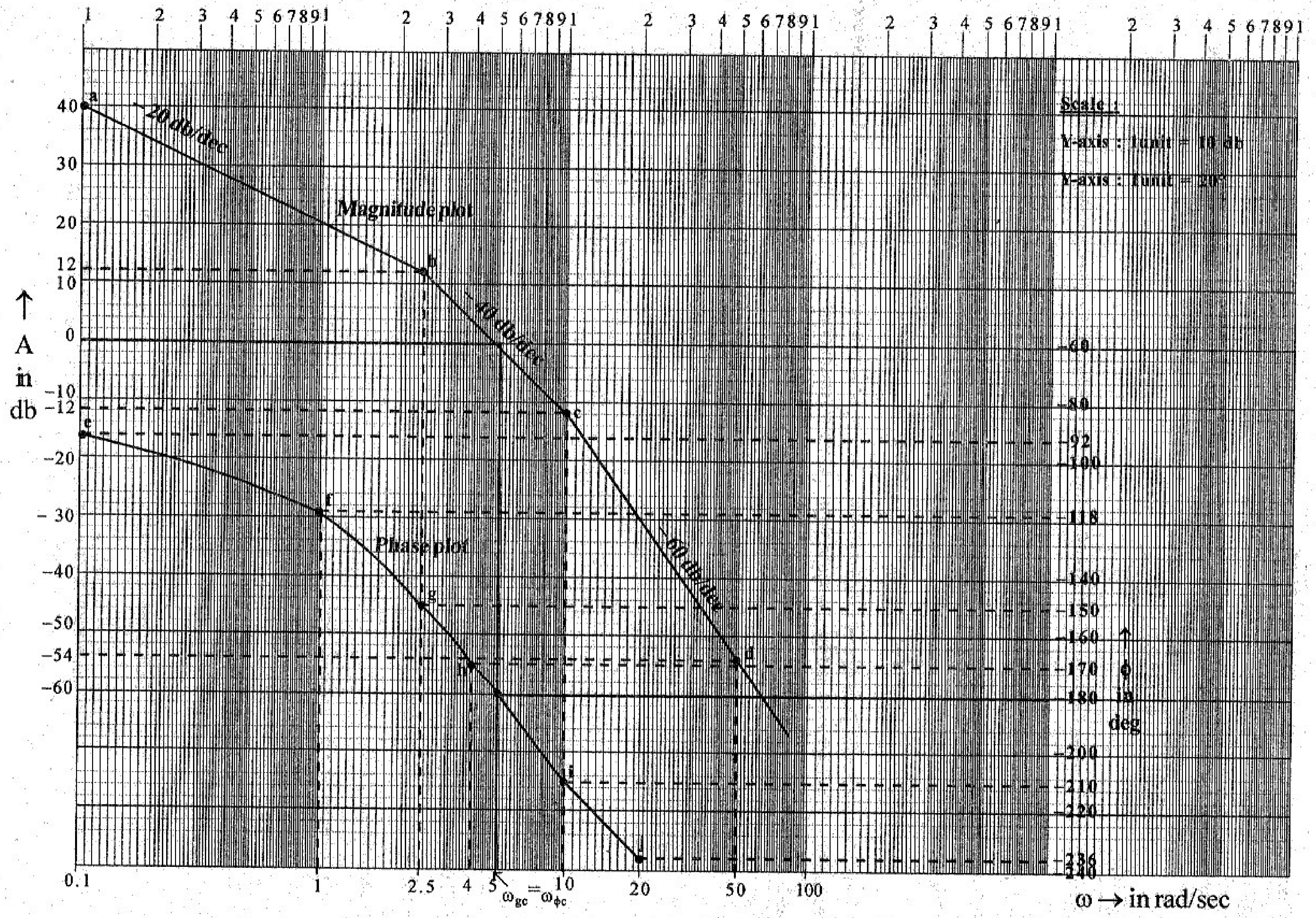


Fig 3.4.1 : *Bode plot of transfer function,* $G(j\omega) = \dfrac{10}{j\omega(1+0.4\omega)(1+j0.1\omega)}$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10 db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by a straight line and mark the slope in the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by,

$$\phi = -90° - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

The phase angle of G(jω) are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$\tan^{-1} 0.4\omega$ deg	$\tan^{-1} 0.1\omega$ deg	$\phi = \angle G(j\omega)$ deg	Points in phase plot
0.1	2.29	0.57	−92.86 ≈ −92	e
1	21.80	5.71	−117.5 ≈ −118	f
2.5	45.0	14.0	−149 ≈ −150	g
4	57.99	21.8	−169.79 ≈ −170	h
10	75.96	45.0	−210.96 ≈ −210	i
20	82.87	63.43	−236.3 ≈ 236	j

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

The magnitude and phase plots are shown in fig 3.4.1.

From the graph, the gain and phase cross over frequencies are found to be 5 rad/sec.

RESULT

Gain cross-over frequency = 5 rad/sec.

Phase cross-over frequency = 5 rad/sec.

EXAMPLE 3.5

For the following transfer function draw bode plot and obtain gain cross-over frequency.

$$G(s) = \frac{20}{s\,(1+3s)\,(1+4s)}$$

SOLUTION

The sinusoidal transfer function of G(jω) is obtained by replacing s by jω in the given transfer function.

$$G(j\omega) = \frac{20}{j\omega\,(1+j3\omega)\,(1+j4\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{4} = 0.25$ rad / sec, $\omega_{c2} = \frac{1}{3} = 0.333$ rad / sec.

The various terms of G(jω) are listed in table-1 in the increasing order of their frequencies. Also the table shows the slope contributed by each term and change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{20}{j\omega}$	–	−20	
$\frac{1}{1+j4\omega}$	$\omega_{c1}=\frac{1}{4}=0.25$	−20	$-20-20=-40$
$\frac{1}{1+j3\omega}$	$\omega_{c2}=\frac{1}{3}=0.33$	−20	$-40-20=-60$

Choose a frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.15$ rad/sec and $\omega_h = 1$ rad/sec.

Let, $A = |G(j\omega)|$ in db.

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At } \omega=\omega_l,\ A=|G(j\omega)| = 20\log\left|\frac{20}{0.15}\right| = 42.5 \text{ db}$$

$$\text{At } \omega=\omega_{c1},\ A=|G(j\omega)| = 20\log\left|\frac{20}{0.25}\right| = 38 \text{ db}$$

$$\text{At } \omega=\omega_{c2},\ A=\left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})}$$

$$= -40\times\log\frac{0.33}{0.25} + 38 = 33 \text{ db}$$

$$\text{At } \omega=\omega_h,\ A=\left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})}$$

$$= -60\times\log\frac{1}{0.33} + 33 = 4 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1 unit = 10 db on y-axis. The frequencies are marked in decades from 0.01 to 10 rad/sec on logarithmic scales on x-axis. Fix the points a, b, c and d on the graph sheet. Join the points by a straight line and mark the slope in the respective region.

PHASE PLOT

The phase angle of $G(j\omega)$, $\phi = -90^0 - \tan^{-1}3\omega - \tan^{-1}4\omega$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

TABLE-2

ω, rad/sec	$\tan^{-1}3\omega$, deg	$\tan^{-1}4\omega$, deg	$\phi = \angle G(j\omega)$, deg	Points in phase plot
0.15	24.22	30.96	$-145.18 \approx -146$	e
0.2	30.96	38.66	$-159.61 \approx -160$	f
0.25	36.86	45.0	$-171.86 \approx -172$	g
0.33	44.7	52.8	$-187.5 \approx -188$	h
0.6	60.14	67.38	$-218.32 \approx -218$	i
1	71.56	75.96	$-237.56 \approx -238$	j

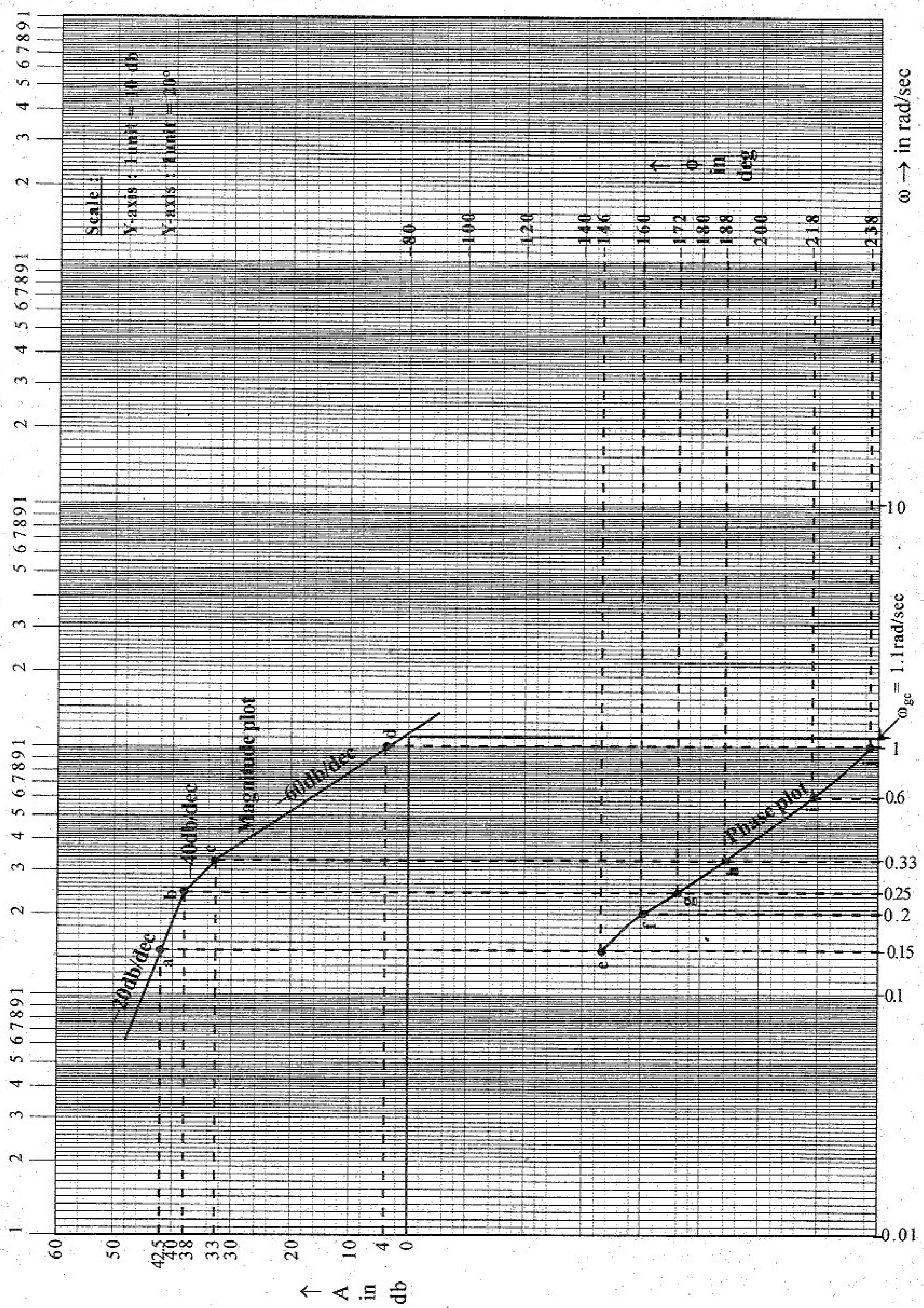


Fig 3.5.1 : *Bode plot for transfer function,* $G(j\omega) = \dfrac{20}{j\omega(1+j3\omega)(1+j4\omega)}$

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve. The magnitude and phase plots are shown in fig 3.5.1. From the graph the gain cross-over frequency is found to be ω_{gc} = 1.1 rad/sec.

EXAMPLE 3.6

For the function, $G(s) = \dfrac{5(1+2s)}{(1+4s)\,(1+0.25s)}$, draw the bode plot.

SOLUTION

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in G(s).

$$\therefore G(j\omega) = \frac{5\,(1+j2\omega)}{(1+j4\omega)\,(1+j0.25\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \dfrac{1}{4} = 0.25$ rad / sec, $\omega_{c2} = \dfrac{1}{2} = 0.5$ rad / sec, $\omega_{c3} = \dfrac{1}{0.25} = 4$ rad / sec

The various terms of G(jω) are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by the each term and the change in slope at the corner frequency.

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$. Let ω_l = 0.1 rad/sec and ω_h = 10 rad/sec.

Let A = |G(jω)| in db and let us calculate A at ω_l, ω_{c1}, ω_{c2}, ω_{c3} and ω_h.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/deg
5	–	0	–
$\dfrac{1}{1+j4\omega}$	$\omega_{c1} = \dfrac{1}{4} = 0.25$	–20	0 – 20 = –20
$1+j2\omega$	$\omega_{c2} = \dfrac{1}{2} = 0.5$	20	–20 + 20 = 0
$\dfrac{1}{1+j0.25\omega}$	$\omega_{c3} = \dfrac{1}{0.25} = 4$	–20	0 – 20 = –20

At $\omega = \omega_l$, $A = |G(j\omega)| = 20\log 5 = +14$ db

At $\omega = \omega_{c1}$, $A = |G(j\omega)| = 20\log 5 = +14$ db

$$\text{At } \omega = \omega_{c2},\ A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})} = -20 \times \log\frac{0.5}{0.25} + 14 = +8\text{db}$$

$$\text{At } \omega = \omega_{c3},\ A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log\frac{\omega_{c3}}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})} = 0 \times \log\frac{4}{0.5} + 8 = +8\text{db}$$

$$\text{At } \omega = \omega_h,\ A = \left[\text{Slope from } \omega_{c3} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c3}}\right] + A_{(\text{at } \omega=\omega_{c3})} = -20\log\frac{10}{4} + 8 = 0\text{ db}$$

Let the points a, b, c, d and e be the points correponding to frequencies ω_l, ω_{c1}, ω_{c2}, ω_{c3} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 5 db on y axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales on x-axis. Fix the points a, b, c, d and e on the graph. Join the points by a straight line and mark the slope in the respective region.

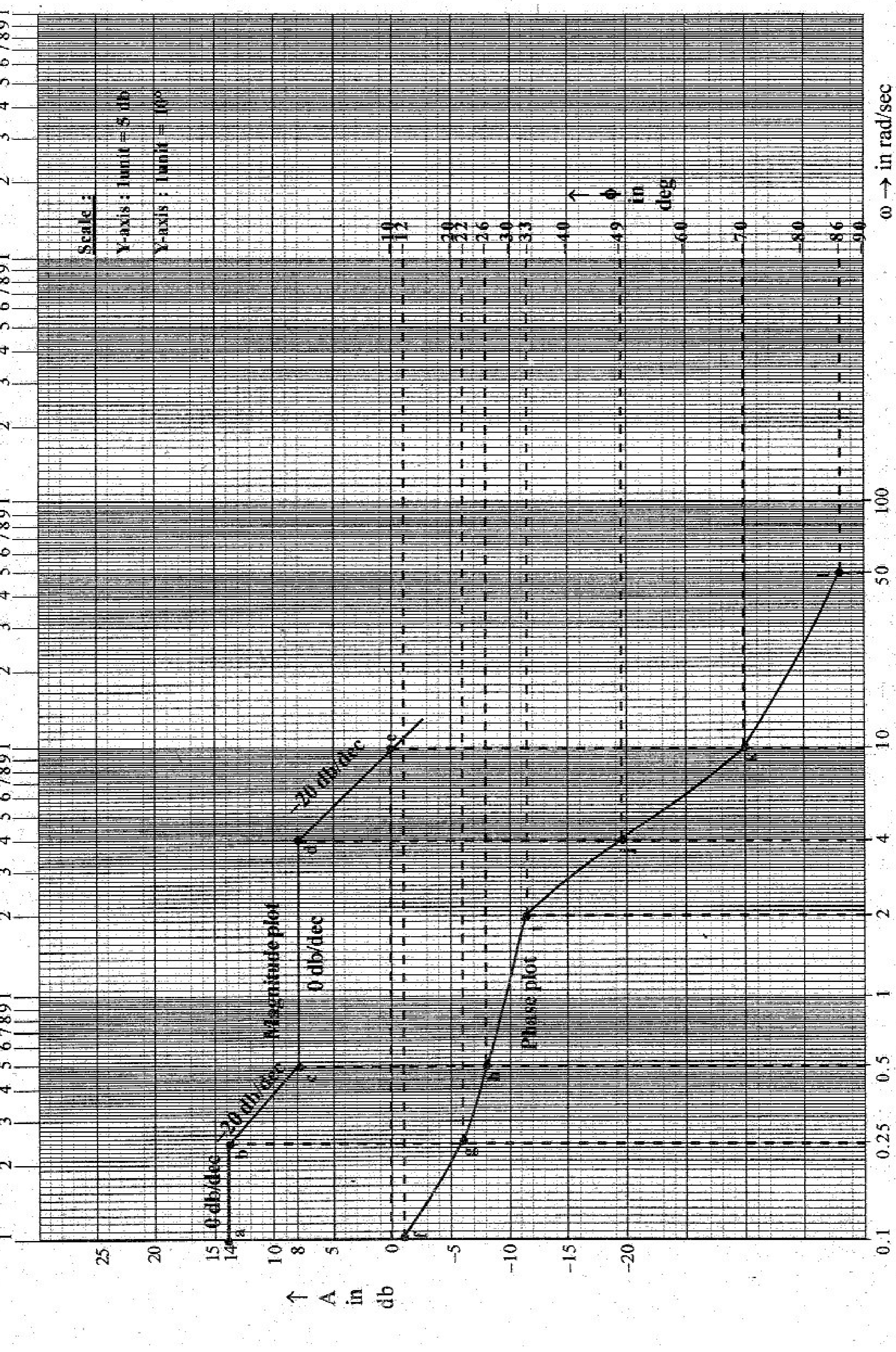


Fig 3.6.1 : *Bode plot of transfer function,* $G(j\omega) = \dfrac{5(1+j2\omega)}{(1+j4\omega)(1+j0.25\omega)}$

PHASE PLOT

The phase angle of G(jω), $\phi = \tan^{-1}(2\omega) - \tan^{-1}(4\omega) - \tan^{-1}(0.25\omega)$

The phase angle of G(jω) are calculated for various values of ω and listed in the table-2.

TABLE-2

ω	$\tan^{-1} 2\omega$ deg	$\tan^{-1} 4\omega$ deg	$\tan^{-1} 0.25\omega$ deg	$\phi = \angle G(j\omega)$	Points in phase plot
0.1	11.3	21.8	1.43	−11.93 ≈ −12	f
0.25	26.56	45.0	3.5	−21.94 ≈ −22	g
0.5	45.0	63.43	7.1	−25.53 ≈ −26	h
2	75.96	82.87	26.56	−33.47 ≈ −33	i
4	82.87	86.42	45.0	−48.55 ≈ −49	j
10	87.13	88.56	68.19	−69.62 ≈ −70	k
50	89.42	89.71	85.42	−85.71 ≈ −86	l

On the same semilog graph sheet choose a scale of 1 unit = 10° on y-axis on the right side of the semilog graph sheet. Mark the calculated phase angle on the graph sheet, Join the points by a smooth curve. The magnitude and phase plots are shown in fig 3.6.1.

3.7 POLAR PLOT

The polar plot of a sinusoidal transfer function G(jω) is a plot of the magnitude of G(jω) versus the phase angle of G(jω) on polar coordinates as ω is varied from zero to infinity. Thus the polar plot is the locus of vectors |G(jω)| ∠G(jω) as ω is varied from zero to infinity. The polar plot is also called ***Nyquist plot.***

The polar plot is usually plotted on a polar graph sheet. The polar graph sheet has concentric circles and radial lines. The circles represent the magnitude and the radial lines represent the phase angles. Each point on the polar graph has a magnitude and phase angle. The magnitude of a point is given by the value of the circle passing through that point and the phase angle is given by the radial line passing through that point. In polar graph sheet a positive phase angle is measured in anticlockwise from the reference axis (0°) and a negative angle is measured clockwise from the reference axis (0°).

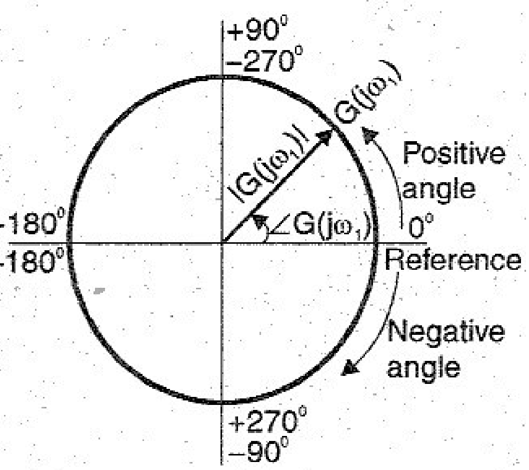


Fig 3.20 : Polar graph.

In order to plot the polar plot, magnitude and phase of G(jω) are computed for various values of ω and tabulated. Usually the choice of frequencies are corner frequencies and frequencies around corner frequencies. Choose proper scale for the magnitude circles. Fix all the points on polar graph sheet and join the points by smooth curve. Write the frequency corresponding to each point of the plot.

Alternatively, if G(jω) can be expressed in rectangular coordinates as,

$$G(j\omega) = G_R(j\omega) + jG_I(j\omega)$$

where, $G_R(j\omega)$ = Real part of G(jω) ; $G_I(j\omega)$ = Imaginary part of G(jω).

then the polar plot can be plotted in ordinary graph sheet between $G_R(j\omega)$ and $G_I(j\omega)$ by varying ω from 0 to ∞. In order to plot the polar plot on ordinary graph sheet, the magnitude and phase of G(jω) are computed for various values of ω. Then convert the polar coordinates to rectangular coordinates using **P → R** conversion (polar to rectangular conversion) in the calculator. Sketch the polar plot using rectangular coordinates.

For minimum phase transfer function with only poles, type number of the system determines the quadrant at which the polar plot starts and the order of the system determines the quadrant at which the polar plot ends. The minimum phase systems are systems with all poles and zeros on left half of s-plane. The start and end of polar plot of all pole minimum phase system are shown in fig 3.21 & 3.22 respectively. Some typical sketches of polar plot are shown in table-3.1.

The change in shape of polar plot can be predicted due to addition of a pole or zero.

1. When a pole is added to a system, the polar plot end point will shift by -90^0.
2. When a zero is added to a system the polar plot end point will shift by $+90^0$.

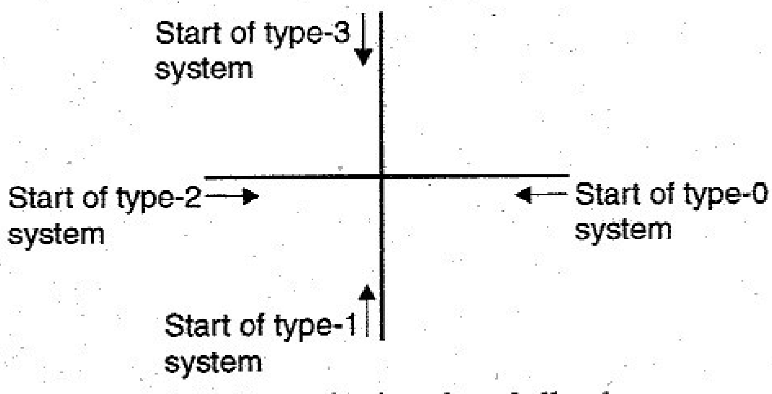


Fig 3.21 : Start of polar plot of all pole minimum phase system.

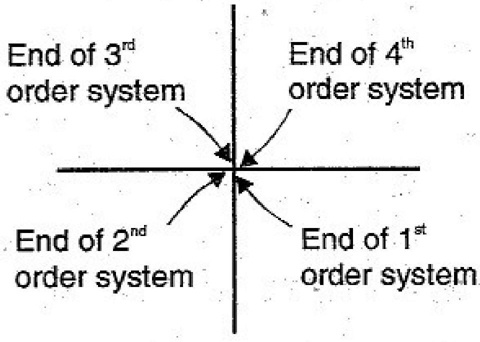


Fig 3.21 : Start of polar plot of all pole minimum phase system.

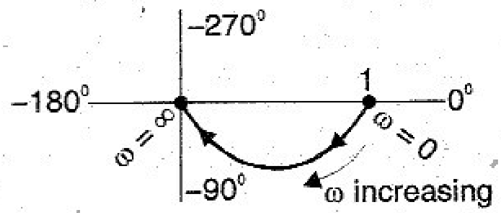
TABLE-3.1 : Typical Sketches of Polar Plot

Type : 0, Order : 1 $\quad G(s)=\dfrac{1}{1+sT}$

$$G(j\omega)=\frac{1}{1+j\omega T}=\frac{1}{\sqrt{1+\omega^2T^2}\ \angle\tan^{-1}\omega T}=\frac{1}{\sqrt{1+\omega^2T^2}}\angle-\tan^{-1}\omega T$$

As $\omega\to 0,\quad G(j\omega)\to 1\angle 0^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle-90^\circ$

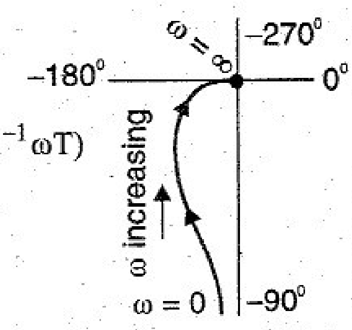


Type : 1, Order : 2 $\quad G(s)=\dfrac{1}{s(1+sT)}$

$$G(j\omega)=\frac{1}{j\omega(1+j\omega T)}=\frac{1}{\omega\angle 90^\circ\sqrt{1+\omega^2T^2}\angle\tan^{-1}\omega T}=\frac{1}{\omega\sqrt{1+\omega^2T^2}}\angle(-90^\circ-\tan^{-1}\omega T)$$

As $\omega\to 0,\quad G(j\omega)\to\infty\angle-90^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle-180^\circ$



Type : 0, Order : 2 $\quad G(s)=\dfrac{1}{(1+sT_1)(1+sT_2)}$

$$G(j\omega)=\frac{1}{(1+j\omega T_1)\ (1+j\omega T_2)}=\frac{1}{\sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1\ \sqrt{1+\omega^2T_2^2}\ \angle\tan^{-1}\omega T_2}$$

$$=\frac{1}{\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)}}\angle(-\tan^{-1}\omega T_1-\tan^{-1}\omega T_2)$$

As $\omega\to 0,\quad G(j\omega)\to 1\angle 0^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle-180^\circ$

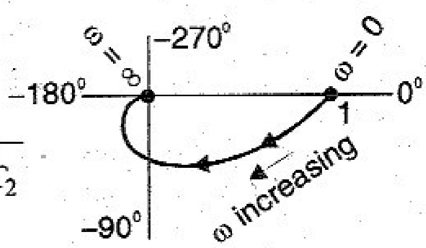
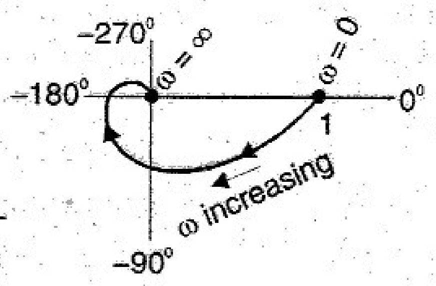


TABLE-3.1 : Typical Sketches of Polar Plot

Type : 0, Order : 3

$$G(s)=\frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$$



$$G(j\omega)=\frac{1}{(1+j\omega T_1)\ (1+j\omega T_2)\ (1+j\omega T_3)}$$

$$=\frac{1}{\sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1\ \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2\ \sqrt{1+\omega^2T_3^2}\angle\tan^{-1}\omega T_3}$$

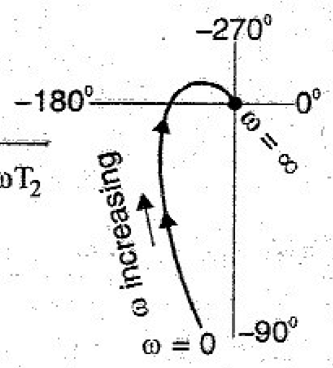
$$=\frac{1}{\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)\left(1+\omega^2T_3^2\right)}}\angle(-\tan^{-1}\omega T_1-\tan^{-1}\omega T_2-\tan^{-1}\omega T_3)$$

As $\omega\to 0,\quad G(j\omega)\to 1\angle 0^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle -270^\circ$

Type : 1, Order : 3

$$G(s)=\frac{1}{s(1+sT_1)(1+sT_2)}$$



$$G(j\omega)=\frac{1}{j\omega(1+j\omega T_1)\ (1+j\omega T_2)}=\frac{1}{\omega\angle 90^\circ\ \sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1\ \sqrt{1+\omega^2T_2^2}\ \angle\tan^{-1}\omega T_2}$$

$$=\frac{1}{\omega\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)}}\angle(-90^\circ-\tan^{-1}\omega T_1-\tan^{-1}\omega T_2)$$

As $\omega\to 0,\quad G(j\omega)\to\infty\angle -90^\circ$

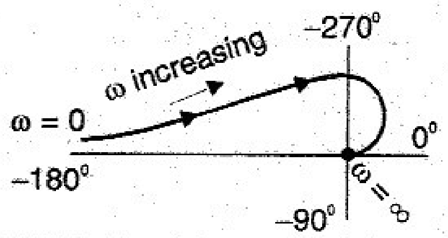
As $\omega\to\infty,\quad G(j\omega)\to 0\angle -270^\circ$

Type : 2, Order : 4

$$G(s)=\frac{1}{s^2(1+sT_1)(1+sT_2)}$$

$$G(j\omega)=\frac{1}{(j\omega)^2\ (1+j\omega T_1)\ (1+j\omega T_2)}=\frac{1}{\omega^2\angle -180^\circ\ \sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1\ \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2}$$

$$=\frac{1}{\omega^2\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)}}\angle(-180^\circ-\tan^{-1}\omega T_1-\tan^{-1}\omega T_2)$$

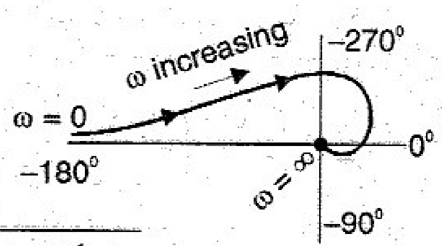


As $\omega\to 0,\quad G(j\omega)\to\infty\angle -180^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle -360^\circ$

Type : 2, Order : 5

$$G(s)=\frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$$



$$G(j\omega)=\frac{1}{(j\omega)^2\ (1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$=\frac{1}{\omega^2\angle -180^\circ\ \sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1\ \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2\ \sqrt{1+\omega^2T_3^2}\angle\tan^{-1}\omega T_3}$$

$$=\frac{1}{\omega^2\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)\left(1+\omega^2T_3^2\right)}}\angle(-180^\circ-\tan^{-1}\omega T_1-\tan^{-1}\omega T_2-\tan^{-1}\omega T_3)$$

As $\omega\to 0,\quad G(j\omega)\to\infty\angle -180^\circ$

As $\omega\to\infty,\quad G(j\omega)\to 0\angle -450^\circ=0\angle -90^\circ$

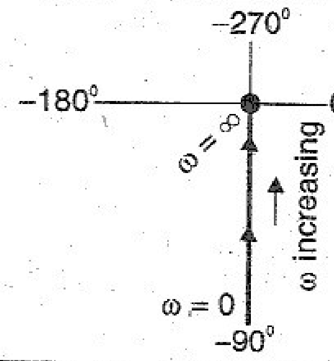
TABLE-3.1 : Typical Sketches of Polar Plot

Type : 1, Order : 1 $G(s)=\frac{1}{s}$

$$G(j\omega)=\frac{1}{j\omega}=\frac{1}{\omega\angle 90^\circ}=\frac{1}{\omega}\angle -90^\circ$$

As $\omega \to 0$, $G(j\omega) \to \infty\angle -90^\circ$

As $\omega \to \infty$, $G(j\omega) \to 0\angle -90^\circ$

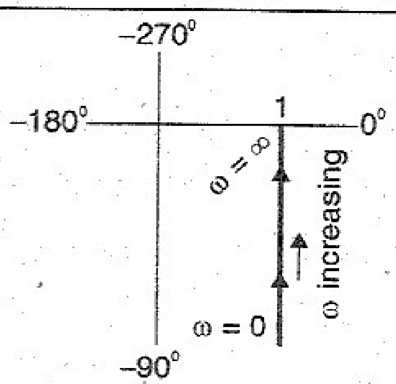


$$G(s)=\frac{1+sT}{sT}$$

$$G(j\omega) = \frac{1+j\omega T}{j\omega T}=\frac{1}{j\omega T}+1 =\frac{1}{\omega T\angle 90^\circ}+1 =\frac{1}{\omega T}\angle -90^\circ +1$$

As $\omega \to 0$, $G(j\omega) \to \infty\angle -90^\circ +1$

As $\omega \to \infty$, $G(j\omega) \to 0\angle -90^\circ +1$

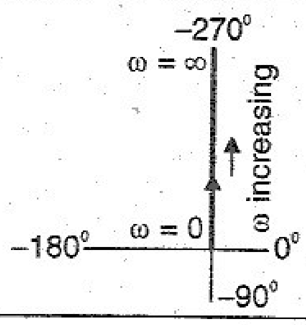


$G(s) = s$

$G(j\omega) = j\omega = \omega\angle 90^\circ$

As $\omega \to 0$, $G(j\omega) \to 0\angle 90^\circ$

As $\omega \to \infty$, $G(j\omega) \to \infty\angle 90^\circ$

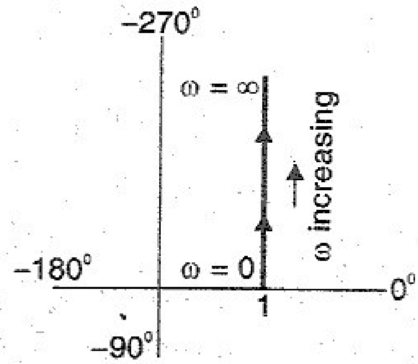


$G(s) = 1+sT$

$G(j\omega) = 1+j\omega T = 1+\omega T\angle 90^\circ$

As $\omega \to 0$, $G(j\omega) \to 1+0\angle 90^\circ$

As $\omega \to \infty$, $G(j\omega) \to 1+\infty\angle 90^\circ$



DETERMINATION OF GAIN MARGIN AND PHASE MARGIN FROM POLAR PLOT

The **gain margin** is defined as the inverse of the magnitude of G(jω) at phase crossover frequency. The **phase crossover frequency** is the frequency at which the phase of G(jω) is 180°.

Let the polar plot cut the 180° axis at point **B** and the magnitude circle passing through the point B be G_B. Now the Gain margin, $K_g = 1/G_B$. If the point B lies within unity circle, then the Gain margin is positive otherwise negative. (If the polar plot is drawn in ordinary graph sheet using rectangular coordinates then the point B is the cutting point of G(jω) locus with negative real axis and $K_g = 1/|G_B|$ where G_B is the magnitude corresponding to point B).

The **phase margin** is defined as, phase margin, $\gamma = 180^0 + \phi_{gc}$ where ϕ_{gc} is the phase angle of G(jω) at gain crossover frequency. The **gain crossover frequency** is the frequency at which the magnitude of G(jω) is unity.

Let the polar plot cut the unity circle at point A as shown in fig 3.23 and 3.24. Now the phase margin, γ is given by $\angle AOP$, i.e. if $\angle AOP$ is below $-180°$ axis then the phase margin is positive and if it is above $-180°$ axis then the phase margin is negative.

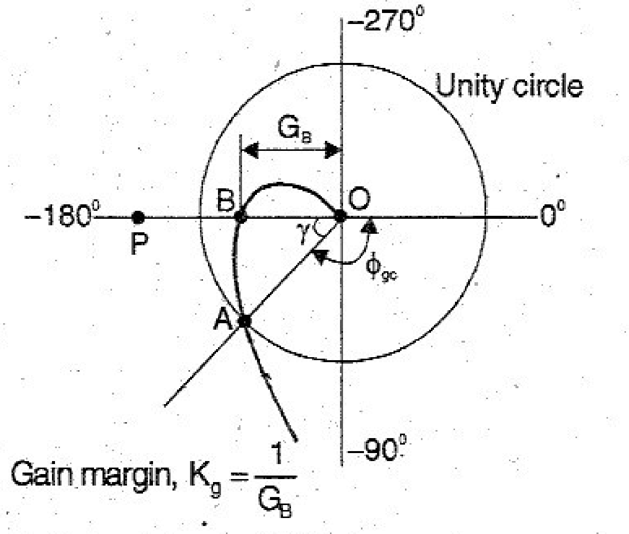


Fig 3.23 : Polar plot showing positive gain margin and phase margin.

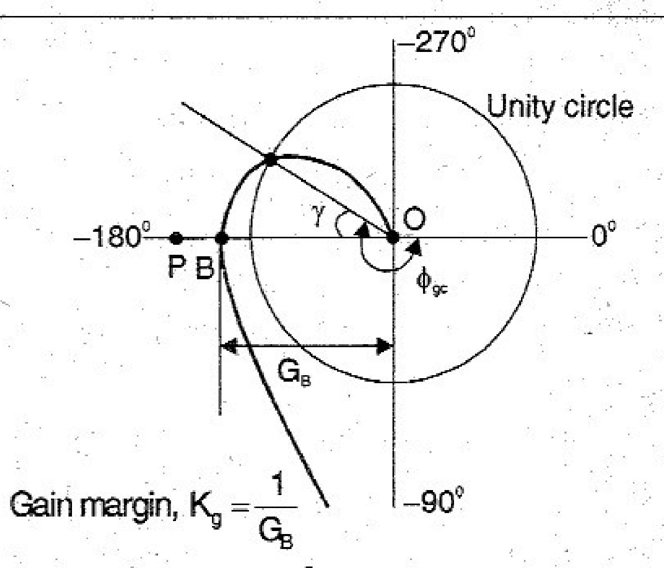


Fig 3.24 : Polar plot showing negative gain margin and phase margin.

GAIN ADJUSTMENT USING POLAR PLOT

To Determine K for Specified GM

Draw G(jω) locus with K =1. Let it cut the $-180°$ axis at point B corresponding to a gain of G_B. Let the specified gain margin be x db. For this gain margin, the G(jω) locus will cut $-180°$ at point A whose magnitude is G_A.

$$\text{Now, } 20\log\frac{1}{G_A} = x \Rightarrow \log\frac{1}{G_A} = \frac{x}{20} \Rightarrow \frac{1}{G_A} = 10^{x/20}$$

$$\therefore G_A = \frac{1}{10^{x/20}}$$

Now the value of K is given by, $K = \frac{G_A}{G_B}$.

If, K >1, then the system gain should be increased.

If, K < 1, then the system gain should be reduced.

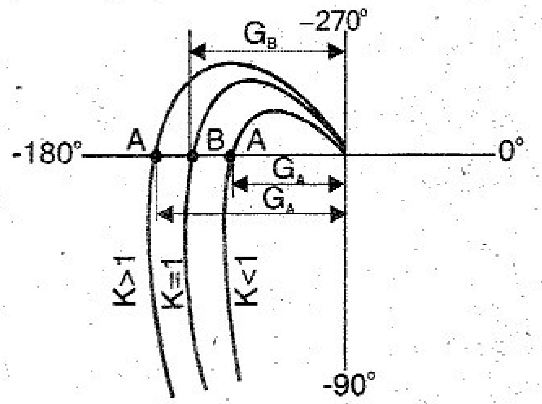


Fig 3.25 : Polar plot for different values of K

To Determine K for Specified PM

Draw G(jω) locus with K = 1. Let it cut the unity circle at point B. (The gain at point B is G_B and equal to unity). Let the specified phase margin be x°

For a phase margin of x°, let ϕ_{gcx} be the phase angle of G(jω) at gain crossover frequency.

$$\therefore x° = 180° + \phi_{gcx} \quad \Rightarrow \quad \phi_{gcx} = x° - 180°$$

In the polar plot, the radial line corresponding to ϕ_{gcx} will cut the locus of G(jω) with K = 1 at point A and the magnitude corresponding to that point be G_A

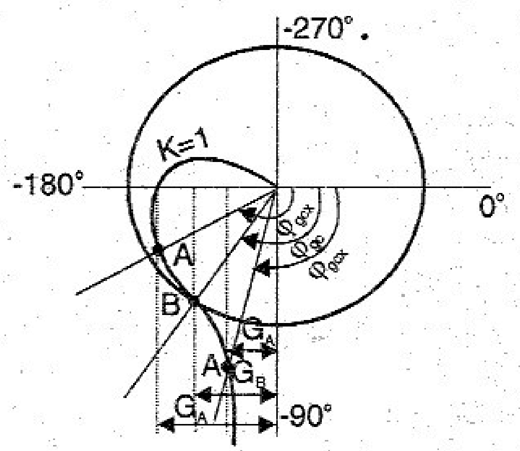


Fig 3.26 : Gain adjustment for required phase margin.

Now, $K = \frac{G_B}{G_A} = \frac{1}{G_A}$ $(\because G_B = 1)$

EXAMPLE 3.7

The open loop transfer function of a unity feedback system is given by G(s) = 1/s(1+s) (1+2s). Sketch the polar plot and determine the gain margin and phase margin.

SOLUTION

Given that, G(s) = 1/s(1+s) (1+2s)

Put s = jω.

$$\therefore G(j\omega) = \frac{1}{j\omega\,(1+j\omega)\,(1+j2\omega)}$$

The corner frequencies are $\omega_{c1} = 1/2 = 0.5$ rad/sec and $\omega_{c2} = 1$ rad/sec. The magnitude and phase angle of G(jω) are calculated for the corner frequencies and for frequencies around corner frequencies and tabulated in table-1. Using polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.7.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.7.2.

$$G(j\omega) = \frac{1}{(j\omega)\,(1+j\omega)\,(1+j2\omega)} = \frac{1}{\omega\angle 90^\circ\ \sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\ \sqrt{1+4\omega^2}\ \angle\tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)\,(1+4\omega^2)}}\ \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)\,(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}} = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

TABLE-1 : Magnitude and phase of G(jω) at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
|G(jω)|	2.2	1.8	1.5	1.2	0.9	0.7	0.3
∠G(jω) deg	-144	-150	-156	-162	-171	-179.5 ≈ -180	-198

TABLE-2 : Real and imaginary part of G(jω) at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.61	-0.37	-0.14	0	0.09

RESULT

Gain margin, $K_g = 1.4286$

Phase margin, $\gamma = +12^\circ$