open loop system given in problem 3.1.*

The Bode plot of program 3.1 is shown in fig p3.1.

PROGRAM 3.2

Write a MATLAB program to draw the Bode plot and to calculate gain margin, phase margin, gain crossover frequency & phase crossover frequency for the open loop system governed by the following transfer function.

$$G(s)=10/(0.04s^3+0.5s^2+s)$$

```
%program to find gain & phase margins using bode plot

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=10/((0.04*s^3)+(0.5*s^2)+s)

bode(Gs,'k')
grid

[GM,PM,wgc,wpc]=margin(Gs);
GMdB=20*log10(GM);
disp('Gain margin in dB,'); GMdB
disp('Phase margin in deg,');PM
disp('Gain cross over frequency in rad/sec,');wgc
disp('Phase cross over frequency in rad/sec,');wpc
```

OUTPUT

```
The given transfer function is,
Transfer function:
                        10
           ----------------------
           0.04 s^3 + 0.5 s^2 + s

Gain margin in dB,
              GMdB =
                  1.9382

Phase margin in deg,
              PM =
                  5.2057

Gain cross over frequency in rad/sec,
                                wgc =
                                    5.0000
```

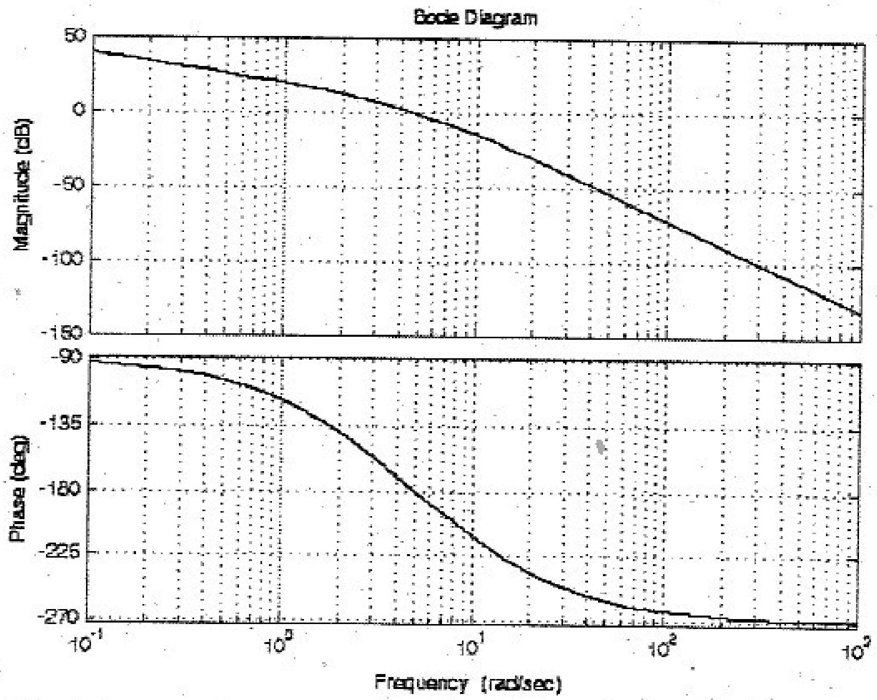

Fig P3.2 : Bode plot of the open loop system given in problem 3.2.

```
Phase cross over frequency in rad/sec,
                                    wpc =
                                          4.4629

The Bode plot of program 3.2 is shown in fig p3.2.
```

PROGRAM 3.3

Write a MATLAB program to draw the Bode plot for the open loop system governed by the following transfer function. The program should take care of drawing magnitude plot in the range +10 to -50 dB, and phase plot in the range -60 to -180 deg.

$G(s)=(75(1+0.2s))/(s(s^2+16s+100))$.

```
%Bode plot with magnitude and phase scaling

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=(75*(1+0.2*s))/(s*(s^2+16*s+100))
[num_cof den_cof]=tfdata(Gs); %determine numerator and denominator
                               %coefficients of G(s)

w=logspace(-1,2,200);
[Mag,Phase,w]=bode(num_cof,den_cof,w);
MagdB=20*log10(Mag);

Mscale1=10*ones(1,200);
Mscale2=-50*ones(1,200);
subplot(2,1,1);semilogx(w,MagdB,'-k',w,Mscale1,'k',w,Mscale2,'k')
grid;
xlabel('Frequency in rad/sec'); ylabel('Magnitude in dB');
Pscale1=-60*ones(1,200);
Pscale2=-180*ones(1,200);
subplot(2,1,2);semilogx(w,Phase,'k',w,Pscale1,'-k',w,Pscale2,'-k')
grid;
xlabel('Frequency in rad/sec'); ylabel('Phase in deg');
```

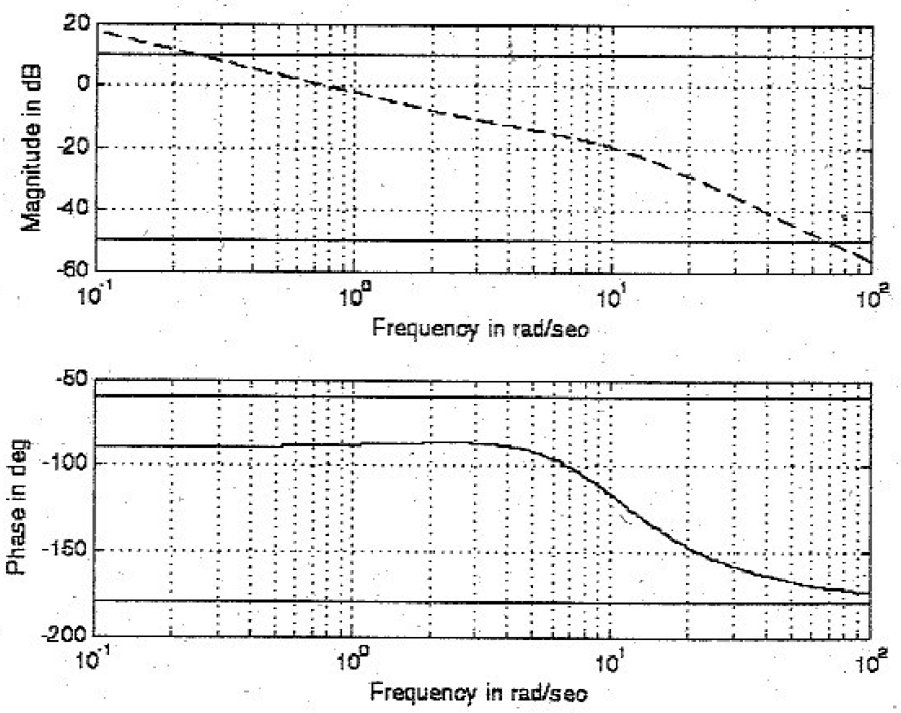

Fig P3.3 : Bode plot of the open loop system given in problem 3.3.

OUTPUT

The given transfer function is,

Transfer function:

```
              15 s + 75
          --------------------
          s^3 + 16 s^2 + 100 s
```

The Bode plot of program 3.3 is shown in fig p3.3.

PROGRAM 3.4

Consider the transfer function of open loop system given below.

$G(s)=20/(s^3+7s^2+10s)$

Write a MATLAB program to determine the transfer function of closed loop system with unity feedback, to plot the Bode plot of closed loop system, and to calculate resonant peak, resonant frequency and bandwidth.

```
%Bode plot of unity feedback closed loop system

clear all
clc
s=tf('s');
disp('The given open loop transfer function G(S) is,');
Gs=20/(s^3+(7*s^2)+10*s)

disp('The closed loop transfer function M(s) is,');
Ms=feedback(Gs,1)
[num_cof den_cof]=tfdata(Ms); %determine numerator & denominator
                                %coefficients of M(s)
w=logspace(-1,1);               %specify frequency range
bode(Ms,w)
grid

[Mag,Phase,w]=bode(Ms,w);
[PeakMag,k]=max(Mag);
disp('Resonant peak in dB,');
Mp=20*log10(PeakMag)

disp('Resonant frequency in rad/sec,');Wr=w(k)

n=1; while 20*log(Mag(n))>=-3;n=n+1; end
disp('Bandwidth in rad/sec,');Wb=w(n)
```

OUTPUT

The given open loop transfer function G(S) is,

Transfer function:

```
                 20
         ------------------
         s^3 + 7 s^2 + 10 s
```

The closed loop transfer function M(s) is,

Transfer function:

```
                   20
         -----------------------
         s^3 + 7 s^2 + 10 s + 20
```

```
Resonant  peak  in  dB,
                         Mp =
                                4.3953

Resonant  frequency  in  rad/sec,
                         Wr =
                                1.6768

Bandwidth  in  rad/sec,
                         Wb =
                                2.4421
```

The Bode plot of program 3.4 is shown in fig p3.4.

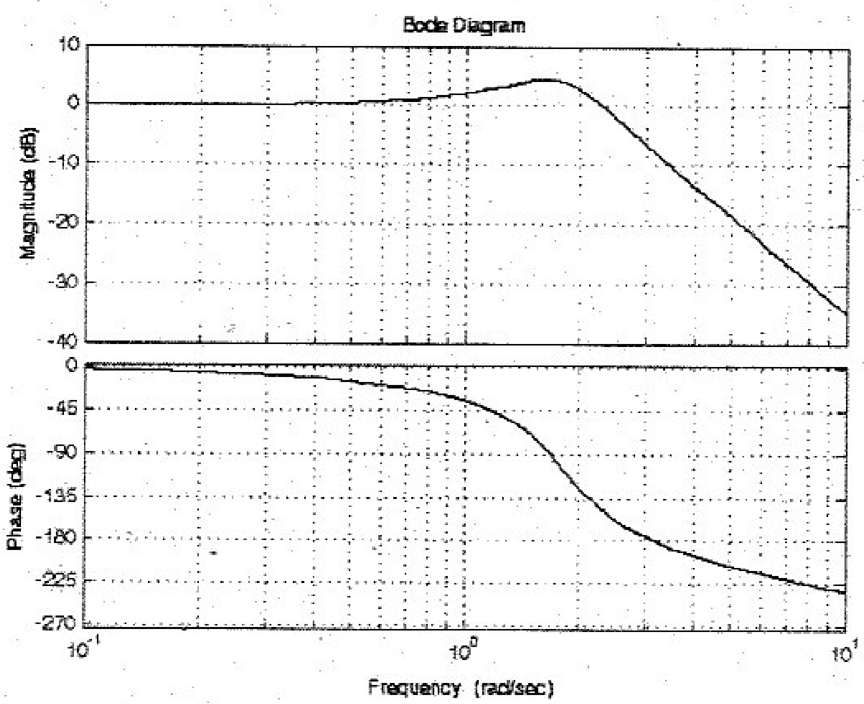

Fig P3.4 : *Bode plot of the closed loop system of program 3.4.*

PROGRAM 3.5

Write a MATLAB program to draw the polar plot and to calculate gain margin and phase margin for the open loop system governed by the following transfer function.

$G(s)=1/(s(1+s)^2)$

```
%Program to draw polar plot and compute gain & phase margins

clear all
clc
s=tf('s');
disp('The given transfer function is,');
Gs=1/(s*(1+s)*(1+s))

[num_cof den_cof]=tfdata(Gs);            %determine numerator and
                                         %denominator coeff. of G(s)
w=0.4 : 0.01 : 4;                        %specify frequency range
[re,im,w]=nyquist(num_cof,den_cof,w);    %determine the real and
                                         %imaginary parts of G(jw)

[GM PM]=margin(num_cof, den_cof);        %compute gain & phase margins
disp('Gain margin,');GM
disp('Phase margin in deg,');PM
```

```
z=re+i*im;                                     %convert rectangular
                                               %coordinates to polar
r=abs(z);
theta=angle(z);
polar(theta,r,'k')                             %draw polar plot
```

OUTPUT

```
The given transfer function is,

Transfer function:
                        1
                 ---------------
                 s^3 + 2 s^2 + s

Gain margin,
          GM =
                2

Phase margin in deg,
                PM =
                      21.3877
```

The Polar plot of program 3.5 is shown in fig p3.5.

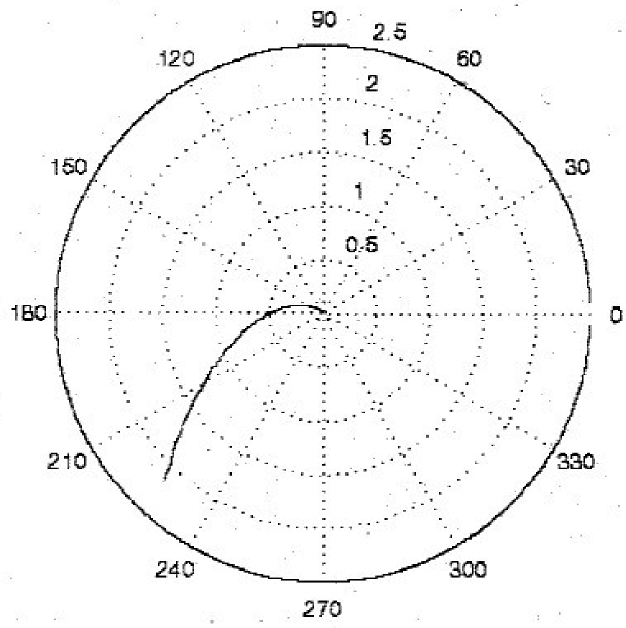

Fig P3.5 : *Polar plot of the open loop system given in problem 3.5.*

PROGRAM 3.6

Write a MATLAB program to draw the polar plot for the open loop system governed by the following transfer function. a) In the frequency range 0.6 to 8 rad/sec b)In the frequency range 5 to 18 rad/sec.

G(s)=1/(S(1+0.2s)(1+0.05s))

```
%program to draw polar plot for two different frequency ranges

clear all
clc
```

```
s=tf('s');
disp('The given transfer function is,');
Gs=1/(s*(1+0.2*s)*(1+0.05*s))

[num_cof den_cof]=tfdata(Gs);   %determine numerator & denominator
                                 %coefficients of G(s)

w1=0.6 :0.001: 8;                                %specify frequency range1
[re1,im1,w1]=nyquist(num_cof,den_cof,w1);%determine the real and
                                                  %imaginary part of G(jw)
z1=re1+i*im1;                                     %convert rectangular
r1=abs(z1);                                      %coordinates to polar
theta1=angle(z1);

subplot(2,1,1);polar(theta1,r1,'k');             %draw polar plot for
                                                 %frequency range1

w2= 5 :0.001: 18;                                %specify frequency range2
[re2,im2,w2]=nyquist(num_cof,den_cof,w2);%determine the real and
                                                  %imaginary part of G(jw)
z2=re2+i*im2;                                     %convert rectangular
r2=abs(z2);                                      %coordinates to polar
theta2=angle(z2);

subplot(2,1,2);polar(theta2,r2,'k');             %draw polar plot for
                                                 %frequency range2
```

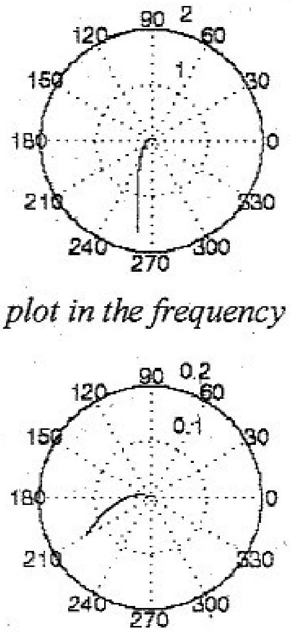

Fig P3.6a : *Polar plot in the frequency range 0.6 to 8 rad/sec.*

Fig P3.6b : *Polar plot in the frequency range 5 to 18 rad/sec.*

Fig P3.6 : *Polar plots of the open loop system given in problem 3.6.*

OUTPUT

```
The given transfer function is,

Transfer function:
                          1
              -----------------------
              0.01 s^3 + 0.25 s^2 + s

The Polar plot of program 3.6 is shown in fig p3.6.
```

PROGRAM 3.7

Write a MATLAB program to draw the polar plot using rectangular to polar coordinates for various values of K for the open loop system governed by the following transfer function.

G(s)=K/(S(1+s)(1+2S))

```
%polar plot for various values of gain,K

clc
s=tf('s');
K=1;
disp('When K=1,the given transfer function is,');
Gs=K/(s*(1+s)*(1+2*s))
[num_cof den_cof]=tfdata(Gs);

for i=1:3;
     if i==1;K=1;[re1,im1]=nyquist([num_cof den_cof],K); end;
     if i==2;K=2;[re2,im2]=nyquist([num_cof den_cof],K);end;
     if i==3;K=25;[re3,im3]=nyquist([num_cof den_cof],K);end;
end

plot(re1,im1,'-k',re2,im2,'-.k',re3,im3,'-k')
axis([-5 1 -5 1]); grid
xlabel('Real axis'); ylabel('Imaginary axis');
text(-3.8,-1.3,'K=1')
text(-3.8,-2.7,'K=2')
text(-1.8,-2.7,'K=25')
```

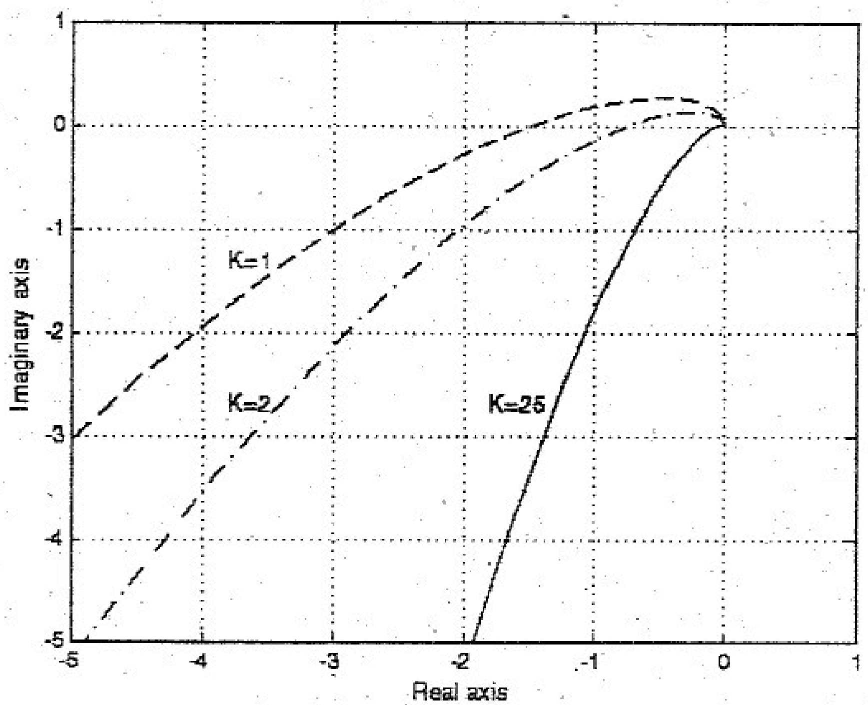

Fig P3.7 : Polar plot of the open loop system for various values of K.

OUTPUT

When K=1, the given transfer function is,

Transfer function:

```
         1
------------------
2 s^3 + 3 s^2 + s
```