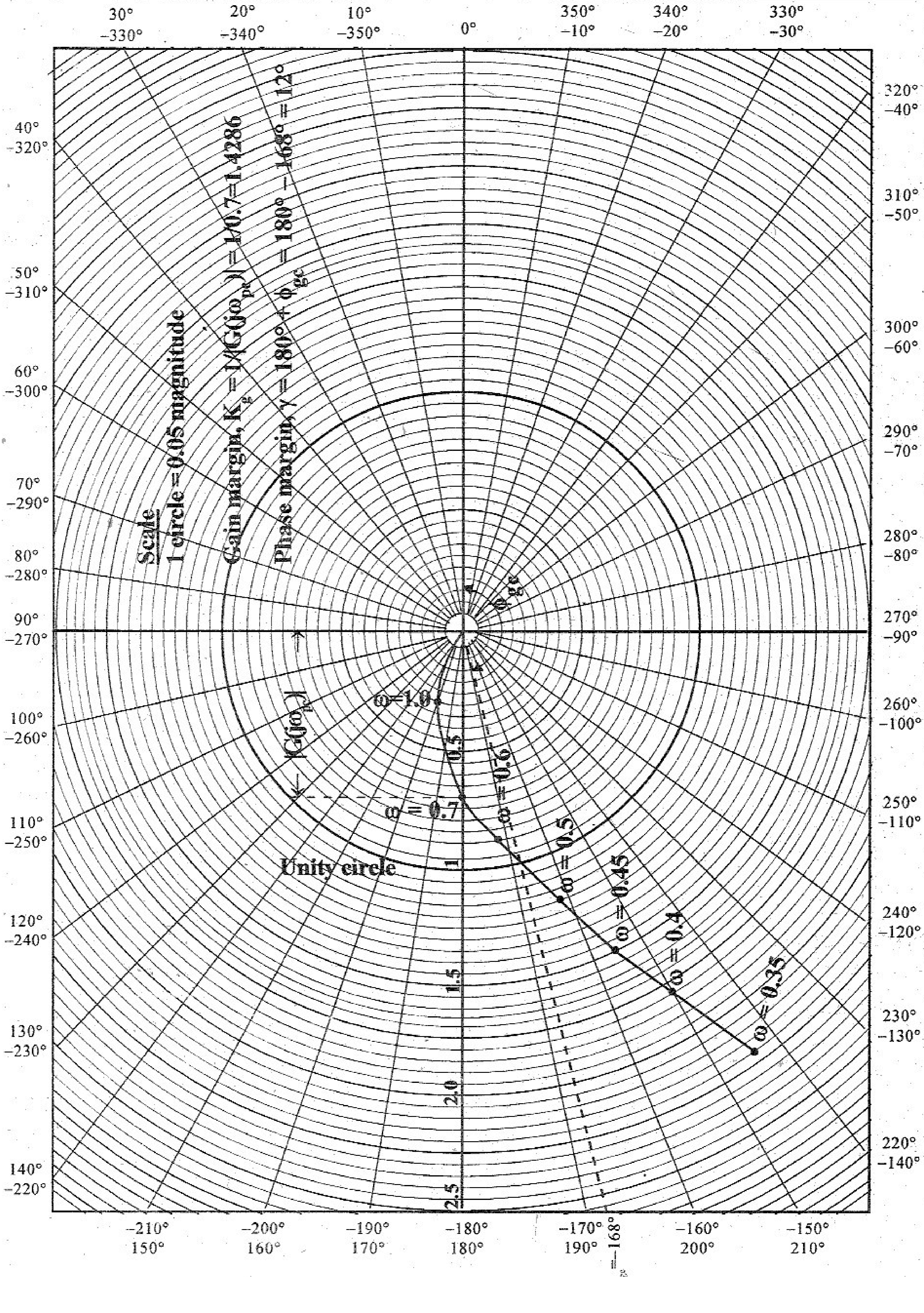

Fig 3.7.1: Polar plot of $G(j\omega) = 1/j\omega\,(1+j\omega)\,(1+j2\omega)$ (using polar coordinates).

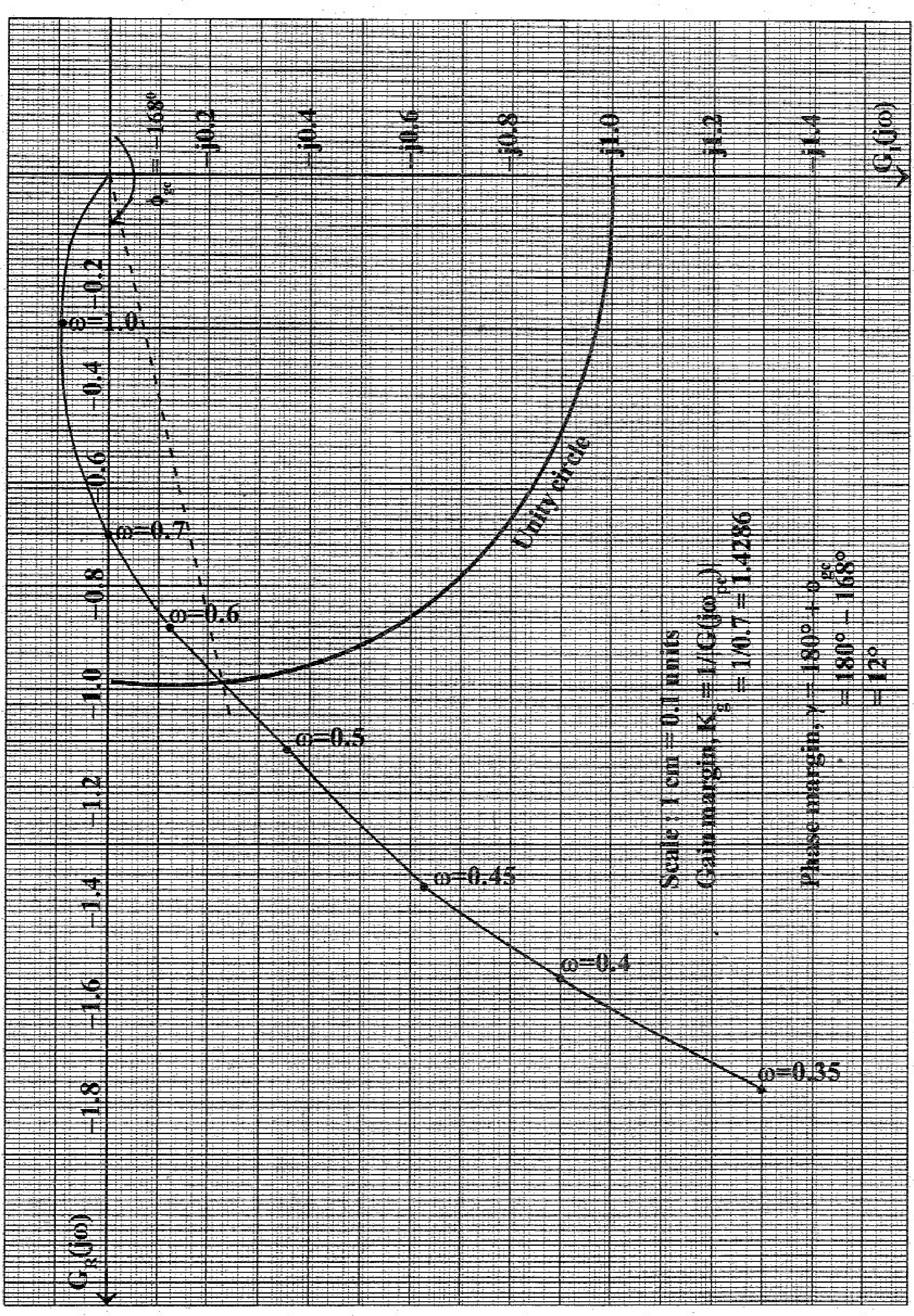

Fig 3.7.2: Polar plot of, $G(j\omega) = \dfrac{1}{j\omega\,(1+j\omega)\,(1+j2\omega)}$ *(using rectangular coordinates).*

EXAMPLE 3.8

The open loop transfer function of a unity feedback system is given by G(s) = 1/ s²(1+s) (1+2s). Sketch the polar plot and determine the gain margin and phase margin.

SOLUTION

Given that, $G(s) = 1/ s^2(1+s)(1+2s)$

Put $s = j\omega$, $\therefore\ G(j\omega) = \dfrac{1}{(j\omega)^2\ (1+j\omega)\ (1+j2\omega)}$

The corner frequencies are ω_{c1} = 0.5 rad/sec and ω_{c2} = 1 rad/sec. The magnitude and phase angle of $G(j\omega)$ are calculated for the corner frequencies and frequencies around corner frequencies are tabulated in table-1. Using the polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.8.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.8.2.

$$G(j\omega) = \frac{1}{(j\omega)^2\ (1+j\omega)\ (1+j2\omega)}$$

$$= \frac{1}{\omega^2\angle 180^\circ\ \sqrt{1+\omega^2}\ \angle \tan^{-1}\omega\ \sqrt{1+4\omega^2}\ \angle\tan^{-1}2\omega}$$

$$G(j\omega) = \frac{1}{\omega^2\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}\ \angle(-180 - \tan^{-1}\omega - \tan^{-1}2\omega)$$

$$|G(j\omega)| = \frac{1}{\omega^2\ \sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}} = \frac{1}{\omega^2\sqrt{(1+\omega^2)\ (1+4\omega^2)}}$$

$$= \frac{1}{\omega^2\ \sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

TABLE-1 : Magnitude and phase plot of G(jω) at various frequencies

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$\lvert G(j\omega)\rvert$	3.3	2.5	1.9	1.5	1.2	0.97≈1	0.8	0.3
$\angle G(j\omega)$ deg	–246	–251	–256	–261	–265	–269	–273	–288

TABLE-2 : Real and imaginary parts of G(jω)

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$G_R(j\omega)$	–1.34	–0.81	–0.46	–0.23	–0.1	–0.02	0.04	0.09
$G_I(j\omega)$	3.01	2.36	1.84	1.48	1.2	1.0	0.8	0.29

RESULT

Gain margin, $K_g = 0$

Phase margin, $\gamma = -90^\circ$

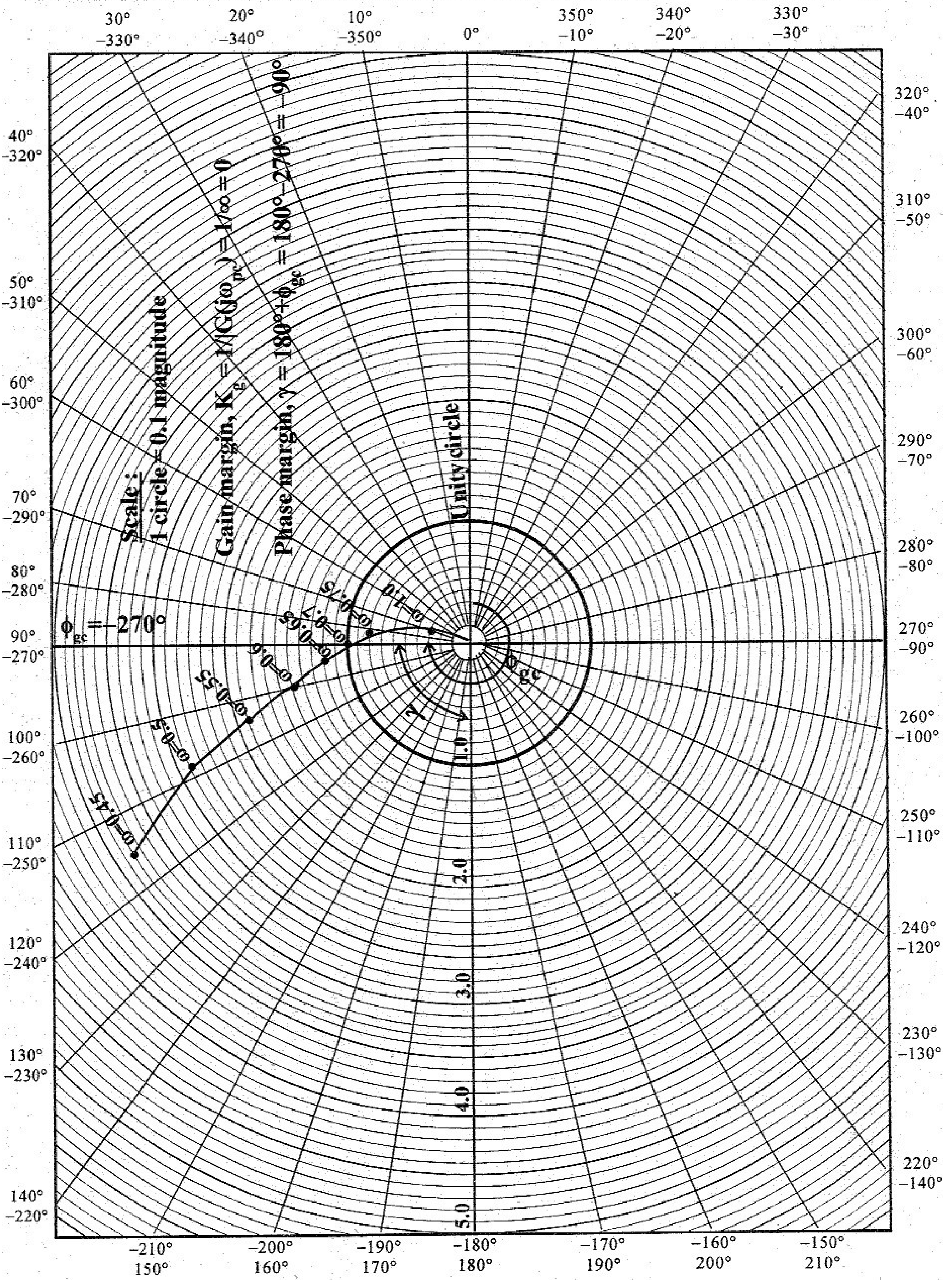

Fig 3.8.1: Polar plot of $G(j\omega) = 1/(j\omega)^2(1+j\omega)(1+j2\omega)$, *(using polar coordinates)*

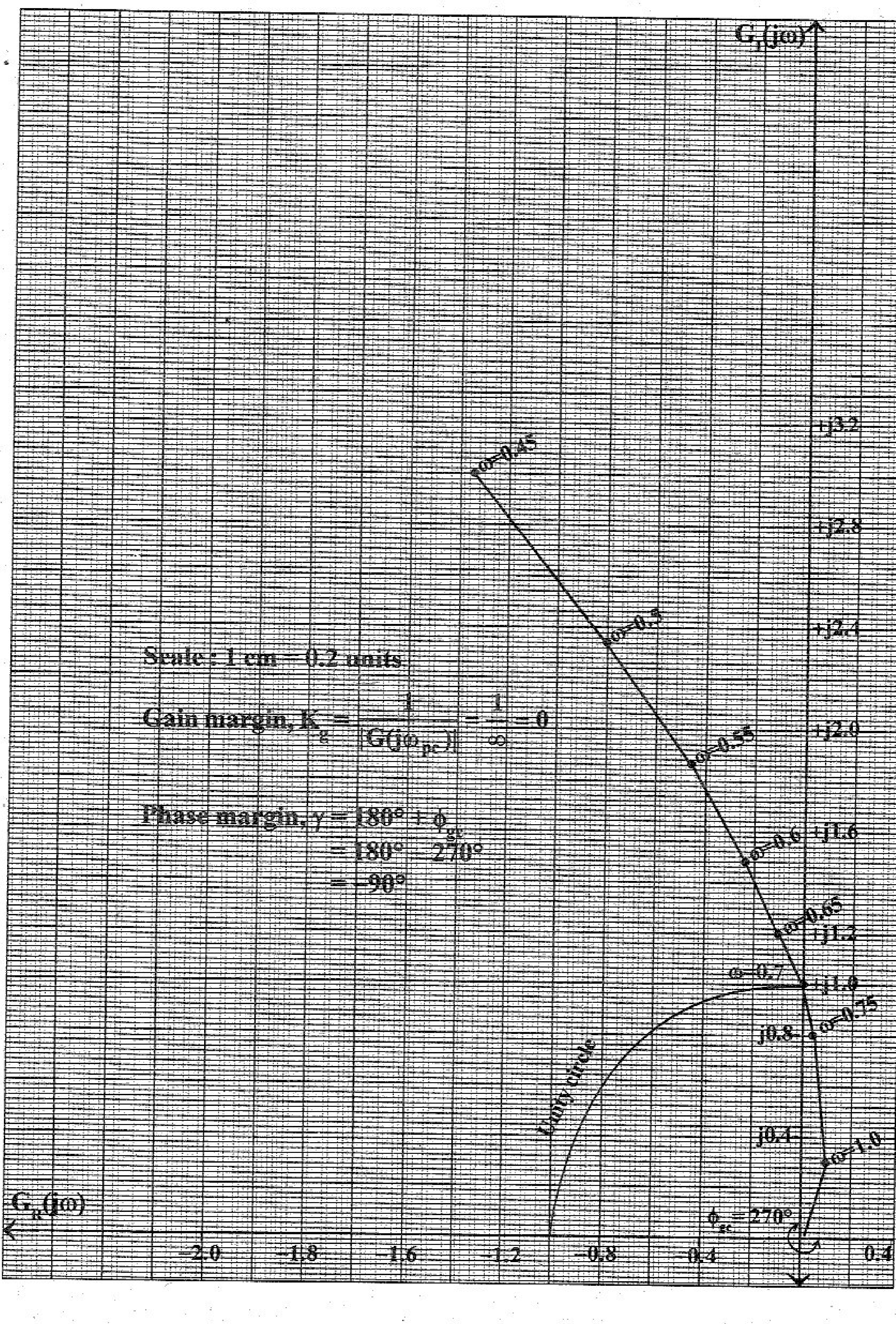

Fig 3.8.2: *Polar plot of* $G(j\omega) = 1/(j\omega)^2\,(1+j\omega)\,(1+j2\omega)$, *(using rectangular coordinates)*

EXAMPLE 3.9

The open loop transfer function of a unity feedback system is given by,

$$G(s) = \frac{(1+0.2s)\,(1+0.025s)}{s^3(1+0.005s)\,(1+0.001s)}.$$

Sketch the polar plot and determine the phase margin.

SOLUTION

$$\text{Given that } G(s) = \frac{(1+0.2s)\,(1+0.025s)}{s^3(1+0.005s)\,(1+0.001s)}$$

$$\therefore G(j\omega) = \frac{(1+j0.2\omega)\,(1+j0.025\omega)}{(j\omega)^3\,(1+j0.005\omega)\,(1+j0.001\omega)}$$

$$= \frac{\sqrt{1+(0.2\omega)^2}\,\angle\tan^{-1}0.2\omega\ \sqrt{1+(0.025\omega)^2}\,\angle\tan^{-1}0.025\omega}{\omega^3\,\angle 270°\ \sqrt{1+(0.005\omega)^2}\,\angle\tan^{-1}0.005\omega\ \sqrt{1+(0.001\omega)^2}\,\angle\tan^{-1}0.001\omega}$$

$$|G(j\omega)| = \frac{\sqrt{1+(0.2\omega)^2}\ \sqrt{1+(0.025\omega)^2}}{\omega^3\ \sqrt{1+(0.005\omega)^2}\ \sqrt{1+(0.001\omega)^2}}$$

$$\angle G(j\omega) = \tan^{-1}0.2\omega + \tan^{-1}0.025\omega - 270° - \tan^{-1}0.005\omega - \tan^{-1}0.001\omega$$

The magnitude and phase angle of $G(j\omega)$ are calculated for various frequencies and listed in table-1. Using the polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.9.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.9.2.

TABLE-1 : Magnitude and phase of G(jω)

ω,rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$\lvert G(j\omega)\rvert$	1.4	1.2	1.0	0.8	0.6	0.4	0.2
$\angle G(j\omega)$, deg	–259	–258	–257	–256	–255	–253	–249

TABLE-2 : Real and imaginary part of G(jω)

ω, rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$G_R(j\omega)$	–0.27	–0.25	–0.22	–0.19	–0.16	–0.12	–0.07
$G_I(j\omega)$	1.37	1.17	0.97	0.78	0.58	0.38	0.19

RESULT

Phase margin, $\gamma = -77°$

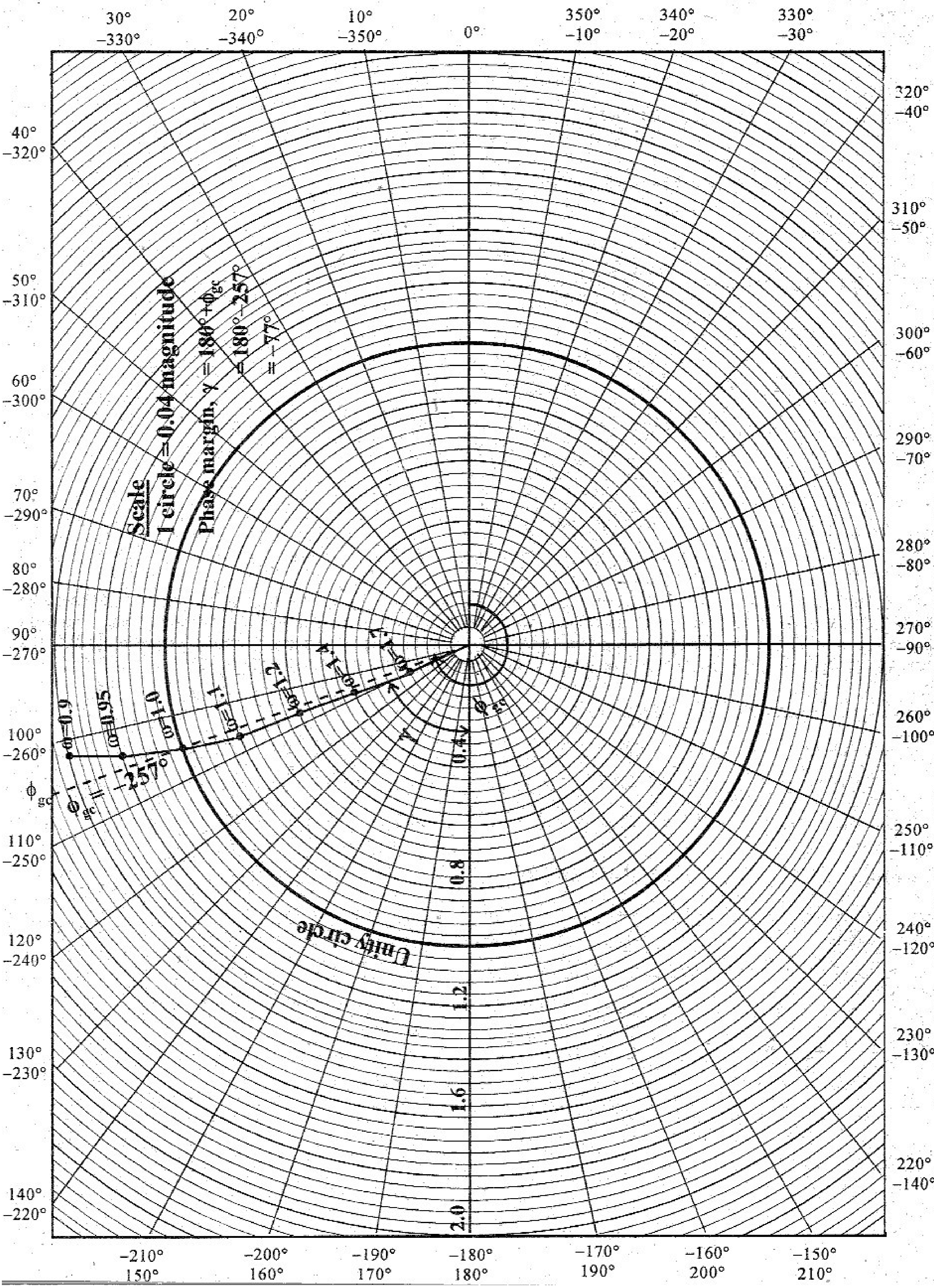

Fig 3.9.1: *Polar plot of* $G(j\omega) = (1+j0.2\omega)(1+j0.025\omega) / (j\omega)^3(1+j0.005\omega)(1+j0.001\omega)$, *(using polar coordinates)*

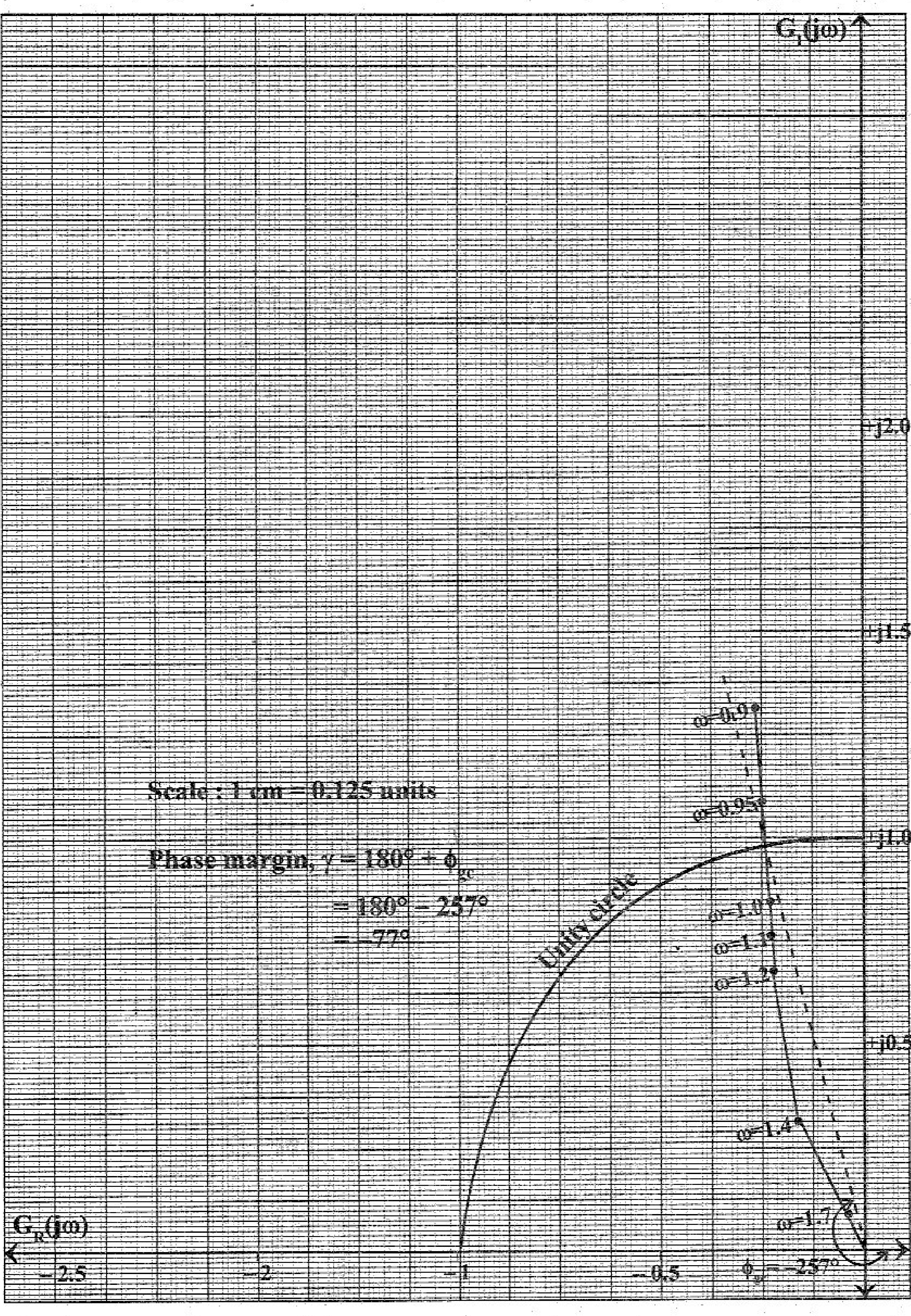

***Fig 3.9.2:** Polar plot of* $G(j\omega) = (1+j0.2\omega)(1+j0.025\omega) / (j\omega)^3(1+j0.005\omega)(1+j0.001\omega)$, *(using rectangular coordinates)*

EXAMPLE 3.10

The open loop transfer function of a unity feedback system is given by G(s) = 1/s(1+s)². Sketch the polar plot and determine the gain and phase margin.

SOLUTION

Given that, $G(s) = 1/s(1+s)^2$.

Put $s = j\omega$,

$$\therefore G(j\omega) = \frac{1}{j\omega\,(1+j\omega)^2} = \frac{1}{j\omega\,(1+j\omega)\,(1+j\omega)}$$

The corner frequency is ω_{c1} = 1 rad/sec. The magnitude and phase angle of G(jω) are calculated for corner frequency and frequencies around corner frequency and tabulated in table-1. Using polar to rectangular conversion the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.10.1. The polar plot using rectangular coordinates are sketched on an ordinary graph sheet as shown in fig 3.10.2.

$$G(j\omega) = \frac{1}{j\omega\,(1+j\omega)^2} = \frac{1}{j\omega\,(1+j\omega)\,(1+j\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ\ \sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\ \sqrt{1+\omega^2}\ \angle\tan^{-1}\omega}$$

$$= \frac{1}{\omega\left(\sqrt{1+\omega^2}\right)^2}\quad \angle(-90^\circ - 2\tan^{-1}\omega)$$

$$|G(j\omega)| = \frac{1}{\omega(1+\omega^2)} = \frac{1}{\omega+\omega^3}$$

$$\angle G(j\omega) = -90^\circ - 2\tan^{-1}\omega$$

TABLE-1: Magnitude and phase of G(jω) at various frequencies

ω rad/sec	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
|G(jω)|	2.2	1.6	1.2	1	0.8	0.6	0.5	0.4
∠G(jω) deg	–134	–143	–151	–159	–167	–174	–180	–185

TABLE-2 : Real and imaginary parts of G(jω) at various frequencies

ω rad/sec	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
$G_R(j\omega)$	–1.53	–1.28	–1.05	–0.93	–0.78	–0.6	–0.5	–0.4
$G_I(j\omega)$	–1.58	–0.96	–0.58	–0.36	–0.18	0.06	0	0.03

RESULT

Gain margin, $K_g = 2$

Phase margin, $\gamma = 21^\circ$