10/08/11.

Expressions for frequency domain Specifications for 2nd order system:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$T(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2+2\zeta\omega_n(j\omega)+\omega_n^2}$$

$$\frac{\omega_n^2}{(j\omega)^2} = \frac{\omega_n^2}{-\omega^2+2j\zeta\omega_n\omega+\omega_n^2}$$

$$= \frac{\omega_n^2}{\omega_n^2\left[\frac{-\omega^2}{\omega_n^2}+j2\zeta\frac{\omega}{\omega_n}+1\right]} = \frac{1}{-\left(\frac{\omega}{\omega_n}\right)^2+2j\zeta\frac{\omega}{\omega_n}+1}$$

Let Normalised frequency, $u = \frac{\omega}{\omega_n}$

$$T(j\omega) = \frac{1}{(1-u^2)+j2\zeta u}$$

Let M = Magnitude of closed loop transfer function.

α = phase angle of transfer function.

$$M = |T(j\omega)| = \left[\frac{1}{(1-u^2)^2+(2\zeta u)^2}\right]^{1/2}$$

$a+jb$

$r\angle\theta$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}(b/a)$

$$\frac{dM}{du} = \left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{-1/2}$$

1] Resonant peak, $M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

2] Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$$u_r = \frac{\omega_r}{\omega_n} = \sqrt{1-2\zeta^2}$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

3] Bandwidth, $\omega_b = \omega_n u_b$

where u_b = Normalized Bandwidth $= \dfrac{\omega_b}{\omega_n}$

$$\omega_b = \omega_n\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]^{1/2}$$

$\frac{\omega}{\omega_n}+1$

4] Phase Margin (γ)

$$\gamma = 180^\circ + \tan^{-1}\left(\frac{2\zeta}{u_{gc}}\right)$$

$$= 180^\circ + \tan^{-1}\left[\frac{2\zeta}{\left(-2\zeta^2+\sqrt{4\zeta^4+1}\right)^{1/2}}\right]$$

where, u_{gc} = normalized gain cross over frequency.

$$= \frac{\omega_{gc}}{\omega_n}$$

Bode plot is a frequency plot of the transfer function of a system.

Two graphs:

1. Magnitude plot (M Vs $\log \omega$)
2. phase plot. (Phase angle Vs $\log \omega$)

16/08/11.

The gain margin is defined as the reciprocal of the magnitude of $G(j\omega)$ at phase cross over frequency (ω_{pc}).

The phase cross over frequency is the frequency at which the phase of $G(j\omega)$ is $180°$.

The phase margin is defined as $\gamma = 180° + \phi_{gc}$ where ϕ_{gc} is the phase angle of $G(j\omega)$ at gain crossover frequency.

The gain cross over frequency is the frequency at which the amplitude of $G(j\omega)$ is unity (or db magnitude is zero).

Q] Plot the diagram

$$G(s) = \frac{10}{s(1+0.4s)(1+0.8s)}$$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.8j\omega)}$$

$$G(j\omega) = \frac{10}{j\omega(1+j0.4\omega)(1+j0.8\omega)}$$

$$\phi = -90° - \tan^{-1} 0.4\omega - \tan^{-1} 0.8\omega$$

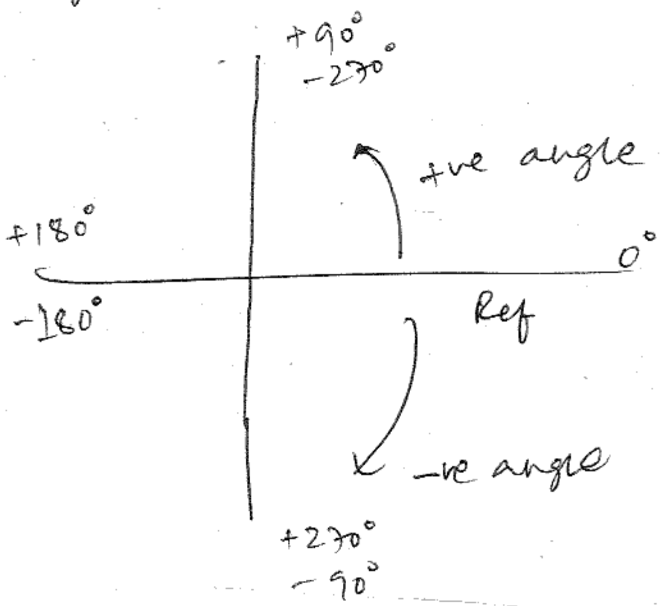
polar plot

Magnitude of $G(j\omega)$ Vs Phase angle of $G(j\omega)$ on polar co-ordinates as ω is varied from 0 to infinity.

$|G(j\omega)| \angle G(j\omega)$

$G(j\omega) = G_R(j\omega) + j\,G_i(j\omega)$

17/08/11. To plot the polar compute the Magnitude and phase of $G(j\omega)$ values for various values of ω and calculate them.



To plot the polar

ω rad/sec.	
$G(j\omega)$	
$\angle G(j\omega)$ deg.	

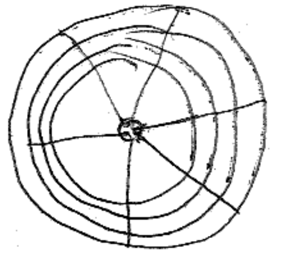
The polar plot is

The circles represent the magnitude and the radial line represent the phase angle.

TYPE-0 :

$G(s) = \frac{1}{1+sT}$

$G(j\omega) = \frac{1}{1+j\omega T}$



24/08/11.

N- circle

Let $N = \tan\alpha$

where, $\alpha = \tan^{-1} Y/X - \tan^{-1} Y/_{1+X}$

$$N = \tan\left[\tan^{-1} Y/X - \tan^{-1} Y/_{1+X}\right]$$

$$\left[\because \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B}\right]$$

$$= \frac{\tan(\tan^{-1} Y/X) - \tan(\tan^{-1} Y/_{1+X})}{1+\tan.\tan^{-1}(Y/X)\cdot\tan^{-1}(\tan^{-1} Y/_{1+X})}$$

$$= \frac{Y/X - Y/_{1+X}}{1+Y/X \cdot Y/_{1+X}} = \frac{\frac{Y+\cancel{XY}-\cancel{XY}}{\cancel{X(1+X)}}}{\frac{X+X^2+Y^2}{\cancel{(1+X)X}}}$$

$$N = \frac{Y}{X+X^2+Y^2}$$

Rearranging the terms,

$$X+X^2+Y^2 = Y/N$$

$$X+X^2+Y^2 - Y/N = 0$$

Add the term $\frac{1}{4}+\left(\frac{1}{2N}\right)^2$ to both sides,

$$X+X^2+Y^2 - Y/N + \frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(X^2+\frac{1}{4}+X\right) + \left(Y^2+\frac{1}{(2N)^2} - Y/N\right) = \frac{1}{4}+\frac{1}{(2N)^2}$$

$$(X+1/2)^2 + (Y-1/2N)^2 = \frac{1}{4}+\frac{1}{(2N)^2}$$

The equation of a circle with centre at (x_1, y_1) and radius r is,

$$(x-x_1)^2 + (y-y_1)^2 = r^2$$

Centre of the radius is $\left(-\frac{1}{2}, \frac{1}{2N}\right)$

and radius $\sqrt{\frac{1}{4} + \frac{1}{(2N)^2}}$

Frequency domain.

13/07/11.

UNIT-II.

STANDARD TEST SIGNALS

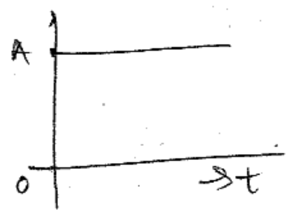
Time Response Analysis.

1) Transient response.
2) Steady state response:

Standard test signals

1. Step Signal - Sudden Change
2. Impulse - Shock
3. Ramp - Constant velocity
4. parabolic - Constant acceleration.

Step Signal.



$r(t)$

$r(t) = 0$ at $t < 0$

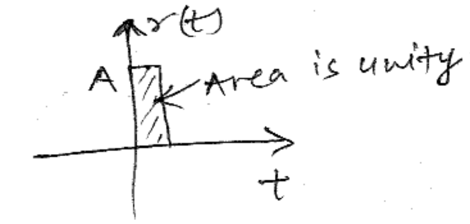
$r(t) = A$ at $t > 0$

unit step. $A = 1$

$r(t) = 1$ at $t > 0$

$R(s) = 1/s$

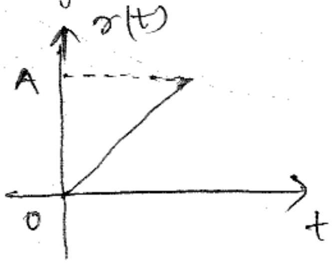
Impulse signal



$r(t) = \infty$ at $t = 0$

$r(t) = 0$ at $t \neq 0$.

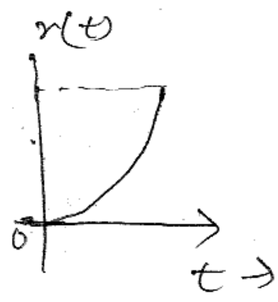
Ramp



$r(t) = At$ for $t > 0$

$= 0$ for $t < 0$

parabolic

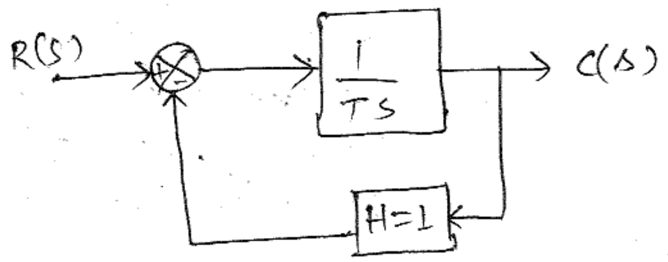


$r(t) = \frac{At^2}{2}$ for $t > 0$

$= 0$ for $t < 0$

Q) Find Time response analysis of first order.

Consider a 1st order system.



The closed loop transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \frac{1/Ts}{1+\frac{1}{Ts}\times 1} = \frac{1}{1+Ts}$$

where, $T \rightarrow$ The time constant of the system.

For unit step input, $r(t) = 1$

$$R(s) = 1/s$$

$$C(s) = \frac{1}{1+Ts} \cdot R(s)$$

$$= \frac{1}{1+Ts} \times \frac{1}{s} = \frac{1}{s(1+Ts)}$$

$$C(s) = \frac{1}{s(1+Ts)} = \frac{A}{s} + \frac{B}{1+Ts}$$

$$1 = A(1+Ts) + Bs$$

Equating the constant term & 's' term

$$\boxed{A = 1}$$

$$B + T = 0$$

$$\boxed{B = -T}$$

$$c(s) = \frac{1}{s} - \frac{T}{1+Ts}$$

$$\boxed{c(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}}$$ //

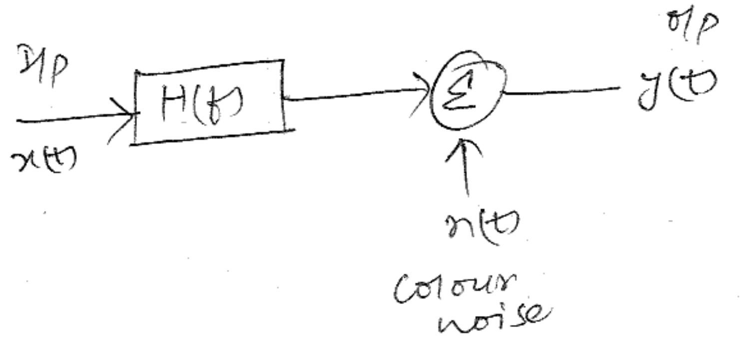
$$c(t) = L^{-1}[1/s] - L^{-1}\left[\frac{1}{s+1/T}\right]$$ [∵ Using Inverse Laplace Transform]

$$\boxed{c(t) = 1 - e^{-1/T\,t}.}$$ //

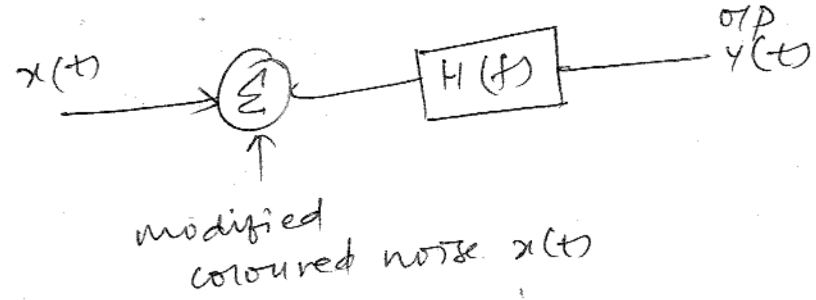
Information Capacity of Coloured Noise Channel

The additive noise is not white (signal to Noise ratio is not constant with frequency over the bandwidth).

Let $n(t)$ be the channel noise additive of the channel output is needed at sampling function of stationary gaussian processes of zero mean and power spectral density, $S_n(f)$.



Bandwidth power links noise channel.



Equivalent model of channel.

1] Find the i/p symbols described by the power spectrum density $S_x(f)$ that maximize the mutual information b/w channel o/p $y(t)$ and channel $x(t)$, subject to the constraint that average power $x(t)$ is fixed at the constant value 'p'.

2] Hence, determine the optimum information capacity of the channel.

$$S'_n(f) := \frac{S_n(f)}{H(f)} \quad \text{--- (1)}$$

where,

$S'_n(f) \to$ power density of noise.

$H(f) \to$ magnitude response of channel.

Channel is divided into large no. of adj. frequency, is loss.