Q1. What is a control System:-

Soln A system consists of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity, then the System is called control system. The output quantity is called controlled variable or response and input quantity is called command signal or excitation.

Q2. Define transfer function?

Soln The transfer function of a system is defined as the ratio of Laplace transform of output to Laplace transform of input with zero initial conditions. (It is also defined as the Laplace transform of the impulse response of System with zero initial condition)

Q3. What is a State variable mode?

Soln

Q4 What are the advantage and disadvantage of open loop system.

Soln Advantage of open loop System.

1> The open loop system are simple and economical.

2> The open loop system are easier to construct.

3> Generally the open loop system are stable

Disadvantage

1> The open loop systems are inaccurate and unreliable

2> The change in the output due to external disturbance are not corrected automatically.

Q5 What are the advantage and disadvantage of closed loop system

Soln Advantage

1> The closed loop system are accurate.

2> The closed loop system are accurate even in the presence of non-linearities

3> The closed loop system are less affected by noise.

4> The sensitivity of the System may be made small to make the system more stable.

Disadvantage

1> The closed loop system are complex and costly.

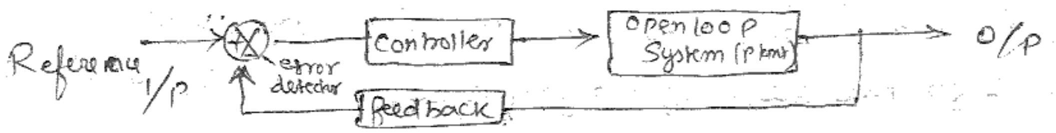
2> The feedback in closed loop System may lead to oscillatory response.

3> The feedback reduces the overall gain of the system.

4> Stability is a major problem in a closed loop system.

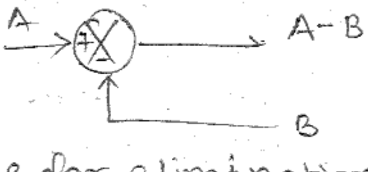
Q6 What are the components of feedback control system?

Soln The component of feedback control system are plant, feedback path element, error detector and controller.



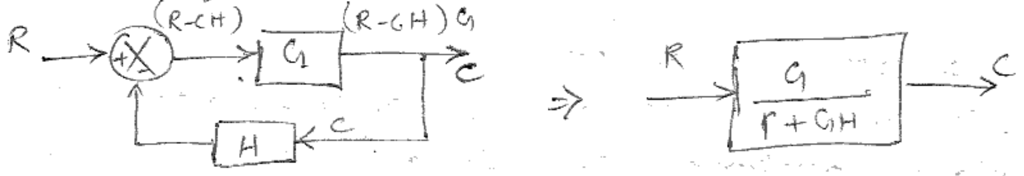
Q7 What is a summing point?

Soln Summing point are used to add two or more signals in the system.

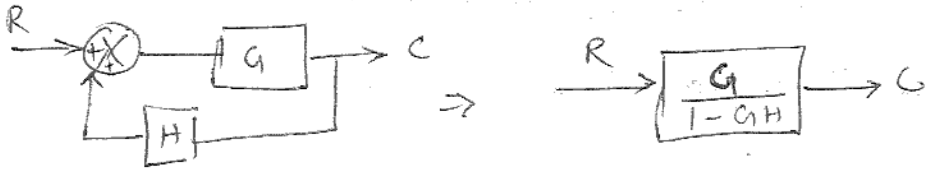


Q8 Write the rule for elimination of a feedback loop in a block diagram.

Soln Elimination of (-ve) feedback loop.



Elimination of (+ve) feedback loop.



Q10 Define loop gain.

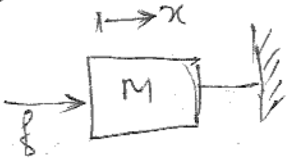
Soln It is the product of the branch transmittance (gains) of a loop.

Q11 What is sink and source?

Soln Source is the input node in the signal flow graph and it has only outgoing branches. Sink is a output node in the signal flow graph and it has only incoming branches.

Q12 Write the force balance equation of ideal mass element.

Soln Let a force f be applied to an ideal mass M. The mass will offer an opposing force, f_m which is proportional to acceleration.



$$\therefore \; f = f_m = M \frac{d^2x}{dt^2}$$

Q15 Write the Mason's Gain formula.

Soln Mason's gain formula states that the overall gain of the system [transfer function] as follow.

$$\text{Over all gain} = T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

T → Transfer f^n.

K = Number of forward paths in the signal flow graph.

P_K = Forward path gain of K^{th} forward path.

Δ_K = Δ for the part of the graph which is not touching K^{th} F.P.

Q16 What are the characteristics of negative feedback.?

Sol^n The characteristics of negative feedback as as follows.

i) Accuracy in tracking steady state value.

ii) rejection of disturbance signal.

iii) low sensitivity to parameter variation.

iv) reduction in gain at the expense of better stability.

Q18 Write the torque balance Eq^n of ideal rotational spring.

Sol^n Let a torque T be applied to an ideal rotational spring constant K. The spring will offer an opposing torque T_K which is proportional to angular displacement

$T_K = K\theta$ $\qquad$ $T = T_K = K(\theta_1 - \theta_2)$

Q21 Define non-touching loop.

Sol^n The loop are said to be non touching if they donot have common nodes.

Q22 Name the two types of electrical analogues for mechanical system.

Sol^n The two types of analogous for Mechanical System are

* Force-voltage and Force current analog.

Q23 ~~What~~ What is signal flow graph.

Sol^n A Signal flow graph is a diagram that represents a Set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in S-domain. The Signal flow graph of the system can be constructed using these equations.

Unit II Part-A

Q1 Define Peak time?

Soln It is the time taken for the response to reach the peak value, the very first time (or) It is the time taken for the response to reach peak over shoot, M_p.

Q2 Define Peak Over shoot?

Soln It is defined as the ratio of the maximum peak value to final value, where maximum peak value is measured from final value.

Let final value $= C(\infty)$, Maxm value $= c(t_p)$

$\therefore$ Peak Over Shoot, $M_p = \dfrac{C(t_p) - C(\infty)}{C(\infty)}$

Q3 Define Settling Time.

Soln It is defined as the time taken by the response to reach and stay within a specified error and the error is usually specified as % of final value. The usual tolerable error is 2% or 5% of the final value.

Q4 What is the order of a system.

Soln The order of the System is given by the order of the differential Equation govering the System. It is also given by the maximum power of s in the denominator polynomial of transfer function. The maxm power of s also gives the number of poles of the System and so the order of the System is also given by number of poles of the transfer function.

Q6 How the System is classified depending on the value of damping?

Soln Depending on the value of damping, the System can be classified into the following four cases.

Case 1: Undamped System, $\zeta = 0$

Case 2: Underdamped " , $0 < \zeta < 1$

Case 3: Critically damped system, $\zeta = 1$

Case 4: Over damped system, $\zeta > 1$

Q7 What is transient and steady State response?

Soln The transient response is the response of the system when the input changes from one state to another. The response of the system as $t \to \infty$ is called Steady State response.

Q8 Name the test signal used for control system.
Sol^n The commonly used test input signals in control system are Impulses, Step, Ramp, Acceleration are sinusoidal signal.

Q.10. Define pole.
Sol The pole of a function F(S) is the value at which the function, F(S) becomes infinite, where F(S) is a function of complex variable S.

Q11) What is damped frequency of oscillation.
Sol^n In underdamped system the response is damped oscillatory. The frequency of damped oscillation is given by,

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

Q12 Define rise time.
Sol^n It is the time taken for response to raise from 0 to 100% the very first time. For underdamped system the rise time is calculated from 0 to 100%. But the overdamped systems it is the time taken by the response to raise from 10% to 90%. for critically damped system. It is the time taken for response to raise from 5% to 95%.

Q13 Distinguish b/w order and type of system.
Sol^n i) Type number is specified for loop transfer f^n but order can be specified for any transfer function. (open loop or closed loop transfer function).
ii) The type number is given by number of poles of loop transfer function lying at origin of s-plane but the order is given by the number of poles of transfer function.

Q14 What is steady state error.
Sol^n The steady state error is the value of error signal e(t) when t tends to infinity. The steady state error is a measure of system accuracy. These error arise from the nature of inputs, type of system and from non-linearity of system components.

Q15 What is time response.
Sol^n The time response is the output of the closed loop system as a function of time. It is denoted by c(t). It is given by inverse Laplace of the product of i/p and transfer f^n of the system.

The closed loop T.F $\dfrac{C(S)}{R(S)} = \dfrac{G(S)}{1+G(S)H(S)}$

Response in s-domain $C(S) = \dfrac{R(S)G(S)}{1+G(S)H(S)}$

Response in time domain $c(t) = L^{-1}\{C(S)\} = L^{-1}\left\{\dfrac{R(S)G(S)}{1+G(S)H(S)}\right\}$

Q16 What is a PID controller?

Solⁿ PID controller is a device which produces a control signal consisting of three terms: One proportional to error signal another one proportional to integral of error signal and the third one proportional to derivative of error signal.

Q17 What are generalized error coefficients.

Solⁿ They are the coefficents of generalized error series. The generalized error series is given by.

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + \dots + \frac{C_n}{n!} r^{n}(t)$$

The Coefficient $C_0, C_1, C_2 \dots C_n$ are called generalized error coefficients or dynamic error coefficients

The nth Coefficient $C_n = \underset{s\to 0}{Lt} \frac{d^n}{ds^n} F(s)$, where $F(s) = \frac{1}{1+G(s)H(s)}$

Q18 Mention two adv. of generalized error constant over static error Constant.

Solⁿ i) Generalized error series gives error signal as a fn of time.

ii) Using generalized error constants the steady state error can be determined for any type of input but static error constant are used to determine steady state error when the i/p is anyone of the standard input.

Q19 Define Velocity error Constant.

Solⁿ The velocity error constant $K_v = \underset{s\to 0}{Lt}\, sG(s)H(s)$. The steady state error in type-1 system for unit ramp input is given by $1/K_v$

Q20 A unity feed back system has a open loop transfer fn of $G(s) = 10/(s+1)(s+2)$. determine the steady state error for unit step input.

Solⁿ The Steady state error for unit step i/p

$$e_{ss} = \frac{1}{1+K_p}, \text{ where } K_p = \underset{s\to 0}{Lt}\, G(s)H(s)$$

For unity feed back system $H(s) = 1$

$$\therefore K_p = \underset{s\to 0}{Lt}\, G(s) = \underset{s\to 0}{Lt} \frac{10}{(s+1)(s+2)}$$

$$= 5$$

$$\text{and } e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5} = \frac{1}{6}$$

Q21 Why derivative controller is not used in the control system?

Soln The derivative controller produces a control action based on rate of change of error signal and it does not produce corrective measure for any constant error. Hence derivative controller is not used in control system.

Q23 What is an impulse signal.

Soln A signal which is available for very short duration is called impulse signal. Ideal impulse signal is a unit impulse signal which is defined as a signal having zero values at all time except at $t=0$, At $t=0$ the magnitude becomes infinite. It is denoted by $\delta(t)$ and Mathematically expressed as.

$$\delta(t) = \infty \;; t=0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 \;; t \neq 0$$

Q24 What are static error constants?

Soln The K_p, K_v and K_a are called static error constants. These constants are associated with steady state error in a particular type of system and for a standard i/p.

Part-A Unit-III

Q1. What is frequency response?

Sol^n The magnitude and phase function of sinusoidal transfer f^n. of a system are real function of frequency ω, and so they are called frequency response.

Q2 What are the frequency domain specifications?

Sol^n The frequency domain specifications indicates the performance of the System in frequency domain and they are.

1> Resonant Peak, M_r. 2> Resonant frequency, ω_r
3> Bandwidth, ω_b (4> Cut-off rate (5> Gain Margin, K_g.
6> Phase Margin, γ.

Q4 Define resonant peak.

Sol^n The maximum value of the magnitude of closed loop transfer function is called Resonant Peak.

Q5. What is resonance frequency.

Sol^n The frequency at which the resonant peak occurs is called Resonant frequency. The resonant peak is the max^m value of the magnitude of closed loop transfer f^n.

Q6. Define gain margin.

Sol^n The gain margin, K_g. Is defined as the value by which gain of the system has to be increased to drive system to be verge of instability. It is given by the reciprocal of the magnitude of open loop transfer function, at phase cross-over frequency, ω_{pc} when expressed in decibels. It is given by the negative of db magnitude of $G(j\omega)$ at phase cross over.

$$\text{Gain Margin, } K_g = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} \quad \text{and } K_g \text{ in db} = 20\log\frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$= -20\log|G(j\omega)|_{\omega=\omega_{pc}}$$

Q7. Define phase margin?

Sol^n The phase margin, γ is that amount of additional phase lag to the gain cross-over frequency, ω_{gc} required to bring the System to the verge of instability. It is given by, $180° + \phi_{gc}$ where ϕ_{gc} is the phase of $G(j\omega)$ at the gain cross over frequency.

$$\text{Phase margin } \gamma = 180° + \phi_{gc} \quad \text{where } \phi_{gc} = \angle G(j\omega)\big|_{\omega=\omega_{gc}}.$$

Q8 What is Nichol's Chart.

Sol^n The Nichols Chart consists of M and N contours superimposed on ordinary graph. Analog each M-contour the magnitude of Closed loop system. M will be constant. Along each N-Contour, the phase α of closed loop system will be constant. The ordinary graph consist of magnitude in db, marked on the y-axis and the phase in degrees marked on axis. The Nicholos Chart is used to find the closed loop frequency response from the open loop frequency response.

Q9 What are the advantages of Nichols chart?

Sol^n i) It is used to find closed loop frequency response from open loop frequency response.

ii) The frequency domain specification can be determined from Nichol's Chart.

iii) The gain of the system can be adjusted to satisfy the given specification.

Q10 Define polar plot.

Sol^n The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle/argument of $G(j\omega)$ on polar or rectangular co-ordinate as ω is varied from zero to infinity.

Q11 Define bode plot?

Sol^n The bode plot is a frequency response plot of the transfer function of a system. It consists of two plots Magnitude & phase. The magnitude plot is a graph b/w magnitude of a system transfer f^n in db and freq ω. The phase plot is a graph b/w the phase or argument of a system transfer f^n in degrees and the frequency, ω. Usually both the plots are plotted on a common axis in which the frequencies are expressed in logrithim scale.

Q13 What is cut off rate?

Sol^n The slope of the log-magnitude curve near the cut-off frequency is called cut-off rate.

Q14 What is gain cross over frequency?

Sol^n The gain cross over frequency is the frequency at which the magnitude of the open loop transfer f^n is unity.

Q15 What is phase cross over frequency?

Sol^n The phase cross over frequency is the freq at which the phase of the open loop transfer function is 180°

Q16. Define corner frequency?

Solⁿ The magnitude plot can be approximated by asymptotic straight lines. The frequencies corresponding to the meeting point of asymptotes are called corner frequency. The slope of the magnitude plot changes at every corner freq.

Q17. What are M & N circles?

Solⁿ The magnitude, M of closed loop transfer fⁿ with unity feedback will be in the form of circle in complex plane for each constant value of M. The family of these circles are called M-circles.

Let $N = \tan\alpha$ where α is the phase of closed loop transfer fⁿ with unity feedback. For each constant value of N, a circle can be drawn in the complex plane. The family of these circles are called N-circles.

Q18 Define Bandwidth.

Solⁿ The bandwidth is the range of frequencies for which the system gain is more than -3db.

Q19 Enumerate the advantages of frequencies response.

Solⁿ 1) The absolute and relative stability of the closed loop system can be estimated from the knowledge of the open loop frequency response.

2) The practical testing of system can be easily carried with available sinusoidal signal generators and precise measurement equipments.

3) The Transfer function of complicated functions can be determined experimentally by frequency response tests.

4) The design the parameter adjustment can be carried more easily.

5) The corrective measure for noise disturbance and parameter variation can be easily carried.

6) It can be extended to certain non-linear systems.

Q22 Write the formula for gain margin & phase margin.

Solⁿ Gain Margin $= K_g = \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$

and K_g in db $= 20\log\frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = -20\log|G(j\omega)|_{\omega=\omega_{pc}}$

Phase Margin $= \gamma = 180^\circ + \phi_{gc}$: where $\phi_{gc} = \angle G(j\omega)|_{\omega=gc}$

Q23 Write the expression for resonant peak and resonant frequency.

Sol^n

Resonant Peak, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

Resonant frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

Q24 What is All-pass Systems?

Sol^n The all pass systems are systems with all pass transfer f^n. In all pass transfer functions, the magnitude is unity at all frequencies and the transfer function will have anti-symmetric pole zero pattern (i.e. for every pole in the left half s-plane, there is a zero on the mirror image position with respect to imaginary axis).

Q25 What is non-minimum phase transfer function?

Sol^n A transfer f^n which has one or more zeros in the right half of s-plane is known as non-minimum phase transfer function.

Q1 Define BIBO stability

Soln A linear relaxed system is said to have BIBO stability if every bounded (finite) input results in a bounded (finite) output.

Q2 What is the requirement for BIBO stability?

Soln The requirement for BIBO stability is that $\int_0^\infty m(t)\,dt < \infty$ when $m(t)$ is impulse response of the system.

Q4 What is characteristics Eqn.

Soln The denominator polynomial of $C(s)/R(s)$ is the characteristics equation of the system

Q5. Define impulse response.

Soln The impulse of a system is the response of a system for impulse input and it is given by inverse laplace transform of the system transfer function.

Q8 How the roots of characteristics equation are related stability?

Soln If the roots of characteristics Eqn has +ve real part then the impulse response of the system is not bounded (the impulse response will be infinite at $t \to \infty$) Hence the system will be unstable. If the roots have negative real part then the impulse response is bounded (the impulse response becomes 0 as $t \to \infty$) Hence the system will be stable.

Q9 What is the relation between stability and coefficient of characteristics polynomial?

Soln If the coefficient of characteristics polynomial are negative or zero, then some of roots lie on right half of s-plane. Hence the system is unstable. If the coefficients of characteristics polynomial are positive and if no coefficient is zero then there is a possibility of the system to be stable provided all the roots are lying on left half of s-plane.

Q10 What is the necessary condition for stability?

Soln The necessary condition for stability is that all the coefficients of the characteristics polynomial must be positive.

Q11 What is Routh stability criterion?

Soln Routh criterion states that the necessary and sufficient condition for stability is that all of the elements in the first column of the routh array be positive.

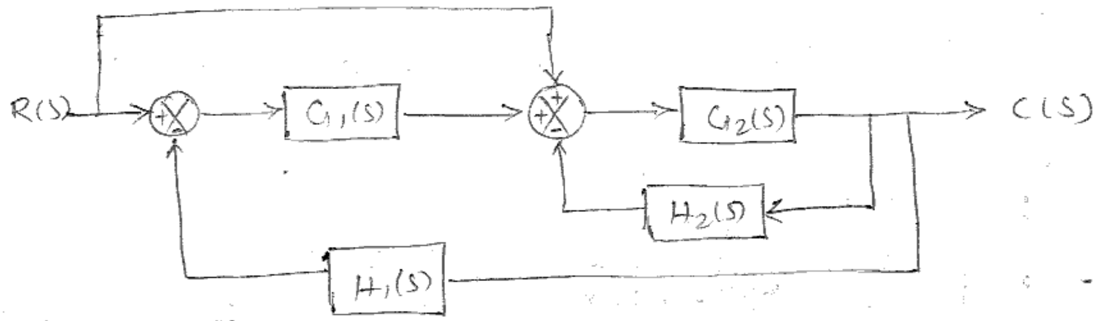
In this condition is not met the system is unstable and the number of sign changes in the elements of the first Column of routh array corresponds to the number of roots of charactoristics Equation in the right half of the S-plane.

Q12

University Question Bank Solution

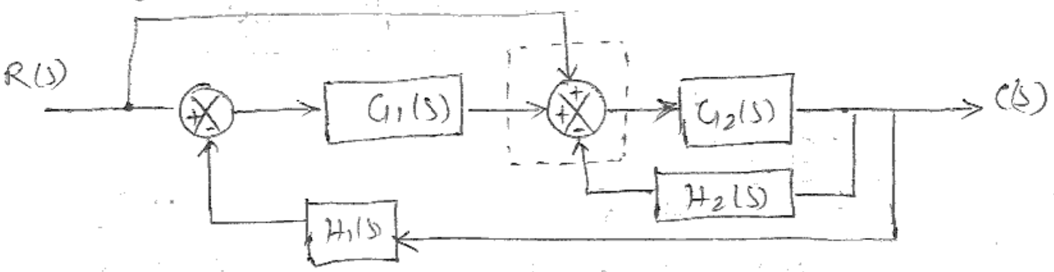
Unit - I   Control System   Part - B

Q1. The Block diagram of a closed loop System is shown in the fig. using the block reduction technique determine the closed loop transfer function. C(s) / R(s)

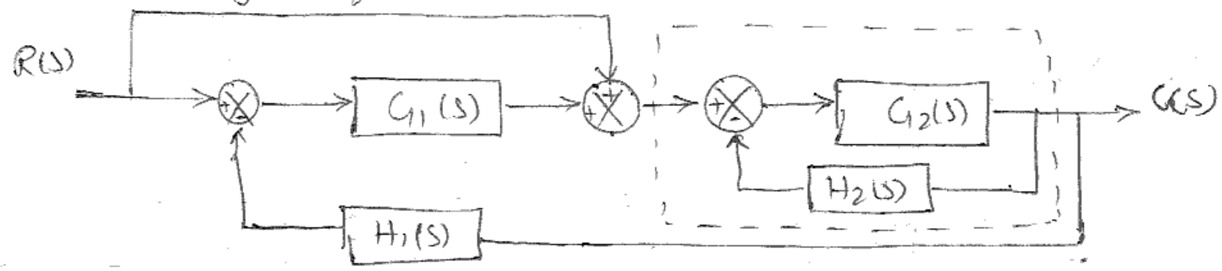


Sol^n

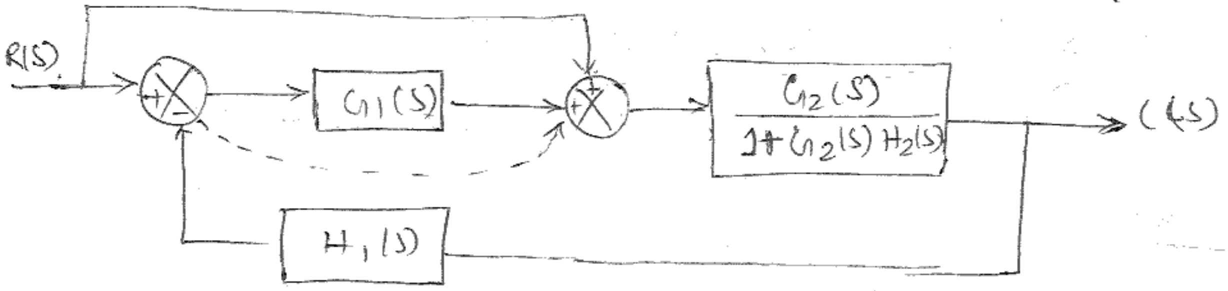
Step 1 Splitting the Summing point.



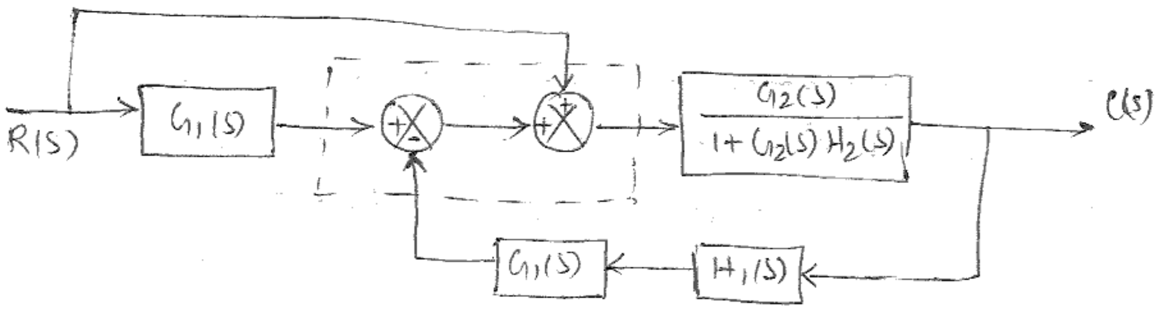
Step 2 Eliminating the feedback path.



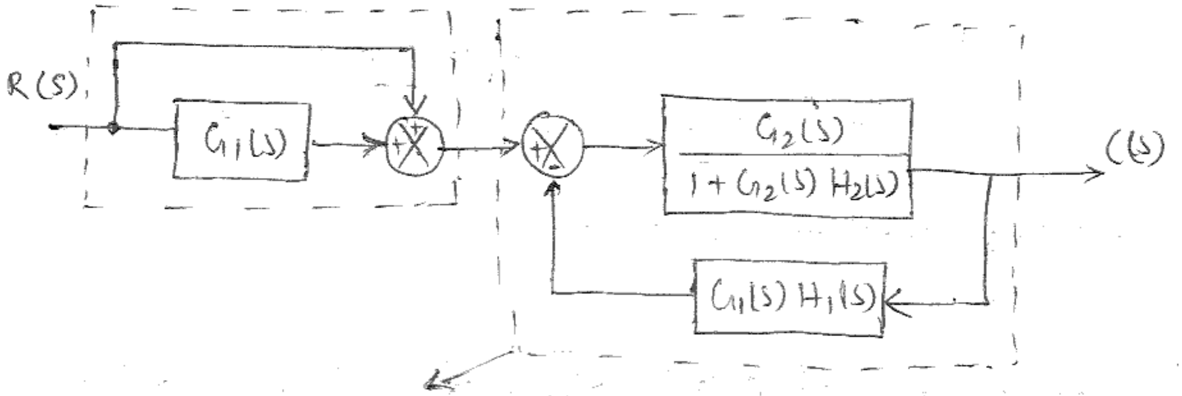
Step 3 : Moving the summing point after the block.



Step 4 Interchanging the summing point and combining the block in cascade:

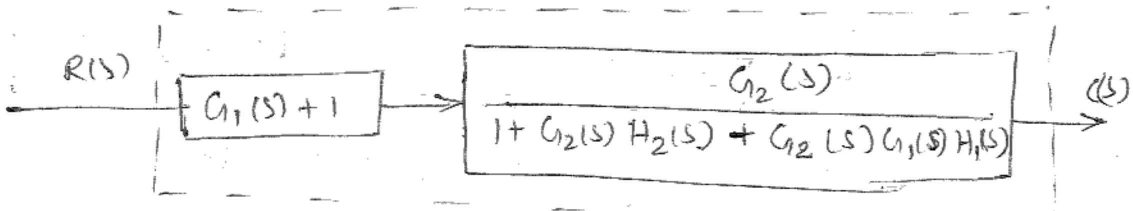


Step 5 Eliminating the feedback path and feed forward path.



$$\frac{\dfrac{G_2(S)}{1+G_2(S)H_2(S)}}{1+\dfrac{G_2(S)}{1+G_2(S)H_2(S)}\,G_1(S)H_1(S)} = \frac{G_2(S)}{1+G_2(S)H_2(S)+G_2(S)H_2(S)G_1(S)}$$

Step 6.
Comining the block in cascade.



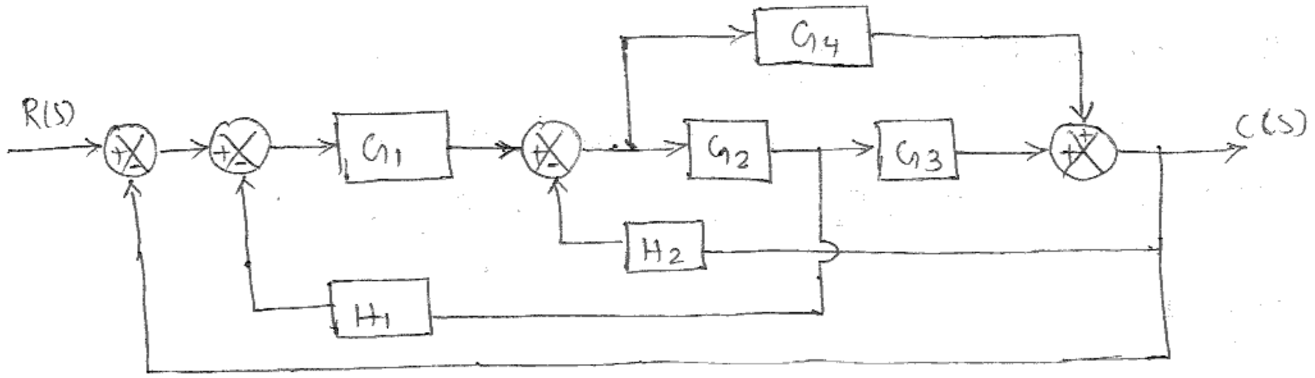
$$\therefore \frac{C(S)}{R(S)} = \frac{G_2(S)\,[G_1(S)+1]}{1+G_2(S)H_2(S)+G_1(S)G_2(S).H_1(S)}$$

Result

The transfer function of the system is,

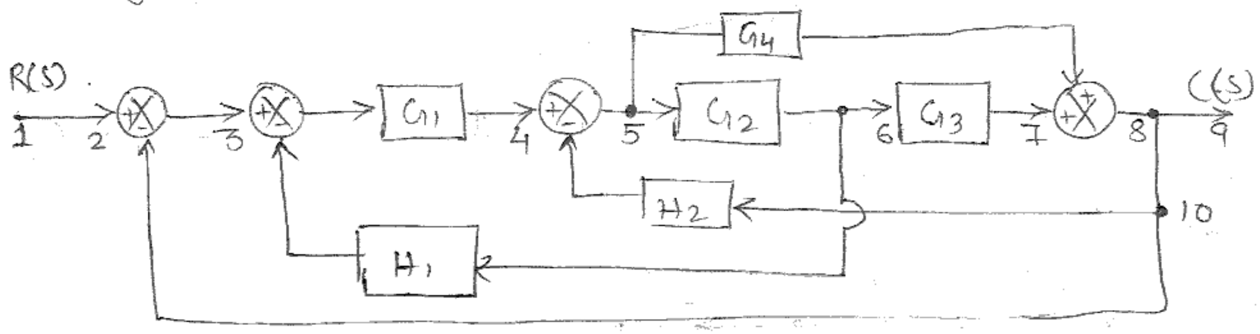
$$\frac{C(S)}{R(S)} = \frac{G_2(S)\,[G_1(S)+1]}{1+G_2(S)H_2(S)+G_1(S)G_2(S)H_1(S)}$$

Q. Convert the block Diagram to Signal flow graph and determine the transfer function using mason's gain formula.

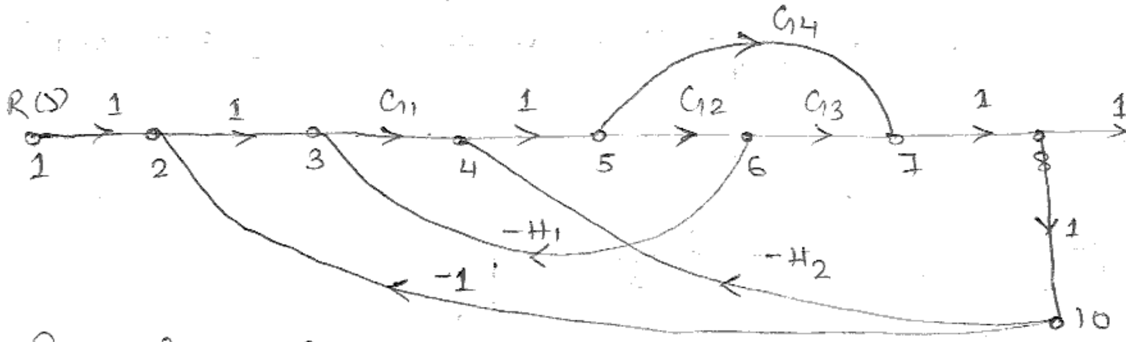


Sol^n

The nodes are assigned at input, output at every Summing point & branch point as shown in fig 2.



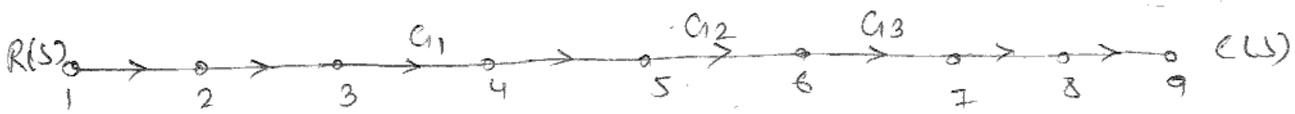
The signal flow graph for the above block Diagram



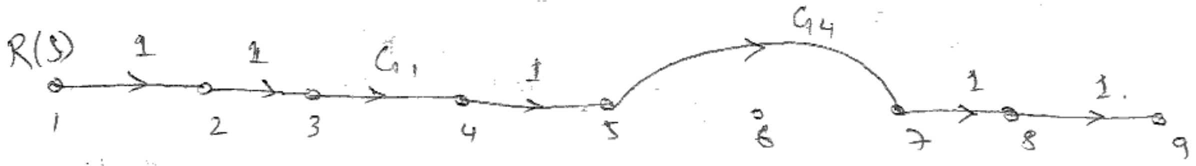
I Forward path Gain.

There are two forward path ∴ K = 2

let the gain of the forward path be P_1 and P_2.



Forward path 1



Forward path 2

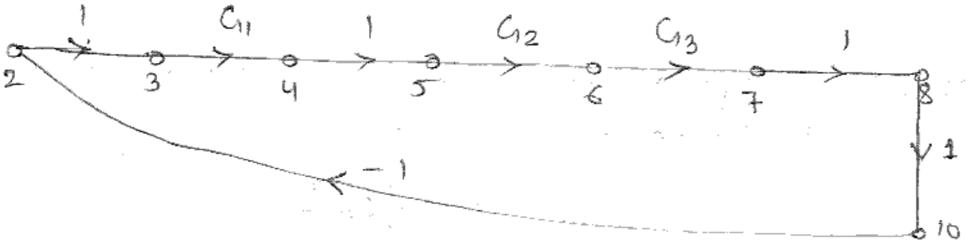
Gain of forward path - 1 , $P_1 = G_1 G_2 G_3$

Gain of Forward path - 2 , $P_2 = G_1 G_4$

II. Individual Loop Gain:-

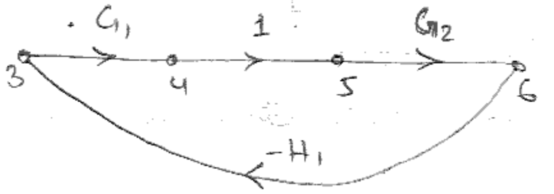
There are five individual loops. let the individual loop gain be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

For Loop 1



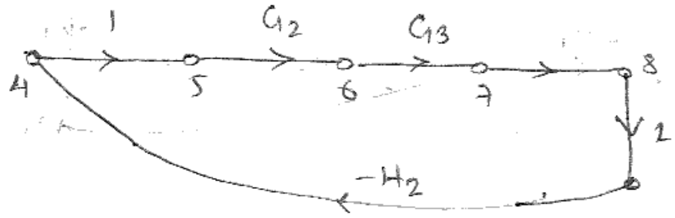
Loop Gain of individual loop 1 = $P_{11} = -G_1 G_2 G_3$

For loop 2



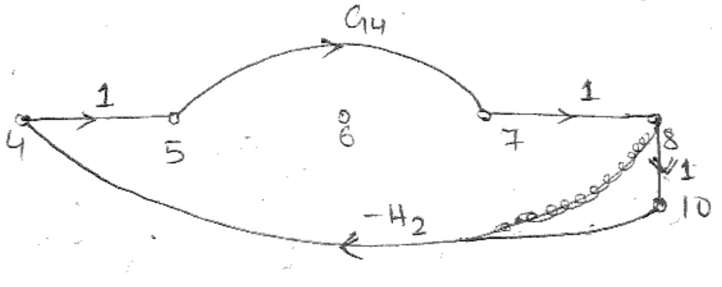
Loop Gain of individual loop 2 = $P_{21} = -G_2 G_1 H_1$

For Loop 3.



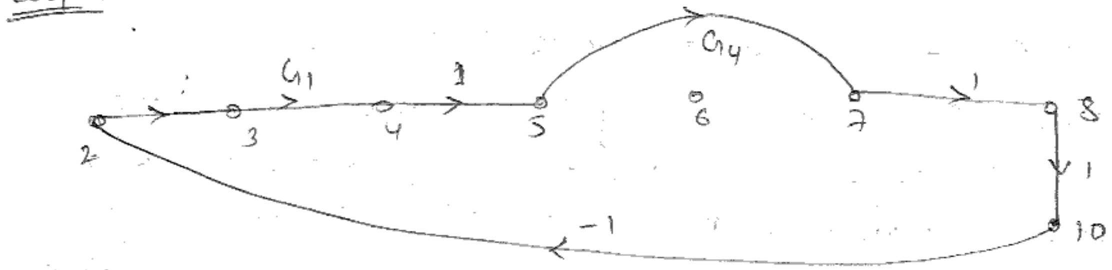
loop gain of individual loop 3 = $P_{31} = -G_2 G_3 H_2$

For loop 5.



loop gain of individual loop 4 = $P_{51} = -G_4 H_2$

Loop 4



Loop gain of individual loop-4

$$P_{41} = -G_1 G_4$$

III Gain product of two Non-Touching loop:

There are no possible cominations of two non-touching loops, three non-touching loops etc. ..

IV Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$$

$$= 1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2$$

Since no part of graph is non touching with forward path - 1 and 2, $\therefore$ $\Delta_1 = \Delta_2 = 1$

V Transfer function, T

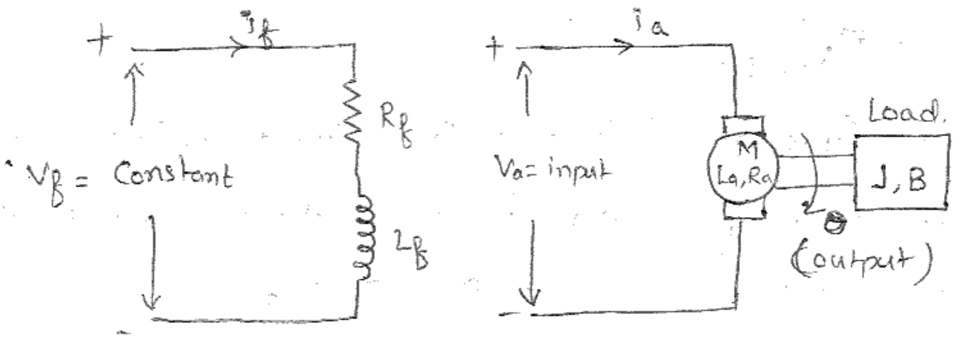
By Mason's gain formula the transfer f^n. T is given by.

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2}$$

Q3 Derive the transfer function of an armature controlled dc motor and hence develop its block Diagram.

Sol^n The Speed of DC motor is directly proportional to armature Voltage and inversely proportional to flux in field winding. In armature controlled DC motor the desired speed is obtained by varying the armature voltage. This speed control system is an electro-mechanical control System. The electrical system consist of the armature and the field circuit but for analysis purpose, only the armature circuit is considered because the field is excited by a constant voltage. The mechanical system consists of the rotating part of the motor and load connected to the shaft of the motor. The armature controlled DC motor speed control System is Shown in fig.



Armature controlled DC motor.

let,

R_a = Armature resistance, Ω

L_a = " inductance, H

i_a = " current, A

V_a = " voltage, V

e_b = Back emf, V

K_t = Torque constant, N-M/A

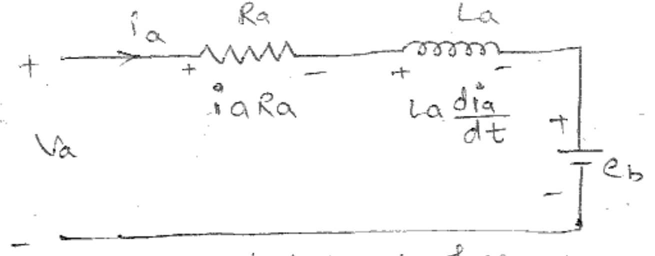
T = torque developed by Motor, N-M

θ = Angular displacement of shaft, rad.

J = Moment of inertia of motor and load, kg-m²/rad.

B = Frictional coefficient of motor and load, N-m/(rad/sec)

K_b = Back emf constant, V/(rad/sec)



Equivalent ckt of armature.

By Kirchoff's Voltage law, we can write,

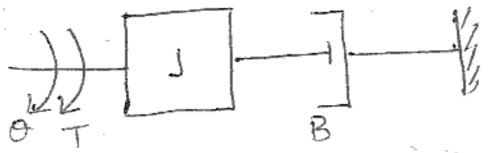
$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad \text{——} \; (1)$$

Torque of DC motor is proportional to the product of flux and current. Since flux is constant in this System, the torque is proportional to i_a alone.

$$T \propto i_a$$

$$\therefore \text{Torque}, \; T = K_t i_a \quad \text{——} \; (2)$$

The Mechanical System of the Motor is shown in fig.



The differential eqn governing the mechanical system of motor is given by.

$$j \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T \quad \text{——} \; (3)$$

The back emf of DC machine is proportional to speed (angular velocity) of shaft.

$$\therefore e_b \propto \frac{d\theta}{dt} \quad \text{or} \quad \text{Back emf}, \; e_b = K_b \frac{d\theta}{dt} \quad \text{——} \; (4)$$

The laplace transform of various time domain signals involved in this system are shown below.

$$L\{V_a\} = V_a(s) \; ; \; L\{e_b\} = E_b(s) \; ; \; L\{T\} = T(s)$$

$$L\{i_a\} = I_a(s) \; ; \; L\{\theta\} = \theta(s)$$

The differential Equation governing the armature controlled DC motor speed control system are :

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \; ; \quad T = K_t i_a \; ; \quad j \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T$$

$$e_b = K_b \frac{d\theta}{dt}$$

Taking Laplace transform of the above Equation with zero initial conditions we get.

$$I_a(s)\,R_a + L_a s\,I_a(s) + E_b(s) = V_a(s) \quad \text{--- (5)}$$

$$T(s) = K_t\,I_a(s) \quad \text{--- (6)}$$

$$Js^2\theta(s) + Bs\,\theta(s) = T(s) \quad \text{--- (7)}$$

$$E_b(s) = K_b\,s\,\theta(s) \quad \text{--- (8)}$$

On Equating Eqn (6) & (7) we get.

$$K_t I_a(s) = (Js^2 + Bs)\,\theta(s)$$

$$I_a(s) = \frac{(Js^2 + Bs)}{K_t}\cdot\theta(s) \quad \text{--- (9)}$$

Equation (5) can be written as:

$$(R_a + sL_a)\,I_a(s) + E_b(s) = V_a(s) \quad \text{--- (10)}$$

Substituting for $E_b(s)$ and $I_a(s)$ from Eqn (9) & (8) respect. in eqn (10)

$$(R_a + sL_a)\cdot\frac{(Js^2 + Bs)}{K_t}\,\theta(s) + K_b\,s\,\theta(s) = V_a(s)$$

$$\left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

The required transfer f^n. is $\dfrac{\theta(s)}{V_a(s)}$

$$\therefore \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)\cdot(Js^2 + Bs) + K_b K_t s} \quad \text{--- (11)}$$

$$= \frac{K_t}{R_a Js^2 + R_a Bs + L_a Js^3 + L_a Bs^2 + K_b K_t s}$$

$$= \frac{K_t}{s\left[JL_a s^2 + (JR_a + BL_a)s + (BR_a + K_b K_t)\right]}$$

$$= \frac{K_t / JL_a}{s\left[s^2 + \left(\dfrac{JR_a + BL_a}{JL_a}\right)s + \left(\dfrac{BR_a + K_b K_t}{JL_a}\right)\right]} \quad \text{--- (12)}$$

The transfer function of armature controlled dc motor can be expressed in another standard form as shown below. From Eqn (1·22) we get.

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}$$

$$= \frac{K_t}{R_a\left(\frac{sL_a}{R_a} + 1\right) Bs\left(1 + \frac{Js^2}{Bs}\right) + K_b K_t s}$$

$$= \frac{K_t / R_a B}{s\left[(1 + sT_a)(1 + sT_m) + \frac{K_b K_t}{R_a B}\right]} \quad \text{— (13)}$$

Where,

$\frac{L_a}{R_a} = T_a$ = Electrical time constant

$\frac{J}{B} = T_m$ = Mechanical time constant.

Unit - II   Part - B

Q1 Derive the expression for unit step response of a second order critically damped system.

$$G(s) = \omega_n^2 / s^2 + 2\zeta\omega_n s + \omega_n^2$$

Sol^n

The standard form of closed loop transfer function of second order system is.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For critical damping $\zeta = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2} \quad \text{--- (1)}$$

When input is unit step, $r(t) = 1$ and $R(s) = 1/s$

$\therefore$ The response in s-domain,

$$C(s) = R(s) \frac{\omega_n^2}{(s+\omega_n)^2} = \frac{1}{s} \frac{\omega_n^2}{(s+\omega_n)^2} = \frac{\omega_n^2}{s(s+\omega_n)^2} \quad \text{--- (2)}$$

By partial fraction expansion, we can write.

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$A = s \times C(s)\Big|_{s=0} = \frac{\omega_n^2}{(s+\omega_n)^2}\Big|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$B = (s+\omega_n)^2 \times C(s)\Big|_{s=-\omega_n} = \frac{\omega_n^2}{s}\Big|_{s=-\omega_n} = -\omega_n$$

$$C = \frac{d}{ds}\left[(s+\omega_n)^2 \times C(s)\right]\Big|_{s=-\omega_n} = \frac{d}{ds}\left(\frac{\omega_n^2}{s}\right)\Big|_{s=-\omega_n}$$

$$= -\frac{\omega_n^2}{s^2}\Big|_{s=-\omega_n} = -1$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)}$$

$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

The response in time domain,

$$c(t) = L^{-1}\{C(s)\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}\right\}$$

$$L\{1\} = 1/s$$

$$L\{te^{-at}\} = \frac{1}{(s+a)^2}$$

$$L\{e^{-at}\} = \frac{1}{s+a}$$

$$c(t) = 1 - \omega_n t e^{-\omega_n t} - e^{-\omega_n t}$$

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t) \quad \text{—— (3)}$$

The eq. (3) is the response of critically damped closed loop second order system for unit step input. For step input of step value the equation (3) should be multiplied by A.

∴ For closed loop critically damped second order system.

$$\text{Unit Step response} = 1 - e^{-\omega_n t}(1+\omega_n t)$$

$$\text{Step Response} = A\left[1 - e^{-\omega_n t}(1+\omega_n t)\right]$$

Using Eqn (3) the response of critically damped second order system is sketched as shown in fig. and observed that the response has no oscillations.

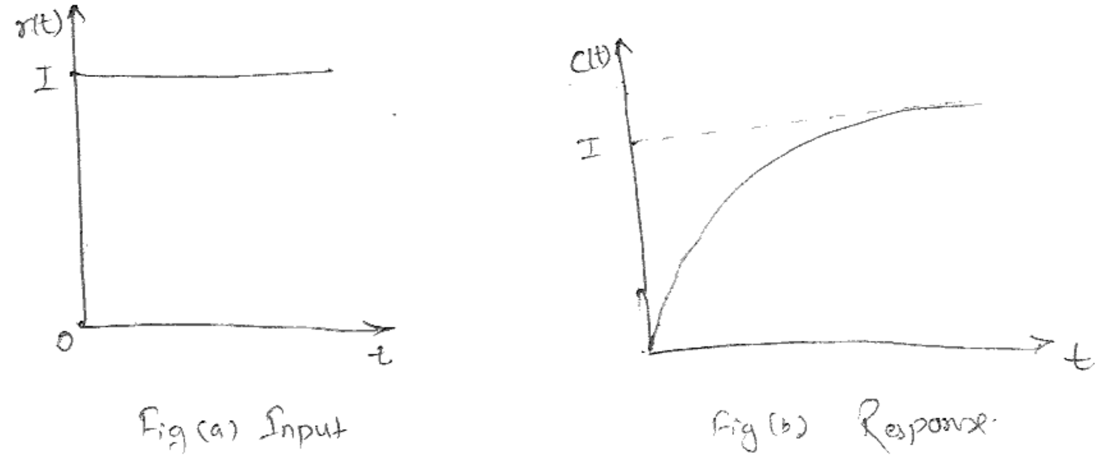


Fig (a) Input

Fig (b) Response

Fig Response of critically damped second order system for unit step input.

Q3 Derive the expression for steady state error of the closed loop system in terms of generalized error coefficients.

Soln The drawback in static error coefficients is that it does not show the variation of error with time and input should be a standard input. The generalized error coefficients gives the steady state error as a function of time. Also using the generalized error coefficients, the steady state error can be found for any type of input.

The error signal in s-domain, $E(s)$ can be expressed as a product of two s-domain functions.

$$E(S) = \frac{R(S)}{1+G(S)H(S)} = \frac{1}{1+G(S)H(S)} R(S) = F(S)R(S) \quad \text{--- (1)}$$

$$\text{where, } F(S) = \frac{1}{1+G(S)H(S)}$$

let $e(t) = L^{-1}\{E(S)\}$ (error signal in time domain)

$f(t) = L^{-1}\{F(S)\}$

$r(t) = L^{-1}\{R(S)\}$ (input signal in time domain)

The convolution theorem of laplace transform states that the laplace transform of the convolution of two time domain signals is equal to the product of their individual Laplace transform.

i.e., $L\{f(t) * r(t)\} = F(S)\, R(S)$

where $*$ is the symbol of convolution operation.

$$\therefore L^{-1}\{F(S)\, R(S)\} = f(t) * r(t) \quad \text{--- (2)}$$

From eqn (1) & (2) we can write.

$$e(t) = f(t) * r(t)$$

Mathematically the convolution of $f(t)$ and $r(t)$ is defined as

$$f(t) * r(t) = \int_{-\infty}^{+\infty} f(T)\, r(t-T)\, dT \; ; \quad \text{where } T \text{ is a dummy variable.}$$

$$\therefore e(t) = \int_{-\infty}^{+\infty} f(T)\, r(t-T)\, dT$$

It is assumed that the input signal starts at t = 0 and does not exist before t = 0. Also we are interested in finding error signal at any time t after t = 0 (i.e. for t > 0). Hence in the above equation the limit of integral can be changed as 0 to t

$$\therefore e(t) = \int_0^t f(T)\, r(t-T)\, dT$$

Using Taylor's series expansion the signal $r(t-T)$ can be expressed as,

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \dots + (-1)^n \frac{T^n}{n!} \overset{n\cdot}{r}(t)$$

where $\dot{r}(t)$ = 1st derivative of $r(t)$

$\ddot{r}(t)$ = 2nd " " "

$\overset{n\cdot}{r}(t)$ = nth " " "

On substituting the Taylor's series expansion of $r(t-T)$ the error $e(t)$ can be written as

$$e(t) = \int_0^t f(T)\left[r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \dots + (-1)^n \frac{T^n}{n!}\overset{n\cdot}{r}(t)\right] dT$$

$$e(t) = \int_0^t f(T)\, r(t) - \int_0^t f(T)\, T\dot{r}(t)\, dT + \int_0^t f(T)\frac{T^2}{2!}\ddot{r}(t)\, dT$$

$$- \int_0^t f(T)\frac{T^3}{3!}\dddot{r}(t) + \dots + \int_0^t f(t)(-1)^n \frac{T^n}{n!}\overset{n\cdot}{r}(t)\, dT \dots \infty$$

Since $r(t), \dot{r}(t), \ddot{r}(t) \dots \overset{n\cdot}{r}(t)$ are constants when the integration is done with respect to T, the error signal can be written as,

$$e(t) = r(t)\int_0^t f(T)\,dT - \dot{r}(t)\int_0^t T f(T)\,dt + \frac{\ddot{r}(t)}{2!}\int_0^t T^2 f(T)\,dt$$

$$- \frac{\dddot{r}(t)}{3!}\int_0^t T^3 f(T)\,dt + \dots + (-1)^n \frac{\overset{n\cdot}{r}(t)}{n!}\int_0^t T^n f(T)\,dt$$

let $C_0 = +\int_0^t f(T)\,dT$ $\qquad C_2 = +\int_0^t T^2 f(T)\,dT$

$C_1 = -\int_0^t T f(T)\,dT$ $\qquad C_3 = -\int_0^t T^3 f(T)\,dT$

$\vdots$

$C_n = (-1)^n \int_0^t T^n f(T)\,dT$

$$e(t) = r(t)C_0 + \dot{r}(t)C_1 + \ddot{r}(t)\frac{C_2}{2!} + \dddot{r}(t)\frac{C_3}{3!} + \dots + \overset{n}{r}(t)\frac{C_n}{n!} + \dots$$

$$= C_0 r(t) + C_1\dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + \dots + \frac{C_n}{n!}\overset{n}{r}(t) \dots \quad (3)$$

The Equation (3) is the general Equation for error signal $e(t)$

The coefficients $C_0, C_1, C_2 \dots C_n$ are called the generalized error coefficients or dynamic error. Coefficients

The Steady State error e_{ss} is obtained by taking limit $t \to \infty$ on $e(t)$

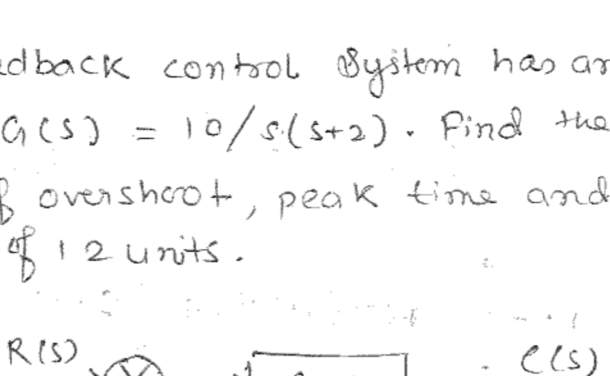
$\therefore$ Steady State error,

$$e_{ss} = \underset{t\to\infty}{Lt}\left[r(t)C_0 + \dot{r}(t)C_1 + \ddot{r}(t)\frac{C_2}{2!} + \dddot{r}(t)\frac{C_3}{3!} + \dots + \overset{n}{r}(t)\frac{C_n}{n!} + \dots\right]$$

$$= C_0 r(t) + C_1\dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + \dots + \frac{C_n}{n!}\overset{n}{r}(t) \dots \quad (4)$$

Q4 A unity feedback control system has an open loop transfer function $G(s) = 10/s(s+2)$. Find the rise time, percentage of overshoot, peak time and settling time for a step input of 12 units.

Sol^n



The unity feedback System is shown in fig.

The closed loop transfer function, $\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$

The Closed loop transfer function.

Given that, $G(S) = 10/s(s+2)$

$$\therefore \frac{C(S)}{R(S)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s(s+2)+10} = \frac{10}{s^2+2s+10} \quad (1)$$

The value of damping ratio ζ and natural frequency of oscillation ω_n are obtained by comparing the system transfer function with standard form of second order transfer function.

Standard form of Second order transfer f^n } $\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$ —— (2)

On Comparing (1) & (2) we get.

$\omega_n^2 = 10$

$\therefore \omega_n = \sqrt{10} = 3.162$ rad/sec

$2\zeta\omega_n = 2$

$\therefore \zeta = \frac{2}{2\omega_n} = \frac{1}{3.162} = 0.316$

$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{1-(0.316)^2}}{0.316} = 1.249$ rad.

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.162\sqrt{1-(0.316)^2} = 3$ rad/sec.

Rise time $t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.249}{3} = 0.63$ sec.

Percentage overshoot $\% M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$= e^{\frac{-0.316\pi}{\sqrt{1-(0.316)^2}}} \times 100$

$= 0.3512 \times 100 = 35.12\%$

Peak overshoot $= \frac{35.12}{100} \times 12$ units

$= 4.2144$ units

Peak time $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.047$ sec.

Time constant, $T = \frac{1}{\zeta\omega_n} = \frac{1}{0.316 \times 3.162} = 1$ sec.

$\therefore$ For 5% error, Settling time, $t_s = 3T = 3$ sec

For 2% error, Settling time, $t_s = 4T = 4$ sec

Result

Rise time, t_r = 0.63 sec.

Percentage overshoot $\% M_p$ = 35.12%

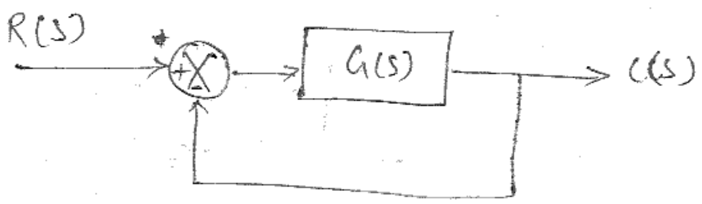
Peak overshoot = 4.2144 Units (for a i/p of 12 units)

Peak time, t_p = 1.047 sec.

Settling time, t_s = 3 sec for 5% error

4 sec for 2% error

Q5 Obtain the response of unity feedback System whose open loop transfer function is $G(s) = 4/s(s+5)$ and when the input is unit step.



Soln

The closed loop transfer f^n $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{\frac{4}{s(s+5)}}{\frac{s(s+5)+4}{s(s+5)}} = \frac{4}{s(s+5)+4}$$

$$= \frac{4}{s^2+5s+4} = \frac{4}{(s+4)(s+1)}$$

The response in s-domain, $C(s) = R(s)\,\frac{4}{(s+1)(s+4)}$

Since the input is unit step, $R(s) = 1/s$,

$$\therefore C(s) = \frac{4}{s(s+1)(s+4)}$$

$$C(s) = \frac{4}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+4)}$$

$$A = C(s) \times s \big|_{s=0} = \frac{4}{(s+1)(s+4)}\Big|_{s=0} = \frac{4}{1\times 4} = 1$$

$$B = C(s) \times (s+1)\big|_{s=-1} = \frac{4}{s(s+4)}\Big|_{s=-1} = \frac{4}{-1(-1+4)} = \frac{-4}{3}$$

$$C = C(s) \times (s+4)\big|_{s=-4} = \frac{4}{s(s+1)}\Big|_{s=-4} = \frac{4}{-4(-4+1)} = \frac{1}{3}$$

The time domain response $c(t)$ is obtained by taking inverse laplace transform of $C(s)$

Response of time domain, $c(t) = L^{-1}\{C(s)\}$

$$= L^{-1}\left\{\frac{1}{s} - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s+4}\right\}$$

$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}$$

$$= 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

Result

Response of unity feedback System, $c(t) = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$

Part B

Unit - III

Q1. Draw the bode plot for the following transfer f^n. And obtain gain cross over frequency $G(s) = \frac{20}{s(1+3s)(1+4s)}$

Sol^n The sinusoidal transfer function of $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given transfer function.

$$G(j\omega) = \frac{20}{j\omega(1+3j\omega)(1+j4\omega)}$$

Magnitude plot

The corner frequency are. $\omega c_1 = \frac{1}{4} = 0.25$ rad/sec

$\omega c_2 = \frac{1}{3} = 0.333$ rad/sec.

The various terms of $G(j\omega)$ are listed in table 1 in the increasing order of their frequencies. Also the table shows the slope contributed by each term and change in the slope at the corner frequency

Table-1

Term	Corner frequency	Slope dB/dec	Change in slope db/dec
$\frac{20}{j\omega}$	—	-20	
$\frac{1}{1+j4\omega}$	$\omega c_1 = \frac{1}{4} = 0.25$	-20	$-20-20=-40$
$\frac{1}{1+j3\omega}$	$\omega c_2 = \frac{1}{3} = 0.33$	-20	$-40-20=-60$

Choose a frequency ω_1 such that $\omega_1 < \omega c_1$ and choose a high frequency ω_h such that $\omega_h > \omega c_2$

Let $\omega_1 = 0.15$ rad/sec and $\omega_h = 1$ rad/sec.

Let $A = |G(j\omega)|$ in db

Let us calculate A at ω_1, ωc_1, ωc_2 and ω_h

At $\omega = \omega_1$, $A = |G(j\omega)| = 20\log\left|\frac{20}{0.15}\right| = 42.5$ db

At $\omega = \omega_{c1}$, $A = |G(j\omega)| = 20\log\left|\frac{20}{0.25}\right| = 38$ db.

At $\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})}$

$$= -40 \times \log \frac{0.33}{0.25} + 38 \Rightarrow 33\,db$$

At $\omega = \omega_n$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_n \times \log \frac{\omega_n}{\omega_{c2}}\right] + A\,(\text{at } \omega = \omega_{c2})$

$$= -60 \times \log \frac{1}{0.33} + 33 \Rightarrow 4\,db.$$

Phase Plot

The Phase angle of $G(j\omega)$, $\phi = -90^\circ - \tan^{-1} 3\omega - \tan^{-1} 4\omega$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table 2.

Table 2

ω, rad/sec	$\tan^{-1} 3\omega$, deg	$\tan^{-1} 4\omega$, deg	$\phi = \angle G(j\omega)$, deg	Points in phase plot
0.15	24.22	30.96	$-145.18 \approx -146$	e
0.2	30.96	38.66	$-159.61 \approx -160$	f
0.25	36.86	45.0	$-171.86 \approx -172$	g
0.33	44.7	52.8	$-187.5 \approx -188$	h
0.6	60.14	67.38	$-218.32 \approx -218$	i
1	71.56	75.96	$-237.56 \approx -238$	j

Q2 Write the procedure for bode plot?

Sol^n Procedure for plotting the magnitude plot is given below.

Step 1: Convert the transfer into Bode form or time constant form. The Bode form of the transfer function is.

$$G(S) = \frac{K(1+ST_1)}{S(1+ST_2)\left(1+\frac{S^2}{\omega_n^2}+2\zeta\frac{S}{\omega_n}\right)}$$

$S \to j\omega$.

$$G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)\left[1-\frac{\omega^2}{\omega_n^2}+j\,2\zeta\frac{\omega}{\omega_n}\right]}$$

Step 2: List the corner frequencies in the increasing order and prepare a table as shown below.

Term	Corner frequency rad/sec.	Slope db/dec	Change in slope db/dec

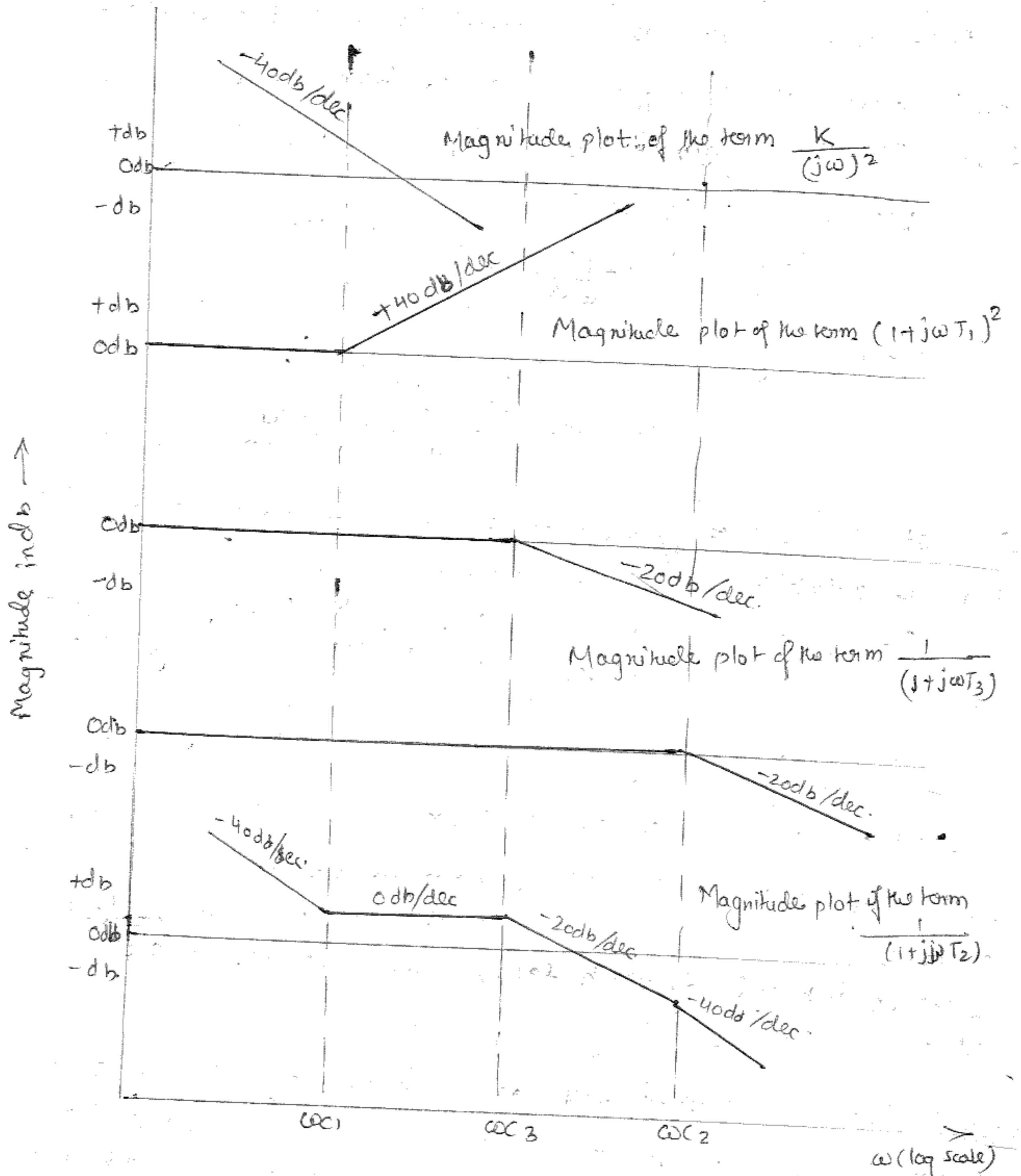
In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)^n$ as the first term and the other term in the increasing order of corner frequencies. Then enter the corner frequency, slope contributed by each term and change in slope at every corner frequency.

Step 3: Choose an arbitrary frequency ω_l which is lesser than the lowest corner frequency. Calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)^n$ at ω_l and at the lowest corner frequency.

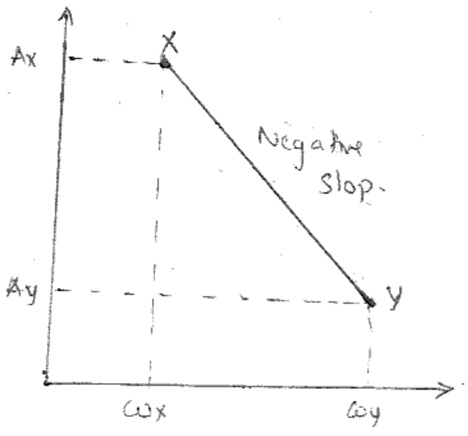
Step 4: Then calculate the gain (dB) magnitude at every corner frequency one by one by using the formula.

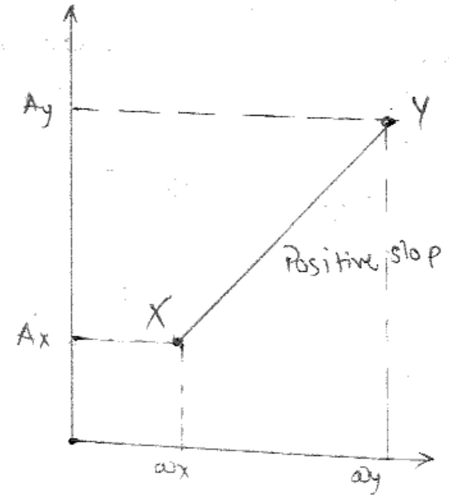
Gain at ω_y = Change in gain from ω_x to ω_y + Gain at ω_x

$$= \left[\text{slope from } \omega_x \text{ to } \omega_y \times \log\frac{\omega_y}{\omega_x}\right] + \text{Gain at } \omega_x$$



Magnitude plot of bode plot of $G(j\omega) = \dfrac{K(1+j\omega T_1)^2}{(j\omega)^2(1+j\omega T_2)(1+j\omega T_3)}$





Step 5 Choose an arbitary frequency ω_n which is greater than the highest corner frequency calculated the gain at ω_n by using the formula in step 4.

Step 6: In a semilog graph sheet mark the required range of frequency on x-axis (log scale) and the range of db magnitude on y-axis (ordinary scale) after choosing proper units

Step 7 Mark all the points obtained in steps 3, 4 and 5 on the graph and join the points by straight lines. Mark the slope at every part of the graph.

Procedure for Phase Plot of Bode Plot :-

The phase plot is an exact plot and no approximations are made while drawing the phase plot, Hence the exact phase angles of $G(j\omega)$ are computed for various values of ω and tabulated. The choice of frequencies are preferably the frequencies chosen for magnitude plot. Usually the magnitude plot and phase plot are drawn in a single semilog - sheet on a common frequency scale.

Take another y-axis in the graph where the magnitude plot is drawn and in the y-axis mark the desired range of phase angles after choosing proper units. From the tabulated values of ω and phase angles mark all the points on the graph. Join the points by a smooth curve.

Q3 Consider a unity feedback system having open loop transfer function $G(s) = 1/s^2(1+s)(1+2s)$. Using polar plot find the gain and phase margin.

Sol^n Given that, $G(s) = 1/s^2(1+s)(1+2s)$

Put $s = j\omega$ $\therefore$ $G(j\omega) = \dfrac{1}{(j\omega)^2\,(1+j\omega)\,(1+j2\omega)}$

The corner frequencies are

$\omega c_1 = 0.5$ rad/sec and $\omega c_2 = 1$ rad/sec.

$$G(j\omega) = \frac{1}{(j\omega)^2 \cdot (1+j\omega)(1+j2\omega)}$$

$$= \frac{1}{\omega^2 \angle 180^\circ \sqrt{1+\omega^2} \angle \tan^{-1}\omega \sqrt{1+4\omega^2} \angle \tan^{-1} 2\omega}$$

$$G(j\omega) = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \angle(-180 - \tan^{-1}\omega - \tan^{-1} 2\omega).$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = \frac{1}{\omega^2\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1} 2\omega$$

Table 1 Magnitude and phase plot of $G(j\omega)$ at various frequencies.

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$\lvert G(j\omega)\rvert$	3.3	2.5	1.9	1.5	1.2	$0.97 \simeq 1$	0.8	0.3
$\angle G(j\omega)$ deg	-246	-251	-256	-261	-265	-269	-273	-288

Table 2. Real and imaginary parts of $G(j\omega)$

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$G_R(j\omega)$	-1.34	-0.81	-0.46	-0.23	-0.1	-0.02	0.04	0.09
$G_I(j\omega)$	3.01	2.36	1.84	1.48	1.2	1.0	0.8	0.29

Result

- Gain Margin $K_g = 0$

Phase Margin, $\gamma = -90^\circ$

Graph included.

Q4. Derive the expression for M circle.

Sol^n The magnitude of closed loop transfer function with unity feedback can be shown to be in the form of circle for every value of M. These circles are called M-circles.

Derive M circle:

Consider the closed loop transfer function of unity feed back system, $M(s) = \frac{G(s)}{1+G(s)}$

Put $s = j\omega$ $\therefore M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

Let $G(j\omega) = X + jY$

where X = Real part of $G(j\omega)$

Y = Imaginary part of $G(j\omega)$

$$\therefore M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\ \angle \tan^{-1}\frac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\ \angle \tan^{-1}\frac{Y}{1+X}}$$

$$= \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle \left\{ \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X} \right\}$$

Let, M = Magnitude of $M(j\omega)$

$$\therefore M = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

On squaring the above Equation we get

$$M^2 = \frac{X^2+Y^2}{(1+X)^2+Y^2} \Rightarrow M^2((1+X)^2+Y^2) = X^2+Y^2$$

$$\Rightarrow M^2(1+X^2+2X+Y^2) = X^2+Y^2$$

$$M^2 + M^2X^2 + M^2 2X + M^2Y^2 - X^2 - Y^2 = 0$$

$$X^2(M^2-1) + M^2 2X + M^2 + Y^2(M^2-1) = 0 \quad \text{—— (1)}$$

When $M=1$, the equation (1) represents a straight line.

when $M=1$, the equation (1) is.

$$X^2(1-1) + 2X + 1 + Y^2(1-1) = 0$$

$$\Rightarrow 2X + 1 = 0$$

$$\Rightarrow X = -1/2$$

Hence when $M=1$, equation (1) represent a Straight line passing through $X=-1/2$ & $Y=0$

when $M \neq 1$, the equation (1) represents a ~~straight line passing through~~ family of circles.

when $M \neq 1$ eqn (1) can be rearranged in the form of equation of a circle as shown below.

$$X^2(M^2-1)+M^2 2X+M^2+Y^2(M^2-1)=0$$

Divide the above equation throughout by (M^2-1)

$$\therefore X^2+\frac{M^2}{M^2-1}2X+\frac{M^2}{M^2-1}+Y^2=0$$

~~On rearranging the above eqn we get~~

$$\cancel{X+X^2+Y^2=\frac{Y}{N}}$$

$$\therefore \cancel{X+X^2+Y^2-\frac{Y}{N}}=0$$

~~In the above equation add the term~~ $1/4+\left(\frac{1}{2N}\right)^2$ on both sides.

$$\cancel{X+X^2+Y^2-\frac{Y}{N}+\frac{1}{4}+\left(\frac{1}{2N}\right)^2=\frac{1}{4}+\left(\frac{1}{2N}\right)^2}$$

$$\left(X^2+\frac{1}{4}+X\right)+\left(Y^2+\frac{1}{(2N)^2}-\frac{Y}{N}\right)=\frac{1}{4}+\frac{1}{(2N)^2}$$

$$\left(X+\frac{1}{2}\right)^2+\left(Y-\frac{1}{2N}\right)^2=\frac{1}{4}+\frac{1}{(2N)^2}$$

Add $\frac{M^2}{(M^2-1)^2}$ on both sides of the above equation

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^2}{M^2-1}+\frac{M^2}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^2}$$

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^2(M^2-1)+M^2}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^2}$$

$$X^2+\frac{M^2}{M^2-1}2X+\frac{M^4}{(M^2-1)^2}+Y^2=\frac{M^2}{(M^2-1)^3}$$

$$\boxed{a^2+2ab+b^2=(a+b)^2}$$

$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{(M^2-1)^2} \quad \text{—— (2)}$$

The Equation of circle with centre at (X_1, Y_1) and radius r is given by.

$$(X - X_1)^2 + (Y - Y_1)^2 = r^2 \quad \text{③}$$

On comparing eqn ② and ③ it can concluded that the equation ② represent a family circle with centre at $(-M^2/(M^2-1), 0)$ and with radius $r = M/(M^2-1)$ for various values of M. The circles given by eqn ② are called M circles.

When M = 0

Centre = (X_1, Y_1)

$$X_1 = -\frac{M^2}{M^2-1} = 0$$

$$Y_1 = 0$$

$$\text{Radius}, r = \frac{M^2}{M^2-1} = 0$$

Hence when M = 0, the magnitude circle becomes a point at (0,0)

When M = ∞

Centre = (X_1, Y_1)

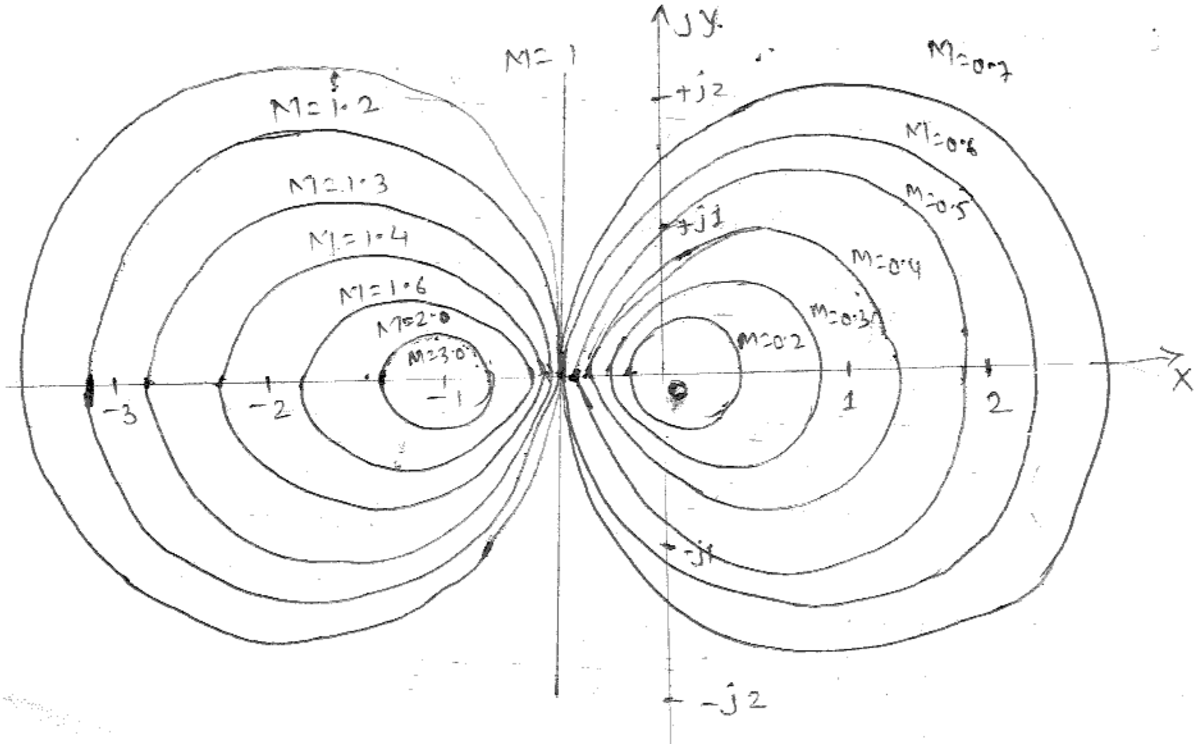
$$X_1 = \frac{-M^2}{M^2-1} \approx -\frac{M^2}{M^2} = -1$$

$$Y_1 = 0$$

$$\text{Radius}, r = \frac{M}{M^2-1} \approx \frac{M}{M^2} = \frac{1}{M} = \frac{1}{\infty} = 0$$

Hence when M = ∞, the magnitude circle becomes a point at (−1,0)

From the above analysis it is clear that the magnitude of closed loop transfer function will be in the form of circles when $M \neq 1$ and $M = 1$, the magnitude is a straight line passing through (−1/2, 0)



The family of constant M-circle.

Q5 Derive the expression for N circle!

Soln If the phase of closed loop transfer function with unity feedback is α, then it can be shown that $\tan\alpha$ will be in the form of circle for every value of α. These circles are called N-circles.

Derive N-circles

Consider the closed loop transfer function of unity feedback system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = M(s)$$

Put $s = j\omega$, $M(j\omega) = \dfrac{G(j\omega)}{1+G(j\omega)}$

Let $G(j\omega) = X + jY$,

where X = Real part of $G(j\omega)$

Y = Imaginary part of $G(j\omega)$

$$\therefore M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\ \angle \tan^{-1}\frac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\ \angle \tan^{-1}\frac{Y}{1+X}}$$

$$= \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}} \angle\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

Let α = Phase of $M(j\omega)$;

$$\alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

Let $N = \tan\alpha$.

$$\therefore N = \tan\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

$$\therefore N = \frac{\tan\left(\tan^{-1}\frac{Y}{X}\right) - \tan\left(\tan^{-1}\frac{Y}{1+X}\right)}{1+\tan\left(\tan^{-1}\frac{Y}{X}\right)\tan\left(\tan^{-1}\frac{Y}{1+X}\right)}$$

$$= \frac{\frac{Y}{X} - \frac{Y}{1+X}}{1+\frac{Y}{X}\times\frac{Y}{1+X}} = \frac{\frac{Y(1+X) - XY}{X(1+X)}}{\frac{X(1+X)+Y^2}{X(1+X)}}$$

$$= \frac{Y+XY-XY}{X^2+X+Y^2} = \frac{Y}{X+X^2+Y^2}$$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \times \tan B}$

$$\therefore N = \frac{Y}{X + X^2 + Y^2}$$

On rearranging the above Equation we get.

$$X + X^2 + Y^2 = \frac{Y}{N}$$

$$\therefore X + X^2 + Y^2 - \frac{Y}{N} = 0$$

In the above Equation add the term $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on both Sides.

$$X + X^2 + Y^2 - \frac{Y}{N} + \frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(X^2 + \frac{1}{4} + X\right) + \left(Y^2 + \frac{1}{(2N)^2} - \frac{Y}{N}\right) = \frac{1}{4} + \frac{1}{(2N)^2}$$

$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2} \quad \text{——— (1)}$$

The Eqn of circle with centre at (X_1, Y_1) and radius r is.

$$(X - X_1)^2 + (Y - Y_1)^2 = r^2 \quad \text{——— (2)}$$

On comparing eqn (1) & (2) it can be concluded that that the Eqn (1) represents a family of circle with centre at $(-1/2, 1/2N)$ and with radius $\sqrt{\frac{1}{4} + \frac{1}{(2N)^2}}$ for Various values of N. The circles given by the equation are called N-circles.

For any value of N, the equation of N circles is Satisfied at two points (0,0) and (-1,0). Hence the N-circle passes through these two points for all values of α ($N = \tan\alpha$)

Consider the equation of N-circle when $X = 0$ and $Y = 0$

$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2}$$

$$\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2}$$

$$\frac{1}{4} + \frac{1}{4N^2} = \frac{1}{4} + \frac{1}{4N^2}$$

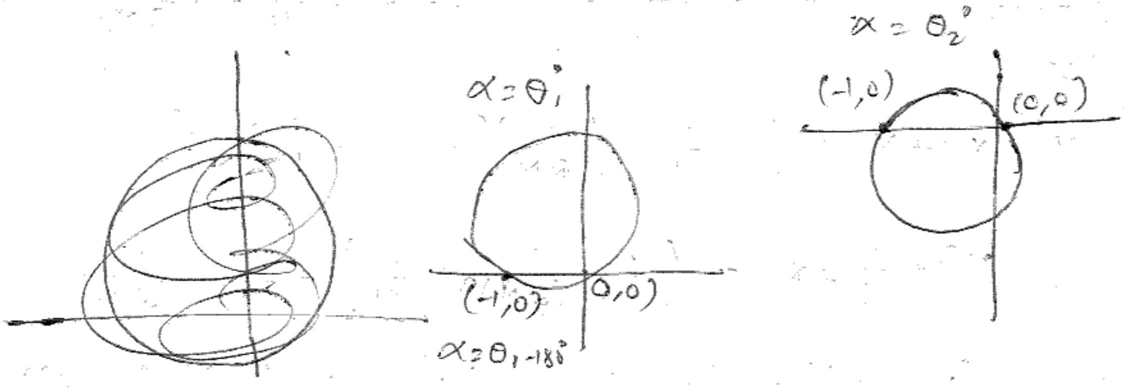
Consider the Equation of N-circle when $X = -1$ and $Y = 0$

$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2}$$

$$\left(-1 + \frac{1}{2}\right)^2 + \left(-\frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(4N)^2}$$

$$\frac{1}{4} + \frac{1}{4N^2} = \frac{1}{4} + \frac{1}{4N^2}$$

The above analysis shows that the eqn of N-circle is satisfied at points (0,0) and (-1,0). When $\alpha = 180°$ the circle becomes a straight line passing through real axis. It is also observed that the circle for $\alpha = \theta° - 180°$ above the real axis will be a part of circle for $\alpha = \theta°$ below the real axis as shown in fig. The family of N are shown in fig.



Part-B Unit-IV

Q1 Using Routh criterion, Determine the Stability of the system represented by the characteristics equation, $s^4+8s^3+18s^2+16s+5=0$ Comment on the location of the roots of the roots of characteristics equation.

Sol^n The characteristics equation of the System is $s^4+8s^3+18s^2+16s+5=0$

The given characteristics Eqn is 4th order Equation and so it has 4 roots. Since the highest power of S is even number from the first row of routh array using the coefficient of even powers of s and form the second row using the coefficient of odd power of s.

s^4 :	1	18	5
s^3 :	8	16	0

The element of s^3 row can be divided by 8 to simplify the Computation.

s^3 =	1	2	0
s^2 =	16	5	0
s^1 =	1.7	0	
s^0 =	0		

→ Column-1.

$$s^2 = \frac{1\times18-2\times1}{1} \qquad \frac{1\times5-0\times1}{1}$$

$$s^2 = 16 \qquad 5$$

$$s^1 = \frac{16\times2-5\times1}{16}$$

$$s^1 = 1.6875 \approx 1.7$$

$$s^0 = \frac{1.7\times5-0\times16}{1.7}$$

$$s^0 = 5$$

On Examining the elements of first column of routh array it is observed that all the elements are positive and there is no sign change. Hence all the roots are lying on the left half of s-plane and the System is stable.

Result

1) Stable System

2) All the four roots are lying on the left half of s-plane.

Q2 Construct Routh array and determine the stability of the system represented by the characteristics equation $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$ Comment on the location of the roots of characteristics equation.

Soln The characteristics Equation of system is,

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

The given characteristics polynomial is 5th order equation and so it has 5 roots, Since the highest power of s is odd number, form the first row of routh array using the coefficient of odd powers of s and form the second row using the coefficient of even power of s.

$$\begin{array}{lccc}
s^5 = & 1 & 2 & 3 \\
s^4 = & 1 & 2 & 5 \\
s^3 = & \epsilon & -2 & \\
s^2 = & \frac{2\epsilon+2}{\epsilon} & 5 & \\
s^1 = & \frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2} & & \\
s^0 = & 5 & &
\end{array}$$

$$s^3 = \frac{1\times 2 - 2\times 1}{1} \qquad \frac{1\times 3 - 5\times 1}{1}$$

$$s^3 = 0 \qquad -2$$

Replace 0 by ϵ

$$s^3 = \epsilon \qquad -2$$

$$s^2 = \frac{\epsilon\times 2 - (-2\times 1)}{\epsilon} \qquad \frac{\epsilon\times 5 - 0\times 1}{\epsilon}$$

$$s^2 = \frac{2\epsilon+2}{\epsilon} \qquad 5$$

$$s^1 = \frac{\frac{2\epsilon+2}{\epsilon}\times(-2) - (5\times\epsilon)}{2\epsilon+2}$$

$$s^1 = \frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}$$

$$s^0 = \frac{\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}\times 5 - 0\times\frac{2\epsilon+2}{\epsilon}}{\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}}$$

On letting $\epsilon \to 0$ we get

$$\begin{array}{lccc}
s^5 = & 1 & 2 & 3 \\
s^4 = & 1 & 2 & 5 \\
s^3 = & 0 & -2 & \\
s^2 = & \infty & 5 & \\
s^1 = & -2 & & \\
s^0 = & 5 & &
\end{array}$$

$\to$ Column 1

On observing the elements of first column of routh array, it is found that there are two sign changes. Hence two roots are lying on the right half of s-plane and the system is unstable. The remaining three roots are lying on the left half of s-plane.

Result :-

1) The system is unstable.

2) Two roots are lying on right half of s-plane & three roots are

lying on left half of s-plane.

Q3 Construct Routh array and determine the stability of the system whose characteristics equation,

$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$. Also determine the number of roots lying on the half of s-plane and on imaginary axis.

Sol^n

The characteristics Eq of the system is

$$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0.$$

The given characteristic polynomial is 6th order ~~polynomial~~ Equation and so it has 6 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^6 =	1	8	20	16
s^5 =	2	12	16	0

The element of s^5 row can be devided by 2 to simplify the calculations.

s^6 =	1	8	20	16
s^5 =	1	6	8	
s^4 =	1	6	8	
s^3 =	0	0		
s^3 =	1	3		
s^2 =	3	8		
s^1 =	0.33			
s^0	8			

$s^4 = \frac{1\times8-6\times1}{1} \quad \frac{1\times20-8\times1}{1} \quad \frac{1\times16-0\times1}{1}$

$s^4 = 2 \quad 12 \quad 16$

divide by 2

$s^4 = 1 \quad 6 \quad 8$

$s^3 = \frac{1\times6-6\times1}{1} \quad \frac{1\times8-8\times1}{1}$

$s^3 = 0 \quad 0$

The auxiliary Eq^n is $A = s^4+6s^2+8$.

On differentiating A with respect to s we get

$$\frac{dA}{ds} = 4s^3+12s$$

The coefficient of $\frac{dA}{ds}$ are used to form s^3 row

$s^3 = 4 \quad 12$

divide by 4

$s^3 = 1 \quad 3$

$s^2 = \frac{1\times6-3\times1}{1} \quad \frac{1\times8-0\times1}{1}$

$s^2 = 3 \quad 8$

The auxiliarly polynomial.

$$s^4+6s^2+8=0$$

Let $s^2 = x$

$$\therefore x^2+6x+8=0$$

The roots of quadratic are.

$$x = \frac{-6 \pm \sqrt{6^2-4\times 8}}{2}$$

$$= -3 \pm 1 = -2 \text{ or } -4$$

The roots of auxilary polynomial is.

$$s = \pm\sqrt{x} = \pm\sqrt{-2} \text{ and } \pm\sqrt{-4}$$

$$= +j\sqrt{2},\ -j\sqrt{2},\ +j2 \text{ and } -j2.$$

s^1	=	$\frac{3\times 3 - 8\times 1}{3}$
s^1	=	0.33
s^0	=	$\frac{0.33\times 8 - 0\times 3}{0.33}$
s^0	=	8

The roots of auxilary polynomial are also roots of characteristic Eqn. Hence 4 roots are lying on imaganary axis and the remaning two roots are lying on the left half of s-plane

Result:-

1> The system is limitedly or marginally stable.

2> Four roots are lying on imaginary axis and remaining two roots are lying on left half of s-plane.

Q4. What are the necessary condition for stability? Explain briefly.

Soln Necessary Condition for Stability are!

Case 1 :! Normal Routh array (Non-zero elements in the first column of routh array)

Case 2 ! A row of all zeros.

Case 3 ! First element of a row is zero but some or other elements are not zero.

Case 1 ! Normal routh Array

In this case there is no difficulty in forming routh array. The routh array can be constructed as explained above. The sign changes are noted to find the number of roots lying on the right half of s-plane and the stability of the system can be estimated.

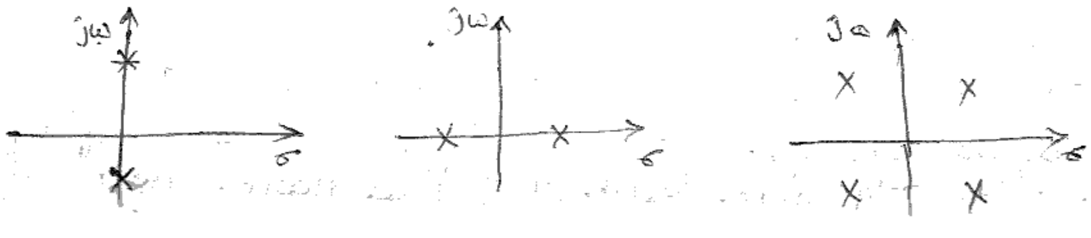
In this case.

1. If there is no sign changes in the first column of Routh array then all the roots are lying on left half of s-plane and the system is stable.
2. If there is sign change in the first column of routh array then the system is unstable and the number of roots lying on the right half of s-plane is equal to number of sign changes. The remaining roots are lying on the left half of s-plane.

<u>Case II</u> :- <u>A row of all Zeros</u>

An all zero row indicates the existance of an even polynomial as a factor of the given characteristic equation. In an even polynomial the exponents of S are even integers or zero only. This even polynomial factor is also called auxillary polynomial. The coefficient of the auxiliary polynomial will always be the elements of the row directly above the row of zeros in the array.

The roots of an even polynomial occur in pairs that are equal in magnitude and opposite in sign. Hence, these roots can be purely imaginary, purely real or complex. The purely imaginary and purely real root occur in pairs. The complex roots occur in groups of four and the complex roots have quadrantal symmetry, that is the roots are symmetrical with respect to both the real and imaginary axes. The fig. 4.1 shows the roots of an even polynomial.



Method - I

1> Determine the auxilary polynomial, A(s)

2> Differentiate the auxilary polygnomial with respect to s, to get dA(S)/ds

3> The row of zeros is replaced with coefficients of dA(s)/ds.

4> Continue the construction of array in the usual manner and the array to interpreted as follows.

a> If there are sign changes in the first column of routh array then the system is unstable.

The number of roots lying on right half of s-plane is equal to number of sign changes. The number of roots on imaginary axis can be estimated from the roots of auxiliary polynomial. The remaining roots are lying on the left of s-plane.

b> If there are no sign changes in the first column of routh array then all zeros row indicates the existence of purely imaginary roots and so the system is limited or marginally stable. The roots of auxilary equation lies on imaginary axix and the remaining roots lies on left half of s-plane.

Method-2

1> Determine the auxiliary polynomial, A(S)

2> Divide the characteristics equation by auxilary polynomial.

3> Construct Routh array using the coefficient of quotient polynomial.

4> The array is interpretted as follows.

a> If there are sign changes in the first column of routh array of quotient polynomial then the system is unstable. The no. of roots of quotient polynomial lying on right half of s-plane is given by number of sign changes in first column of routh array.

The roots of auxilary polynomial are directly calculated to find whether they are purely imaginary or purely real or complex.

The total no. of roots on right half of s-plane is given by the sum of no. of sign changes and the no. of roots of auxiliary polynomial with +ve real part. The no. of roots on imaginary axis can be estimated from the roots of auxiliary polynomial. The remaining roots are lying on the left half of s-plane.

b) If there is no sign change in the first column of routh array of quotient polynomial then the system is limitedly or marginally stable. Since there is no sign change all the roots of quotient polynomial are lying on the left half of s-plane.

The roots of auxilary polynomial are directly calculated to find whether they are purely imaginary or purely real or complex. The number of roots lying on imaginary axis and on the right half of s-plane can be estimated from the roots of auxiliary polynomial. The remaining roots are lying on the left half of s-plane.

Case-III First element of a row is zero

While constructing routh array, if a zero is encountered as first element of a row then all the elements of the next row will be infinite. To overcome this problem let $0 \rightarrow \epsilon$ and complete the construction of array in the usual way (as that of case-I).

Finally let $\epsilon \rightarrow 0$ and determine the values of the elements of the array which are functions of ϵ. The resultant array is interpreted as follows.

a) If there is no sign change in first column of routh array and if there is no row with all zeros, then all the roots are lying on left half of s-plane and the system is stable.

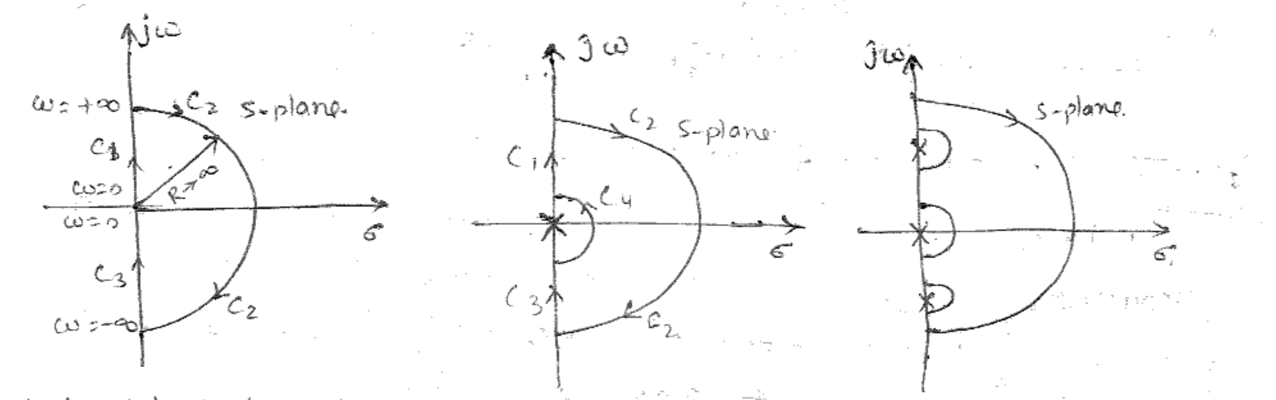
b) If there are sign changes in first column of routh array and there is no row with all zeros, then some of the roots are lying on the right half of s-plane and the system is unstable. The no. of roots lying on the right half of s plane is equal to no. of sign changes and the remaining roots are lying on the left half of s-plane.

c) If there is a row of all zeros after letting $\epsilon \rightarrow 0$ then there is a possibility of roots on imaginary axis. Determine the auxiliary polymial and divide the Characteristic Eqn by auxilary polynomial to eliminate the imaginary roots. The routh array is constructed using the coefficient of quotient polynomial and the Characteristic Eqn is interpreted as explained in method-2 of case-II polynomial.

Q5. Write the procedure for investigating the Stability Using Nyquist criterion.

Soln The following procedure can be followed to investigate the stability of closed loop System from the knowledge of open loop system, using Nyquist Stability criterion.

1. Choose a Nyquist contour as shown in fig. which encloses the entire right half s-plane except the singular points. The Nyquist contor encloses all the right half s-plane poles and zeros of $G(s)H(s)$.



a) Nyquist contour when there is no pole on imaginary axis.

(b) Nyquist contour when there are poles at origin

c) Nyquist contour when there are poles on imaginary axis and at origin

2) The Nyquist contour should be mapped in the $G(s)H(s)$-plane Using the function $G(s)H(s)$ to determine the encirclement $-1+j0$ point in the $G(s)H(s)$-plane. The Nyquist contour of fig (b) can be divided into four section C_1, C_2, C_3 & C_4. The mapping of the four Section in the $G(s)H(s)$-plane. can be carried sectionwise and then combined together to get entire $G(s)H(s)$ contour.

3) In Section C_1 the value of ω varies from 0 to +∞. The mapping of section C_1 is obtained by letting $s = j\omega$ in $G(s)H(s)$ and varying ω from 0 to +∞

$$\text{i.e. } G(s)H(s)\Big|_{\substack{s=j\omega \\ \omega=0 \text{ to } \infty}} = G(j\omega)H(j\omega)\Big|_{\omega=0 \text{ to } \infty}$$

The locous of $G(j\omega)H(j\omega)$ as ω is varied from 0 to +∞ will be the $G(s)H(s)$-contour in $G(s)H(s)$-plane Corresponding to section C_1 in s-plane. This locous is the polar plot of $G(j\omega) H(j\omega)$. There are three ways of mapping this section of $G(s) H(s)$ - contour, they are.

i) Calculate the values of $G(j\omega) H(j\omega)$ for various Values of ω and Sketch the actual locus of $G(j\omega)H(j\omega)$

(or)

ii) Separate the real part and imaginary part of $G(j\omega) H(j\omega)$. Equate the imaginary part to zero, to find the frequency at which the $G(j\omega) H(j\omega)$ locus crosses real axis (to find phase crossover freq). Substitute this frequency on real part and find the crossing point of the locus on real axis. Sketch the approximate locus of $G(j\omega) H(j\omega)$ from the Knowledge of type number and order of the System (or from the value of $G(j\omega) H(j\omega)$ at $\omega = 0$ and $\omega = \infty$).

iii) Separate the magnitude and phase of $G(j\omega) H(j\omega)$ Equate the phase of $G(j\omega) H(j\omega)$ to $180°$ and solve for ω. This value of ω is the phase crossover frequency and the magnitude at this frequency is the crossing point on real axis. Sketch the approximate root locus as mentioned in method (ii)

4. The Section C_2 of Nyquist contour has a Semicircle of infinite radius. Therefore, every point on section C_2 has infinite magnitude but the argument varies from $+\pi/2$ to $-\pi/2$. Hence the mapping of section C_2 from s-plane to $G(s) H(s)$ plane can be obtained by letting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s) H(s)$. and varying θ from $+\pi/2$ to $-\pi/2$.

Consider the loop transfer function in time constant form and with y number of poles at origin as such below.

$$G(s)\,H(s) = \frac{K(1+sT_1)(1+sT_2)(1+sT_3)\ldots}{s^y(1+sT_a)(1+sT_b)(1+sT_c)\ldots}$$

let $G(s) H(s)$ has m zeros & n poles including poles at origin. for practical System $n > m$.

Since $s \to Re^{j\theta}$ and $R \to \infty$ the term $(1+sT)$ can be ~~exposed~~ approximated to sT,

$$\therefore G(S)H(S) \approx K \frac{ST_1 \times ST_2 \times ST_3 \ldots}{S^y \times ST_a \times ST_b \times ST_c \ldots}$$

$$= K_1 \frac{S^m}{S^n} = \frac{K_1}{S^{n-m}}$$

On letting $s = \underset{R\to\infty}{Lt} Re^{j\theta}$ we get,

$$G(S)H(S)\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K_1}{\underset{R\to\infty}{Lt}(Re^{j\theta})^{n-m}} = 0e^{-j\theta(n-m)}$$

when $\theta = \frac{\pi}{2}$, $G(S)H(S) = 0e^{-j\pi/2(n-m)}$

when $\theta = -\frac{\pi}{2}$, $G(S)H(S) = 0e^{+j\frac{\pi}{2}(n-m)}$

From the above two Eqn we can conclude that the section C_2 of Nyquist contour in s-plane is mapped as circles/circular arc around origin with radius tending to zero in the $G(S)H(S)$-plane.

5. In Section C_3, the value of ω varies from $-\infty$ to 0. The mapping of section C_3 is obtained by letting $s = +j\omega$ in $G(S)H(S)$ and varying ω from $-\infty$ to 0.

$$\text{i.e } G(S)H(S)\Big|_{\substack{s=+j\omega \\ \omega=-\infty \text{ to } 0}} = G(j\omega)H(j\omega)\Big|_{\omega=-\infty \text{ to } 0}$$

The locus of $G(j\omega)H(j\omega)$ as ω is varied from $-\infty$ to 0 will be the $G(S)H(S)$ - contour in $G(S)H(S)$ - plane corresponding to section C_3 in S-plane. This locus in the inverse polar plot of $G(j\omega)H(j\omega)$. The inverse polar plot is given by the mirror image of polar plot with respect to real axis.

6. The Section C_4 of Nyquist contour has semicircle of zero radius. Therefore every point on semicircle has zero magnitude but the argument varies from $-\pi/2$ to $\pi/2$. Hence the mapping of section C_4 from S-plane to $G(S)H(S)$ plane can be obtained by letting $S = \underset{R\to 0}{Lt} Re^{j\theta}$ in as shown below:

$$G(S)H(S) = \frac{K(1+ST_1)(1+ST_2)(1+ST_3)\ldots}{S^y(1+ST_a)(1+ST_b)(1+ST_c)\ldots}$$

Let $G(S)\,H(S)$ has m zeros & n-poles including poles at origin. For practical system $n > m$. Since $s \to Re^{j\theta}$ and $R \to 0$, the term $1+sT$ can be approximated to 1.

$$\therefore \; G(s)\,H(s) \approx K \frac{1}{s^{y}}$$

On letting, $s = \underset{R\to 0}{Lt} \; Re^{j\theta}$ we get

$$G(s)\,H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to 0}{Lt}\,(Re^{j\theta})^{y}} = \infty\, e^{-j\theta y}$$

When $\theta = -\frac{\pi}{2}$, $G(s)\,H(s) = \infty\, e^{j\pi/2\, y}$

When $\theta = \pi/2$, $G(s)\,H(s) = \infty\, e^{-j\pi/2\, y}$

From the above two Eq^n we can conclude that the Section C_4 of Nyquist contour in s-plane is mapped as circle/circular arc in $G(s)\,H(s)$ plane with origin as centre and infinite radius.