Signal 2/7/11

UNIT-2

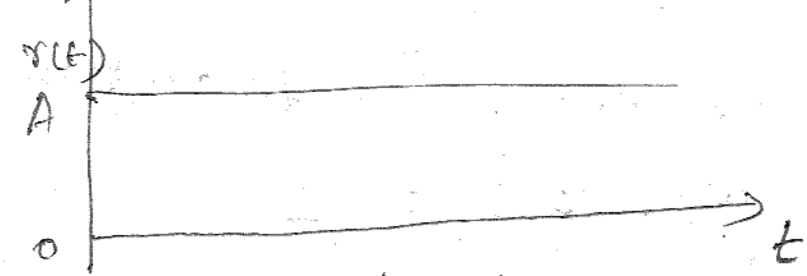
Time Response Analysis

Step Signal :- The step signal whose value changes from 0 to another level in negligible time, the functional representation step signal is given by

$r(t) = A\,u(t)$

$u(t) = 1 \;,\; t > 0$

$\quad\;\; = 0 \;,\; t < 0$

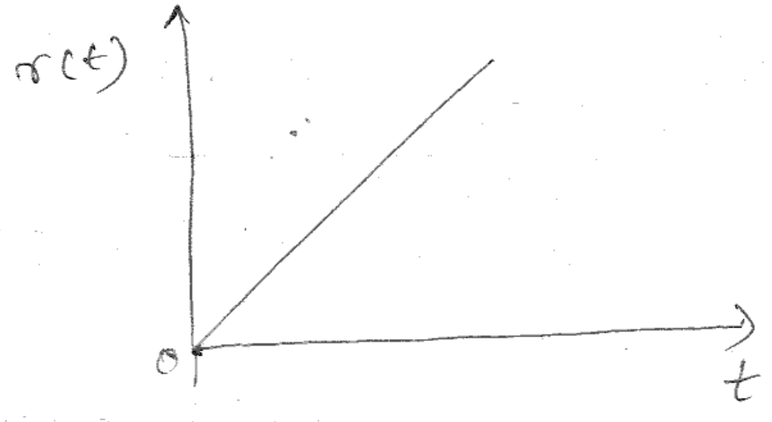


Take Laplace transform

$$R(s) = \frac{A}{s}$$

Ramp Signal :- The Ramp signal starts at the value of zero and increases linearly with the time. The function of ramp signal is represented by

$r(t) = At \cdot u(t)$



Taking laplace transform

$$R(S) = \frac{A}{S^2}$$

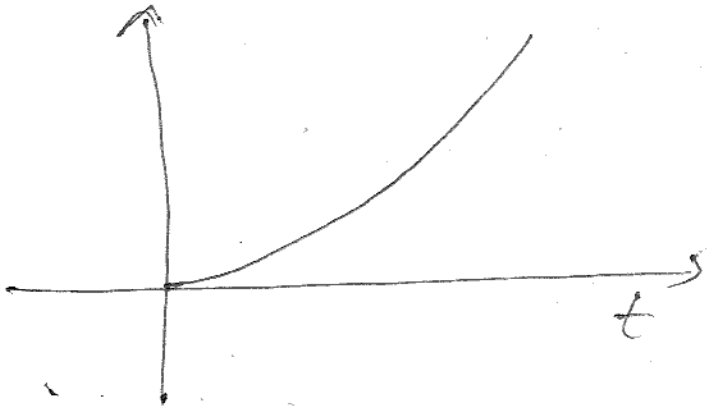
<u>Impulse signal</u> :- An unit impulse is defined mathematically $\delta(t) = 1$ when $t = 0$ & $\delta(t) = 0$; $t =$ Const ($t \neq 0$)

Taking laplace tranform of $\delta(t)$ is 1 i.e. $R(S) = 1$

<u>Parabolic i/p signal</u> :-

The functional representation of parabolic signal is

$$r(t) = A.\frac{t^2}{2}, \quad t > 0$$
$$= 0, \quad t < 0$$



Taking Laplace transform

$R(s) = \frac{A}{2} \cdot \frac{\cancel{(3-1)!} \times 2}{s^3}$ $\qquad L(t^n) = \frac{n!}{s^{n+1}}$

$R(s) = \frac{A}{s^3}$

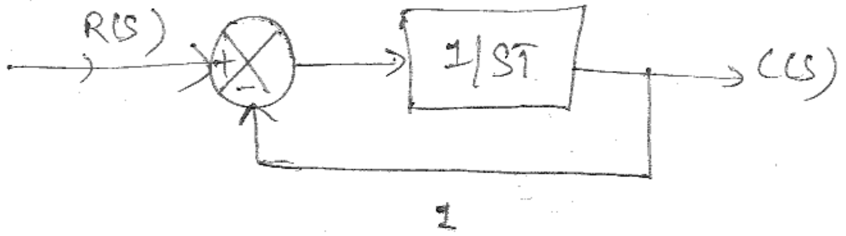
Signal Type	$r(t)$	$R(s)$
Step	$A\,u(t)$	A/s
Ramp	$At\,u(t)$	A/s^2
Impulse	$\delta(t)$	1
Parabolic	$\frac{At^2}{2}$	$\frac{A}{s^3}$

Order :- order of the system is given by highest power of denominator polynomial of closed loop transfer function.

Type :- If a System contain 1 pole at origin it is called Type 1

& if no pole is present at origin it is Type 0.

Time Response for first order System



$$\frac{C(s)}{R(s)} = \frac{1/ST}{1+\frac{1}{ST}\cdot 1} = \frac{1}{ST+1}$$

$$C(s) = R(s)\left(\frac{1}{ST+1}\right)$$

System response for first order system with unit step input.

$$R(s) = \frac{1}{S}$$

$$C(s) = \frac{1}{S}\left(\frac{1}{\cancel{S}1+ST}\right)$$

$$C(s) = \frac{A}{S} + \frac{B}{1+ST}$$

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$$\frac{1}{S(1+ST)} = \frac{A(1+ST)+BS}{S(1+ST)}$$

$$1 = A(1+ST)+BS$$

Put $S = 0$

$$1 = A(1)+0$$

$$\boxed{A = 1} \quad (1)$$

and Put $S = 1$

$$1 = A(1+T)+B \quad (2)$$

$$A(1+T)+B = 1$$

$$A + B = 1$$

$$\cancel{A} + AT - \cancel{A} = 0 \qquad A = 0$$

$$1+T+B = 1$$

$$\boxed{B = -T}$$

$$C(s) = \frac{1}{S} - \frac{T}{1+ST}$$

$$L^{-1}C(s) = 1 - L^{-1}\left(\frac{T}{1+ST}\right)$$

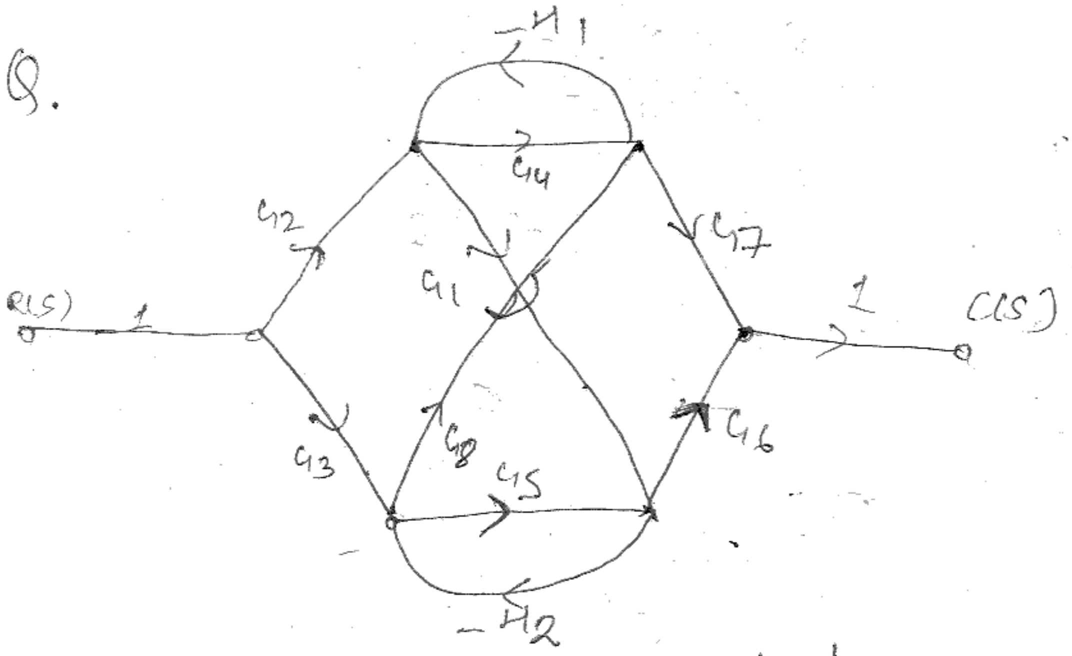
$$C(t) = 1 - Te^{-tT}$$

or

$$C(t) = 1 - L^{-1}\left(\frac{T}{T(S+\frac{1}{T})}\right)$$

$$C(t) = 1 - e^{-t/T}$$

Q.



find $\frac{C(S)}{R(S)}$ using mason's formula

Time Response for second order System with unit Step i/P

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$\xi = 0 \rightarrow$ undamped Response

$0 < \xi < 1 \rightarrow$ Underdamped Response

(In this the System is Stable, it reaches it Steady State value)

$\xi = 1 \rightarrow$ critically (it obtained a Sustained oscillation.

$\xi > 1 \rightarrow$ overdamped condition.
(non linear case) $\xi \rightarrow$ Zita

case 1 :- undamped condition ($\xi = 0$)

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 0 + \omega_n^2}$$

$\omega_n \rightarrow$ Natural frequency.

$\xi \rightarrow$ damping Ratio.

$\omega_d \rightarrow$ Damped freq.

$$C(S) = R(S)\left[\frac{\omega_n^2}{S^2+\omega_n^2}\right]$$

For unit step signal

$$C(S) = \frac{1}{S}\left[\frac{\omega_n^2}{S^2+\omega_n^2}\right]$$

by taking partial fraction

$$\frac{\omega_n^2}{S(S^2+\omega_n^2)} = \frac{A}{S} + \frac{BS+C}{S^2+\omega_n^2}$$

$$\omega_n^2 = A(S^2+\omega_n^2) + (BS^2 + CS)$$

$$\omega_n^2 = (A+B)S^2 + CS + A\omega_n^2$$

Equating coefficients

$A = 1$

$A + B = 0$

$B = -1$

$CS = 0$

$C = 0$

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$$C(s) = \frac{1}{s} + \frac{(-1)s}{s^2+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

Taking inverse laplace transform

$$c(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{s}{s^2+\omega_n^2}\right]$$

$$\boxed{c(t) = 1 - \cos\omega_n t}$$

Case II (underdamped system)

$$0 < \xi < 1$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = R(s)\left[\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right]$$

$$= \frac{1}{s}\left[\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right]$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$\omega_n^2 = A(s^2+2\xi\omega_n s+\omega_n^2) + Bs^2 + Cs$$

$$\omega_n^2 = (A+B)s^2 + (2\xi\omega_n A + C)s + A\omega_n^2$$

$$A = 1.$$

$$A + B = 0 \Rightarrow B = -1$$

$$2\xi\omega_n A + C = 0$$

$$2\xi\omega_n + C = 0$$

$$C = -2\xi\omega_n$$

$$C(s) = \frac{1}{s} + \frac{-s-2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

Taking inverse laplace transform

$$C(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{s}{s^2+2\xi\omega_n s+\omega_n^2}\right] - 2\xi\omega_n L^{-1}\left[\frac{1}{s^2+2\xi\omega_n s+\omega_n^2}\right]$$

$$= 1$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+(\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$

where $\omega_d^2 = \omega_n^2(1-\xi^2)$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

Taking inverse laplace transform

~~$c(t) = 1 - \cos\omega_d t - \sin\omega_d t \cdot \xi\frac{\omega_n}{\omega_d}$~~

~~$L^{-1}C(s) =$~~

$$c(t) = 1 - \left[e^{-\xi\omega_n t}\left[L^{-1}\frac{s}{s^2+\omega_d^2} + L^{-1}\frac{\xi\omega_n}{s^2+\omega_d^2}\right]\right]$$

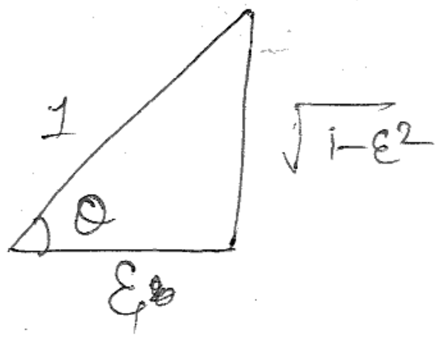
$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\frac{\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t}{\sqrt{1-\xi^2}}\right]$$

let us assume a right angle triangle



$\sin\theta = \sqrt{1-\xi^2}$

$\cos\theta = \xi$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta\cos\omega_d t + \cos\theta.\sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right]$$

17/11

Time Response for 2nd order System under critically damped Condition

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

for critically damped case $\xi = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$C(s) = R(s) \cdot \frac{\omega_n^2}{(s+\omega_n)^2}$$

where unit step i/p $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{(s+\omega_n)^2}$$

$$\cancel{C(s)} \quad \omega_n^2 = A(s+\omega_n)^2 + Bs^2 + Cs$$

$$\omega_n^2 = A(s^2 + \omega_n^2 + 2s\omega_n) + Bs^2 + Cs$$

$$\omega_n^2 = (A+B)s^2 + (2A\omega_n + C)s + A\omega_n^2$$

$$\boxed{A = 1},$$

$$A + B = 0 \Rightarrow \boxed{B = -1}$$

$$2A\omega_n + C = 0$$

$$2\omega_n + C = 0$$

$$\boxed{C = -2\omega_n}$$

$$C(s) = \frac{1}{s} + \frac{-s-2\omega_n}{(s+\omega_n)^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\omega_n}{(s+\omega_n)^2}$$

$$\cancel{C(s) = \frac{1}{s} - \frac{s+\omega_n+\omega_n}{s^2+\omega_n^2+2\omega_n s}}$$

$$C(s) = \frac{1}{s} - \left[\frac{s+\omega_n}{(s+\omega_n)^2} + \frac{\omega_n}{(s+\omega_n)^2}\right]$$

$$L^{-1}[C(s)] = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{s+\omega_n}{(s+\omega_n)^2} + \frac{\omega_n}{(s+\omega_n)^2}\right]$$

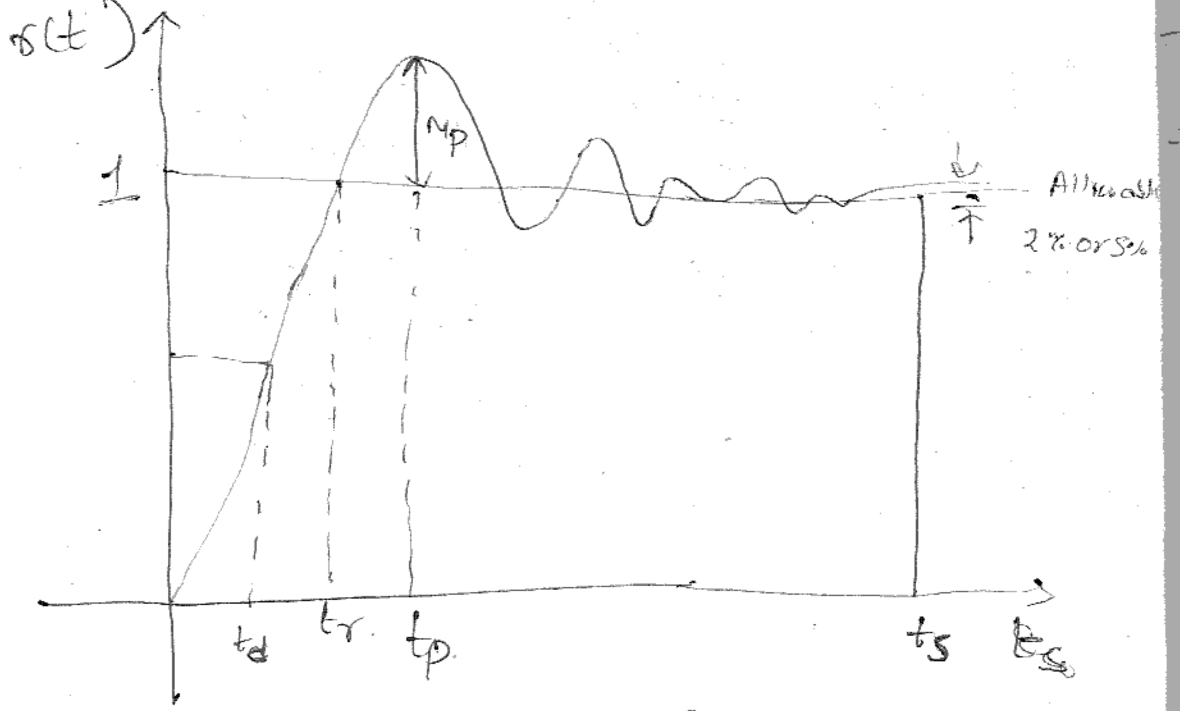
$$C(t) = 1 - L^{-1}\left[\frac{1}{s+\omega_n}\right] - L^{-1}\left[\frac{\omega_n}{(s+\omega_n)^2}\right]$$

$$= 1 - \left[e^{-\omega_n t} + \omega_n . e^{-\omega_n t} . t\right]$$

$$C(t) = 1 - e^{-\omega_n t}\left[1 + \omega_n t\right]$$

Time domain Specification

1. Peak Time. (t_p)
2. Delay Time. (t_d)
3. Rise Time. (t_r)
4. maximum peak overshoot (M_p)
5. Settling Time (t_s)



Time response of System.

Delay time :- It is the time taken for the response to reach 50% of its final value.

Rise Time :- It is time taken for the response to reach 100% of its final value at very first time.

Peak Time :- It is the time taken for the response to reach the maximum value of amplitude.

Settling Time :- The response reaches the steady state value in certain time called Settling Time.

Allowable 2% or 5%

Time Response of underdamped with unit step i/P

Raise Time

$$c(t) = 1 - e^{-\xi \omega_n t}(\sin \omega_d t + \theta)$$

when $c(t) = 1$, $t = t_r$

$$\cancel{1} = \cancel{1} - e^{-\xi \omega_n t_r}(\sin \omega_d t_r + \theta)$$

$$e^{-\xi \omega_n t_r}(\sin \omega_d t_r + \theta) = 0$$

$e^{-\xi \omega_n t_r} \neq 0$ and $\sin \omega_d (t_r + \theta) = 0$

$\sin \pi = 0$

$$(\omega_d t_r + \theta) = \pi$$

$$\boxed{t_r = \left(\frac{\pi - \theta}{\omega_d}\right)}$$

$$\theta = \cos^{-1}(\xi)$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

$$M_p \text{ (maximum peak overshoot)} = e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}$$

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$t_s = \frac{4}{\xi \omega_n}$ for 2 % Error

$= \frac{3}{\xi \omega_n}$ for 5% Error.

t_s

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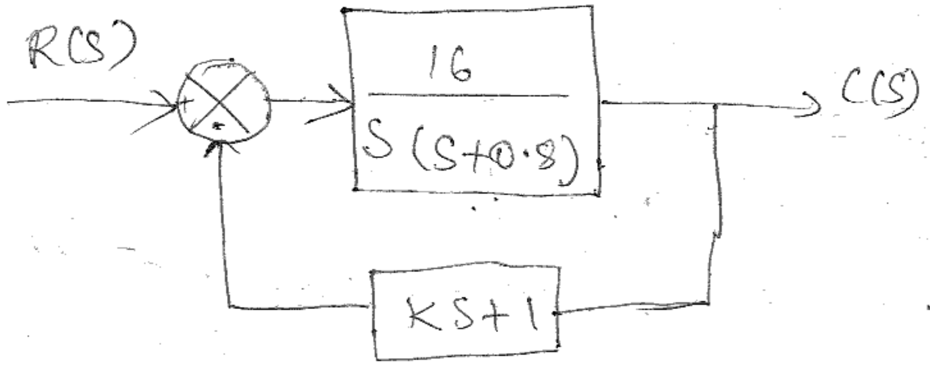
Peak Time

$$c(t) = 1 - e^{-\xi \omega_n t}(\sin(\omega_d t + \theta))$$

diff. $c(t)$ w.r.t t & put $t = t_p$

$$\frac{d\,c(t)}{dt} = 0 = e^{-\xi \omega_n t} \times -\xi \omega_n t (\sin(\omega_d t + \theta)) - \cos(\omega_d t + \theta)$$

Q. To obtain the Time response of the system the system shown in fig. for unit step input.
Given $\xi = 0.5$. Also calculate the rise time, Peak time & Maximum peak overshoot and settling time.



$$\frac{C(S)}{R(S)} = \frac{\frac{16}{S(S+0.8)}}{1 + (KS+1)\frac{16}{S(S+0.8)}}$$

$$= \frac{\dfrac{16}{\cancel{s(s+0.8)}}}{\dfrac{s(s+0.8)+(Ks+1)16}{\cancel{(s+0.8)s}}}$$

$$= \frac{16}{s^2+0.8s+16Ks+16}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+s(16K+0.8)+16}$$

$$C(s) = R(s)\cdot\frac{16}{s^2+s(16K+0.8)+16} \quad -(1)$$

as Input is unit step so $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s}\cdot\frac{16}{[s^2+s(16K+0.8)+16]}$$

The second other equation is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad -(2)$$

Compare (1) & (2)

$\omega_n^2 = 16 \Rightarrow \omega_n = 4$

$2\xi\omega_n = 16K+0.8$

$2\times 0.5\times 4 = 16K+0.8$

$4 = 16K+0.8$

$$\Rightarrow \quad 3.2 = 16K$$

$$K = \frac{3.2}{16} = 0.2$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{16}{s^2+4s+16}$$

$$C(s) = R(s) \cdot \frac{16}{s^2+4s+16}$$

unit step $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s} \cdot \frac{16}{s^2+4s+16}$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+4s+16}$$

$$16 = A(s^2+4s+16) + Bs^2 + Cs$$

$$16 = (A+B)s^2 + (4A+C)s + 16A$$

compare the coeff

$$\boxed{A = 1}$$

$$A + B = 0$$

$$\boxed{B = -1}$$

$$4A + C = 0$$

$$4 + C = 0 \Rightarrow \boxed{C = -4}$$

$$C(s) = \frac{1}{s} + \frac{-s-4}{s^2+4s+16}$$

$$C(s) = \frac{1}{s} - \frac{s+4}{s^2+4s+16}$$

$s^2+4s+16$

$s = \frac{-4 \pm \sqrt{16-32}}{2}$

$= \frac{-4 \pm \sqrt{\ }i}{2}$

$= -2 \pm 25$

$$C(S) = \frac{1}{S} - \cancel{\frac{S+4}{S-(-2+2\sqrt{3}i)}}$$

$$C(S) = \frac{1}{S} - \frac{S+4}{S^2+4S+4+12}$$

$$= \frac{1}{S} - \frac{(S+2)+2}{(S+2)^2+(\sqrt{12})^2}$$

$$C(S) = \frac{1}{S} - \left[\frac{S+2}{(S+2)^2+(\sqrt{12})^2} + \frac{2}{(S+2)^2+(\sqrt{12})^2}\right]$$

$$c(t) = L^{-1}\left[\frac{1}{S}\right] - L^{-1}\left[\frac{S+2}{(S+2)^2+(\sqrt{12})^2}\right] - L^{-1}\left[\frac{2}{(S+2)^2+(\sqrt{12})^2}\right]$$

$$= 1 - e^{-2t}\left(L^{-1}\left[\frac{S}{S^2+(\sqrt{12})^2}\right] + \frac{2}{\sqrt{12}}L^{-1}\left[\frac{\sqrt{12}}{S^2+(\sqrt{12})^2}\right]\right)$$

$$c(t) = 1 - e^{-2t}\left[\cos\sqrt{12}t + \frac{2}{\sqrt{12}}\sin\sqrt{12}t\right]$$

$$\theta = \cos^{-1}(\xi)$$

$$\theta = 60^\circ = 1.047 \text{ rad.}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 4\sqrt{1-0.25} = 4\times 0.86$$

$$\boxed{\omega_d = 3.46}$$

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.047}{3.46}$$

$$\boxed{t_r = 0.605 \text{ sec}}.$$

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{3.46} = 0907$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= e^{-0.5\times3.14/\sqrt{1-0.5^2}}$$

$$= e^{-1.57/0.86}$$

$$M_p = e^{-1.82}$$

$$M_p = 0.16$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{\cancel{4}}{0.5\times\cancel{4}} = 2 \text{ sec.}$$

$$t_s = \frac{3}{\xi\omega_n} = \frac{3}{0.5\times4} = \frac{3}{2} = 1.5$$

UNIT-II

Root locus

Stability Analysis

minimize Error

{Break away point lies b/w two poles

{Break in point lies b/w two zeros.

1) $G(s) = \dfrac{K}{s(s^2+2s+9)}$

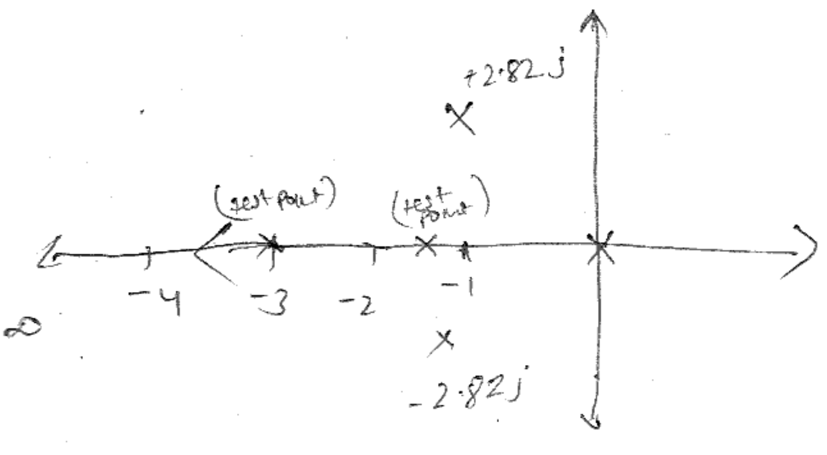
$s(s^2+2s+9) = 0$

$s = 0, \quad -1 \pm \sqrt{8}j$

No of Roots $(n) = 3$

" " Zeros $(m) = 0$

2) Locate the Root locus path.



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