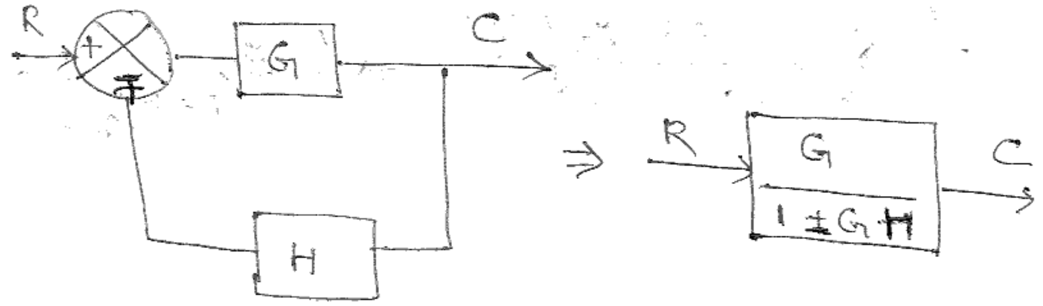
x) Elimination of feedback loop.



J.S.
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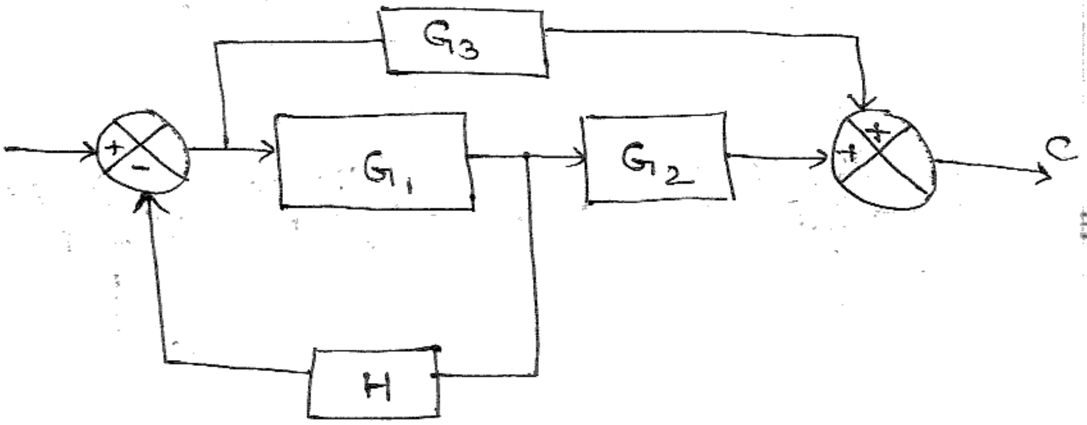
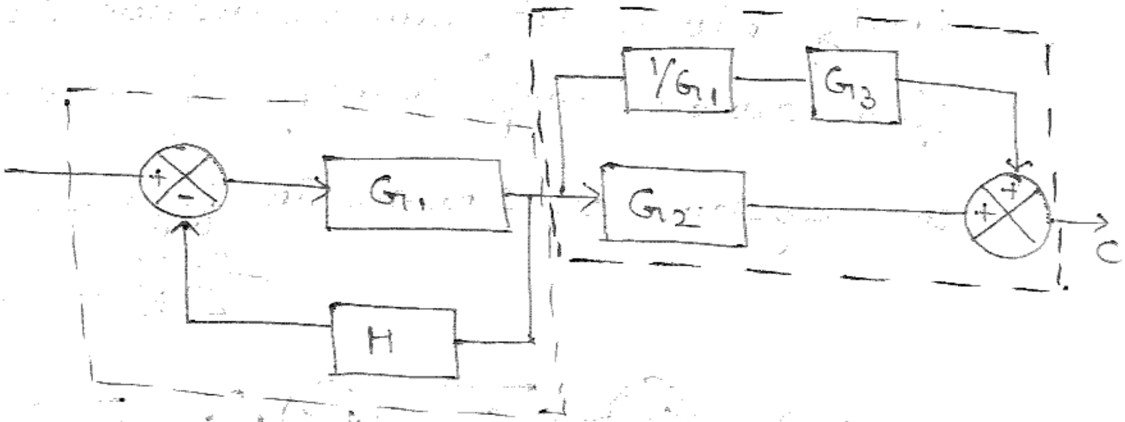
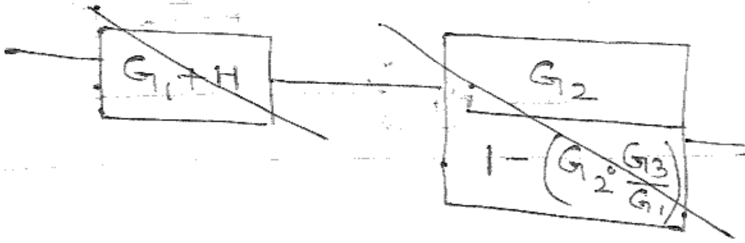
Q- Reduce the block diagram shown in figure and find $\frac{C}{R}$.



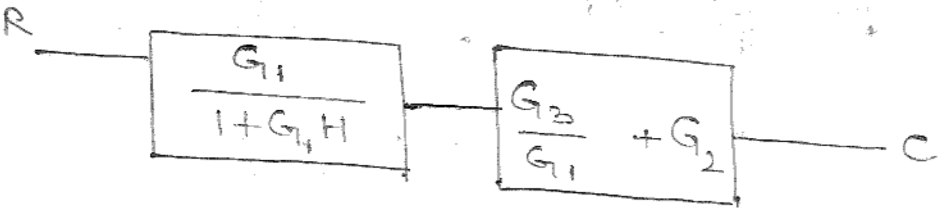
Step1. Moving the branch pt ahead of the block



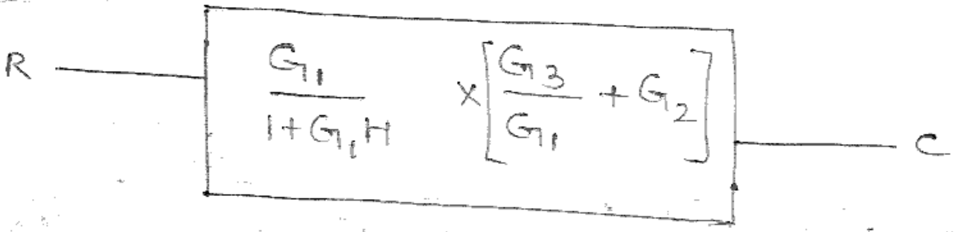
~~Step 2.~~

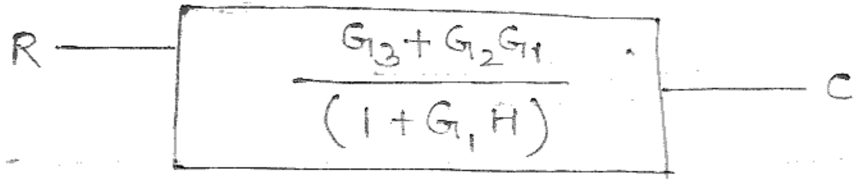


Step 2.



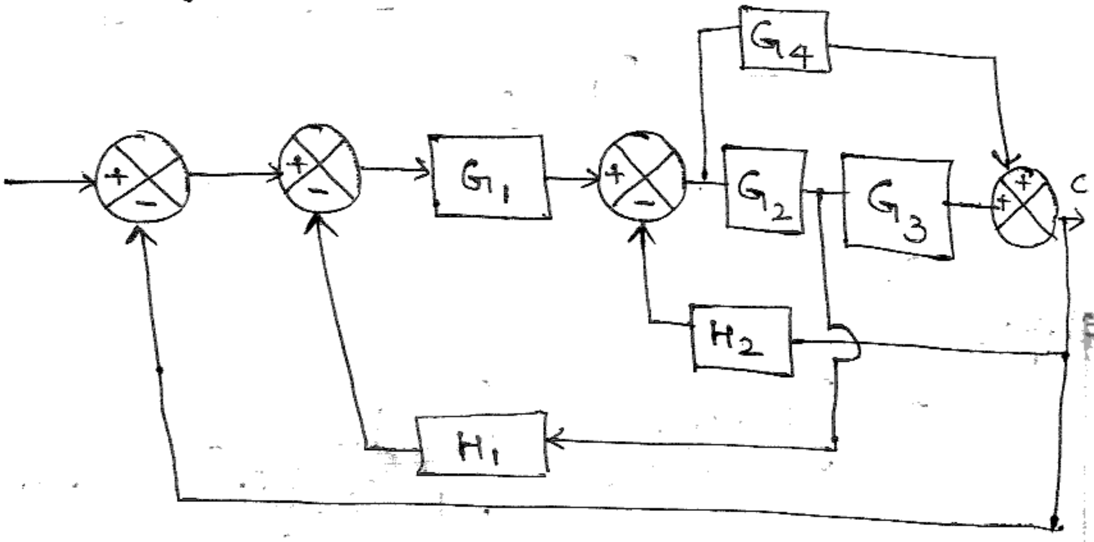
Step 3.



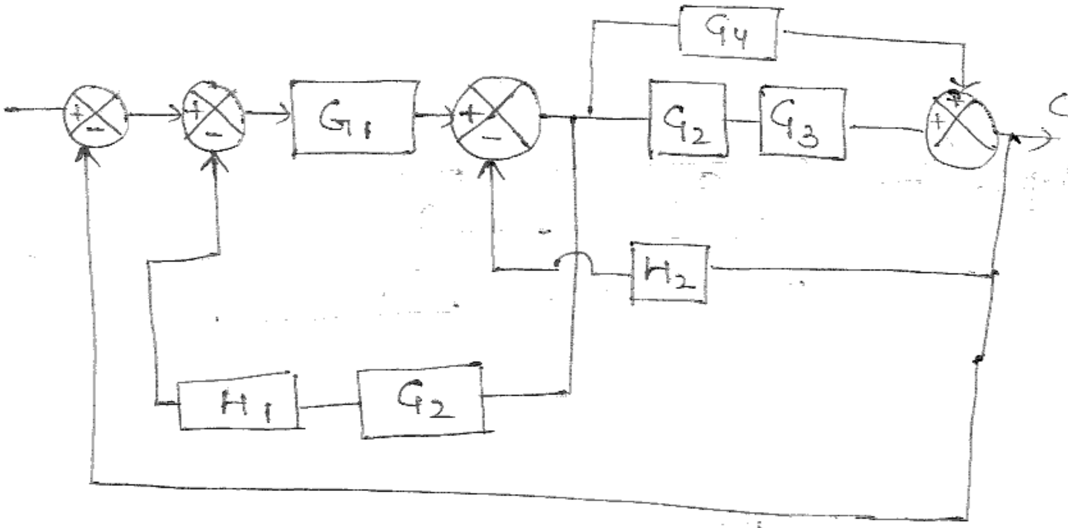


$$\therefore \; C/R = \frac{G_1 G_2 + G_3}{1 + G_1 H}$$

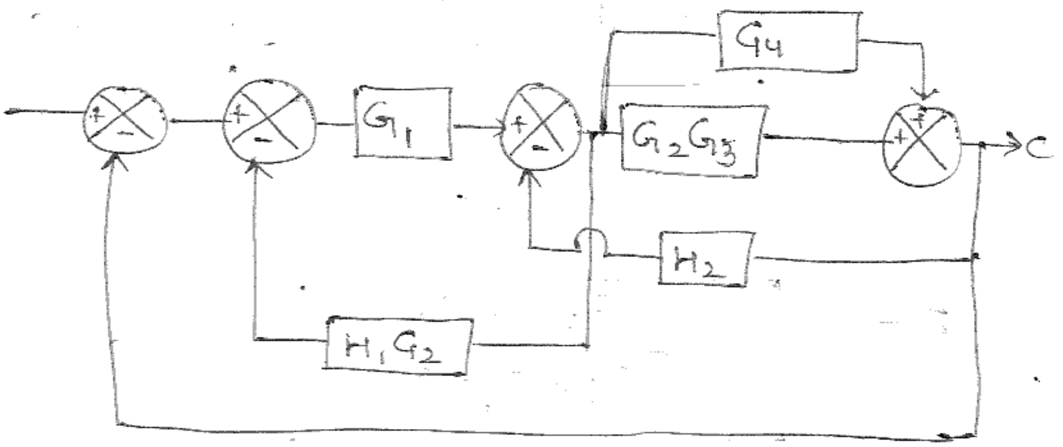
Q- Using block diagram reduction technique find closed loop transfer function of the system as shown in figure.



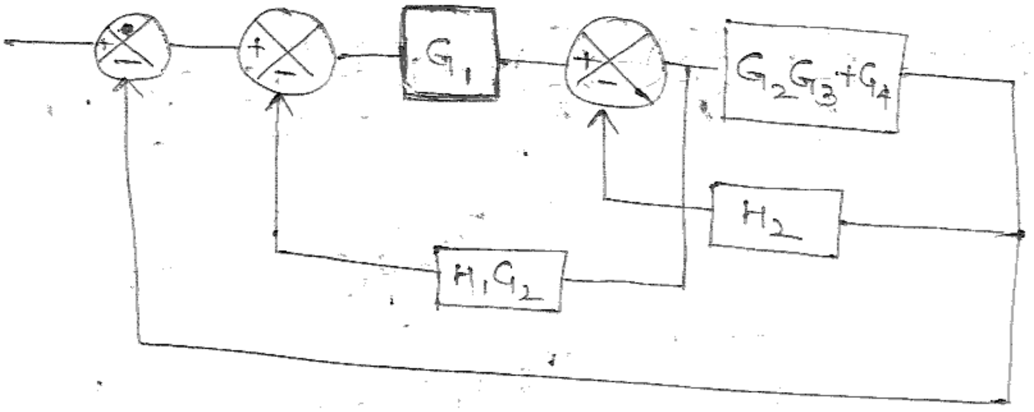
Step 1.



Step 2.

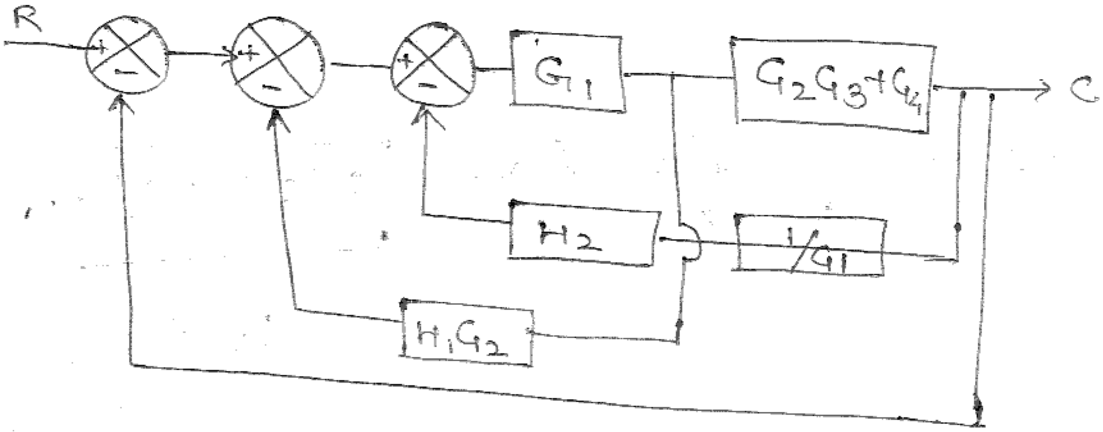


chnique step3.

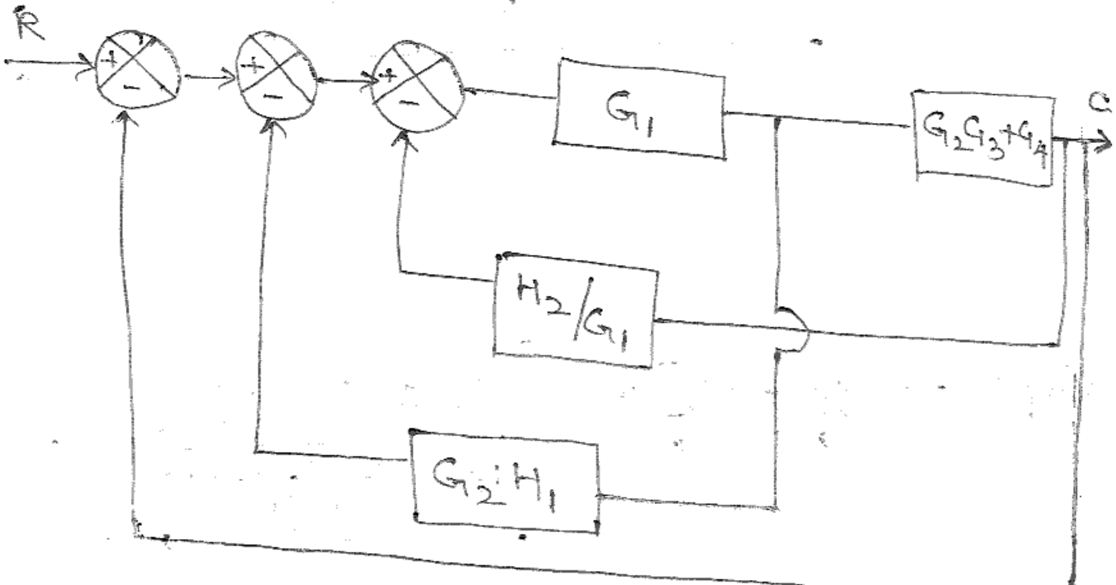


of

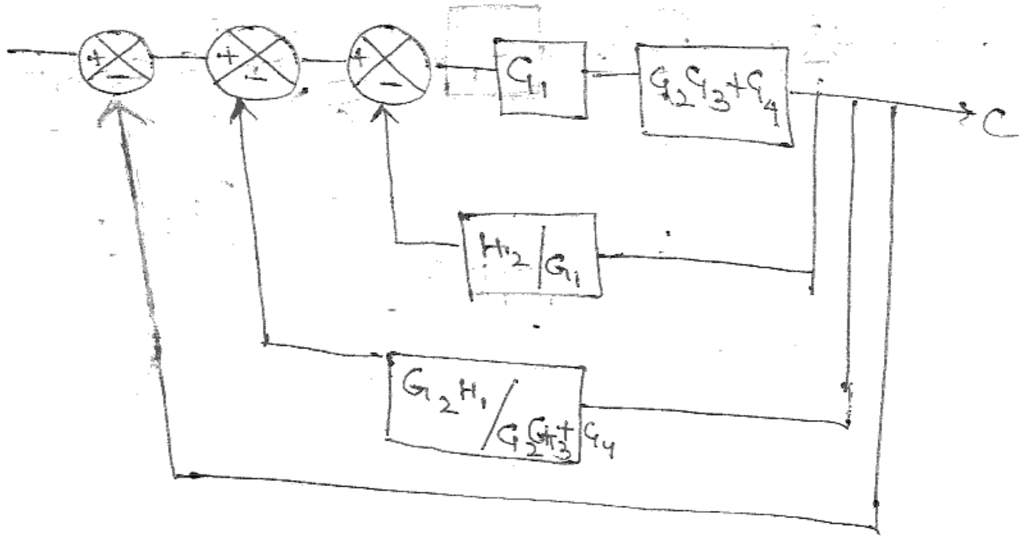
step4.



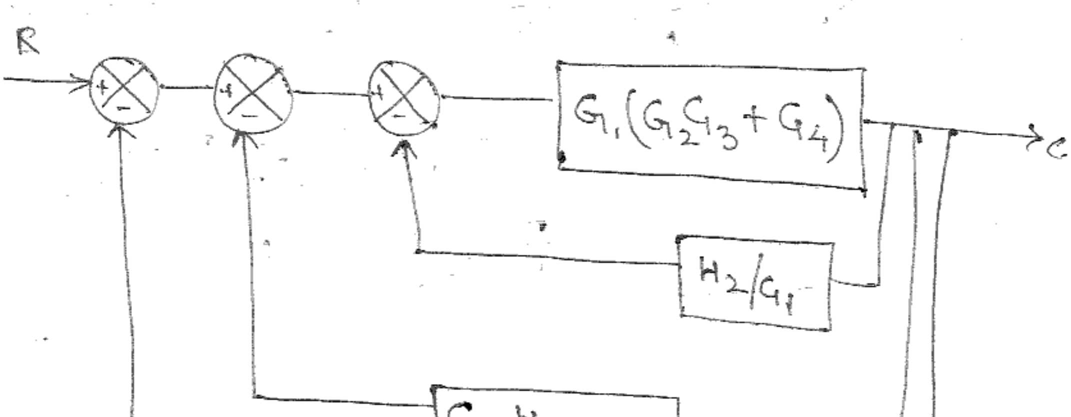
Step5.



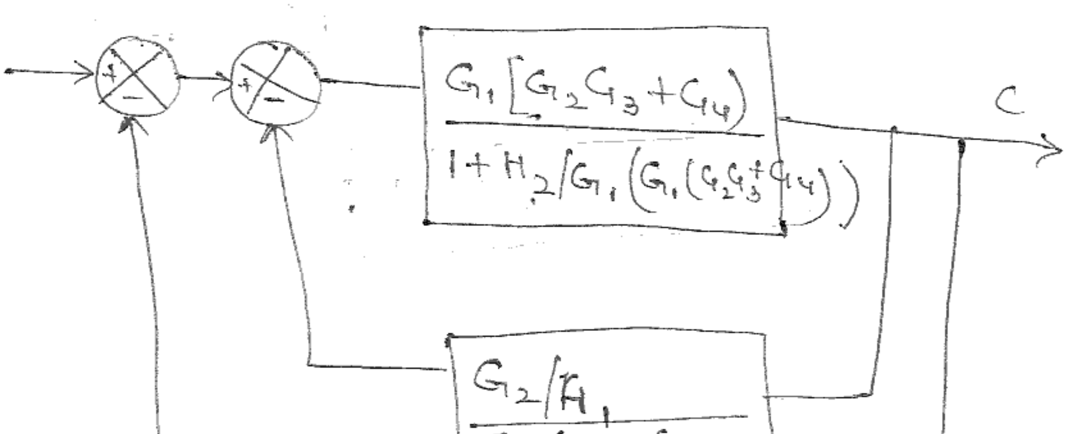
Step 6.



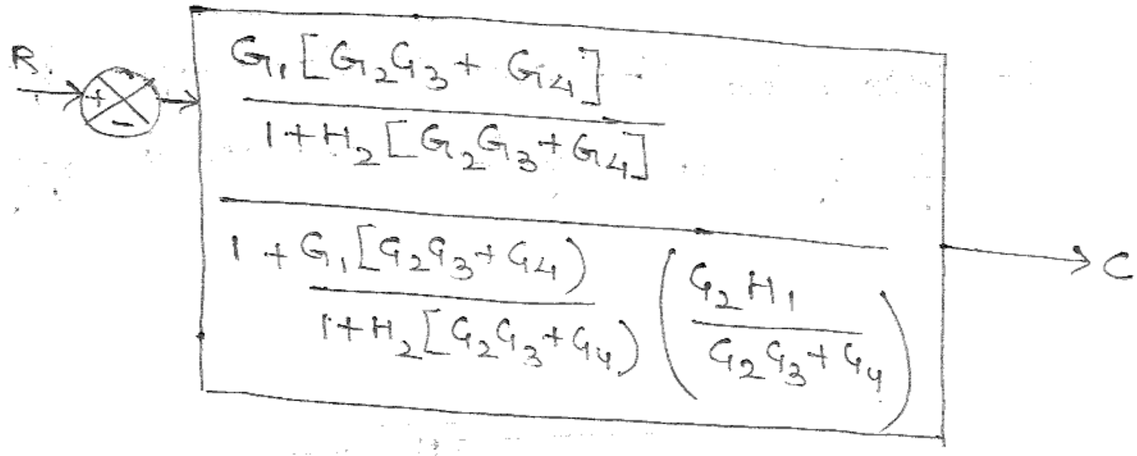
Step 7.



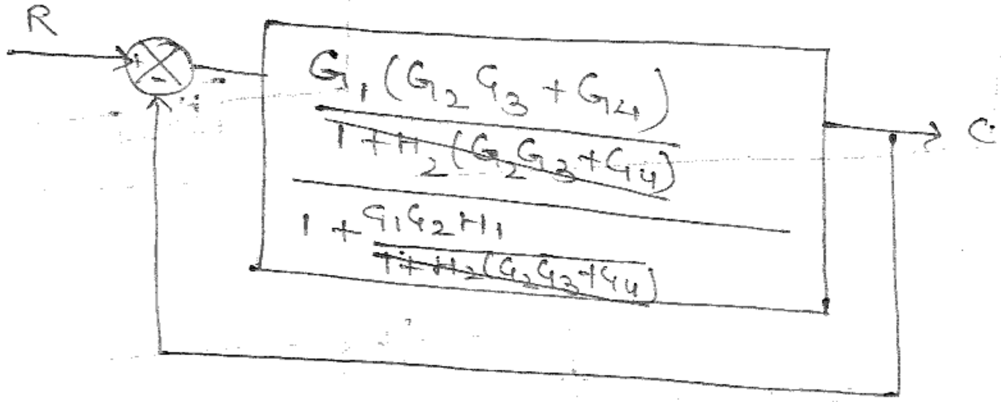
Step 8.



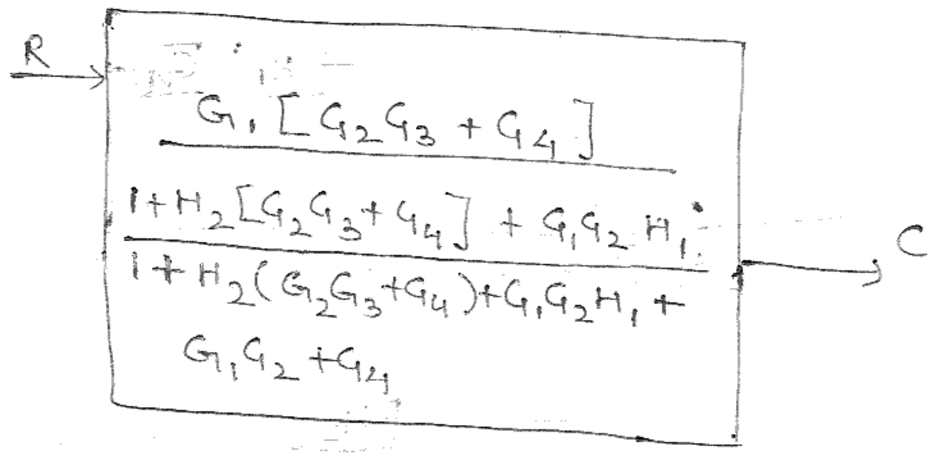
Step 9.



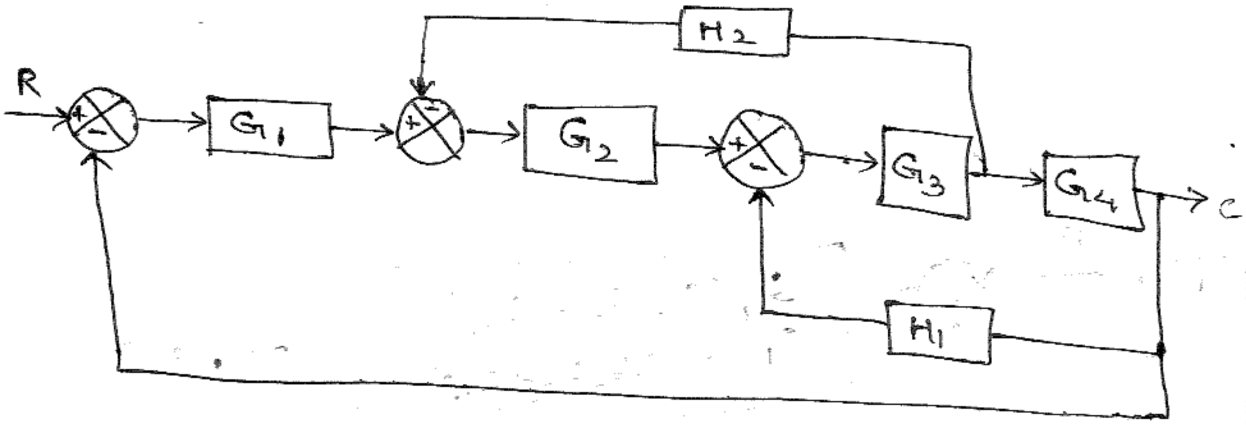
Step 10.



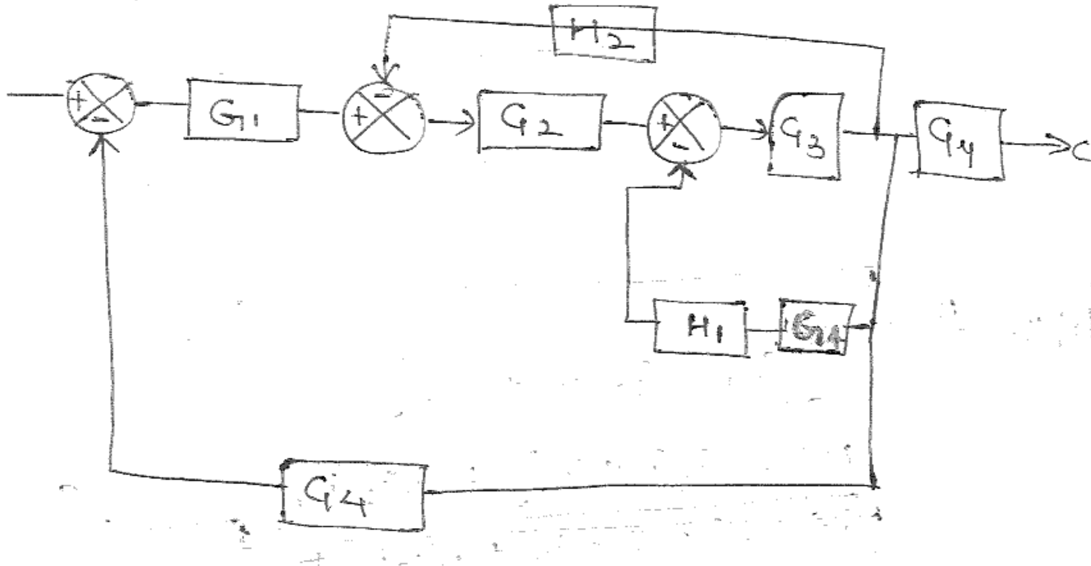
Step 11.



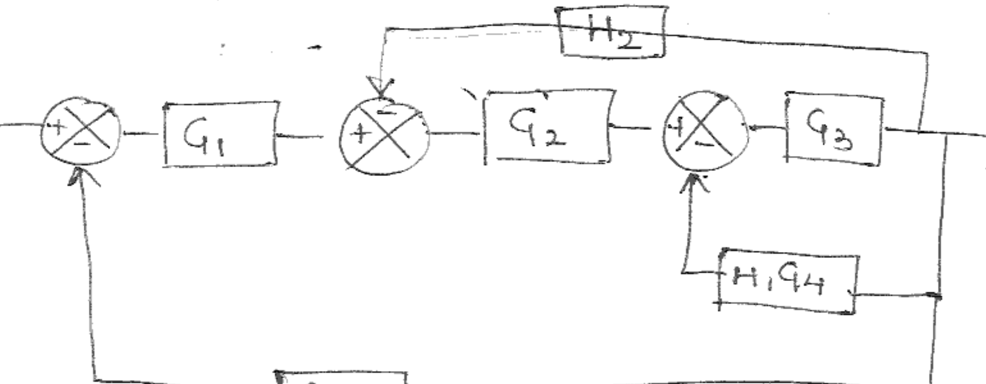
Q- Determine the overall transfer func $\frac{C}{R}$ for the system shown in fig.



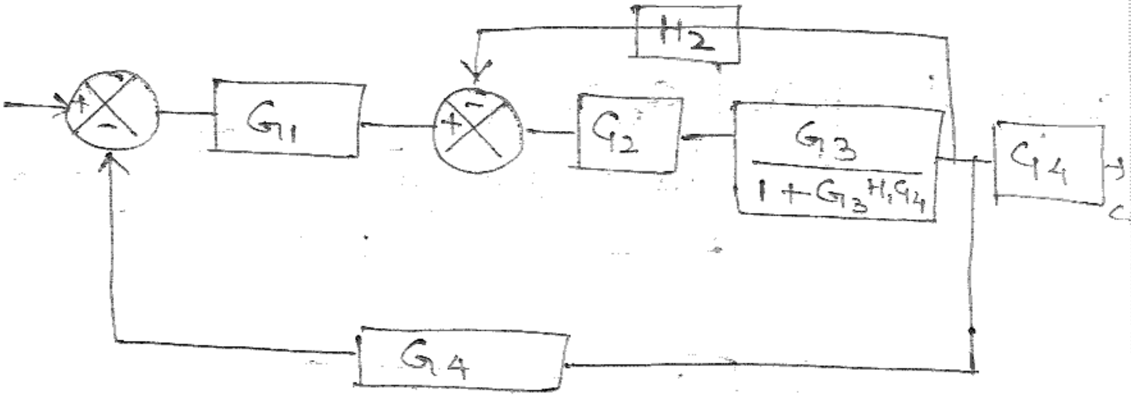
Sol-



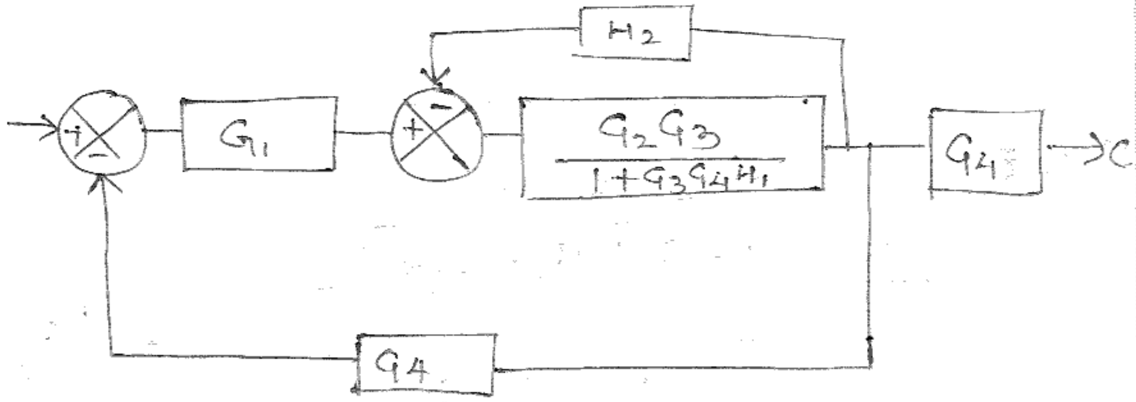
Step 2.



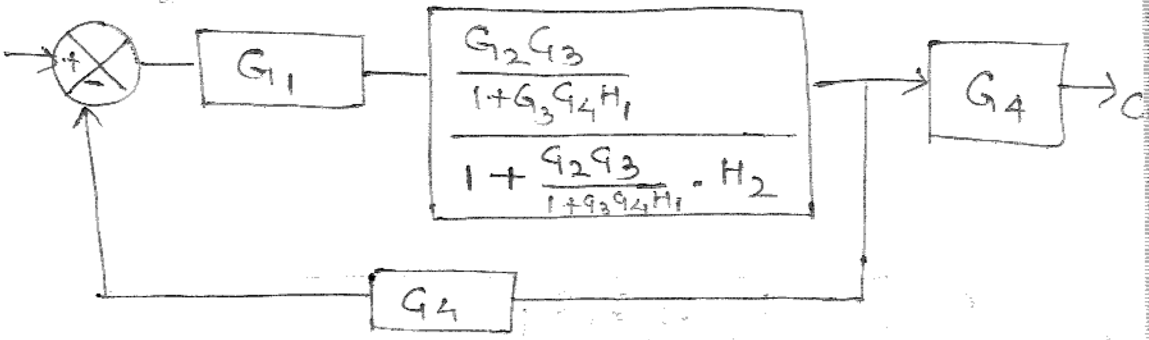
step 3.



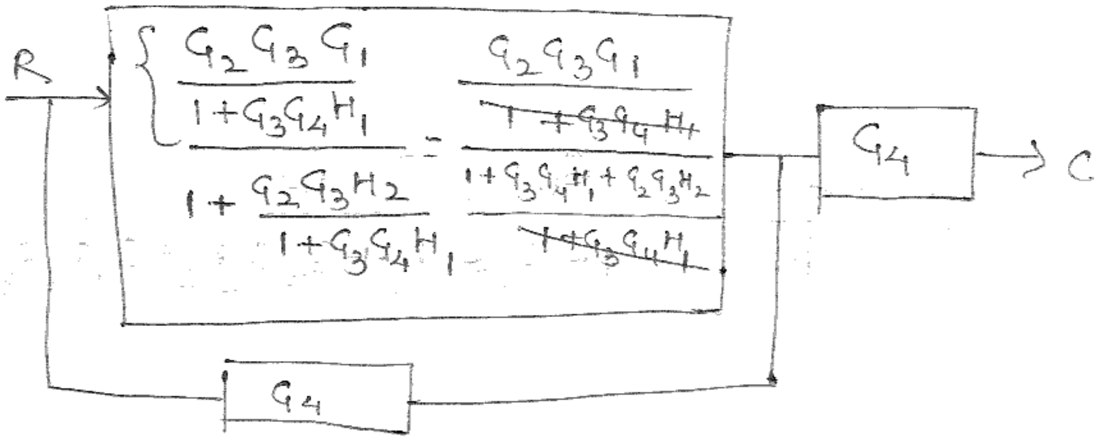
step 4.



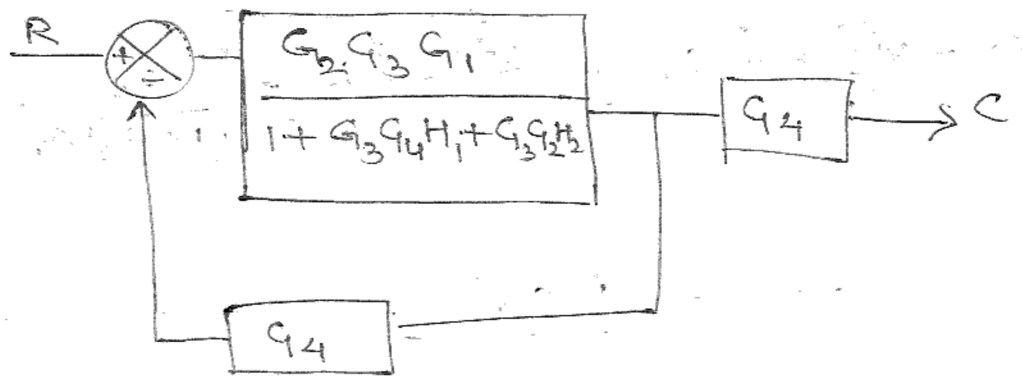
step 5.



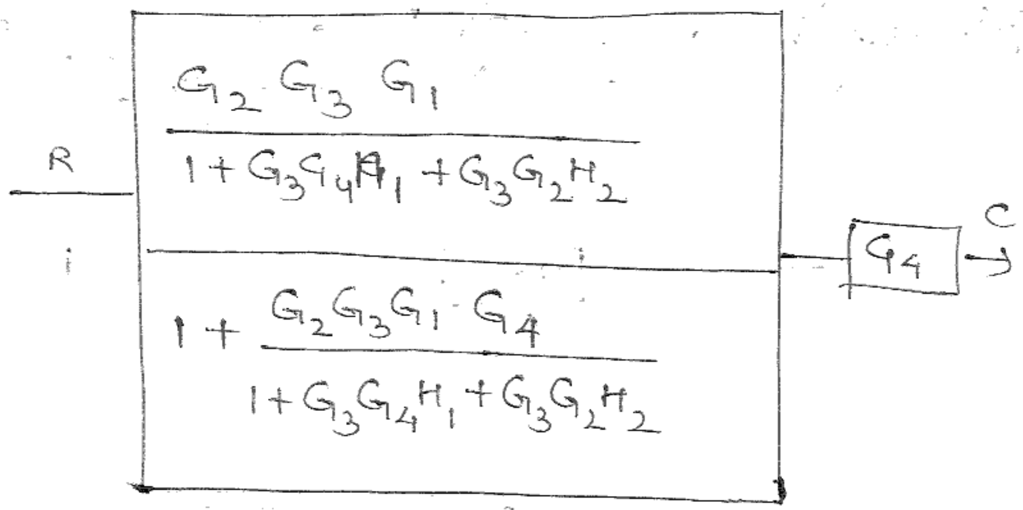
step 6.



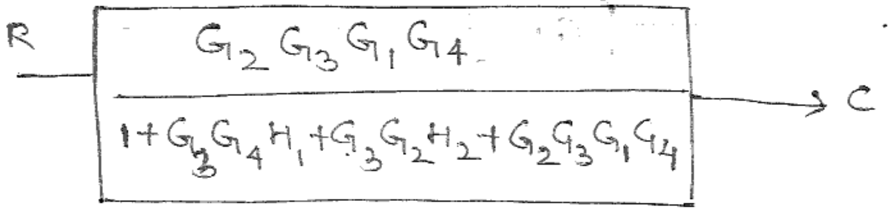
Step 7.



Step 8



Step 9.



$$\therefore \frac{C}{R} = \frac{G_1\,G_2\,G_3\,G_4}{1+G_3G_4H_1+G_3G_2H_2+G_1G_2G_3G_4}$$

7.11

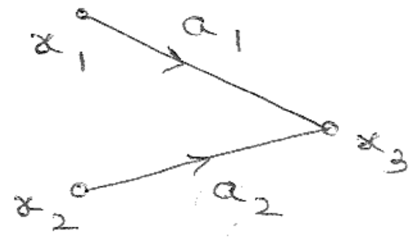
★ Signal Flow Graph

It is used to represent the control system graphically.

→ Rules

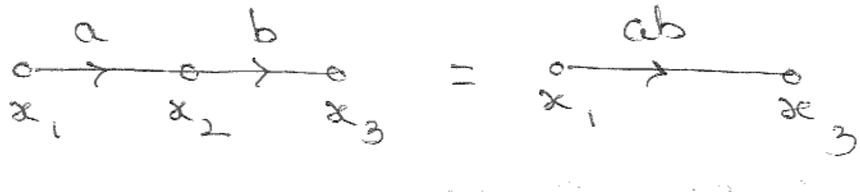
1) Incoming signal to a node ~~to~~ through a branch is given by the product of a signal at ~~and~~ previous node and gain of the branch

$$x_2 = a x_1$$

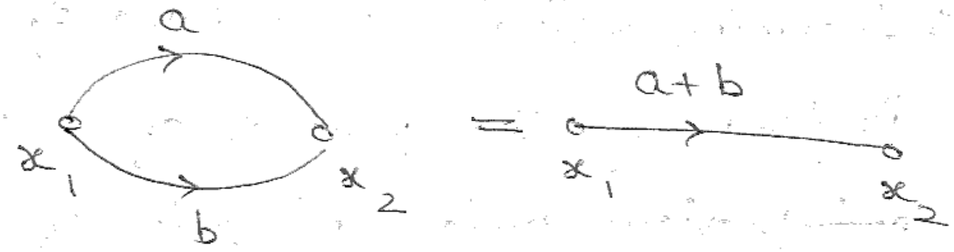


$$x_3 = a_1 x_1 + a_2 x_2$$

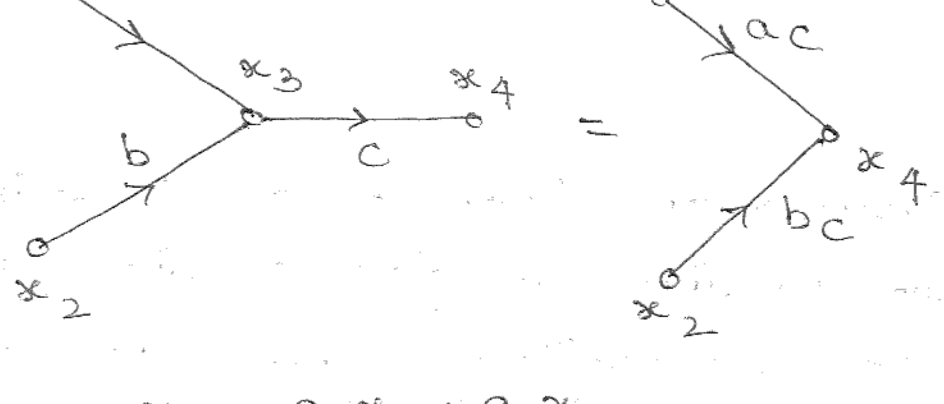
2) Cascaded and branches can be ~~eliminated~~ ~~to~~ combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.



3) Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittance.

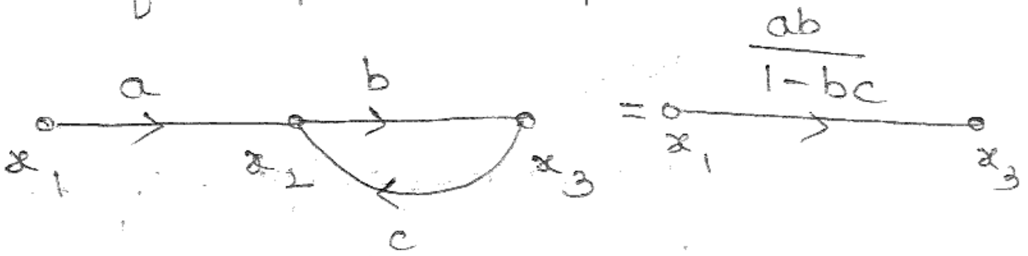


4) Mixed load can be eliminated by multiplying the Transmittance of out going branch to the transmittance of all incoming branches to the mixed load.



$$x_3 = a_1 x_1 + a_2 x_2$$

5) Loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input.



$$x_2 = ax_1 + cx_3$$

$$x_3 = bx_2$$

$$\frac{x_3}{b} = ax_1 + cx_3$$

$$ax_1 = \frac{x_3}{b} - cx_3$$

$$ax_1 = \frac{x_3 - bcx_3}{b}$$

$$abx_1 = x_3(1-bc)$$

$$\frac{x_3}{x_1} = \cancel{\frac{1-bc}{ab}} = \frac{ab}{1-bc}$$

$$\frac{x_3}{x_1} = \frac{ab}{1-bc}$$

Mason's Gain Formula

12.7.11

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

Here,

T = Transfer func of the system

P_K = forward path gain of K^{th} forward ~~P_K th~~ path.

$\Delta = 1 -$ [Sum of individual loop gain]

+ [Sum of gain products of all possible combinations of two non-touching loops]

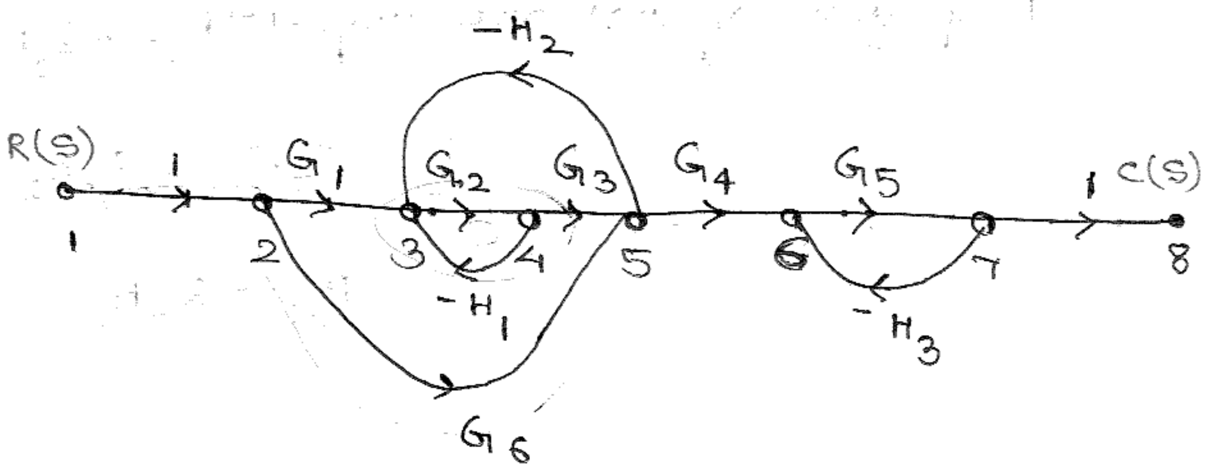
− [Sum of gain products of all possible combinations of three non-touching loops]

+ . . . .

$\Delta_K = \Delta$ for that part of the graph which is non-touching K_{th} forward path.

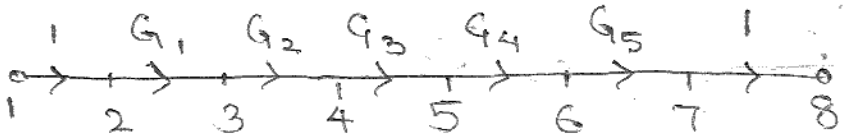
12.7.11

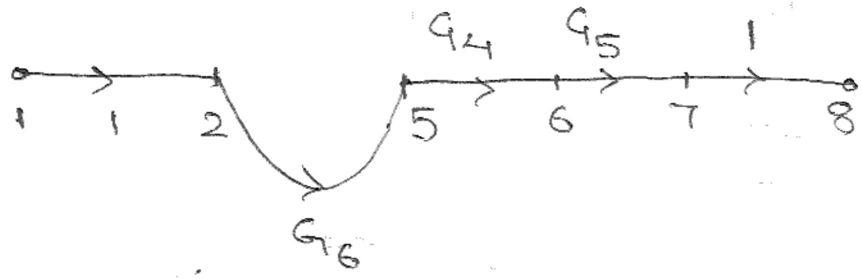
Q- Find the overall transfer function of the system whose signal flow graph is shown in figure.



Sol- 1) Forward path gain

K = 2

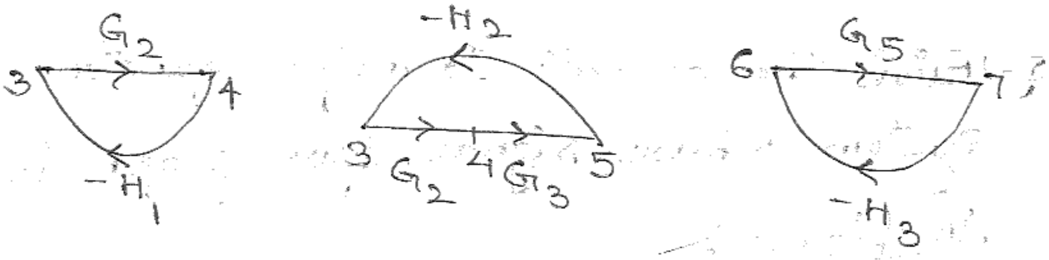




Gain of forward path 1 $\Rightarrow P_1 = G_1 G_2 G_3 G_4 G_5$

lly, $P_2 = G_6 G_4 G_5$

II) Individual loop gain

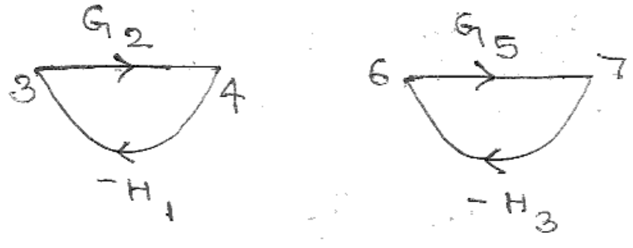


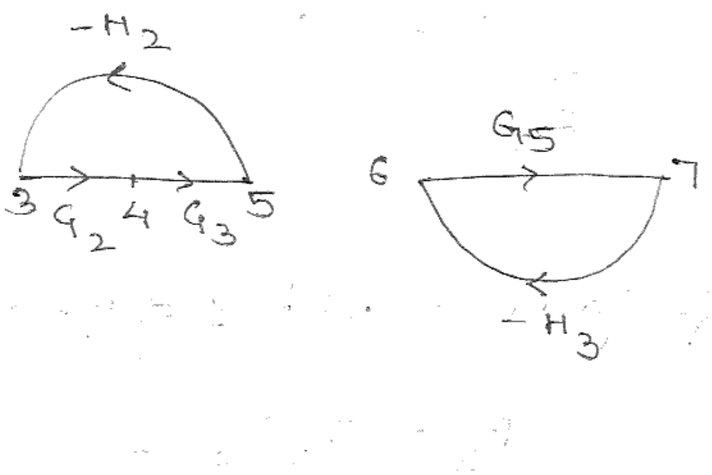
Loop gain of individual loop - 1 $\Rightarrow P_{11} = -G_2H_1$

$P_{21} = -G_2G_3H_2$

$P_{31} = -G_5H_3$

III) Gain product of two non touching loop





Gain product 1st combination of two non touching loop $P_{12} = (-G_2H_1)(-G_5H_3)$

$$= G_2G_5H_1H_3$$

$$P_{22} = G_2G_3G_5H_2H_5$$

IV) Calculate Δ

$$\Delta = 1 - [P_{11} + P_{21} + P_{31}] + [P_{12} + P_{22}]$$

$$= 1 - [-G_2H_1 + G_2G_3H_2 - G_5H_3] + [G_2G_5H_1H_3 + G_2G_3G_5H_2H_5]$$

$$= 1 - [-G_2H_1 - G_2G_3H_2 - G_5H_3] + [G_2G_5H_1H_3 + G_2G_3G_5H_2H_5]$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_5$$

V) Calculate Δ_K

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - (-G_2 H_1)$$

$$= 1 + G_2 H_1$$

$$T = \frac{1}{\Delta} \sum_K [P_K \Delta_K]$$

$$T = \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]$$

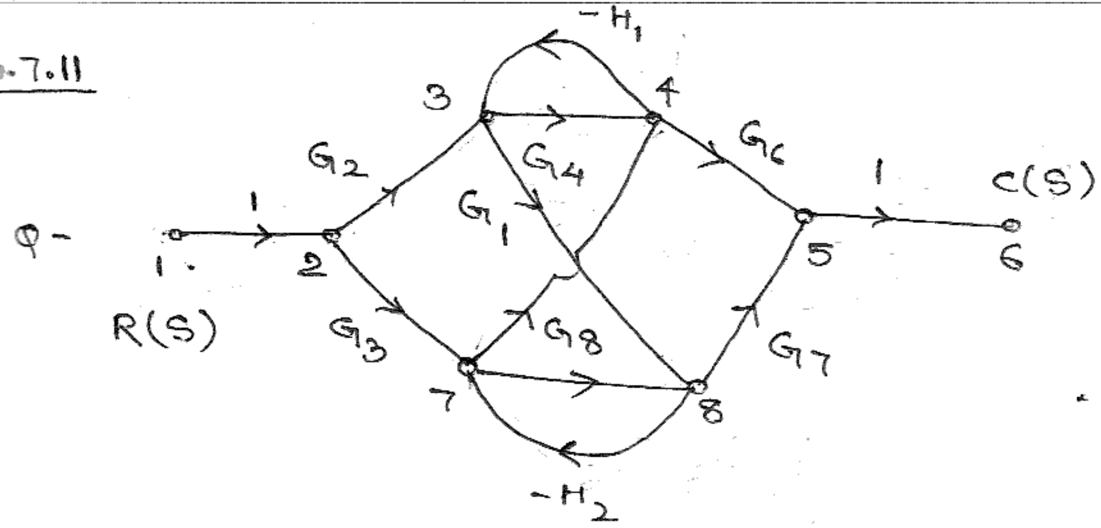
$$= \frac{1}{\Delta} [G_1 G_2 G_3 G_4 G_5 + G_6 G_4 G_5 (1 + G_2 H_1)]$$

$$T = \frac{1}{\Delta} [G_1 G_2 G_3 G_4 G_5 + G_6 G_4 G_5 + G_6 G_4 G_5 G_2 H_1]$$

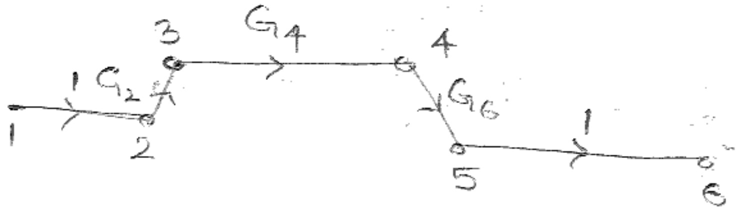
$$T = \frac{G_1 G_2 G_3 G_4 G_5 + G_6 G_4 G_5 + G_6 G_4 G_5 G_2 H_1}{1 + G_2 H_1 + G_2 H_2 G_3 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_5}$$

9 -

P.7.11

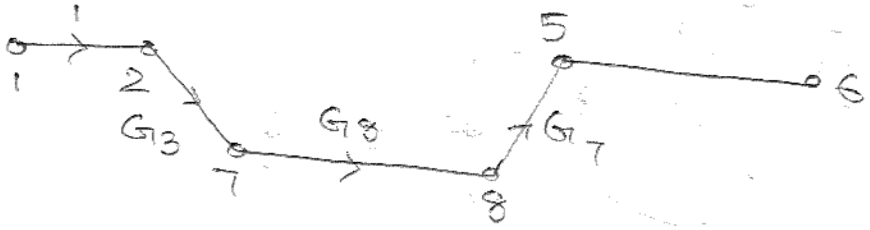


K = 6



$P_1 = G_2 G_4 G_6$

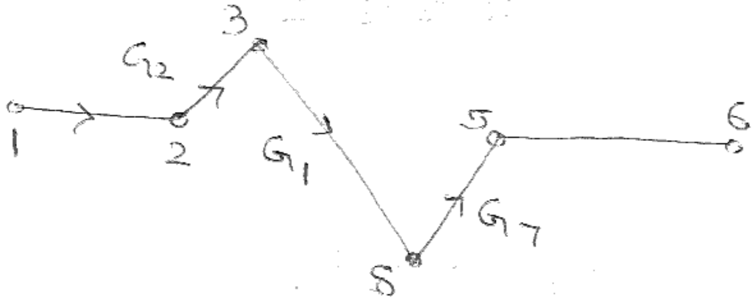
$(1 + G_2 H_1)$



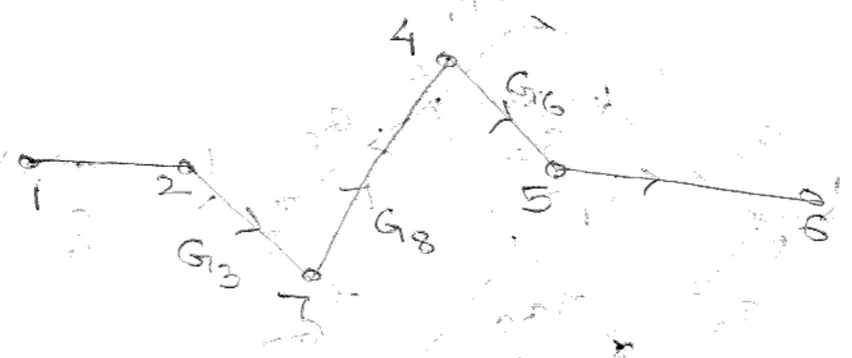
$G_5 G_2 H_1]$

$P_2 = G_3 G_8 G_7$

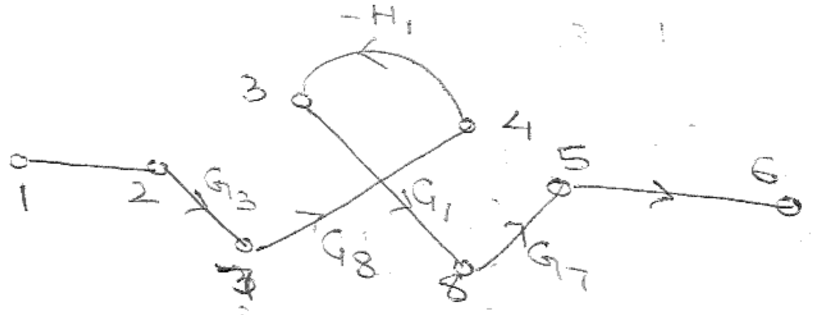
$\frac{5 G_2 H_1}{H_1 H_3}$



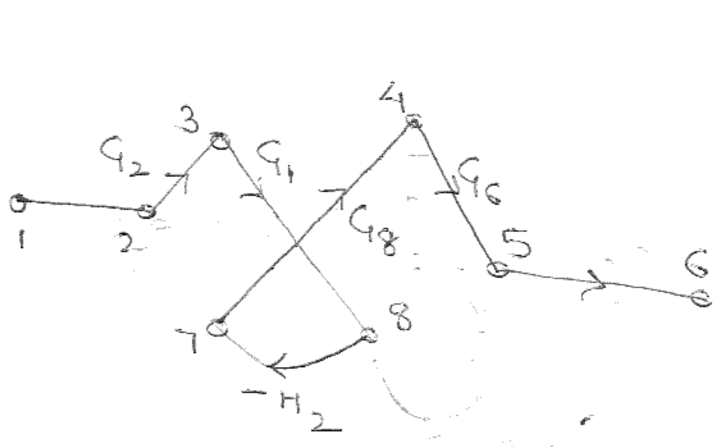
$P_3 = G_2 G_1 G_7$



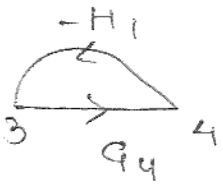
$$P_4 = G_3 G_8 G_6$$



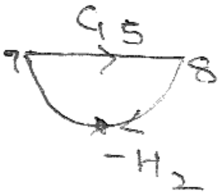
$$P_5 = -G_3 G_8 G_1 G_7 H_1$$



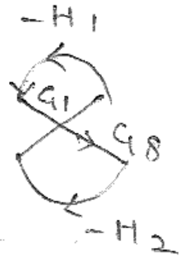
$$P_6 = -G_2 G_1 G_8 G_6 H_2$$



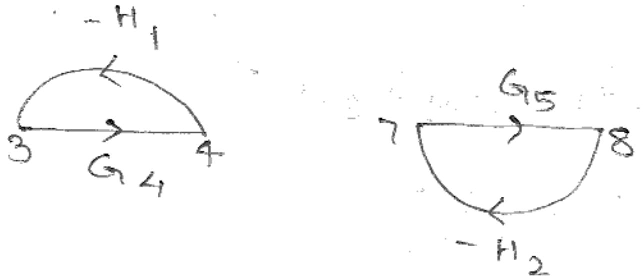
$$P_{11} = -G_4 H_1$$



$$P_{21} = -G_5H_2$$



$$P_{31} = G_1G_8H_1H_2$$



$$= G_5H_1G_4H_2$$

$$\Delta = 1 - [P_{11} + P_{21} + P_{31}] + P[_{12}]$$

$$= 1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_5G_4H_1H_2$$

Δ_K

$$\Delta_1 = 1 + H_2G_5$$

$$\Delta_2 = 1 + H_1G_4$$

$$T = \frac{1}{\Delta} \sum_{K} [P_K \Delta_K]$$

$$= \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 \cdots + P_6 \Delta_6]$$

$$T = \frac{1}{\Delta} [G_2 G_4 G_6 (1 + H_2 G_5) + G_3 G_5 G_7 (1 + H_1 G_4)$$

$$+ G_3 G_8 G_6 (1) + G_2 G_1 G_7 (1) +$$

$$\{- G_2 G_1 H_2 G_8 G_6 (1)\} -$$

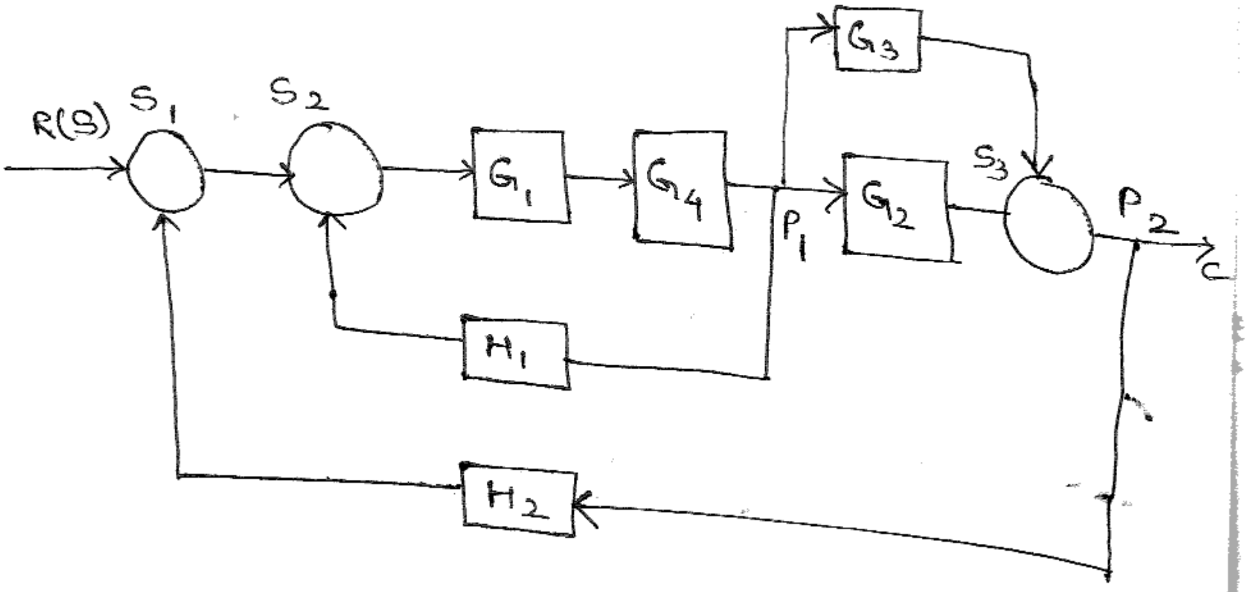
$$G_3 G_8 H_1 G_7 G_1]$$

$$T = \frac{1}{1 + (G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_3 G_4 H_1 H_2)}$$

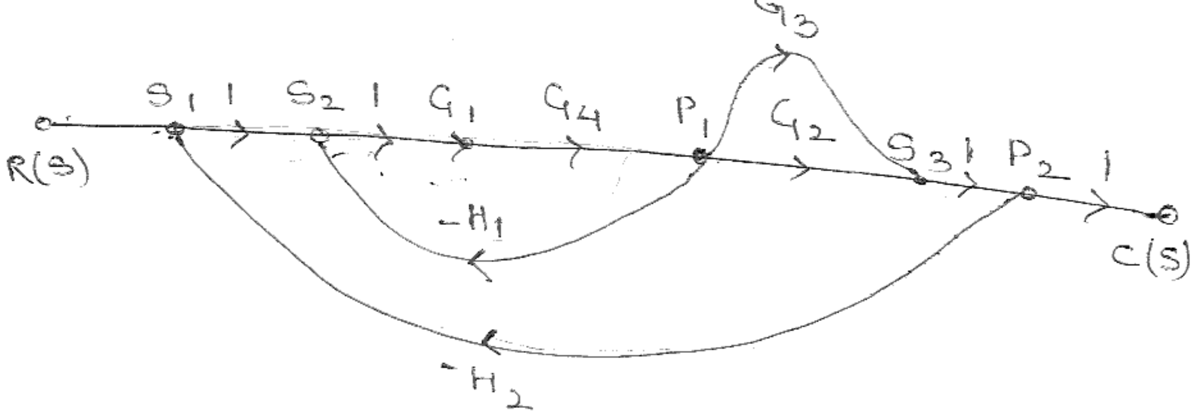
(Ans)

14.7.11

Q- Convert the following block diagram to signal flow graph and find the transfer function.

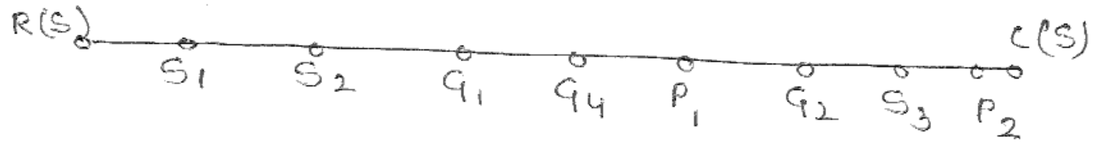


⇓



I) Forward path gain

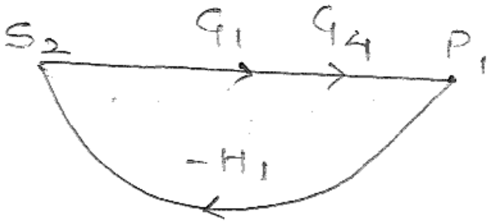
K = 2



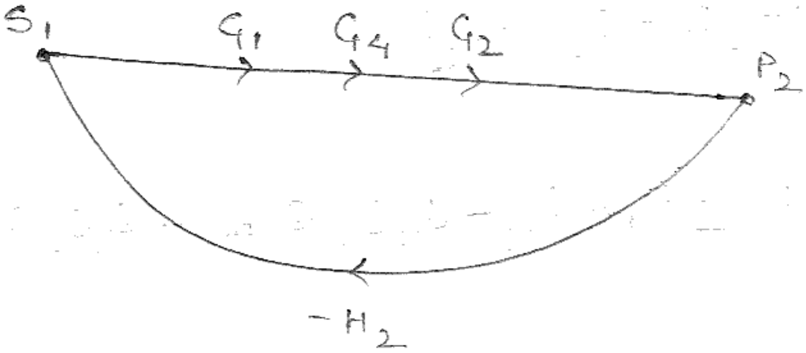
$P_1 = G_1 G_4 G_2$

$P_2 = G_1 G_4 G_3$

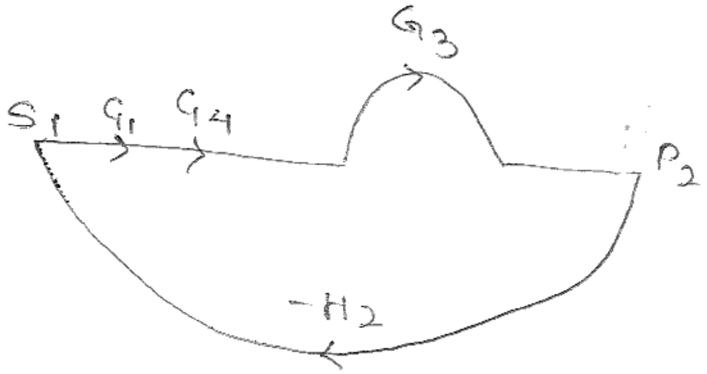
II) Individual loop gain



$P_{11} = -H_1 G_1 G_4$

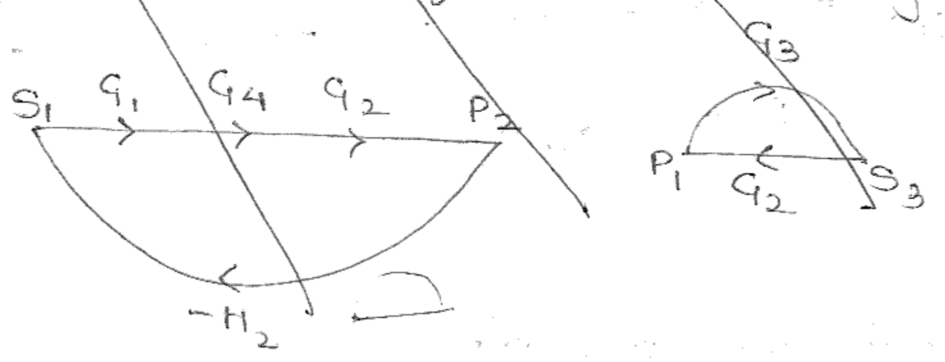


$P_{21} = -G_1 G_4 G_2 H_2$



$P_{31} = -G_1 G_4 G_3 H_2$

III) Gain product of two non-touching loops



III) No. of two non-touching loops = 0

IV) Calculate Δ

$$\Delta = 1 - [P_{11} + P_{21} + P_{31}]$$

$$= 1 - [-H_1 G_1 G_4 - G_1 G_4 G_2 H_2 - G_1 G_4 G_3 H_2]$$

$$\Delta = 1 + H_1 G_1 G_4 + G_1 G_4 G_2 H_2 + G_1 H_2 G_3 G_4$$

V) Calculate Δ_K

$K = 2$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$T = \frac{1}{\Delta} \sum_K [P_K \Delta_K]$$

loops

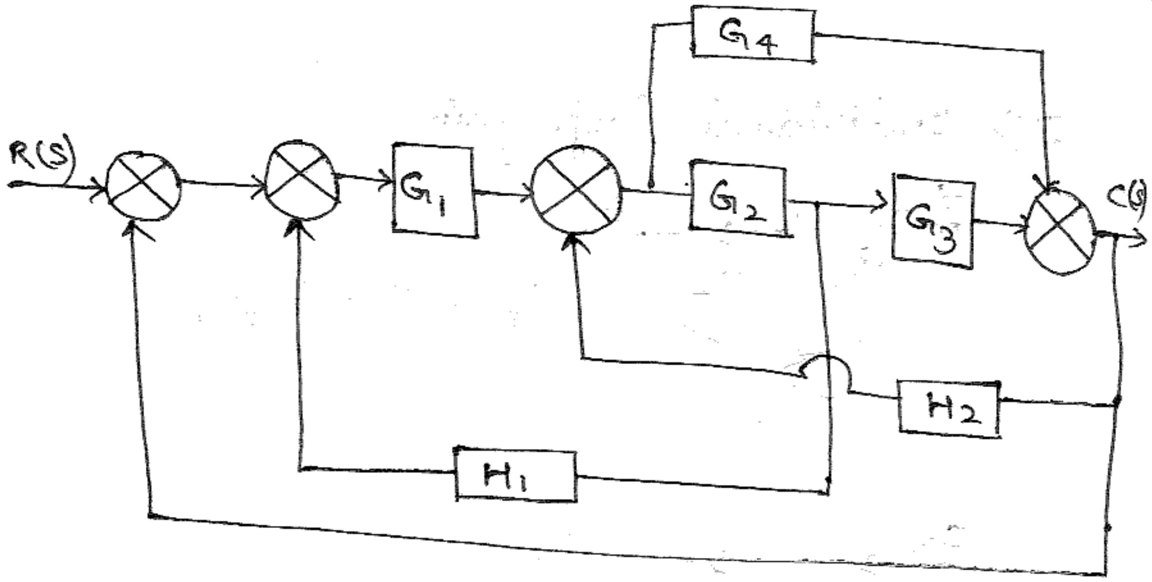
$$T = \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

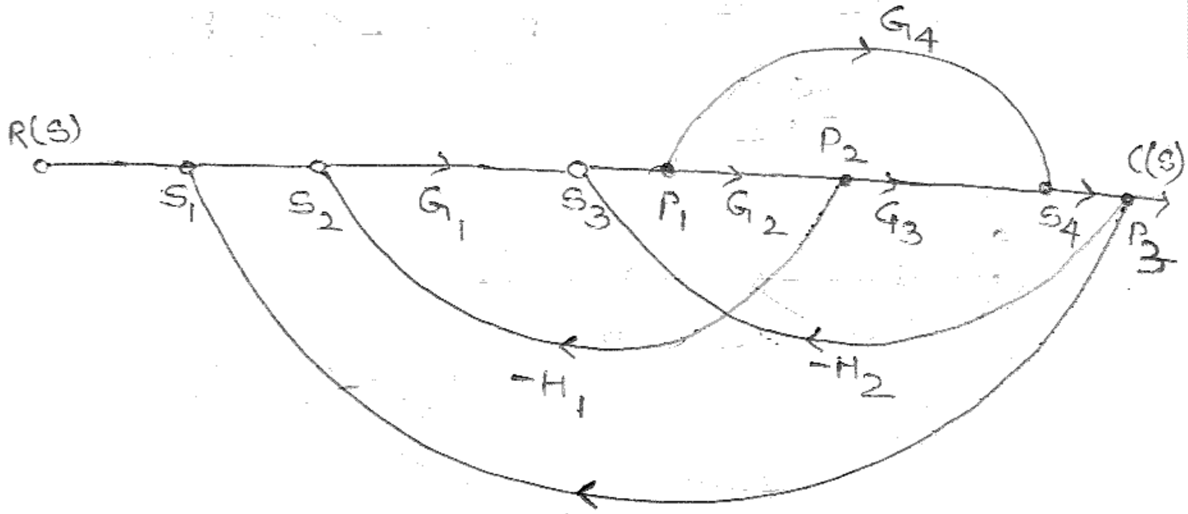
$$= \frac{1}{\Delta}\left[G_1 G_4 G_2 + G_1 G_4 G_3\right]$$

$$T = \frac{G_1 G_4 G_2 + G_1 G_4 G_3}{1 + H_1 G_1 G_4 + G_1 G_4 G_2 H_2 + G_1 G_3 G_4 H_2}$$

5.7.11

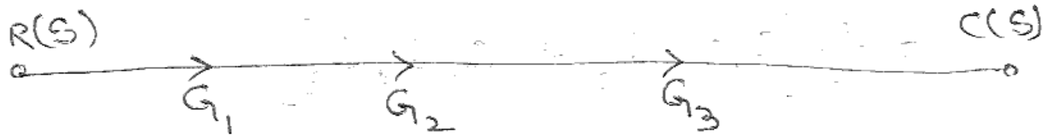
Q-



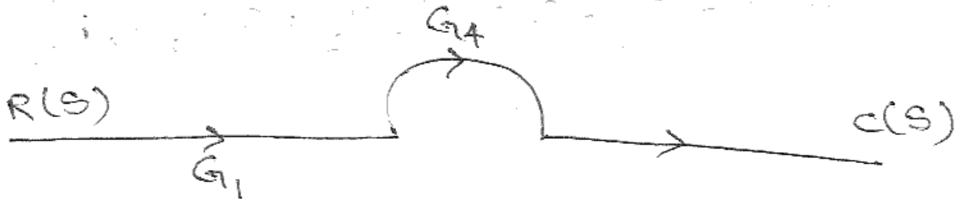


I) Forward path gain

$K = 2$

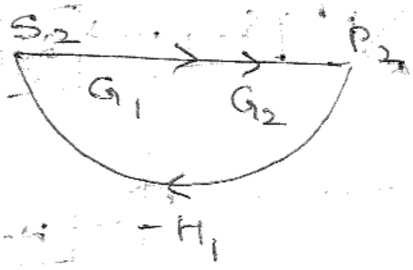


$$P_1 = G_1 G_2 G_3$$

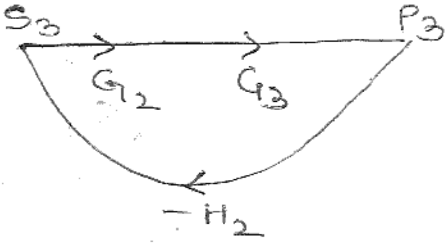


$$P_2 = G_1 G_4$$

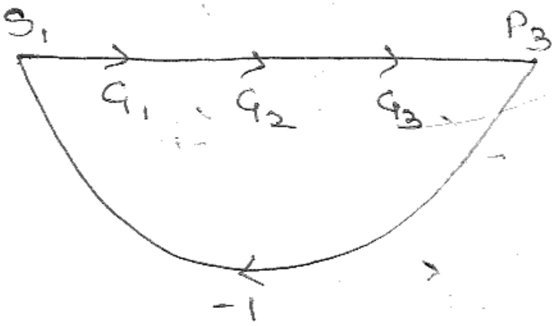
II) Individual loop gain



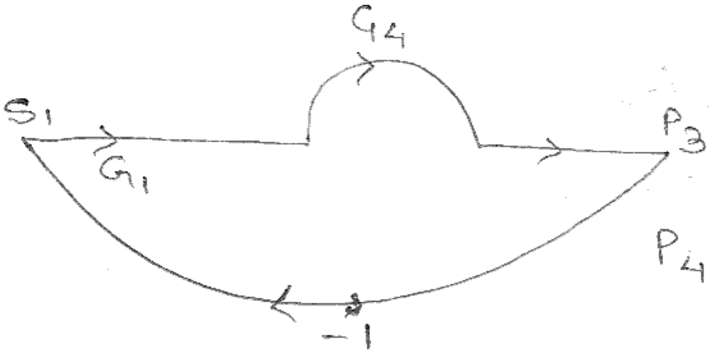
$$P_{11} = -G_1 G_2 H_1$$



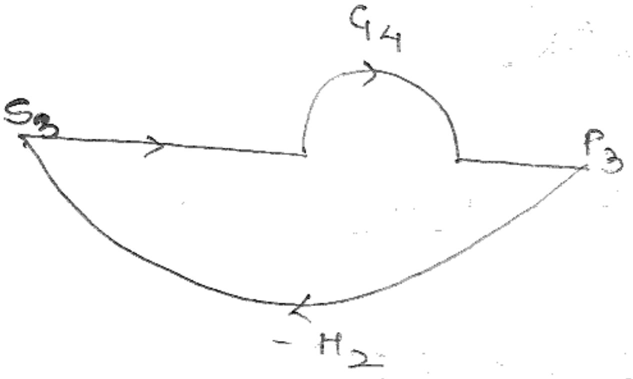
$$P_{21} = -G_2 G_3 H_2$$



$$P_{31} = -G_1 G_2 G_3$$



$P_{41} = -G_1 G_4$



$P_{51} = -G_4 H_2$

III) Gain product of two non touching loop = 0

IV) Calculate Δ

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$$

$$= 1 - [-G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3 - G_1G_4 - G_4H_2]$$

$$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_1G_4 + G_4H_2$$

V) Calculate Δ_K

$$K = 2$$

$$\therefore \Delta_1 = 1$$

$$\Delta_2 = 1.$$

$$T = \frac{1}{\Delta} \sum_K [P_K \Delta_K]$$

$$= \frac{1}{\Delta} [P_1\Delta_1 + P_2\Delta_2]$$

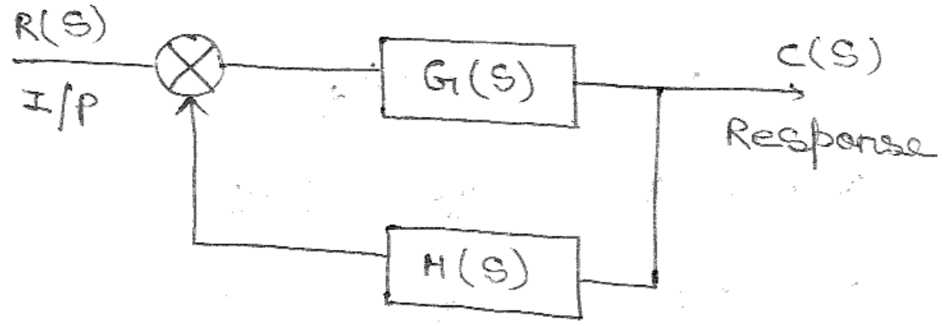
$$= \frac{1}{\Delta} [G_1G_2G_3 + G_1G_4]$$

$$T = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_1G_4 + G \ldots}$$

<Ans>

7.11

Unit - 02. Time Response Analysis



Closed loop transfer function $= \dfrac{C(S)}{R(S)}$

$$= \frac{G(S)}{1 + G(S)H(S)}$$

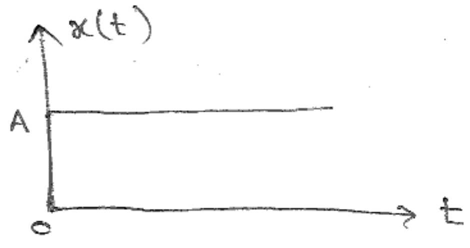
Response in S domain $c(S) = R(S) = \dfrac{G(S)}{1+G(S)H(S)}$

Response in time domain $c(t) = L[c(S)]$

$$= L^{-1}[\,"\,]$$

→ Test Signal

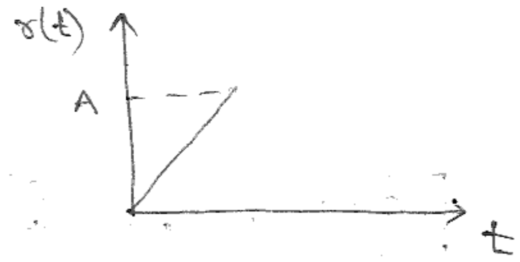
1) Step signal



$x(t) = A$ for $t \geq 0$

$x(t) = 0$ for $t < 0$

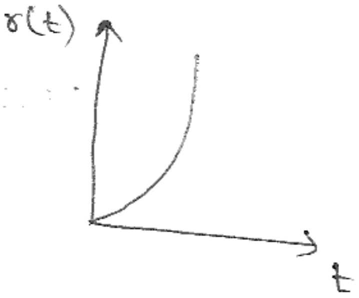
2) Ramp signal



$r(t) = At$ for $t \geq 0$

$r(t) = 0$ for $t < 0$

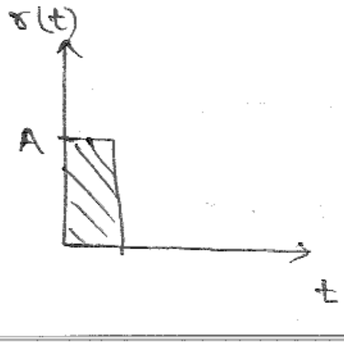
3) Parabolic signal



$r(t) = \frac{At^2}{2}$ $t \geq 0$

$r(t) = 0$ $t < 0$

4) Impulse signal



$r(t) = \infty$ at $t = 0$

$r(t) = 0$ at $t \neq 0$

→ Standard Test Signal

Name of signal	Time domain eq of signal $r(t)$	Laplace transform of signal $R(s)$
Step	A	A/S
unit step	1	$1/S$
Ramp	At	A/s^2
unit ramp	t	$1/s^2$
Parabolic	$At^2/2$	A/s^3
unit parabolic	$t^2/2$	$1/s^3$
Impulse	$\delta(t)$	1

→ order of a system

$$\text{Transfer function } T(S) = K \frac{P(S)}{\phi(S)}$$

$$\phi(S) = a_0 S^n + a_1 S^{n-1} + a_2 S^{n-2} + \dots + a_{n-1} S + a_n$$

~~Res~~

★ Response of 1st order system for unit step input

$$\frac{C(S)}{R(S)} = \frac{1}{1+TS}$$

$$r(t) = 1, \quad R(S) = \frac{1}{S}$$

$$C(S) = R(S) \cdot \frac{1}{1+TS}$$

$$= \frac{1}{S} \frac{1}{1+TS}$$

$$= \frac{1}{S} \cdot \frac{1}{T(S+1/T)}$$

$$C(S) = \frac{1/T}{S(S+1/T)}$$

By using partial fraction

$$C(S) = \frac{1/T}{S(S+1/T)} = \frac{A}{S} + \frac{B}{S+1/T}$$

$$= \frac{1/T}{S(S+1/T)} = \frac{A(S+1/T) + BS}{S(S+1/T)}$$

$$\frac{1}{T} = A\left(S + \frac{1}{T}\right) + BS$$

$$\frac{1}{T} = AS + \frac{A}{T} + BS$$

$$A = 1$$

$$AS + BS = 0$$

$$1 + B = 0$$

$$\Rightarrow B = -1$$

$$\therefore A = 1 \quad \text{and} \quad B = -1$$

$$C(S) = \frac{1}{S} - \frac{1}{S + 1/T}$$

Taking inverse laplace transform

$$\boxed{c(t) = 1 - e^{-t/T}}$$

20.7.11

* Second Order System

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

where,

ω_n = undamped natural freq

ζ = damping ratio

2 marks Damping ratio is defined as the ratio the actual damping to the critical damping.

→ Depending on η types of damping

- Undamped system $\zeta = 0$
- under damped system $0 < \zeta < 1$
- Critically damped " $\zeta = 1$
- Over damped " $\zeta > 1$

Roots of the quadratic equation

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$

$$\Rightarrow s = -2\zeta\omega_n s \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}$$

$$s_1 = \frac{-\cancel{2}\zeta\omega_n + \cancel{2}\omega_n\sqrt{\zeta^2 - 1}}{\cancel{2}}$$

$$\therefore s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

when $\zeta = 0$

$$s_1 = \omega_n\sqrt{-1} \quad = j\omega_n$$

$$s_2 = -\omega_n\sqrt{-1} \quad = -j\omega_n$$

roots are purely imaginary and the system is undamped.

when $\zeta = 1$

$$s_1 = -\omega_n + \omega_n \times 0 \quad = -\omega_n$$

$$s_2 \cancel{= \omega_n + 0} = 0$$

roots are real and equal and the system is critically damped.

when $\zeta > 1$

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

roots are real and unequal so system is over damped.

when $0 < \zeta < 1$

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)}$$

$$= -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

roots are complex conjugate and sys is under damped.

★ Response of undamped second order system for unit step input

standard form of 2nd order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Subt. $S = 0$

$$\therefore \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + \omega_n^2}$$

$$C(S) = \frac{1}{S} \times \frac{\omega_n^2}{S^2 + \omega_n^2}$$

using partial fraction

$$\frac{\omega_n^2}{S(S^2 + \omega_n^2)} = \frac{A}{S} + \frac{B}{S^2 + \omega_n^2}$$

$$\omega_n^2 = A(S^2 + \omega_n^2) + BS$$

$$\omega_n^2 = AS^2 + A\omega_n^2 + BS$$

$$A = 1 \quad ; \quad B = -S$$

$$C(S) = \frac{1}{S} - \frac{S}{S^2 + \omega_n^2}$$

Taking inverse laplace transform

$$c(t) = 1 - \cos \omega_n t$$

Response of critical damped 2nd order system for unit step input

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$$\zeta = 1$$

$$\therefore \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\omega_n S + \omega_n^2}$$

$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\omega_n S + \omega_n^2)}$$

$$C(S) = \frac{\omega_n^2}{S(S+\omega_n)^2}$$

Taking partial fraction

$$C(S) = \frac{A}{S} + \frac{B}{(S+\omega_n)} + \frac{C}{(S+\omega_n)^2}$$

$$\frac{\omega_n^2}{S(S+\omega_n)^2} = \frac{A}{S} + \frac{B}{(S+\omega_n)} + \frac{C}{(S+\omega_n)^2}$$

$$\frac{\omega_n^2}{S(S+\omega_n)^2} = \frac{A}{S} \Bigg|_{S=0}$$

$$\frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2}} = \frac{A}{1} \Rightarrow A = 1$$

$$\frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{B}{(s+\omega_n)} \Bigg|_{s=1}$$

$$\frac{\omega_n^2}{\cancel{(1+\omega_n)^2}} = \frac{B}{\cancel{(1+\omega_n)}}$$

$$\frac{\omega_n^2}{1+\omega_n} = B$$

$$B + B\omega_n = \omega_n^2$$

$$B\omega_n - \omega_n^2 = -B$$

$$\omega_n(B-\omega_n) + B = 0$$

$$B = -\omega_n$$

$$C = -1$$

$$\therefore C(s) = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

Taking inverse laplace

$$c(t) = 1 - e^{-\omega_n t}\left[1+\omega_n t\right]$$