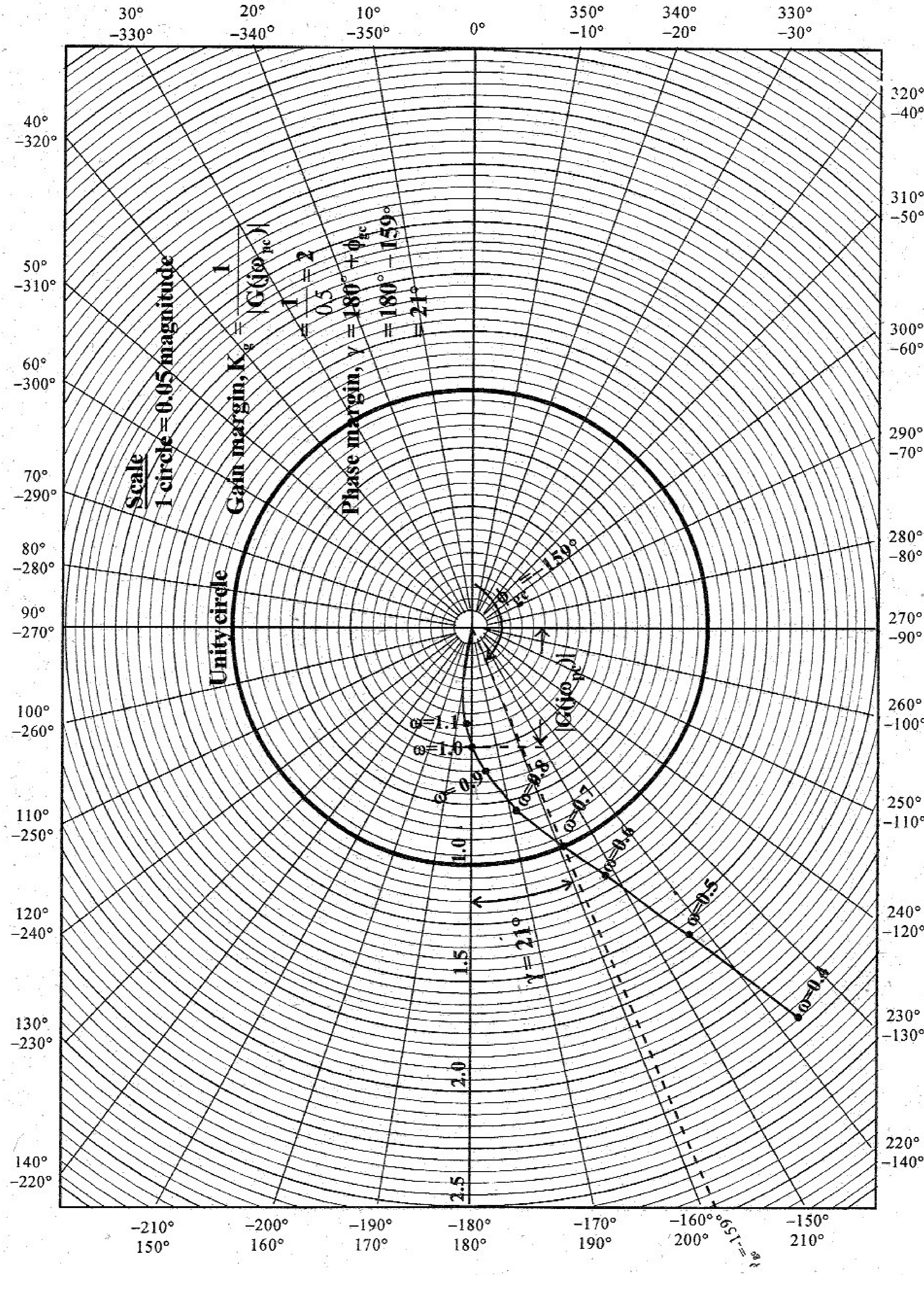

Fig 3.10.1: Polar plot of $G(j\omega) = 1/[j\omega\,(1+j\omega)^2]$ (using polar coordinates)

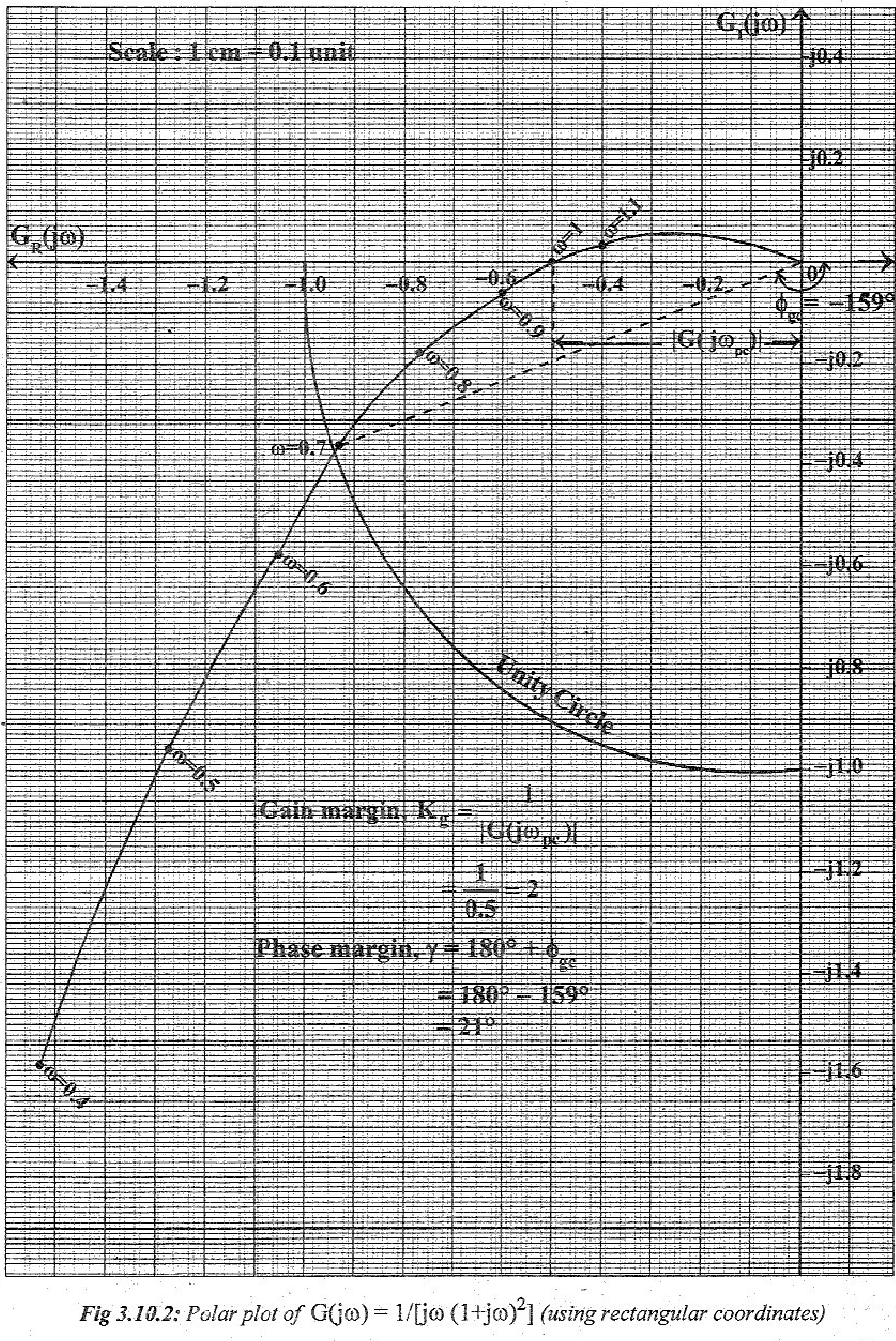

Fig 3.10.2: Polar plot of $G(j\omega) = 1/[j\omega\,(1+j\omega)^2]$ *(using rectangular coordinates)*

EXAMPLE 3.11

Consider a unity feedback system having an open loop transfer function $G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$.

Sketch the polar plot and determine the value of K so that (i) Gain margin is 18 db (ii) Phase margin is 60°.

SOLUTION

Given that, $G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$. The polar plot is sketched by taking K = 1.

$\therefore$ Put K = 1 and s = jω in G(s). $\therefore G(j\omega) = \frac{1}{j\omega(1+j0.2\omega)(1+j0.05\omega)}$

The corner frequencies are $\omega_{c1} = 1/0.2 = 5$ rad/sec and $\omega_{c2} = 1/0.05 = 20$ rad/sec. The magnitude and phase angle of G(jω) are calculated for various frequencies and tabulated in table-1. Using polar to rectangular conversion the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.11.1. Polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.11.2.

$$G(j\omega) = \frac{1}{j\omega(1+j0.2\omega)(1+j0.05\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ \sqrt{1+(0.2\omega)^2}\angle\tan^{-1}0.2\omega \sqrt{1+(0.05\omega)^2}\angle\tan^{-1}0.05\omega}$$

$$= \frac{1}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}}\angle(-90^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.05\omega)$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}} \quad \text{and} \quad \angle G(j\omega) = -90^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.05\omega$$

TABLE-1 : Magnitude and Phase of G(jω) at Various Frequencies

ω rad/sec	0.6	0.8	1	2	3	4
|G(jω)|	1.65	1.23	1.0	0.5	0.3	0.2
∠G(jω) deg	–98	–101	–104	–117.5	–129.4	–140

ω rad/sec	5	6	7	9	10	11	14
|G(jω)|	0.14	0.1	0.07	0.05	0.04	0.03	0.02
∠G(jω) deg	–149	–157	–164	–176	–180	–184	–195

TABLE-2 : Real and Imaginary Parts of G(jω) at Various Frequencies

ω rad/sec	0.6	0.8	1	2	3	4
$G_R(j\omega)$	–0.23	–0.23	–0.24	–0.23	–0.19	–0.15
$G_I(j\omega)$	–1.63	–1.21	–0.97	–0.44	–0.23	–0.13

ω rad/sec	5	6	7	9	10	11	14
$G_R(j\omega)$	–0.120	–0.092	–0.067	–0.050	–0.04	–0.030	–0.019
$G_I(j\omega)$	–0.072	–0.039	–0.019	–0.0034	0	0.002	0.005

In the polar plot shown in fig 3.11.1 and 3.11.2 there are two plots, marked as curve-I and curve-II. These two loci are sketched with different scales to clearly determine the gain margin and phase margin.

From the polar plot, with K = 1,

Gain margin, K_g = 1/0.04 = 25.

Gain margin in db = 20 log 25 = 28 db.

Phase margin, γ = 76°.

Case (i)

With K = 1, let G(jω) cut the –180° axis at point B and gain corresponding to that point be G_B. From the polar plot G_B = 0.04. The gain margin of 28 db with K = 1 has to be reduced to 18 db and so K has to be increased to a value greater than one.

Let G_A be the gain at –180° for a gain margin of 18 db.

$$\text{Now, } 20\log\frac{1}{G_A} = 18 \quad\Rightarrow\quad \log\frac{1}{G_A} = \frac{18}{20} \quad\Rightarrow\quad \frac{1}{G_A} = 10^{18/20}$$

$$\therefore G_A = \frac{1}{10^{18/20}} = 0.125$$

$$\text{The value of K is given by, } K = \frac{G_A}{G_B} = \frac{0.125}{0.04} = 3.125$$

Case (ii)

With K = 1, the phase margin is 76°. This has to be reduced to 60°. Hence gain has to be increased.

Let ϕ_{gc2} be the phase of G(jω) for a phase margin of 60°

$$\therefore 60° = 180° + \phi_{gc2}$$

$$\phi_{gc2} = 60° - 180° = -120°$$

In the polar plot the –120° line cut the locus of G(jω) at point C and cut the unity circle at point D.

Let, G_C = Magnitude of G(jω) at point C.

G_D = Magnitude of G(jω) at point D.

From the polar plot, G_C = 0.425 and G_D = 1.

$$\text{Now, } K = \frac{G_D}{G_C} = \frac{1}{0.425} = 2.353$$

RESULT

(a) When K = 1, Gain margin, K_g = 25

Gain margin in db = 28db

(b) When K = 1, Phase margin, γ = 76°

(c) For a gain margin of 18 db, K = 3.125

(d) For a phase margin of 60°, K = 2.353

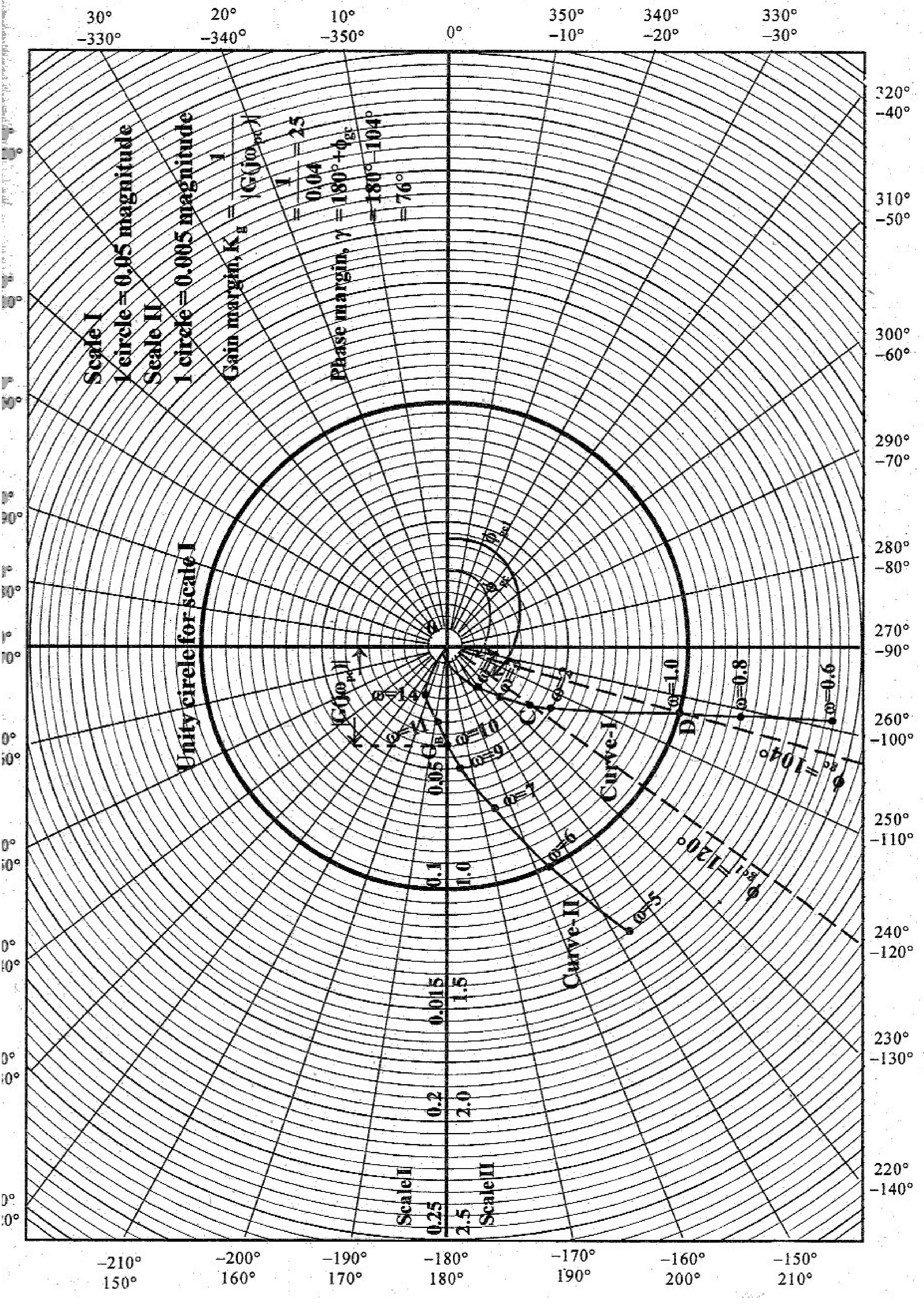

Fig 3.11.1: Polar plot of $G(j\omega) = 1/j\omega\,(1+j0.2\omega)\,(1+j0.05\omega)$, *(using polar coordinates)*

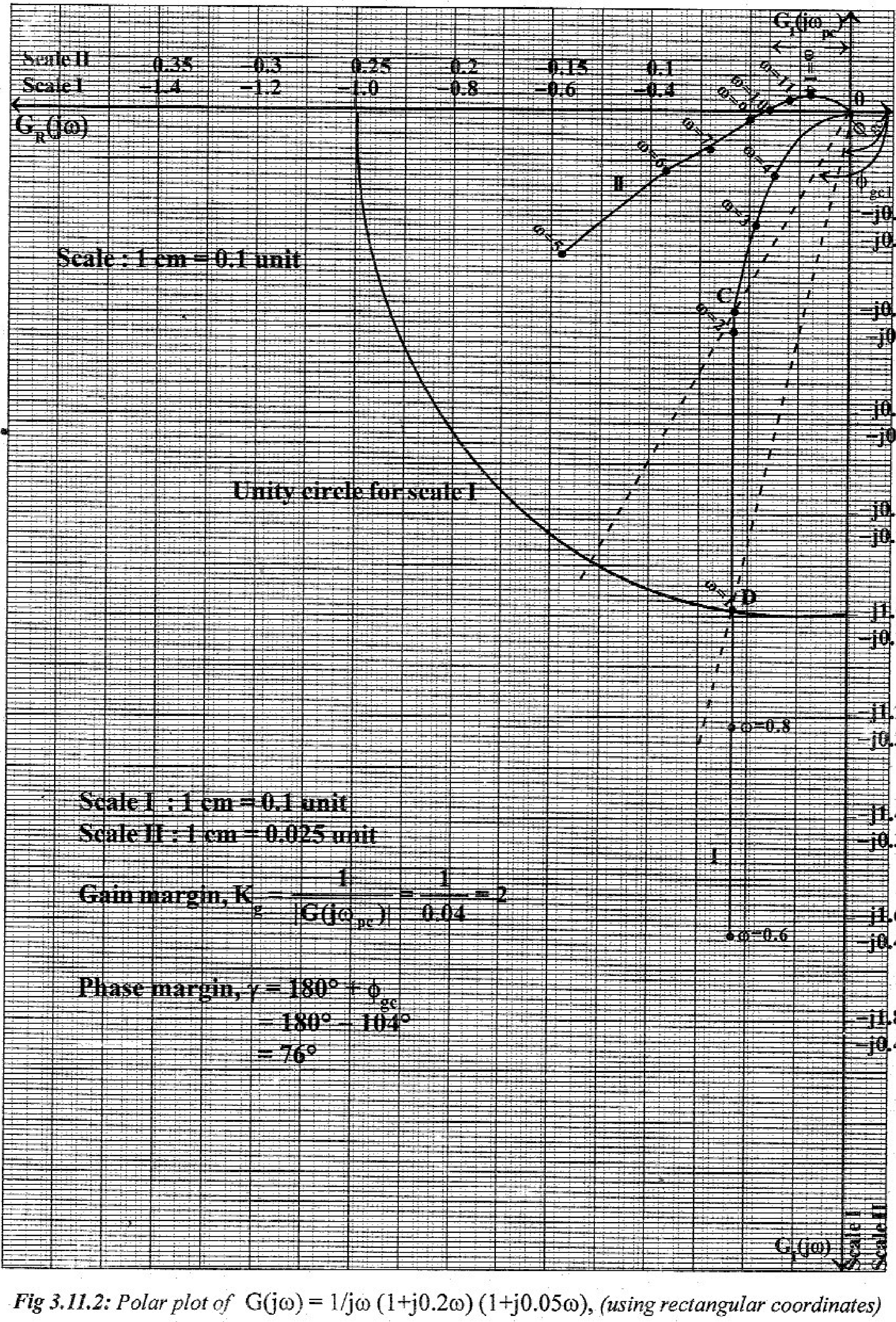

Fig 3.11.2: *Polar plot of* $G(j\omega) = 1/j\omega\,(1+j0.2\omega)\,(1+j0.05\omega)$, *(using rectangular coordinates)*

EXAMPLE 3.12

Consider a unity feedback system having an open loop transfer function, $G(s) = \dfrac{K}{s\,(1+0.5s)\,(1+4s)}$. Sketch the polar plot and determine the value of K so that (i) Gain margin is 20 db and (ii) Phase margin is 30°.

SOLUTION

Given that, G(s) = K/s (1+0.5s) (1+4s)

The polar plot is sketched by taking K = 1.

Put K = 1 and s = jω in G(s).

$$\therefore G(j\omega) = \frac{1}{j\omega\,(1+j0.5\omega)\,(1+j4\omega)}$$

The corner frequencies are ω_{c1} = 1/4 = 0.25 rad/sec and ω_{c2} = 1/0.5 = 2 rad/sec. The magnitude and phase angle of G(jω) are calculated for various frequencies and tabulated in table-1. Using polar to rectangular conversion the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.12.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.12.2.

$$G(j\omega) = \frac{1}{j\omega\,(1+j0.5\omega)\,(1+j4\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ\ \sqrt{1+(0.5\omega)^2}\ \angle\tan^{-1}0.5\omega\ \sqrt{1+(4\omega)^2}\ \angle\tan^{-1}4\omega}$$

$$= \frac{1}{\omega\sqrt{1+0.25\omega^2}\ \sqrt{1+16\omega^2}}\ \angle(-90^\circ-\tan^{-1}0.5\omega-\tan^{-1}4\omega)$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\ \sqrt{1+0.25\omega^2}\ \sqrt{1+16\omega^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}0.5\omega - \tan^{-1}4\omega$$

TABLE-1 : Magnitude and Phase of G(jω) at Various Frequencies

ω rad/sec	0.3	0.4	0.5	0.6	0.8	1.0	1.2
|G(jω)|	2.11	1.3	0.87	0.61	0.35	0.22	0.15
∠G(jω) deg	−149	−159	−167	−174	−184	−193	−199

TABLE-2 : Real part and Imaginary parts of G(jω) at Various Frequencies

ω rad/sec	0.3	0.4	0.5	0.6	0.8	1.0	1.2
$G_R(j\omega)$	−1.8	−1.21	−0.85	−0.61	−0.35	−0.21	−0.14
$G_I(j\omega)$	−1.09	−0.47	−0.2	−0.06	0.02	0.05	0.05

From the polar plot, with K = 1,

Gain margin, K_g = 1/0.44 = 2.27

Gain margin in db = 20 log 2.27 = 7.12 db

Phase margin, $\gamma = 180^\circ + \phi_{gc} = 180^\circ - 165^\circ = 15^\circ$

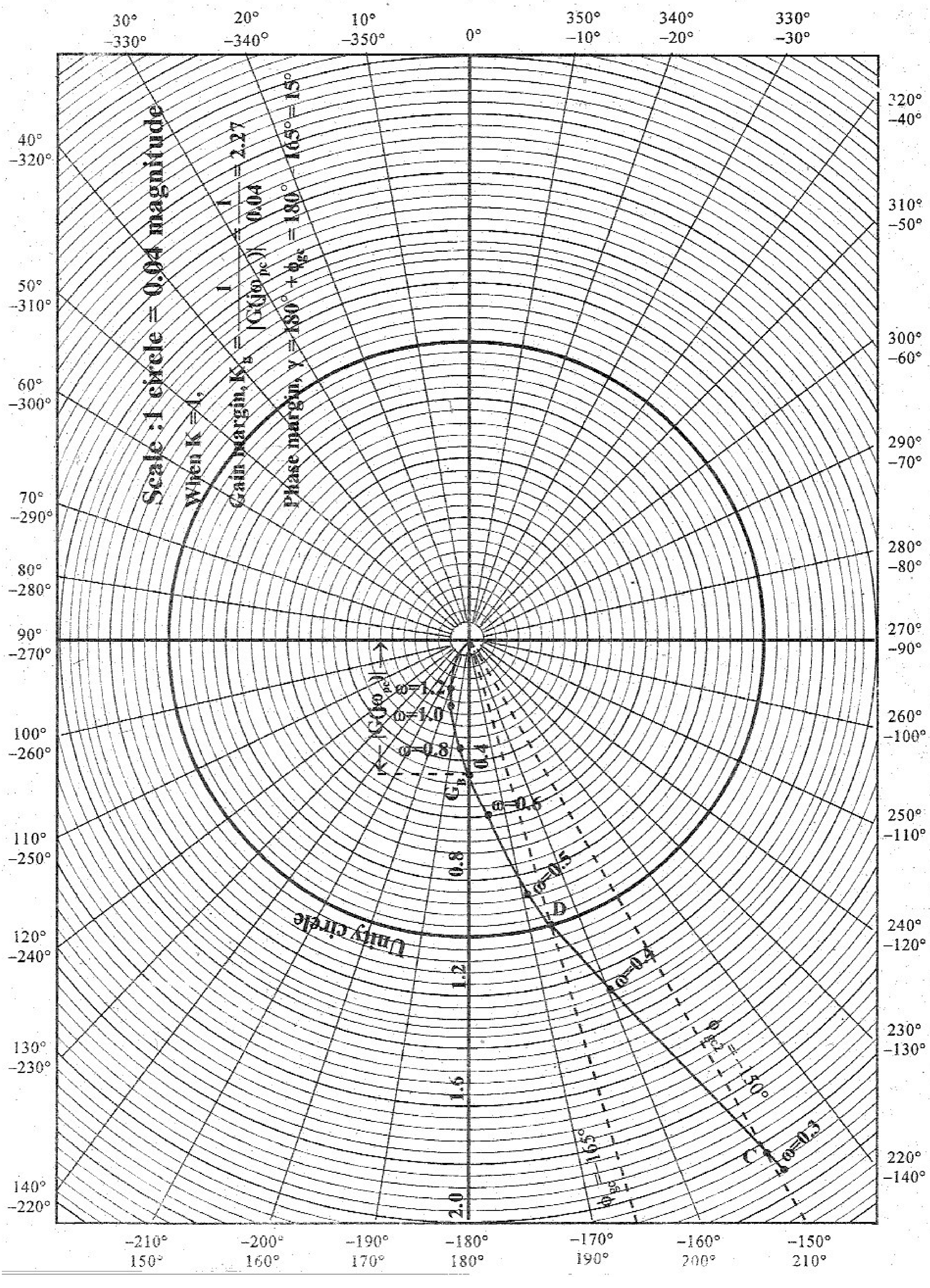

Fig 3.12.1: *Polar plot of* $G(j\omega) = 1/[j\omega\,(1+j0.5\omega)\,(1+j4\omega)]$, *(using polar coordinates)*

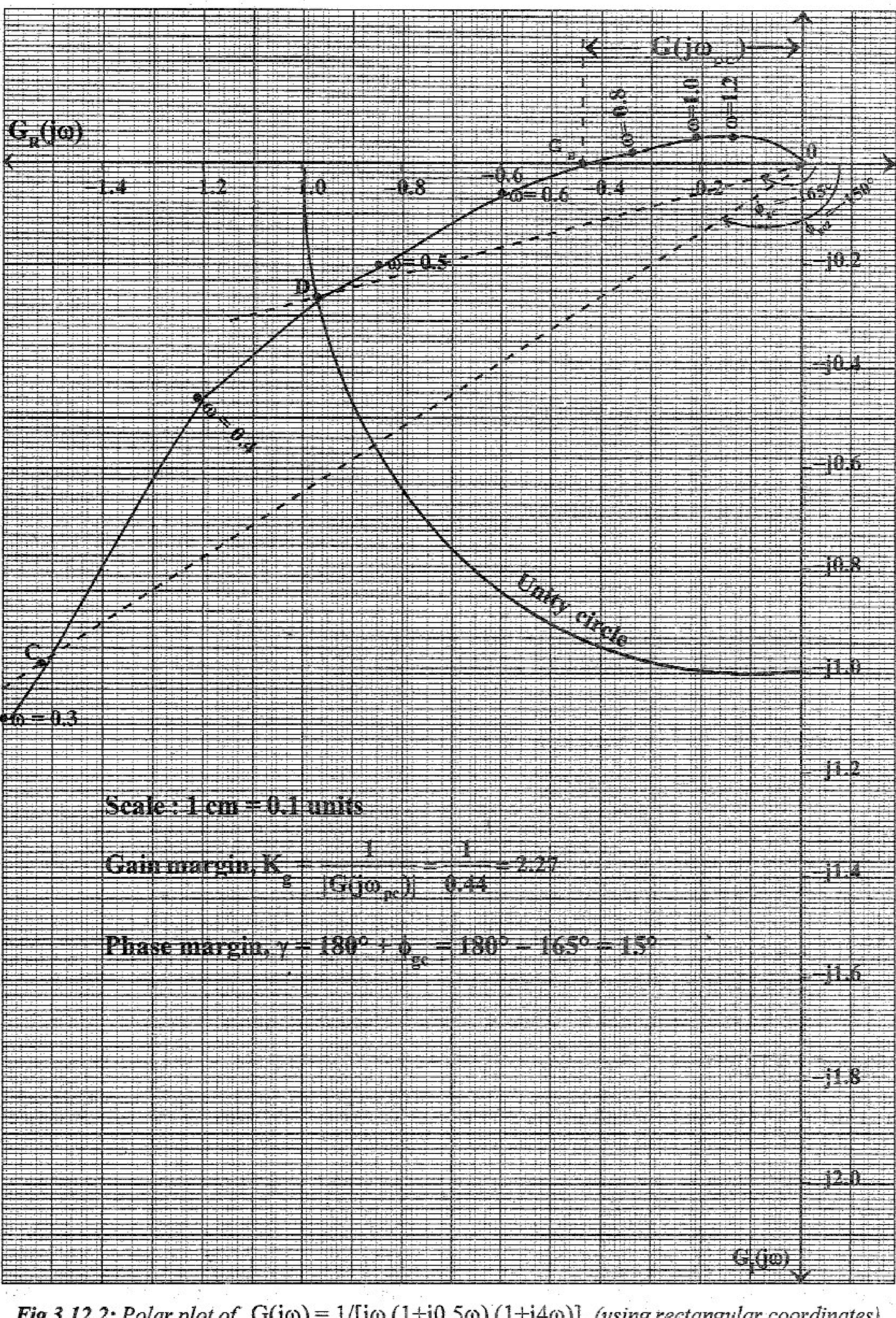

Fig 3.12.2: *Polar plot of* $G(j\omega) = 1/[j\omega\,(1+j0.5\omega)\,(1+j4\omega)]$, *(using rectangular coordinates).*