Case (i)

With K = 1, let G(jω) cut the −180° axis at point B and gain corresponding to that point be G_B. From the polar plot, $G_B = 0.44$. The gain margin of 7.12 db with K = 1 has to be increased to 20 db and so K has to be decreased to a value less than one.

Let G_A be the gain at −180° for a gain margin of 20 db.

$$\text{Now, } 20 \log \frac{1}{G_A} = 20$$

$$\log \frac{1}{G_A} = \frac{20}{20} = 1$$

$$\frac{1}{G_A} = 10^1 = 10$$

$$\therefore G_A = \frac{1}{10} = 0.1$$

The value of K is given by, $K = \frac{G_A}{G_B} = \frac{0.1}{0.44} = 0.227$

Case (ii)

With K = 1, the phase margin is 15°. This has to be increased to 30°. Hence the gain has to be decreased.

Let ϕ_{gc2} be the phase of G(jω) for a phase margin of 30°.

$$\therefore 30° = 180° + \phi_{gc2}$$

$$\phi_{gc2} = 30° - 180° = -150°$$

In the polar plot the −150° line cuts the locus of G(jω) at point C and cut the unity circle at point D.

Let, G_C = Magnitude of G(jω) at point C.

G_D = Magnitude of G(jω) at point D.

From the polar plot, $G_C = 2.04$ and $G_D = 1$

$$\text{Now, } K = \frac{G_D}{G_C} = \frac{1}{2.04} = 0.49$$

RESULT

(a) When K = 1, Gain margin, K_g = 2.27

Gain margin in db = 7.12 db

(b) When K = 1, Phase margin, γ = 15°

(c) For a gain margin of 20 db, K = 0.227

(d) For a phase margin of 30°, K = 0.49

3.8 NICHOLS PLOT

The **Nichols plot** is a frequency response plot of the open loop transfer function of a system. The Nichols plot is a graph between magnitude of G(jω) in db and the phase of G(jω) in degree, plotted on a ordinary graph sheet.

In order to plot the Nichols plot, the magnitude of G(jω) in db and phase of G(jω) in deg are computed for various values of ω and tabulated. Usually the choice of frequencies are corner frequencies. Choose appropriate scales for magnitude on y–axis and phase on x–axis. Fix all the points on ordinary graph sheet and join the points by smooth curve, and mark frequencies corresponding to each point.

In another method, first the Bode plot of G(jω) is sketched. From the Bode plot the magnitude and phase for various values of frequency, ω are noted and tabulated. Using these values the Nichols plot is sketched as explained earlier.

DETERMINATION OF GAIN MARGIN AND PHASE MARGIN FROM NICHOLS PLOT

The gain margin in db is given by the negative of db magnitude of G(jω) at the phase crossover frequency, ω_{pc}. The ω_{pc} is the frequency at which phase of G(jω) is –180°. If the db magnitude of G(jω) at ω_{pc} is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of G(jω) at gain cross over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of G(jω) is zero. Now the phase margin, γ is given by $\gamma = 180° + \phi_{gc}$. If ϕ_{gc} is less negative than –180° then phase margin is positive and vice versa. The positive and negative gain margins are illustrated in fig 3.27.

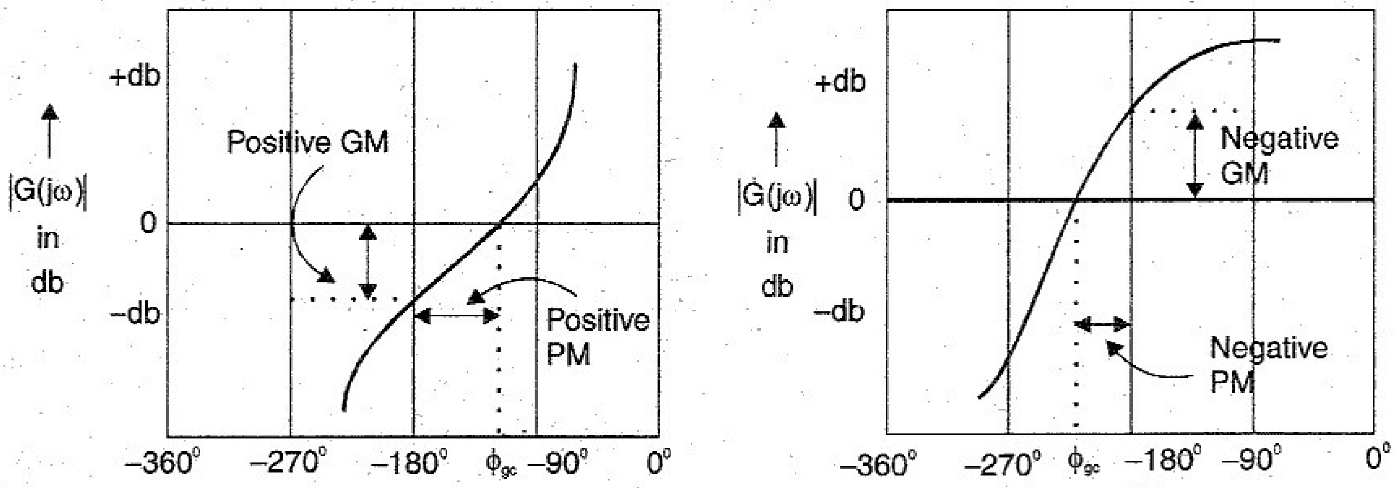


Fig 3.27 : Nichols plot showing phase margin (PM) and gain margin (GM).

GAIN ADJUSTMENT IN NICHOLS PLOT

In the open loop transfer function, G(jω) the constant K contributes only magnitude. Hence by changing the value of K the system gain can be adjusted to meet the desired specifications. The desired specifications are gain margin and phase margin.

In a system transfer function, if the value of K required to be estimated, in order to satisfy a desired specification, then draw the Nichols plot of the system with K=1. The constant K can add 20logK to every point of the plot. Due to this addition, the Nichols plot will shift vertically up or down. Hence shift the plot vertically up or down to meet the desired specification. Equate the vertical distance by which the Nichols plot is shifted to 20logK and solve for K.

Let, x = change in db (x is positive if the plot is shifted up and vice versa).

Now, $20 \log K = x \quad \Rightarrow \quad \log K = \frac{x}{20} \quad \Rightarrow \quad \therefore K = 10^{\frac{x}{20}}$

EXAMPLE 3.13

Consider a unity feedback system having an open loop transfer function $G(s) = \frac{K(1+10s)}{s^2 (1+s)(1+2s)}$. Sketch the Nichols plot and determine the value of K so that (i) Gain margin is 10db, (ii) Phase margin is 10°.

SOLUTION

Given that $G(s) = \frac{K(1+10s)}{s^2(1+s)(1+2s)}$

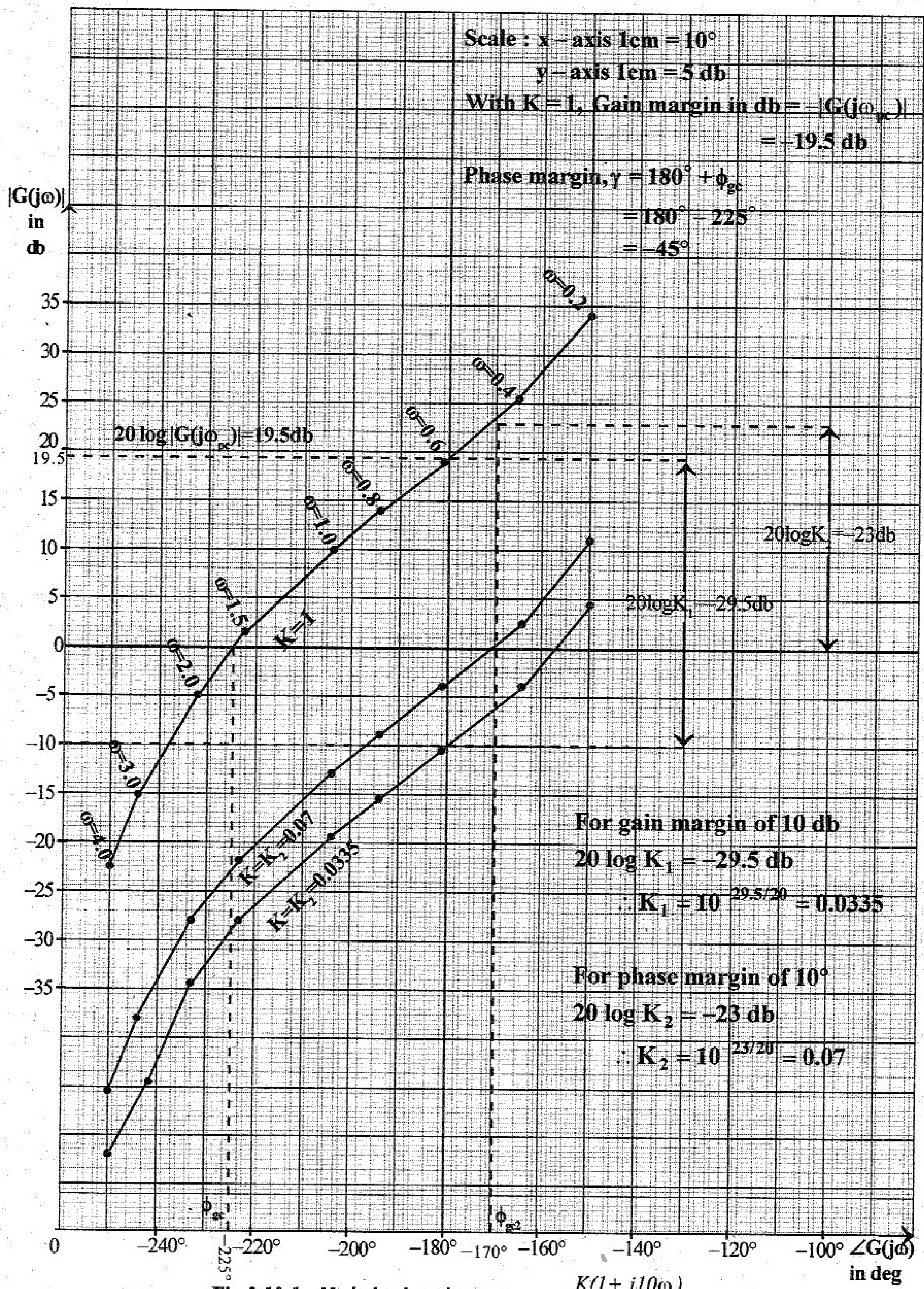


Fig 3.13.1 : Nichols plot of $G(j\omega) = \dfrac{K(1+j10\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)}$.

The sinusoidal transfer function G(jω) is obtained by letting to s = jω. Also put K = 1.

$$\therefore G(j\omega) = \frac{(1+j10\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)} = \frac{\sqrt{1+(10\omega)^2}\ \angle\tan^{-1}10\omega}{\omega^2\angle 180^0\sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\sqrt{1+(2\omega)^2}\ \angle\tan^{-1}2\omega}$$

$$|G(j\omega)| = \frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}; \qquad \therefore |G(j\omega)|_{\text{indb}} = 20\log\left[\frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}\right]$$

$$\angle G(j\omega) = \tan^{-1}10\omega - 180^0 - \tan^{-1}\omega - \tan^{-1}2\omega$$

The magnitude of G(jω) in db and phase of G(jω) in deg are calculated for various values of ω and listed in the following table. The Nichols plot of G(jω) with K = 1 is sketched as shown in fig 3.13.1

ω rad/sec	0.2	0.4	0.6	0.8	1.0	1.5	2.0	3.0	4.0
|G(jω)| db	34.1	25.4	19.3	14.3	10	1.4	–5.3	–15.2	–22.5
∠G(jω) deg	–150	–164	–181	–194	–204	–222	–232	–244	–250

From the Nichols plot the gain margin and phase margin of the system when K=1 are,

Gain margin = – 19.5 db

Phase margin = – 45°

Gain adjustment for required gain margin

For a gain margin of 10 db, the magnitude of G(jω) should be –10db, when the phase is –180°. When K = 1, the magnitude of G(jω) is +19.5db corresponding to phase angle of –180°. Hence if we add –29.5 db to every point of G(jω) ,then the plot shifts downwards and it will cross –180° axis at a magnitude of –10db. The magnitude correction is independent of frequency and so this gain can be contributed by the term K. Let this value of K be K_1. The value of K_1 is calculated by equating $20\log K_1$ to –29.5db.

$$\therefore 20\log K_1 = -29.5\ \text{db} \quad\Rightarrow\quad \log K_1 = \frac{-29.5}{20} \quad\Rightarrow\quad K_1 = 10^{\frac{-29.5}{20}} = 0.0335$$

Gain adjustment for required phase margin

Let ϕ_{gc2} = phase of G(jω) at gain crossover frequency for a phase margin of 10°

$\therefore$ Phase margin, $\gamma_2 = 180° + \phi_{gc2}$

$\therefore \phi_{gc2} = \gamma_2 - 180° = 10° - 180° = -170°$

When K = 1, the magnitude of G(jω) is +23 db corresponding to a phase of –170°. But for a phase margin of 10°, this gain should be made zero. Hence if we add –23db to every point of G(jω) locus then the plot shifts downwards and it will cross –170° axis at magnitude of 0 db. The magnitude correction is independent of frequency and so this gain can be contributed by the term K. Let this value of K be K_2. The value of K_2 is calculated by equating $20\log K_2$ to –23db.

$$\therefore 20\log K_2 = -23 \quad\Rightarrow\quad \log K_2 = -23/20 \quad\Rightarrow\quad K_2 = 10^{\frac{-23}{20}} = 0.07$$

RESULT

(a) When K = 1,

Gain margin = –19.5 db

Phase margin = – 45°

(b) For a gain margin of 10db, $K = K_1$ = 0.0335

(c) For a phase margin of 10°, $K = K_2$ = 0.07

3.9 CLOSED LOOP RESPONSE FROM OPEN LOOP RESPONSE

The closed loop transfer function of the system is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = M(s)$$

The sinusoidal transfer function is obtained by replacing s by jω.

$$\therefore\ M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)\,H(j\omega)}$$

Let, $M(j\omega) = M\angle\alpha$

where, M = Magnitude of closed loop transfer function

α = Phase of closed loop transfer function.

The magnitude and phase of closed loop system are functions of frequency, ω. The sketch of magnitude and phase of closed loop system with respect to ω is closed loop frequency response plot. The magnitude and phase of closed loop system for various values of frequency can be evaluated analytically or graphically. The analytical method of determining the frequency response involves tedious calculations. Two graphical methods are available to determine the closed loop frequency response from open loop frequency response. They are,

1. M and N circles
2. Nichols chart.

3.10 M AND N CIRCLES

The magnitude of closed loop transfer function with unity feedback can be shown to be in the form of circle for every value of M. These circles are called **M-circles.**

If the phase of closed loop transfer function with unity feedback is α, then it can be shown that tan α will be in the form of circle for every value of α. These circles are called **N-circles.**

The M and N circles are used to find the closed loop frequency response graphically from the open loop frequency response G(jω) without calculating the magnitude and phase of the closed loop transfer function at each frequency.

The M and N circles are available as standard chart. The chart consists of M and N circles superimposed on ordinary graph sheet. Using ordinary graph the locus of G(jω) (Polar Plot) is sketched. The locus of G(jω) will cut the M-circles and N-circles at various points. The intersection of G(jω) locus with M and N circles gives the magnitude and phase of the closed loop system at frequencies corresponding to the cutting point of G(jω).

The M and α for various values of ω are tabulated. The magnitude and phase response of closed loop system are sketched on semilog graph sheet by taking ω on the logarithmic scale on x-axis. [The closed loop frequency response has two plots. They are M Vs ω and α Vs ω]

M-CIRCLES

Consider the closed loop transfer function of unity feedback system, $M(s) = \frac{G(s)}{1+G(s)}$

Put $s = j\omega$, $\therefore M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

Let, $G(j\omega) = X + jY$

where, X = Real part of $G(j\omega)$.

Y = Imaginary part of $G(j\omega)$.

$$\therefore M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\angle\tan^{-1}\frac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\angle\tan^{-1}\frac{Y}{1+X}} = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}\angle\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

Let, M = Magnitude of $M(j\omega)$

$$\therefore M = \frac{\sqrt{X^2 + Y^2}}{\sqrt{(1+X)^2 + Y^2}}$$

On squaring the above equation we get,

$$M^2 = \frac{X^2+Y^2}{(1+X)^2+Y^2} \Rightarrow M^2\left((1+X)^2+Y^2\right) = X^2 + Y^2 \Rightarrow M^2\left(1+X^2+2X+Y^2\right) = X^2 + Y^2$$

$$M^2 + M^2X^2 + M^2 2X + M^2Y^2 - X^2 - Y^2 = 0$$

$$X^2(M^2-1) + M^2 2X + M^2 + Y^2(M^2-1) = 0 \qquad .....(3.30)$$

When $M = 1$, the equation (3.30) represents a straight line.

When $M = 1$, the equation (3.30) is,

$$X^2(1-1) + 2X + 1 + Y^2(1-1) = 0 \quad \Rightarrow \quad 2X + 1 = 0 \quad \Rightarrow \quad X = -1/2$$

Hence when $M = 1$, equation (3.30) represents a straight line passing through $X = -1/2$ & $Y = 0$.

When $M \neq 1$, the equation (3.30) represents a family of circles.

When $M \neq 1$, equation (3.30) can be rearranged in the form of equation of a circle as shown below.

$$X^2(M^2-1) + M^2 2X + M^2 + Y^2(M^2-1) = 0$$

Divide the above equation throughout by $(M^2 - 1)$.

$$\therefore X^2 + \frac{M^2}{M^2-1}2X + \frac{M^2}{M^2-1} + Y^2 = 0$$

Add $\frac{M^2}{(M^2-1)^2}$ on both sides of the above equation.

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^2}{M^2-1}+\frac{M^2}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^2}$$

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^2(M^2-1)+M^2}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^2}$$

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^4}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^2}$$

$a^2 + 2ab + b^2 = (a+b)^2$

$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{(M^2-1)^2} \quad .....(3.31)$$

The equation of circle with centre at (X_1, Y_1) and radius r is given by,

$$(X-X_1)^2+(Y-Y_1)^2=r^2 \quad .....(3.32)$$

On comparing equation (3.31) and equation (3.32), it can be concluded that the equation (3.31) represents a family circles with centre at $(-M^2/M^2-1), 0)$ and with radius, $r = M/(M^2-1)$ for various values of M. The circles given by equation (3.31) are called M-circles.

When M = 0

Centre $=(X_1,Y_1)$

$$X_1=-\frac{M^2}{M^2-1}=0$$

$$Y_1=0$$

Radius, $r=\frac{M^2}{M^2-1}=0$

Hence when M = 0, the magnitude circle becomes a point at (0,0).

When M = ∞

Centre $=(X_1,Y_1)$

$$X_1=\frac{-M^2}{M^2-1}\approx\frac{-M^2}{M^2}=-1$$

$$Y_1=0$$

Radius, $r=\frac{M}{M^2-1}\approx\frac{M}{M^2}=\frac{1}{M}=\frac{1}{\infty}=0$

Hence when $M=\infty$, the magnitude circle becomes a point at (−1,0).

From the above analysis it is clear that the magnitude of closed loop transfer function will be in the form of circles when $M \neq 1$ and when M = 1, the magnitude is a straight line passing through (−1/2,0).

For values of M less than 1, the magnitude is a circle to the right of the straight line corresponding to M = 1. It is observed that the circles for M<1 passes through (−1/2,0) and (0,0) on the negative real axis. For decreasing values of M, the radius decreases and the circle, becomes a point at (0,0) when M = 0.

For values of M greater than 1, the magnitude is a circle to the left of the straight line corresponding to M = 1. It is observed that circle passes between the points (−1,0) and (−1/2,0) on the negative real axis. For increasing values of M the radius decreases and the circle becomes a point at (−1,0) when $M=\infty$. The family of M-circles are shown in fig 3.28.

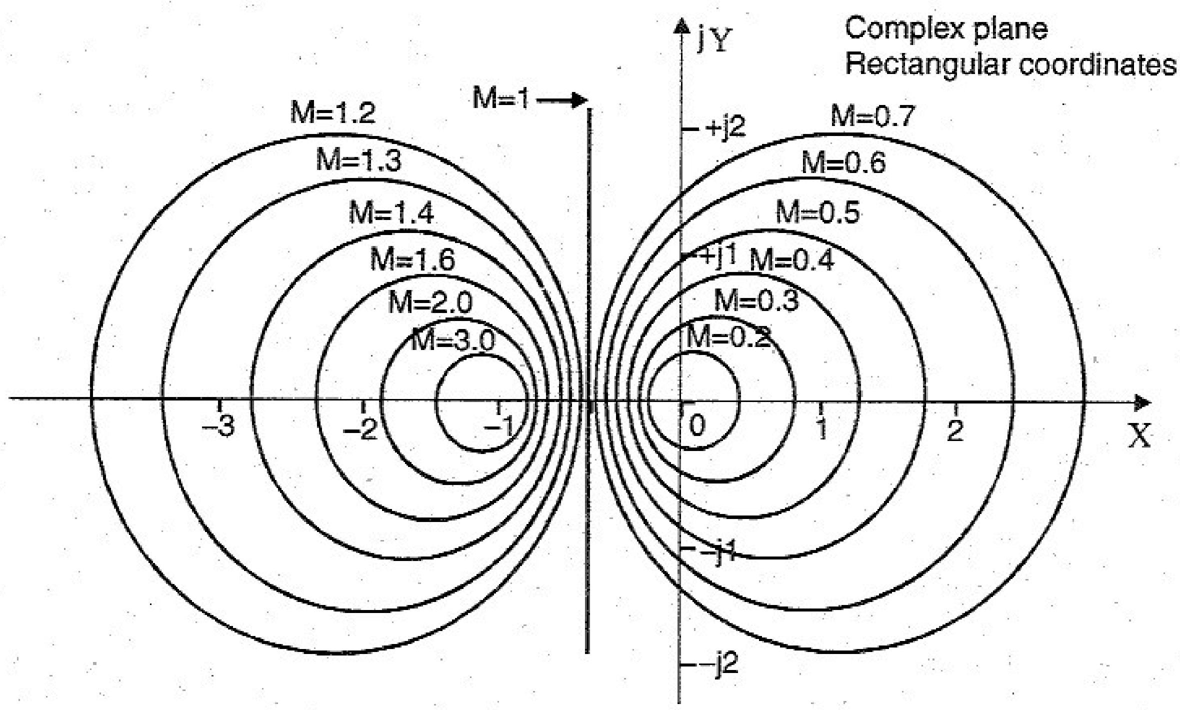


Fig 3.28 : The family of constant M-circles.

N-CIRCLES

Consider the closed loop transfer function of unity feedback system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = M(s)$$

Put $s = j\omega$, $M(j\omega) = \dfrac{G(j\omega)}{1+G(j\omega)}$

Let $G(j\omega) = X + jY$, where, X = Real part of $G(j\omega)$.

Y = Imaginary part of $G(j\omega)$.

$$\therefore M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\angle \tan^{-1}\dfrac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\angle \tan^{-1}\dfrac{Y}{1+X}} = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

Let, α = Phase of $M(j\omega)$; $\therefore \alpha = \tan^{-1}\dfrac{Y}{X} - \tan^{-1}\dfrac{Y}{1+X}$

Let, $N = \tan\alpha$

$$\therefore N = \tan\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

$$\therefore N = \frac{\tan\left(\tan^{-1}\dfrac{Y}{X}\right) - \tan\left(\tan^{-1}\dfrac{Y}{1+X}\right)}{1+\tan\left(\tan^{-1}\dfrac{Y}{X}\right)\tan\left(\tan^{-1}\dfrac{Y}{1+X}\right)} = \frac{\dfrac{Y}{X} - \dfrac{Y}{1+X}}{1+\dfrac{Y}{X}\times\dfrac{Y}{1+X}} = \frac{\dfrac{Y(1+X)-XY}{X(1+X)}}{\dfrac{X(1+X)+Y^2}{X(1+X)}}$$

$$= \frac{Y+XY-XY}{X+X^2+Y^2} = \frac{Y}{X+X^2+Y^2}$$

$$\therefore N = \frac{Y}{X+X^2+Y^2}$$

> ***Note :***
>
> $$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \times \tan B}$$

On rearranging the above equation we get,

$$X+X^2+Y^2=\frac{Y}{N}$$

$$\therefore X+X^2+Y^2-\frac{Y}{N}=0$$

In the above equation add the term $\frac{1}{4}+\left(\frac{1}{2N}\right)^2$ on both sides.

$$X+X^2+Y^2-\frac{Y}{N}+\frac{1}{4}+\left(\frac{1}{2N}\right)^2=\frac{1}{4}+\left(\frac{1}{2N}\right)^2$$

$$\left(X^2+\frac{1}{4}+X\right)+\left(Y^2+\frac{1}{(2N)^2}-\frac{Y}{N}\right)=\frac{1}{4}+\frac{1}{(2N)^2}$$

$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2} \qquad .....(3.33)$$

The equation of circle with centre at (X_1,Y_1) and radius r is,

$$(X-X_1)^2+(Y-Y_1)^2=r^2 \qquad .....(3.34)$$

On comparing equation (3.33) and (3.34), it can be concluded that the equation (3.33) represents a family of circle with centre at (–1/2,1/2N) and with radius $\sqrt{\frac{1}{4}+\frac{1}{(2N)^2}}$ for various values of N. The circles given by the equation (3.30) are called N-circles.

For any value of N, the equation of N-circles is satisfied at two points (0,0) and (–1,0). Hence the N-circles passes through these two points for all values of α. (N = tan α).

Consider the equation of N-circle,

When X = 0 and Y = 0,

$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2}$$

$$\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2}$$

$$\frac{1}{4}+\frac{1}{4N^2}=\frac{1}{4}+\frac{1}{4N^2}$$

Consider the equation of N-circle,

When X = –1 and Y = 0,

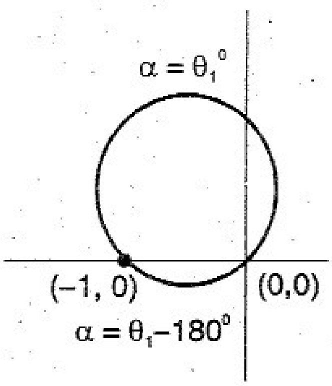
$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2}$$

$$\left(-1+\frac{1}{2}\right)^2+\left(-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(4N)^2}$$

$$\frac{1}{4}+\frac{1}{4N^2}=\frac{1}{4}+\frac{1}{4N^2}$$

The above analysis shows that the equation of N-circle is satisfied at points(0,0) and (–1, 0).

When α = 180° the circle becomes a straight line passing through real axis. It is also observed that the circle for α = θ°–180° above the real axis will be a part of circle for α = θ ° below the real axis, as shown in fig 3.29. The family of N circles are shown in fig 3.30.



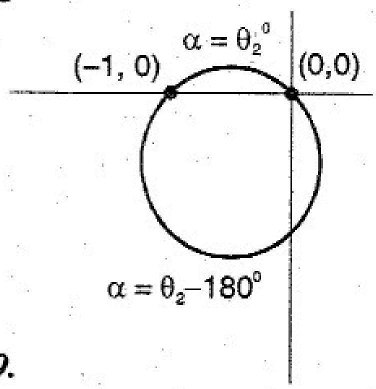


Fig 3.29.

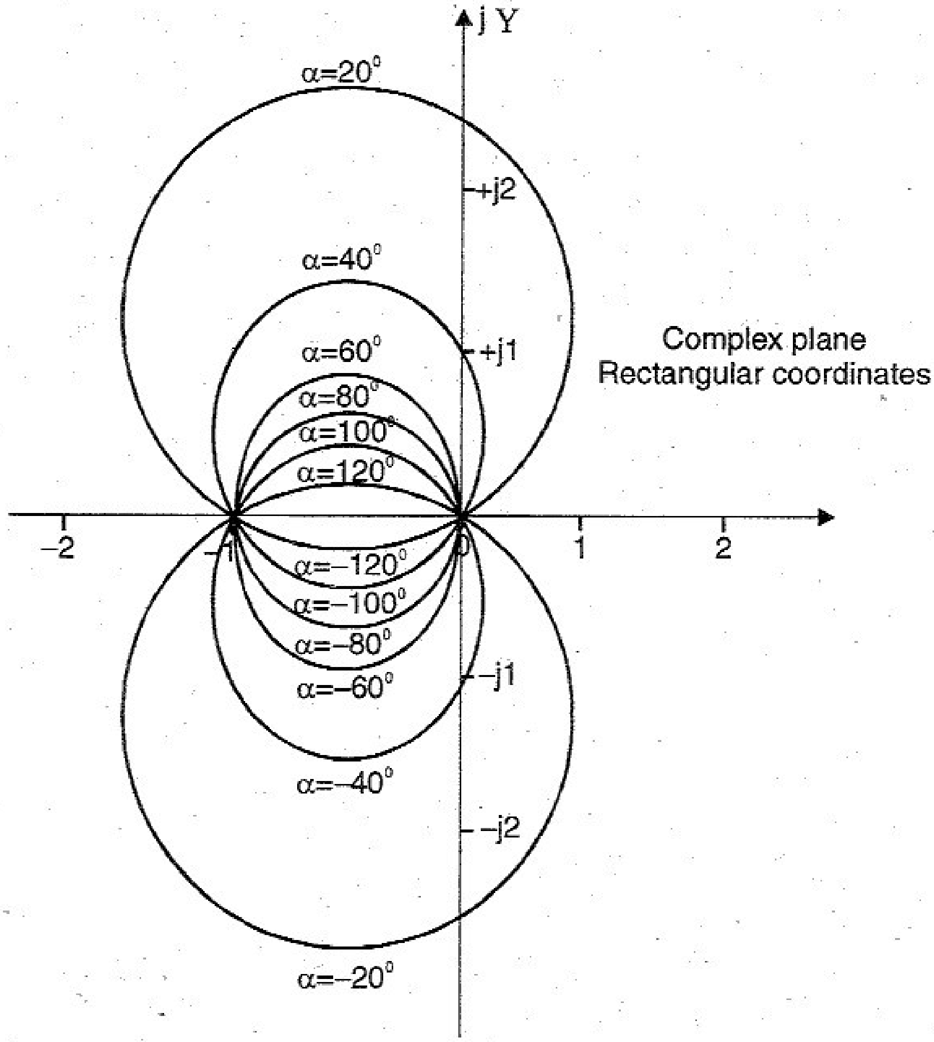


Fig 3.30 : The family of constant N-circles.

3.11 NICHOLS CHART

N.B Nichols transformed the constant M and N circles to log-magnitude and phase angle coordinates and the resulting chart is known as ***Nichols chart.***

Nichols chart consist of M and N contours, superimposed on ordinary graph. The M contours are the magnitude of closed loop system in decibels and the N contours are the corresponding phase angle locus of closed loop system. The ordinary graph consist of magnitude in db marked on the Y-axis and the phase in degrees marked on the X-axis.

The Nichols plot of open loop system can be plotted on the ordinary graph. The Nichols plot is a graph between magnitude of $G(j\omega)$ in db and the phase of $G(j\omega)$ in degree,plotted on an ordinary graph sheet. To draw the Nichols plot the magnitude and phase angle of $G(j\omega)$ are calculated for various values of ω. Alternatively the Bode plot of $G(j\omega)$ is sketched and from Bode plot, the magnitude and phase of $G(j\omega)$ for any frequency can be obtained.

Using Nichols chart the closed loop frequency response can be determined graphically from the locus of open loop frequency response. When the Nichols plot of $G(j\omega)$ is sketched on Nichols chart, the locus of $G(j\omega)$ will cut the M and N contours at various points. The cutting point of locus of $G(j\omega)$ with the M-contour gives the magnitude of closed loop frequency response corresponding to a frequency same as that of $G(j\omega)$ at that point.

The cutting point of locus of $G(j\omega)$ and N contour gives the phase of closed loop frequency response corresponding to a frequency same as that of $G(j\omega)$ at that point. The magnitude M and phase angle α ($N = \tan\alpha$) of closed loop system are tabulated . The closed loop frequency response consists of two plots. They are magnitude M Vs ω and phase angle α Vs ω. Hence using the tabulated values the bode plot of closed of closed loop system can be drawn.

The frequency domain specifications can be determined from Nicols chart. Fig 3.31, shows various frequency domain specifications of a typical $G(j\omega)$ locus. Also the Nichols plot drawn on a Nichols chart can be used for gain adjustment.

ESTIMATION OF FREQUENCY DOMAIN SPECIFICATIONS USING NICHOLS CHART

Resonant Peak (M_r) and Resonant Frequency (ω_r)

The resonant peak is given by the value of M-contour which is tangent to $G(j\omega)$ locus. The resonant frequency is given by the frequency of $G(j\omega)$ at the tangency point.

Bandwidth

The Bandwidth is given by frequency corresponding to the intersection point of $G(j\omega)$ and –3 db M -contour.

Gain Margin

The gain margin is given by negative of magnitude of $G(j\omega)$ in db at phase crossover frequency, ω_{pc}. At phase crossover frequency the phase of $G(j\omega)$ is $-180°$

Gain Margin, K_g in db $= -|G(j\omega_{pc})|_{in\ db}$

Phase Margin

The phase margin,γ is given by $\gamma = 180° + \phi_{gc}$ where ϕ_{gc} is the phase of $G(j\omega)$ at gain crossover frequency. At gain crossover frequency the magnitude of $G(j\omega)$ is zero db.

GAIN ADJUSTMENT USING NICHOLS CHART

Determination of K for Specified Gain Margin

Draw the $G(j\omega)$ locus with K=1. Determine the amount of gain to be added at $\phi = -180°$, so that db magnitude of $G(j\omega)$ locus at $-180°$ is negative of the specified gain margin. Let the db gain to be added be x db. The gain contribution is independent of frequency and so it can be achieved by choosing proper value of K. The value of K is obtained by equating 20logK to x db.

Now, $20\log K = x$

$$\therefore K = 10^{\frac{x}{20}}$$

Determination of K for Specified Phase Margin

Draw the $G(j\omega)$ locus with K= 1. The phase margin, $\gamma = 180° + \phi_{gc}$ where ϕ_{gc}, is phase of $G(j\omega)$ at gain crossover frequency. $\therefore \phi_{gc} = \gamma - 180°$. For specified phase margin, calculate ϕ_{gc} and from the Nichols plot determine the db gain at ϕ_{gc} . Let this gain be y db. For the specified phase margin, this gain should be made zero. Hence –y db should be added to every point of $G(j\omega)$. This is achieved by choosing proper value of K. The value of K is obtained by equating 20logK to –y db.

$$\text{Now, } 20\log K = -y \quad \Rightarrow \quad \log K = -\frac{y}{20} \quad \Rightarrow \quad K = 10^{\frac{-y}{20}}$$

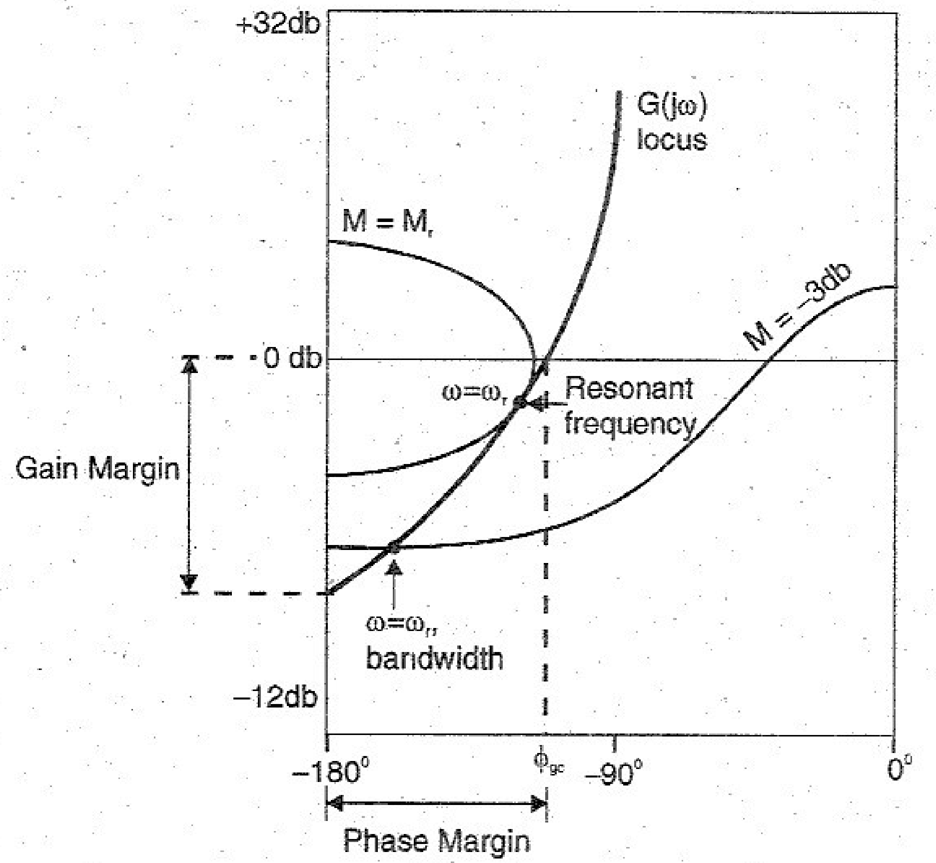


Fig 3.31 : Determination of frequency domain specification from Nichols Chart.

Determination of K for Specified Resonant Peak, M_r

Draw the G(jω) locus with K = 1. Using a tracing paper, trace the locus of G(jω). A standard tracing paper, called Nichols overlay is available). Then shift the locus vertically up or down, so that $M = M_r$ contour is tangent to G(jω) locus. Measure the vertical shift in db. Let the shift be ± x db. (+ for up and –for down).

$$\text{Now,}\ \ 20\log K = \pm x \quad \Rightarrow \quad \log K = \pm\frac{x}{20} \quad \Rightarrow \quad K = 10^{\pm\frac{x}{20}}$$

Determination of K for a Specified Bandwidth

Draw the G(jω) locus with K=1. Determine the open loop gain G(jω) at $\omega = \omega_b$ where, ω_b is the specified bandwith. Determine the point of intersection of -3db M-contour and this open loop gain on the Nichols chart. Let this point be point A. Trace the G(jω) locus. Shift the G(jω) locus vertically up or down, so that it passes through point A. Measure the vertical shift in db. Let the shift be ± x db (+for up and –for down).

$$\text{Now,}\ \ 20\log K = \pm x \quad \Rightarrow \quad \log K = \pm\frac{x}{20} \quad \Rightarrow \quad K = 10^{\pm\frac{x}{20}}$$

EXAMPLE 3.14

The open loop transfer function of unity feedback system is, $G(s) = Ke^{-0.2s}/s(1+0.25s)(1+0.1s)$. Using Nichols chart, determine the following.

(a) The value of K so that the gain margin of the system is 4 db.

(b) The value of K so that the phase margin of the system is 40°

(c) The value of K so that resonant peak M_r of the system is 1 db. What are the corresponding values of ω_r and ω_b?

(d) The value of K so that the bandwidth ω_b of the system is 1.5 rad/sec.

SOLUTION

First the actual bode plot of G(jω) with K=1 is plotted on semilog graph sheet. The magnitude of G(jω) in db and phase of G(jω) for various frequencies are calculated and listed in Table-1. The choice of frequencies are chosen such that the magnitude plot extends in the range of 40 db to –14db and the phase plot extends in the range of 0° to –180°.

Given that, $G(s) = \frac{Ke^{-0.2s}}{s(1+0.25s)(1+0.1s)}$

Let, K = 1 and put, s = jω.

$$\therefore G(j\omega) = \frac{e^{-j0.2\omega}}{j\omega(1+0.25s)(1+0.1s)} = \frac{1\angle -0.2\omega \times \frac{180°}{\pi}}{\omega\angle 90° \sqrt{1+0.0625\omega^2}\angle \tan^{-1}0.25\omega \sqrt{1+0.01\omega^2}\angle \tan^{-1}0.1\omega}$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{1+0.0625\omega^2}\sqrt{1+0.01\omega^2}}; \quad \therefore |G(j\omega)|_{in\ db} = 20\log\left[\frac{1}{\omega\sqrt{1+0.0625\omega^2}\sqrt{1+0.01\omega^2}}\right]$$

$$\angle G(j\omega) = -0.2\omega \times \frac{180°}{\pi} - 90° - \tan^{-1}0.25\omega - \tan-10.1\omega$$

TABLE-1: Calculated values of |G(jω)| and ∠G(jω)

ω rad/sec	0.01	0.02	0.05	0.1	0.2	0.5	1.0	2.0	4.0
|G(jω)| db	40	34	26	20	14	6	0	–7	–16
∠G(jω) deg	–90	–91	–91	–93	–96	–106	–121	–151	–203

The magnitude and phase plot of Bode plot of G(jω) are shown in fig 3.14.1 From the bode plot the phase and frequency for various values of magnitudes are noted and tabulated in table-2. (The choice of magnitudes are 20,16,12,.... i.e, in steps of 4 db, which is convenient for Nichols plot on Nichols chart). Using the values listed in table-2 the locus of G(jω) on Nichols chart is sketched as shown in fig 3.14.2.

TABLE-2: Values of |G(jω)| and ∠G(jω) Noted from Bode Plot

ω rad/sec	0.1	0.16	0.25	0.4	0.64	1.0	1.5	2.2	3.0
|G(jω)| db	20	16	12	8	4	0	– 4	– 8	–12
∠G(jω) deg	–90	–92	–96	–102	–110	–120	–136	–156	–180

Gain Margin and Phase Margin when K = 1

When K = 1, the G(jω) locus cuts the –180° axis at –12db. Hence the magnitude at phase crossover frequency is –12db.

$\therefore$ Gain Margin, $K_g = -|G(j\omega_{pc})|_{indb} = -(-12) = +12$db.

When K = 1, the phase of G(jω) is –120° corresponding to magnitude of 0 db. Hence the phase at gain crossover frequency is –120°.

$\therefore$ Phase margin, $\gamma = 180° + \phi_{gc} = 180° - 120° = 60°$

To find K for a gain margin of 4db

When gain margin is 4 db, the locus of G(jω) should cross the 180° axis at –4 db. When K = 1, the magnitude of G(jω) is –12db corresponding to a phase of 180°. Hence if we add, –4 – (–12) = 8 db to every point of G(jω) then the plot shifts upwards and crosses –180° axis at –4 db. This magnitude correction is achieved by choosing appropriate value of K. The value of K is obtained by equating 20 log K to 8 db.

$\therefore 20\log K = 20\,db \quad \Rightarrow \quad \log K = \frac{8}{20} \quad \Rightarrow \quad K = 10^{\frac{8}{20}} = 2.5$

The G(jω), when K = 2.5 is shown in fig 3.14.2.

To find K for phase margin of 40°

Let ϕ_{gc2} be the phase of G(jω) at gain crossover frequency when the phase margin is 40°.

$\therefore$ Phase margin, $\gamma_2 = 180° + \phi_{gc2}$

$\therefore \phi_{gc2} = \gamma_2 - 180° = 40° - 180° = -140°$

From the above calculation it is evident that for a phase margin of 40°, the magnitude of G(jω) should be 0 db corresponding to a phase of –140°. When K = 1, the magnitude of G(jω) is –5db corresponding to a phase of –140°. Hence if we add +5 db to every point of G(jω) locus then the plot shifts upwards and crosses –140° axis at 0 db. This magnitude correction is achieved by choosing appropriate values of K. The value of K is obtained by equating 20 logK to 5 db.

$\therefore 20\log K = 5 \quad \Rightarrow \quad \log K = \frac{5}{20} \quad \Rightarrow \quad K = 10^{\frac{5}{20}} = 1.78$

The G(jω) locus, when K = 1.78 is shown in fig 3.14.2

To find K for a resonant peak of 1 db

The resonant peak, M_r is given by M-contour which is tangent to G(jω) locus. When K = 1, the G(jω) locus is tangent to M = 0.25 db contour. Hence when, K = 1, resonant peak is 0.25 db.

For a resonant peak of 1 db, the M = 1 db contour should be made tangent to G(jω) locus. For this, G(jω) locus can be shifted vertically up or down so that it becomes tangent to M = 1 db contour. In this problem the G(jω) locus is shifted vertically up to make it tangent to M = 1 db contour. The shifted G(jω) locus is shown in fig 3.14.3.

> *Note : Trace the G(jω) locus when K = 1 on a tracing paper and shift the traced locus over the Nichols chart vertically so that it is tangent to required M-contour. By keeping the tracing paper at the shifted position darken the traced locus, so that it makes an impression on nichols chart.*

The vertical shift is equivalent to adding a magnitude of 20 log K to every point of G(jω) locus. From the shifted locus of G(jω) it is observed that +2db is added to every point of G(jω) locus. Hence the value of K is obtained by equating 20 log K to +2 db.

$20\log K = 2db \quad \Rightarrow \quad \log K = \frac{2}{20} \quad \Rightarrow \quad K = 10^{\frac{2}{20}} = 1.26$

The resonant frequency, ω_r is given by the frequency of G(jω) at the tangency point. The magnitude of G(jω) is 0 db at the tangency point of M = 1 db contour. The corresponding frequency is noted from the bode plot of G(jω). From the bode plot the frequency at 0 db is 1.0 rad/sec. Hence the resonant frequency, ω_r = 1.0 rad/sec.

To find K so that ω_b = 1.5 rad/sec

The bandwidth, ω_b is given by the frequency of G(jω) corresponding to the meeting point of G(jω) locus and M = –3db contour. From the bode plot find the magnitude of G(jω) when ω = 1.5 rad/sec. From fig 3.14.1 it is observed that magnitude of G(jω) is – 4db when ω = 1.5 rad/sec.

In the Nichols chart, find the point where the M = –3db contour passes through – 4db line. Let this point be P. Now the G(jω) locus with K = 1 is shifted vertically down so that is passes through point P. The shifted G(jω) locus is shown in fig 3.1.3.

> *Note : Trace the G(jω) locus when K = 1 on a tracing paper and shift the raced locus over Nichols chart so that is passes through point P. By keeping the tracing paper at the shifted position, darken the traced locus, so that is makes an impression on Nichols chart.*

The vertical shift is equivalent to adding a magnitude of 20logK to every point of G(jω) locus. From the shifted locus of G(jω) it is observed that -6db is added to every point of G(jω) locus. Hence the value of K is obtained by equating 20logK to –6 db.

$20\log K = -6\,db \quad \Rightarrow \quad \log K = -\frac{6}{20} \quad \Rightarrow \quad K = 10^{-6/20} = 0.5$

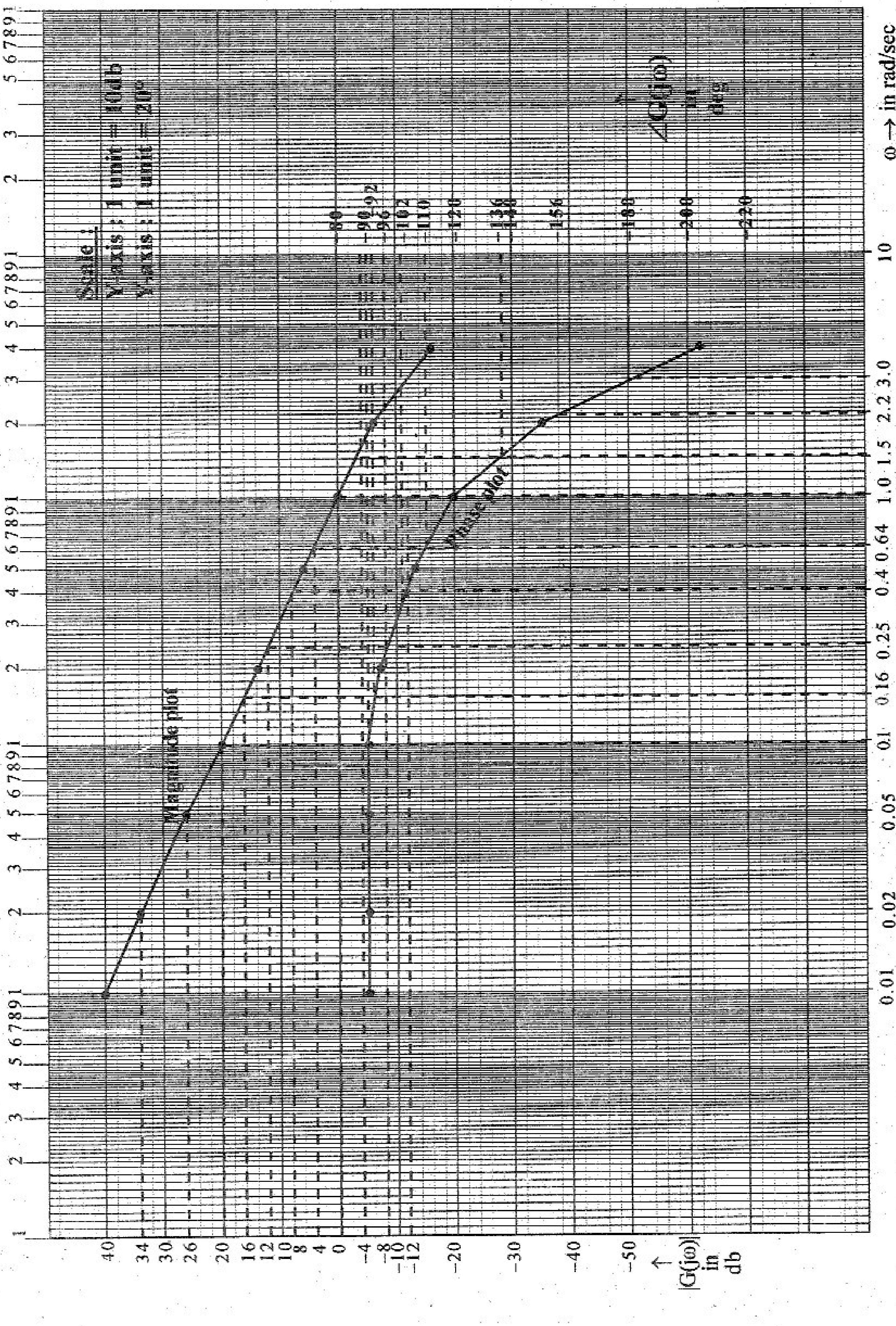


Fig 3.14.1 : Bode plot of $G(j\omega) = e^{-j0.2\omega}/j\omega\,(1 + j0.25\,\omega)\,(1 + j0.1\,\omega)$