PROCEDURE FOR MAGNITUDE PLOT OF BODE PLOT

From the analysis of previous sections the following conclusions can be obtained.

1. The constant gain K, integral and derivative factors contribute gain (magnitude) at all frequencies.
2. In approximate plot the first, quadratic and higher order factors contribute gain (magnitude) only when the frequency is greater than the corner frequency.

Hence the low frequency response upto the lowest corner frequency is decided by K or $K/(j\omega)^n$ or $K(j\omega)^n$ term. Then at every corner frequency the slope of the magnitude plot is altered by the first, quadratic and higher order terms. Therefore the magnitude plot can be started with K or $K/(j\omega)^n$ or $K(j\omega)^n$ term and then the db magnitude of every first and higher order terms are added one by one in the increasing order of the corner frequency.

This is illustrated in the following example.

$$\text{Let, } G(s) = \frac{K(1+sT_1)^2}{s^2(1+sT_2)(1+sT_3)}$$

$$\therefore G(j\omega) = \frac{K(1+j\omega T_1)^2}{(j\omega)^2(1+j\omega T_2)(1+j\omega T_3)}$$

Let, $T_2 < T_3 < T_1$.

The corner frequencies are, $\omega_{c1} = \frac{1}{T_1}$, $\omega_{c2} = \frac{1}{T_2}$, $\omega_{c3} = \frac{1}{T_3}$.

Let, $\omega_{c1} < \omega_{c3} < \omega_{c2}$.

The magnitude plot of the individual terms of $G(j\omega)$, and their combined magnitude plot are shown in fig 3.18.

The step by step procedure for plotting the magnitude plot is given below

Step1 : Convert the transfer function into Bode form or time constant form. The Bode form of the transfer function is

$$G(s) = \frac{K(1+sT_1)}{s(1+sT_2)\left(1+\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}\right)} \xrightarrow{s=j\omega} G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)\left(1-\frac{\omega^2}{\omega_n^2}+j2\zeta\frac{\omega}{\omega_n}\right)}$$

Step 2 : List the corner frequencies in the increasing order and prepare a table as shown below.

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec

In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)^n$ as the first term and the other terms in the increasing order of corner frequencies. Then enter the corner frequency, slope contributed by each term and change in slope at every corner frequency.

Step 3 : Choose an arbitrary frequency ω_l which is lesser than the lowest corner frequency. Calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)^n$ at ω_l and at the lowest corner frequency.

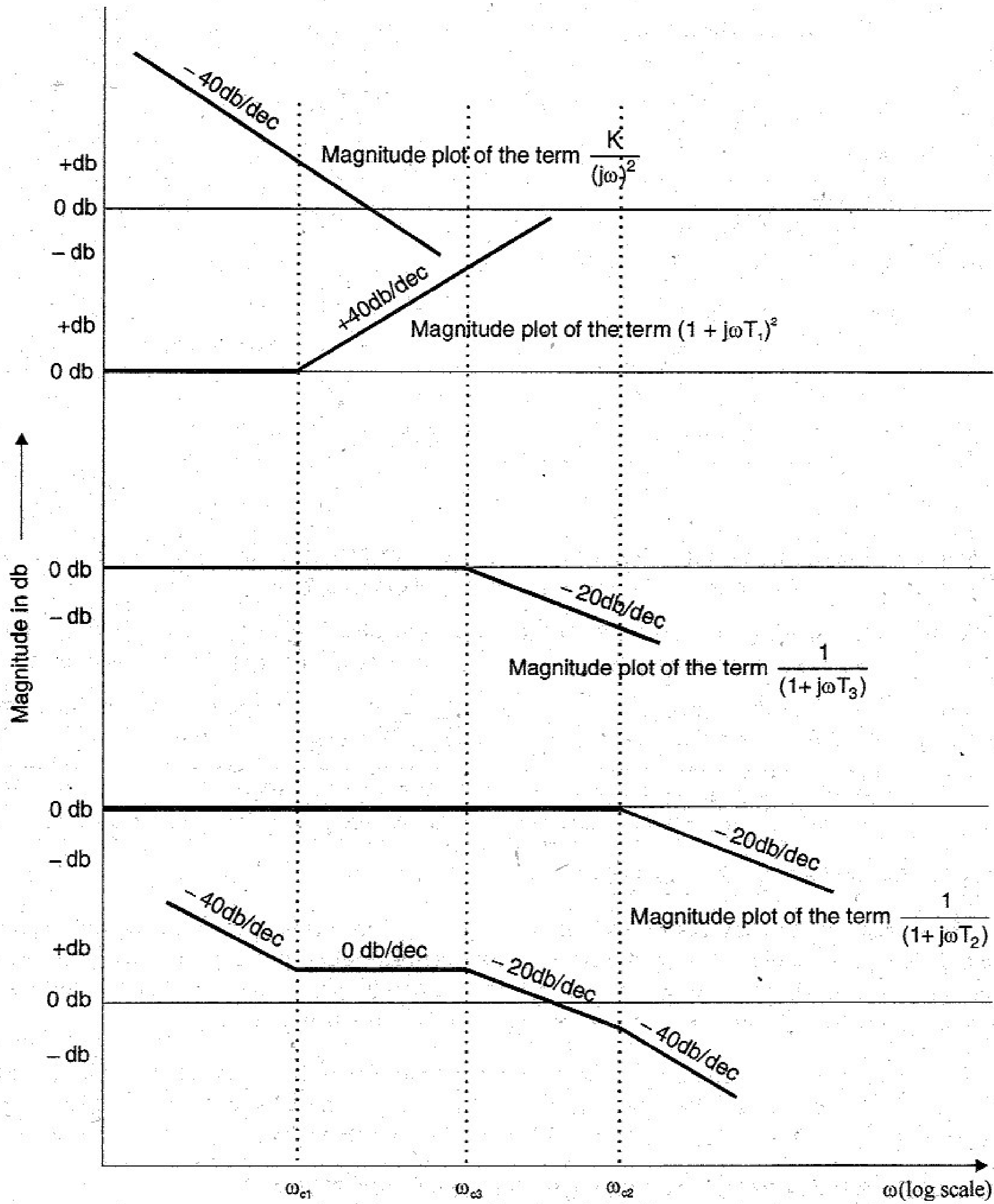
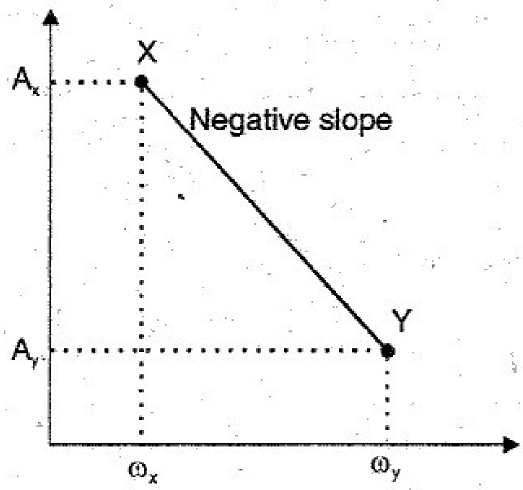


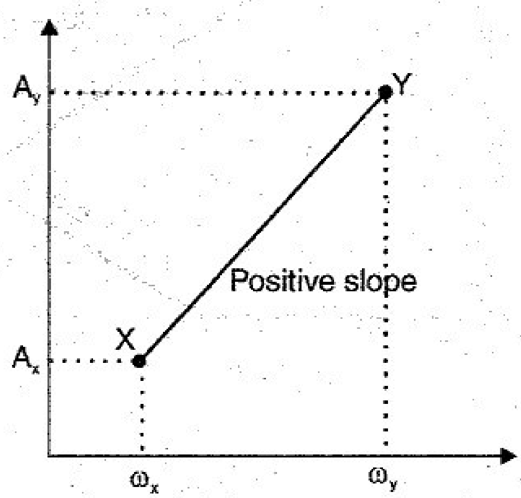
Fig 3.18 : *Magnitude plot of bode plot of,* $G(j\omega) = \dfrac{K(1+j\omega T_1)^2}{(j\omega)^2(1+j\omega T_2)(1+j\omega T_3)}$.

Step 4 : Then calculate the gain (db magnitude) at every corner frequency one by one by using the formula,

Gain at ω_y = change in gain from ω_x to ω_y + Gain at ω_x

$$= \left[\text{Slope from } \omega_x \text{ to } \omega_y \times \log\frac{\omega_y}{\omega_x}\right] + \text{Gain at } \omega_x$$





Step 5 : Choose an arbitrary frequency ω_h which is greater than the highest corner frequency. Calculate the gain at ω_h by using the formula in step 4.

Step 6 : In a semilog graph sheet mark the required range of frequency on x-axis (log scale) and the range of db magnitude on y-axis (ordinary scale) after choosing proper units.

Step 7 : Mark all the points obtained in steps 3, 4, and 5 on the graph and join the points by straight lines. Mark the slope at every part of the graph.

> ***Note :*** *The magnitude plot obtained above is an approximate plot. If an exact plot is needed then appropriate corrections should be made at every corner frequencies.*

PROCEDURE FOR PHASE PLOT OF BODE PLOT

The phase plot is an exact plot and no approximations are made while drawing the phase plot. Hence the exact phase angles of G(jω) are computed for various values of ω and tabulated. The choice of frequencies are preferably the frequencies chosen for magnitude plot. Usually the magnitude plot and phase plot are drawn in a single semilog - sheet on a common frequency scale.

Take another y-axis in the graph where the magnitude plot is drawn and in this y-axis mark the desired range of phase angles after choosing proper units. From the tabulated values of ω and phase angles, mark all the points on the graph. Join the points by a smooth curve.

DETERMINATION OF GAIN MARGIN AND PHASE MARGIN FROM BODE PLOT

The gain margin in db is given by the negative of db magnitude of G(jω) at the phase cross-over frequency, ω_{pc}. The ω_{pc} is the frequency at which phase of G(jω) is $-180°$. If the db magnitude of G(jω) at ω_{pc} is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of G(jω) at gain cross over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of G(jω) is zero. Now the phase margin, γ is given by, $\gamma = 180° + \phi_{gc}$. If ϕ_{gc} is less negative than $-180°$ then phase margin is positive and vice versa.

The positive and negative gain margins and phase margins are illustrated in fig 3.19.

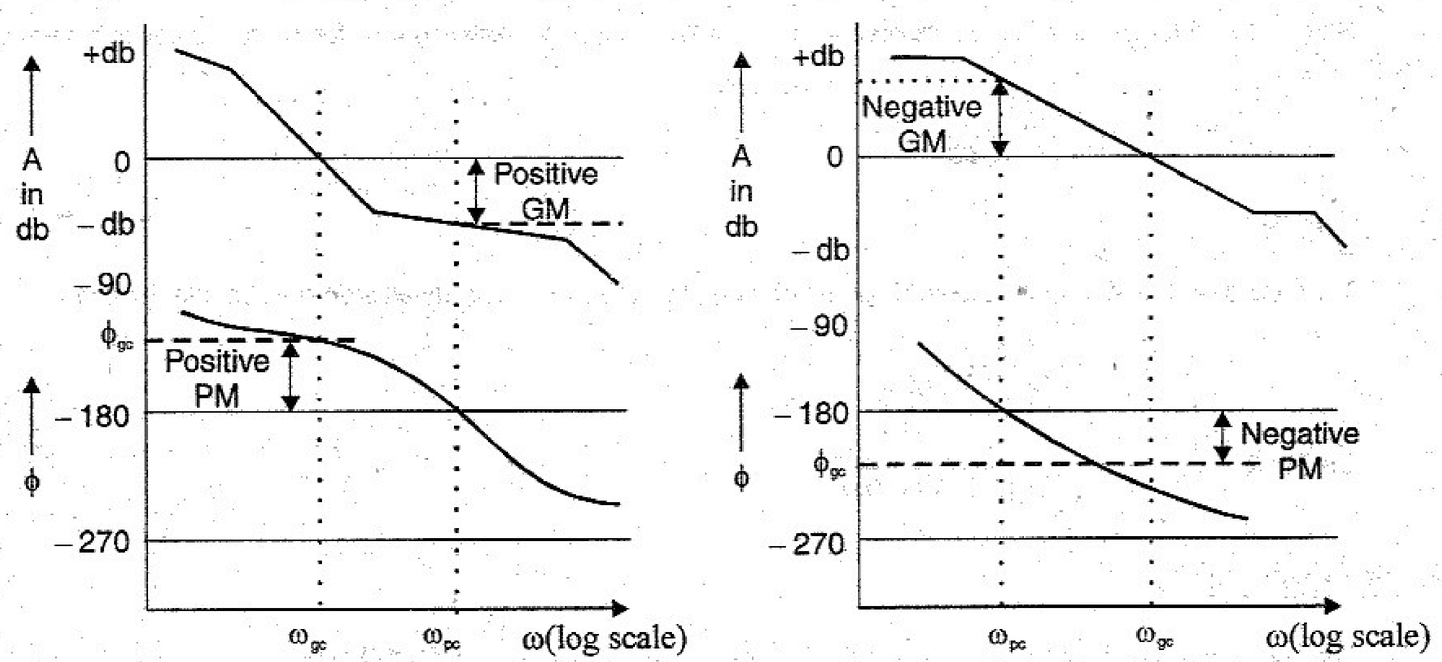


Fig 3.19 : Bode plot showing phase margin (PM) and gain margin (GM).

GAIN ADJUSTMENT IN BODE PLOT

In the open loop transfer function G(jω) the constant K contributes only magnitude. Hence by changing the value of K the system gain can be adjusted to meet the desired specifications. The desired specifications are gain margin, phase margin, ω_{pc} and ω_{gc}. In a system transfer function if the value of K required to be estimated to satisfy a desired specification then draw the bode plot of the system with K = 1. The constant K can add 20 log K to every point of the magnitude plot and due to this addition the magnitude plot will shift vertically up or down. Hence shift the magnitude plot vertically up or down to meet the desired specification. Equate the vertical distance by which the magnitude plot is shifted to 20 log K and solve for K.

Let, x = change in db (x is positive if the plot is shifted up and vice versa).

Now, $20 \log K = x$; $\log K = x/20$; $\therefore K = 10^{x/20}$

> ***Note :** A point in complex plane can be represented by rectangular coordinates or by polar coordinates. Consider a point, $z = a+jb$ in complex plane.*
>
> *Now,* $|z| = \sqrt{a^2+b^2}$ *and* $\angle z = \tan^{-1} b/a$.
>
> *If the point lies in first or fourth quadrant then the argument as calculated by $\tan^{-1} b/a$ will be the correct values. But if it lies either in second or third quadrant then a correction should be made in the calculated values of argument, because the calculator will always give the values of $\tan^{-1} b/a$ either from 0 to + 90° or from 0 to–90°. The corrections to be made while converting from rectangular to polar coordinates is shown below.*
>
> A point in I[st] quadrant, $a + jb = \sqrt{a^2+b^2}\ \angle \tan^{-1} b/a$
>
> A point in II[nd] quadrant, $-a + jb = \sqrt{a^2+b^2}\ \angle(\pi - \tan^{-1} b/a)$
>
> A point in III[rd] quadrant, $-a - jb = \sqrt{a^2+b^2}\ \angle(\pi + \tan^{-1} b/a)$
>
> A point in IV[th] quadrant, $a - jb = \sqrt{a^2+b^2}\ \angle -\tan^{-1} b/a)$

EXAMPLE 3.1

Sketch Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

SOLUTION

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in the given s-domain transfer function.

$$\therefore\ G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

$$\text{Let } K = 1,\quad \therefore\ G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec}$ and $\omega_{c2} = \frac{1}{0.02} = 50 \text{ rad/sec}$

The various terms of G(jω) are listed in Table-1 in the increasing order of their corner frequency. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$(j\omega)^2$	–	+40	
$\frac{1}{1+j0.2\omega}$	$\omega_{c1} = \frac{1}{0.2} = 5$	–20	$40 - 20 = 20$
$\frac{1}{1+j0.02\omega}$	$\omega_{c2} = \frac{1}{0.02} = 50$	20	$20 - 20 = 0$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad/sec and $\omega_h = 100$ rad/sec.

Let, $A = |G(j\omega)|$ in db.

Let us calculate A at $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h.

$$\text{At } \omega = \omega_l,\quad A = 20\log\left|(j\omega)^2\right| = 20\log(\omega)^2 = 20\log(0.5)^2 = -12 \text{ db}$$

$$\text{At } \omega = \omega_{c1},\quad A = 20\log\left|(j\omega)^2\right| = 20\log(\omega)^2 = 20\log(5)^2 = 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2},\quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})} = 20 \times \log\frac{50}{5} + 28 = 48 \text{ db}$$

$$\text{At } \omega = \omega_h,\quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})} = 0 \times \log\frac{100}{50} + 48 = 48 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on, logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by straight lines and mark the slope on the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by,

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$$

The phase angle of G(jω) are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg	Point in phase plot
0.5	5.7	0.6	$173.7 \approx 174$	e
1	11.3	1.1	$167.6 \approx 168$	f
5	45	5.7	$129.3 \approx 130$	g
10	63.4	11.3	$105.3 \approx 106$	h
50	84.3	45	$50.7 \approx 50$	i
100	87.1	63.4	$29.5 \approx 30$	j

On the same semilog graph sheet choose a scale of 1unit = 20°, on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

CALCULATION OF K

Given that the gain crossover frequency is 5 rad/sec. At ω = 5 rad/sec the gain is 28 db. If gain crossover frequency is 5 rad/sec then at that frequency the db gain should be zero. Hence to every point of magnitude plot a db gain of –28db should be added. The addition of –28db shifts the plot downwards. The corrected magnitude plot is obtained by shifting the plot with K = 1 by –28db downwards. The magnitude correction is independent of frequency. Hence the magnitude of –28db is contributed by the term K. The value of K is calculated by equating 20 logK to –28 db.

$$\therefore 20 \log K = -28 \text{ db}$$

$$\log K = \frac{-28}{20} \; ; \; K = 10^{-\left(\frac{28}{20}\right)} = 0.0398$$

The magnitude plot with K = 1 and 0.0398 and the phase plot are shown in fig 3.1.1

> ***Note :** The frequency ω = 5 rad/sec is a corner frequency. Hence in the exact plot the db gain at ω = 5 rad/sec will be 3db less than the actual plot. Therefore for exact plot the 20 log K will contribute a gain of –25db.*
>
> $$\therefore 20 \log K = -25 \text{ db}$$
>
> $$\log K = \frac{-25}{20} \; ; \; K = 10^{-\left(\frac{25}{20}\right)} = 0.0562$$

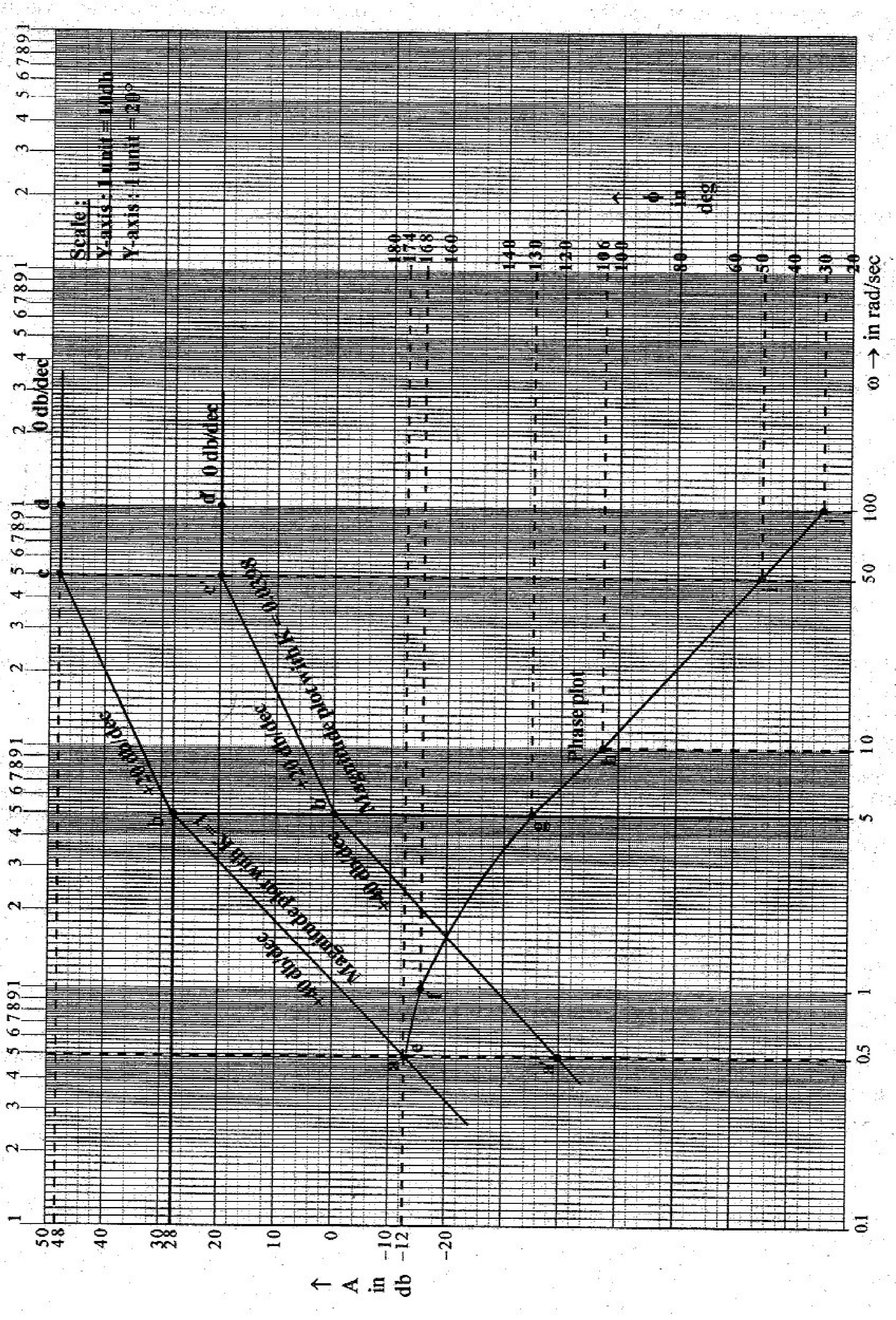


Fig 3.1.1 : Bode plot of transfer function, $G(j\omega) = \dfrac{K(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$

EXAMPLE 3.2

Sketch the bode plot for the following transfer function and determine phase margin and gain margin.

$$G(s)=\frac{75\,(1+0.2s)}{s\left(s^2+16s+100\right)}$$

SOLUTION

On comparing the quadratic factor in the denominator of G(s) with standard form of quadratic factor we can estimate ζ and ω_n.

$$\therefore s^2+16s+100 = s^2+2\zeta\omega_n s+\omega_n^2$$

On comparing we get,

$$\omega_n^2=100 \quad\Rightarrow\quad \omega_n=10$$

$$2\zeta\omega_n=16 \quad\Rightarrow\quad \zeta=\frac{16}{2\omega_n}=\frac{16}{2\times 10}=0.8$$

Let us convert the given s-domain transfer function into bode form or time constant form.

$$\therefore G(s)=\frac{75(1+0.2s)}{s\left(s^2+16s+100\right)}=\frac{75\,(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}=\frac{0.75\,(1+0.2s)}{s\left(1+0.01s^2+0.16s\right)}$$

The sinusoidal transfer function $G(j\omega)$ is obtained by replacing s by $j\omega$ in G(s).

$$\therefore G(j\omega)=\frac{0.75\,(1+0.2\,j\omega)}{j\omega\left(1+0.01(j\omega)^2+0.16j\omega\right)}=\frac{0.75\,(1+j0.2\omega)}{j\omega\left(1-0.01\omega^2+j0.16\omega\right)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1}=\frac{1}{0.2}=5$ rad / sec and $\omega_{c2}=\omega_n=10$ rad / sec

Note : For the quadratic factor the corner frequency is ω_n.

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	–	–20	
$1+j0.2\omega$	$\omega_{c1}=\frac{1}{0.2}=5$	20	$-20+20=0$
$\frac{1}{1-0.01\omega^2+j0.16\omega}$	$\omega_{c2}=\omega_n=10$	– 40	$0-40=-40$

Choose a low frequency ω_l such that $\omega_l<\omega_{c1}$ and choose a high frequency ω_h such that $\omega_h>\omega_{c2}$.

Let, ω_l = 0.5 rad/sec and ω_h = 20 rad/sec.

Let, $A=|G(j\omega)|$ in db.

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At, } \omega=\omega_l,\ A=20\log\left|\frac{0.75}{j\omega}\right|=20\log\frac{0.75}{0.5}=3.5\text{ db}$$

$$\text{At, } \omega=\omega_{c1},\ A=20\log\left|\frac{0.75}{j\omega}\right|=20\log\frac{0.75}{5}=-16.5\text{ db}$$

$$\text{At, } \omega=\omega_{c2},\ A=\left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2}\times\log\frac{\omega_{c2}}{\omega_{c1}}\right]+A_{(\text{at }\omega=\omega_{c1})}$$

$$=0\times\log\frac{10}{5}+(-16.5)=-16.5\text{ db}$$

$$\text{At } \omega=\omega_h,\ A=\left[\text{slope from } \omega_{c2} \text{ to } \omega_h\times\log\frac{\omega_h}{\omega_{c2}}\right]+A_{(\text{at }\omega=\omega_{c2})}$$

$$=-40\times\log\frac{20}{10}+(-16.5)=-28.5\text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 5 db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a,b,c and d on the graph. Join the points by straight lines and mark the slope on the respective region.

> *Note : In quadratic factors the phase varies from 0° to 180°. But calculator calculates $\tan^{-1}$ only between 0° to 90°. Hence a correction of 180° should be added to phase after ω_n.*

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by,

$$\phi=\angle G(j\omega)=\tan^{-1}0.2\omega-90^\circ-\tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}\quad\text{for } \omega\le\omega_n$$

$$\phi=\angle G(j\omega)=\tan^{-1}0.2\omega-90^\circ-\left(\tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}+180^\circ\right)\quad\text{for } \omega>\omega_n$$

The phase angle of G(jω) are calculated for various values of w and listed in Table-2.

TABLE-2

ω rad/sec	$\tan^{-1}0.2\omega$ deg	$\tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}$ deg	$\phi=\angle G(j\omega)$ deg	Points in phase plot
0.5	5.7	4.6	$-88.9\approx-88$	e
1	11.3	9.2	$-87.9\approx-88$	f
5	45	46.8	$-91.8\approx-92$	g
10	63.4	90	$-116.6\approx-116$	h
20	75.9	−46.8+180 =133.2	$-147.3\approx-148$	i
50	84.3	−18.4+180 =161.6	$-167.3\approx-168$	j
100	87.1	−92+180 =170.8	$-173.7\approx-174$	k

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

The magnitude plot and the phase plot are shown in fig 3.2.1.

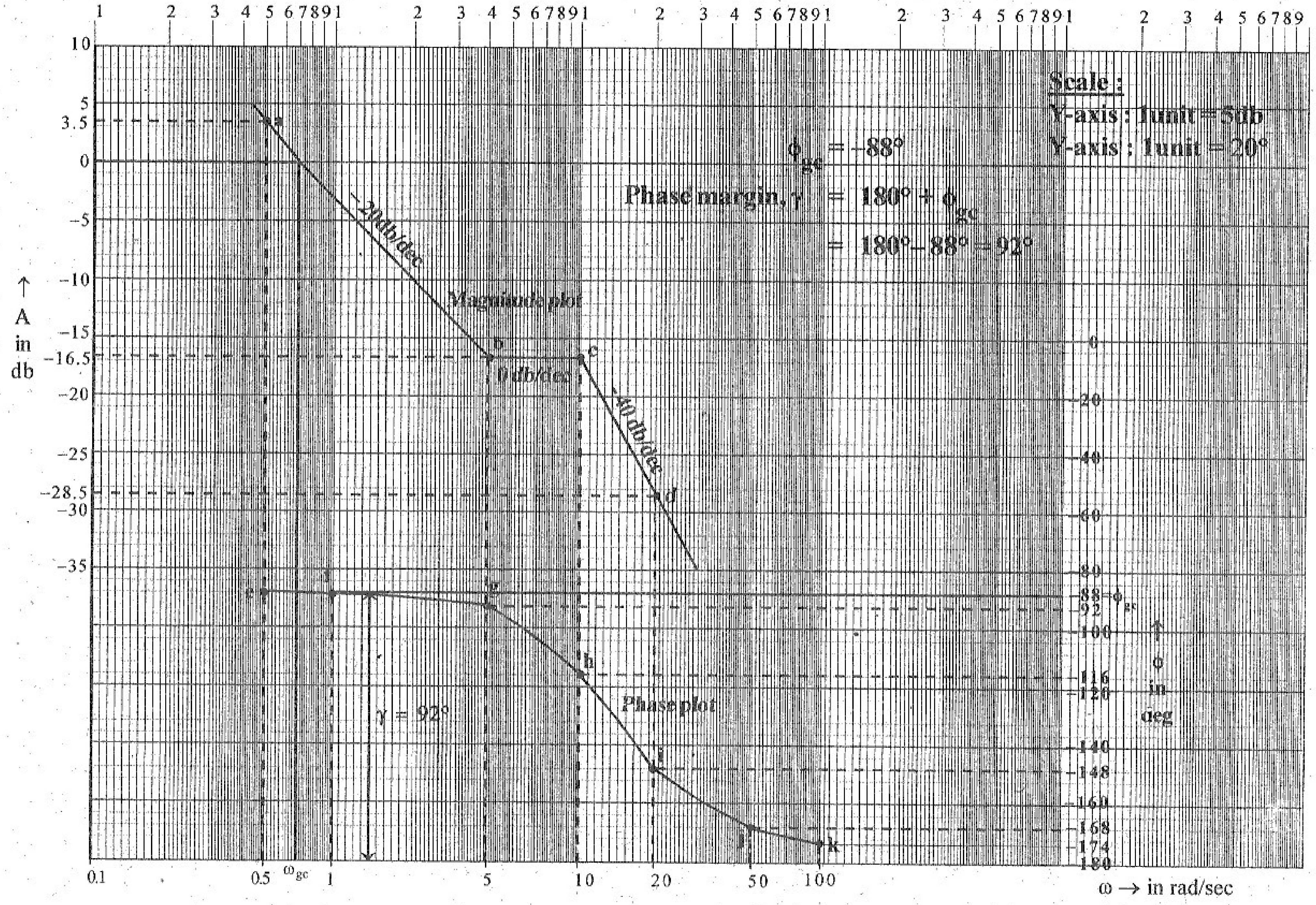


Fig 3.2.1 : Bode plot of transfer function, $G(j\omega) = \dfrac{0.75(1+j0.2\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}$

Let ϕ_{gc} be the phase of $G(j\omega)$ at gain cross-over frequency, ω_{gc}.

From the fig 3.2.1, we get, $\phi_{gc} = 88°$

$$\therefore \text{Phase margin, } g = 180° + \phi_{gc} = 180° - 88° = 92°$$

The phase plot crosses –180° only at infinity. The $|G(j\omega)|$ at infinity is $-\infty$ db.

Hence gain margin is $+\infty$.

EXAMPLE 3.3

Given, $G(s) = \dfrac{K\,e^{-0.2s}}{s\,(s+2)\,(s+8)}$. Find K so that the system is stable with,

(a) gain margin equal to 2db, (b) phase margin equal to 45°.

SOLUTION

Let us take K = 1, and convert the given transfer function to time constant form or bode form.

$$\therefore G(s) = \frac{e^{-0.2s}}{s\,(s+2)(s+8)} = \frac{e^{-0.2s}}{s \times 2\left(1+\dfrac{s}{2}\right) \times 8\left(1+\dfrac{s}{8}\right)} = \frac{0.0625\ e^{-0.2s}}{s\,(1+0.5s)(1+0.125s)}$$

The sinusoidal transfer function $G(j\omega)$ is obtained by replacing s by $j\omega$ in G(s).

$$\therefore G(j\omega) = \frac{0.0625\ e^{-j0.2\omega}}{j\omega\ (1+j0.5\omega)\ (1+j0.125\omega)}$$

Note : $\left|0.0625\,e^{-j0.2\omega}\right| = 0.0625$ *and* $\angle(0.0625e^{-j0.2\omega}) = -0.2\omega$ *radians.*

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \dfrac{1}{0.5} = 2$ rad / sec and $\omega_{c2} = \dfrac{1}{0.125} = 8$ rad / sec

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\dfrac{0.0625}{j\omega}$	–	–20	
$\dfrac{1}{1+j0.5\omega}$	$\omega_{c1} = \dfrac{1}{0.5} = 2$	–20	–20 –20 = –40
$\dfrac{1}{1+j0.125\omega}$	$\omega_{c2} = \dfrac{1}{0.125} = 8$	–20	– 40 –20 = –60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, $\omega_l = 0.5$ rad / sec and $\omega_h = 50$ rad / sec.

Let, $A = |G(j\omega)|$ in db.

Let us calculate A at $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h.

$$\text{At } \omega = \omega_l, \ A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{0.5} = -18\text{db}$$

$$\text{At } \omega = \omega_{c1}, \ A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{2} = -30\text{db}$$

$$\text{At } \omega = \omega_{c2}, \ A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})} = -40 \times \log\frac{8}{2} + (-30) = -54\text{db}$$

$$\text{At } \omega = \omega_h, \ A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})} = -60 \times \log\frac{50}{8} + (-54) = -102\text{db}$$

Let the points a,b,c,d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10db on y-axis. The frequencies are marked in decades from 0.01 to 100 rad/sec on logarithmic scale in x-axis. Fix the points a, b, c, and d on the graph. Join the points by straight line and mark the slope on the respective region.

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = -0.2\omega \times \frac{180^\circ}{\pi} - 90^\circ - \tan^{-1}0.5\omega - \tan^{-1}0.125\omega$$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$-0.2\ \omega\ (180^0/\pi)$ deg	$\tan^{-1} 0.5\ \omega$ deg	$\tan^{-1} 0.125\ \omega$ deg	$\phi = \angle G(j\omega)$ deg	Point in phase plot
0.01	−0.1145	0.2864	0.0716	$-90.4 \approx -90$	e
0.1	−1.145	2.862	0.716	$-94.7 \approx -94$	f
0.5	−5.7	14	3.6	$-113.3 \approx -114$	g
1	−11.4	26	7.12	$-134.4 \approx -134$	h
2	−22.9	45	14	$-171.9 \approx -172$	i
3	−34.37	56.30	20.56	$-201.2 \approx -202$	j
4	−45.84	63.43	26.57	$-225.8 \approx -226$	k

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of the semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by smooth curve.

The magnitude and phase plot are shown in fig 3.3.1.

CALCULATION OF K

Phase margin, $\gamma = 180^0 + \phi_{gc}$, where ϕ_{gc} is the phase of $G(j\omega)$ at $\omega = \omega_{gc}$.

When $\gamma = 45°$, $\phi_{gc} = \gamma - 180^0 = 45^0 - 180^0 = -135°$.

With K = 1, the db gain at $\phi = -135°$ is −24 db. This gain should be made zero to have to PM of 45°. Hence to every point of magnitude plot a db gain of 24 db should be added. The corrected magnitude plot is obtained by shifting the plot with K = 1 by 24 db upwards. The magnitude correction is independent of frequency. Hence the magnitude of 24 db is contributed by the term K. The value of K is calculated by equating 20logK to 24 db.

$$\therefore 20\log K = 24 \quad ; \quad K = 10^{24/20} \quad ; \quad K = 15.84$$

With K = 1, the gain margin = −(−32) = 32 db. But the required gain margin is 2 db. Hence to every point of magnitude plot a db gain of 30 db should be added. This addition of 30 db shifts the plot upwards. The magnitude correction is independent of frequency. Hence the magnitude of 30 db is contributed by the term K. The value of K is calculated by equating 20logK to 30 db.

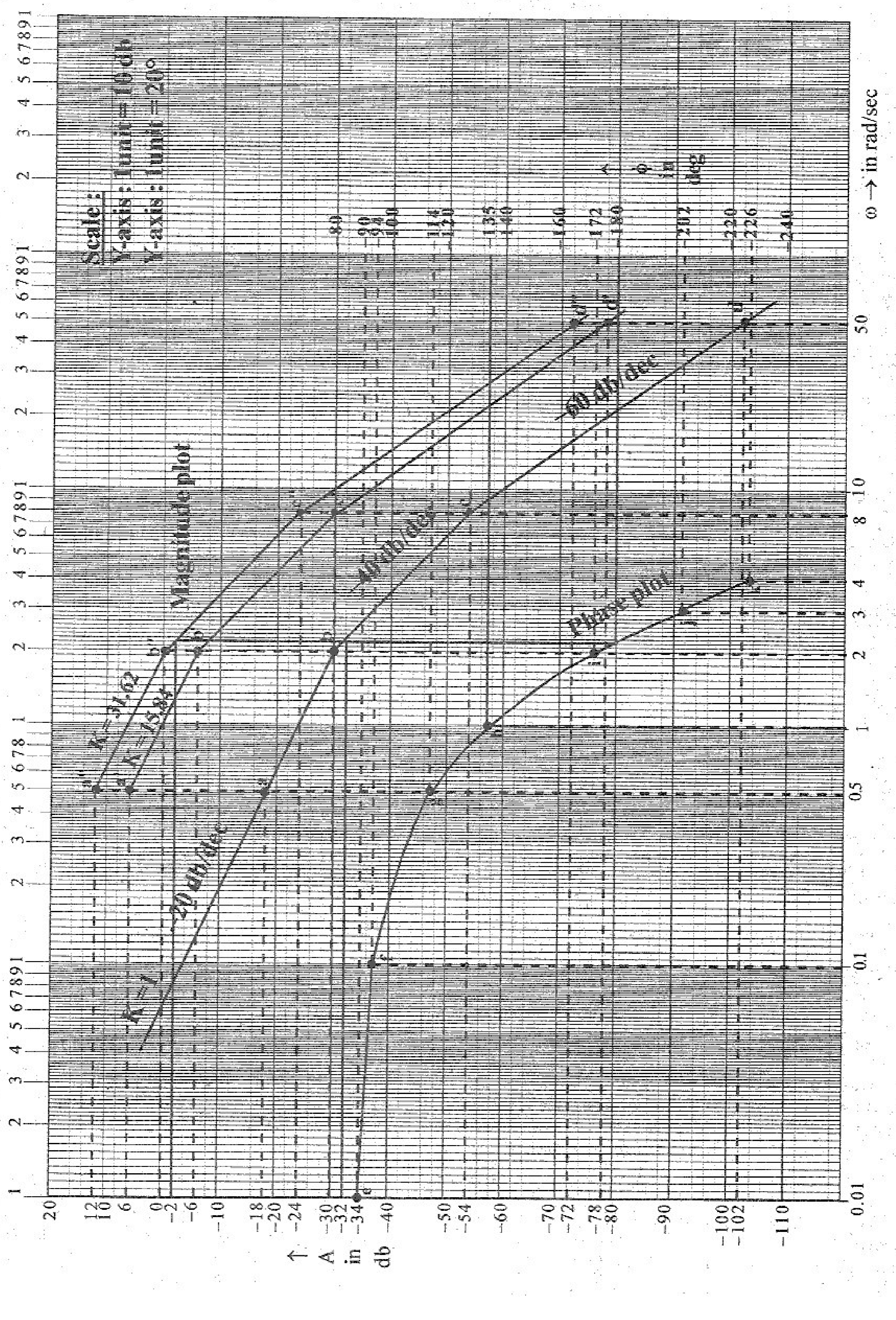


Fig 3.3.1 : *Bode plot of transfer function,* $G(j\omega) = \dfrac{0.0625Ke^{-j0.2\omega}}{j\omega(1+j0.5\omega)(1+j0.125\omega)}$

$\therefore$ 20 log K = 30 ; $K = 10^{30/20}$; K = 31.62

The magnitude plot with K = 15.84 and 31.62 are shown in fig 3.3.1.

EXAMPLE 3.4

Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies.

$$G(s) = \frac{10}{s\,(1+0.4s)\,(1+0.1s)}$$

SOLUTION

The sinusoidal transfer function of G(jω) is obtained by replacing s by jω in the given transfer function.

$$\therefore G(j\omega) = \frac{10}{j\omega\,(1+j0.4\omega)\,(1+j0.1\omega)}$$

MAGNITUDE PLOT

The corner frequencies are,

$$\omega_{c1} = \frac{1}{0.4} = 2.5 \text{ rad/sec} \text{ and } \omega_{c2} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

The various terms of G(jω) are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{10}{j\omega}$	–	– 20	
$\frac{1}{1+j0.4\omega}$	$\omega_{c1} = \frac{1}{0.4} = 2.5$	– 20	–20 –20 = – 40
$\frac{1}{1+j0.1\omega}$	$\omega_{c2} = \frac{1}{0.1} = 10$	– 20	– 40 – 20 = – 60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let, ω_l = 0.1 rad/sec, and ω_h = 50 rad/sec.

Let, A = |G(jω)| in db.

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At } \omega = \omega_l,\quad A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{0.1} = 40 \text{ db}$$

$$\text{At } \omega = \omega_{c1},\quad A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{2.5} = 12 \text{ db}$$

$$\text{At } \omega = \omega_{c2},\quad A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})} = -40 \times \log\frac{10}{2.5} + 12 = -12 \text{ db}$$

$$\text{At } \omega = \omega_h,\quad A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega = \omega_{c2})} = -60 \times \log\frac{50}{10} + (-12) = -54 \text{ db}$$