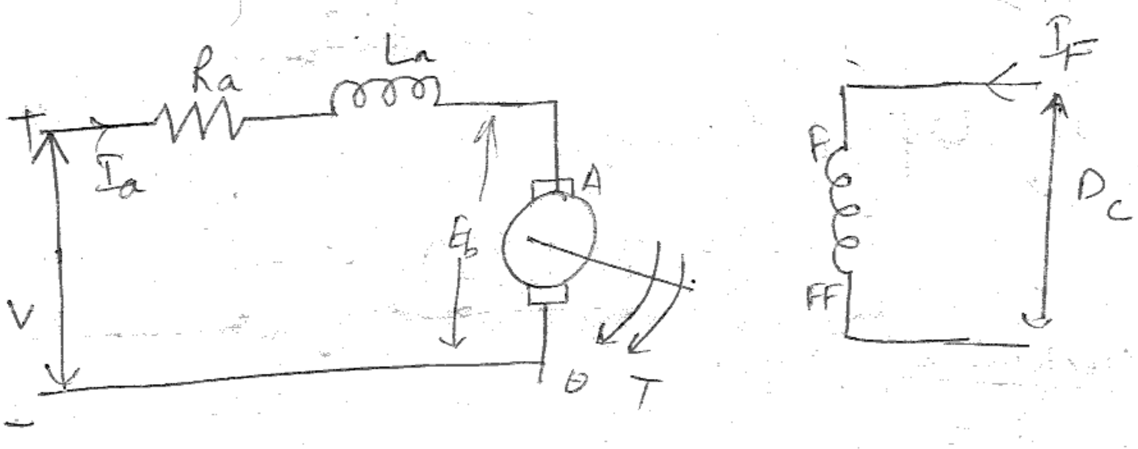
4/7/11 Electromechanical Systems

(1) Armature Control D.c Motor :-



I_F = field current
E_b = back e.m.f.
R_a = Armature Resistance

Applying KVL,

$$V(t) = I_a R_a + L_a \frac{dI_a}{dt} + E(t)$$

Apply laplace transform:-

$$V(s) = I_a(s) R_a + L_a\, s I_a(s) + E(s)$$

$$= I_a(s) \cdot R_a + L_a\, s I_a(s) + E(s) \quad \text{—(2) eq}^n$$

also,

$$E \propto \phi \omega \quad \text{—(3)}$$

where

ϕ = Electric flux

ω = Angular displacement

$$\Rightarrow E = K_b \frac{d\theta}{dt} \quad \text{—(4)}$$

$$E(s) = K_b\, s\,\theta(s) \quad -4(a)$$

Electrical Torque, $T \propto \phi I_a$

$$T = K I_a$$

By laplace transfn,

$$T(s) = K I_a(s) \quad — ⑤$$

Mechanical Torque,

$$T = T_j + T_b$$

$$= J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad — ⑥$$

$$T(s) = J s^2\theta(s) + B s\theta(s) \quad — ⑦$$

~~Equatn eqn ⑤ & ⑦~~

from eqn ②,

$$\frac{V(s) - E(s)}{I_a(s)} = (R_a + sL_a)$$

$$\Rightarrow \boxed{I_a(s) = \frac{V(s) - E(s)}{R_a + sL_a}} \quad — ⑧$$

from ⑤, $$I_a(s) = \frac{T(s)}{K}$$

$$I_a(s) = \frac{J s^2\theta(s) + B s\,\theta(s)}{K} \quad — ⑨$$

from ①,

$$V(s) = I_a(s)\left[R_a + sL_a\right] + K_b\, s\,\theta(s)$$

from 9,

$$I_a(s) = \frac{s\theta(s)\left[sJ + B\right]}{K}$$

$$\therefore V(s) = \frac{s\theta(s)\left[sJ+B\right]}{K}(R_a + sL_a) + K_b\, s\,\theta(s)$$

$$V(s) = \frac{s\theta(s)\left[[sJ+B](R_a+sL_a) + K_b \cdot K\right]}{K}$$

$$\Rightarrow \boxed{\frac{\theta(s)}{V(s)} = \frac{K}{s\left[(sJ+B)(R_a+sL_a)+K_b \cdot K\right]}} \quad (8)$$

$\frac{\theta(s)}{V(s)}$ = Transfer funct^n.

τ_m = mechanical Time Constant $= \frac{J_s}{B}$.

τ_E = Electrical Time Constant, $\frac{L_a}{R_a}$.

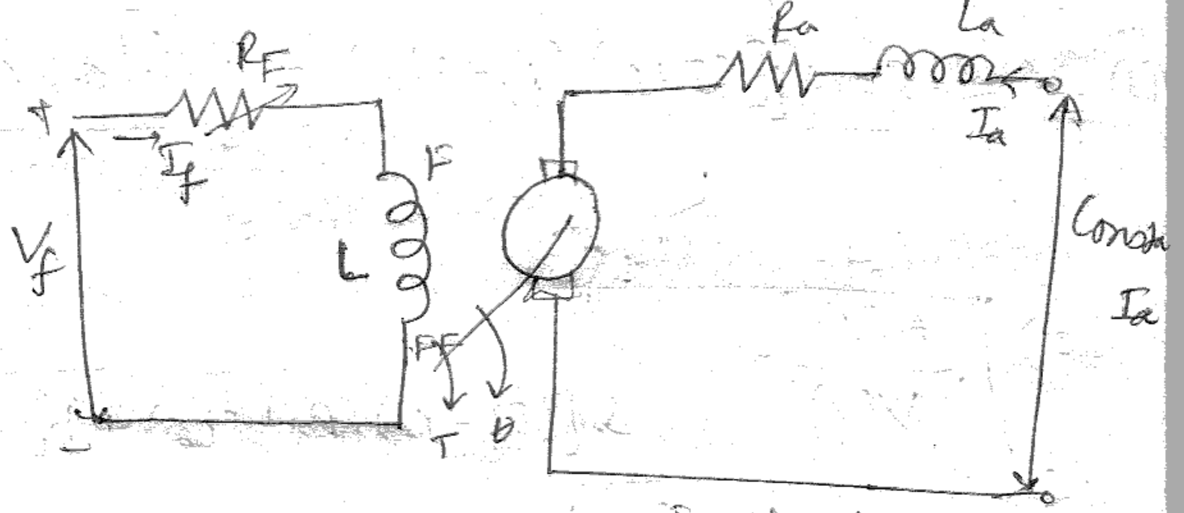
$$\therefore \frac{\theta(s)}{V(s)} = \frac{K}{s\left[R_a B\left(s\frac{J_s}{B}+1\right)\left(1+s\frac{L_a}{R_a}\right)+K_b \cdot K\right]} \quad (9)$$

$$\boxed{\frac{\theta(s)}{V(s)} = \frac{k}{s\left[\cancel{R_aB}(s\tau_m+1)(1+s\tau_E)+k_b \cdot k\right]}}$$

$$\boxed{\frac{\theta(s)}{V(s)} = \frac{k}{s\left[R_aB(s\tau_m+1)(1+s\tau_E)+k_b \cdot k\right]}}$$

$\Rightarrow$ 5/7/11

Field Type speed Control Motor



I_f = field current

we have:-

$\phi \propto I_f \Rightarrow$ flux $\propto$ Field Current

$\phi = k_f I_f$

$$V_f = I_f R_f + L\frac{dI_f}{dt} \quad (1)$$

Torque, $T \propto \phi I_a \Rightarrow T \propto k_f I_f \cdot I_a$

$T = k' K_f I_f \cdot I_a$

$$\boxed{T = k K_f I_f(s)} \quad \text{— (a)} \qquad \text{where } k = k' I_a$$

Mechanical Torque,

$$T = T_m + T_b$$

$$= J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad \text{— (2)}$$

Taking laplace transf of (1) & (2)

$$V_f(s) = R_f I_f(s) + L s I_f(s) \quad \text{— (1)'}$$

$$T(s) = J s^2 \theta(s) + B s \theta(s) \quad \text{— (2)'}$$

from (1)'

$$V_f(s) = I_f(s)[R_f + sL]$$

$$I_f(s) = \frac{V_f(s)}{R_f + Ls} \quad \text{— (3)}$$

~~from (2)~~

T =

Put (3) in (a)

$$T = K K_f \frac{V_f(s)}{R_f + 8Ls} \quad — (4)$$

Eq (2) & (4) are equal

$$J s^2 \theta(s) + B s \theta(s) = K K_f \frac{V_f(s)}{R_f + L_s}$$

$$\Rightarrow \theta(s)\left[Js^2 + Bs\right] = \frac{K K_f V_f(s)}{R_f + L_s}$$

$$\Rightarrow \boxed{\frac{\theta(s)}{V_f(s)} = \frac{K K_f}{(Js^2 + Bs)(R_f + L_s)}}$$

Transfer function.

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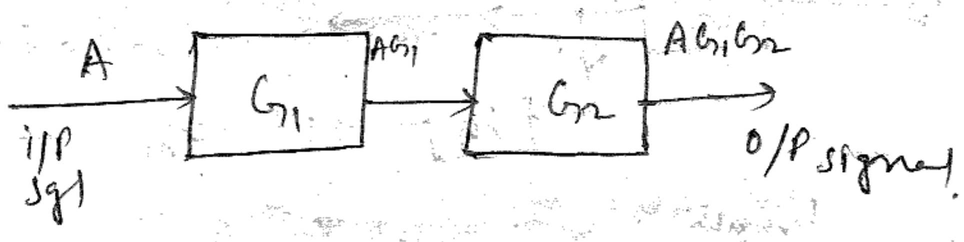
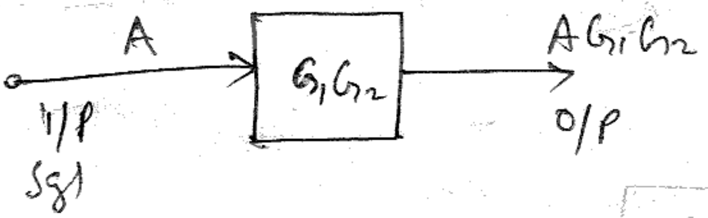
BLOCK DIAGRAM REDUCTION

Cascade Connection (series).

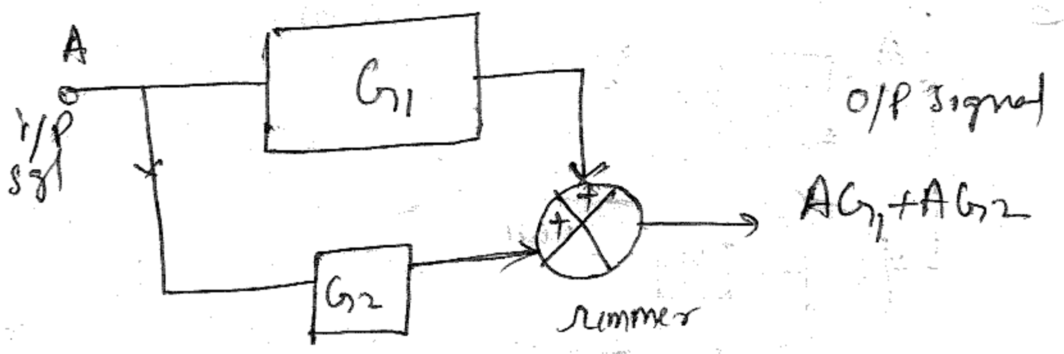
(1)



⇒



(2) Blocks Conected in ‖ :-



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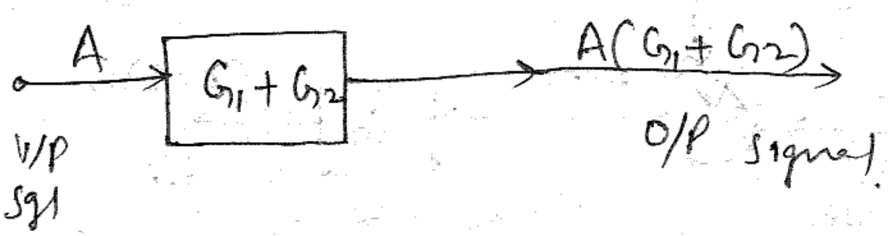
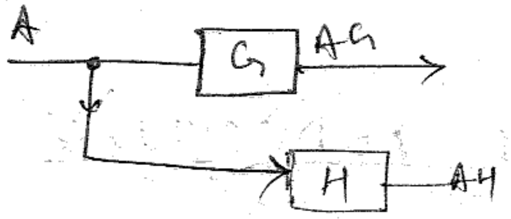
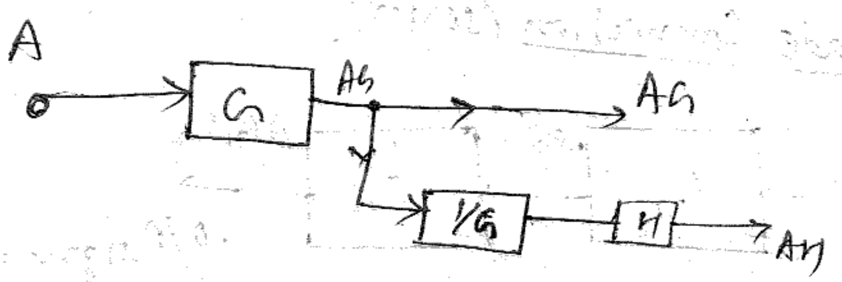


Moving the ~~branch~~ pot node ahead the block

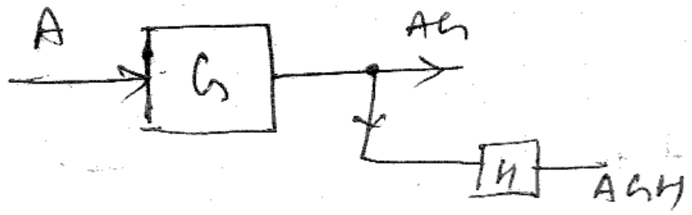
(3)



⇒



(4) Moving the branch pot before the block,,



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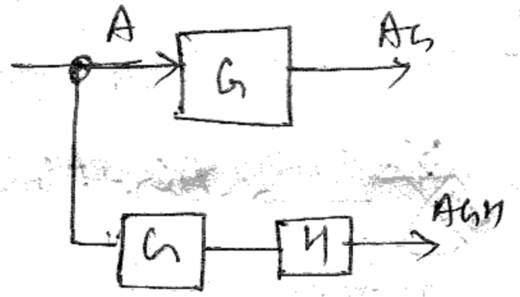
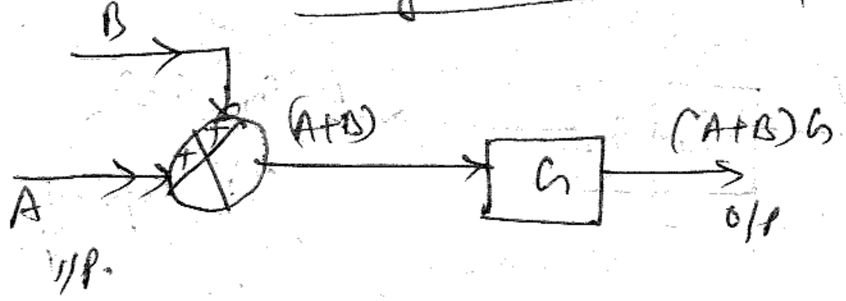


(5) Moving Summer ahead the block



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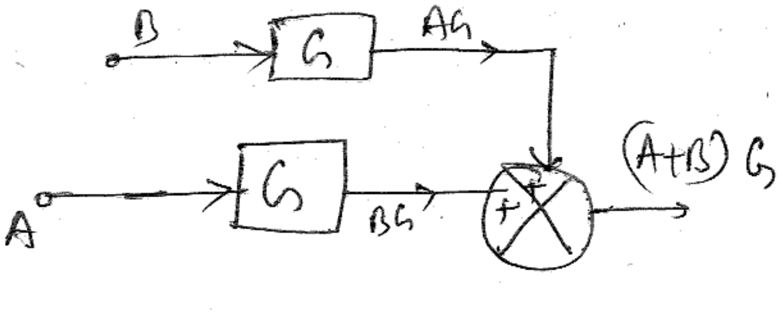
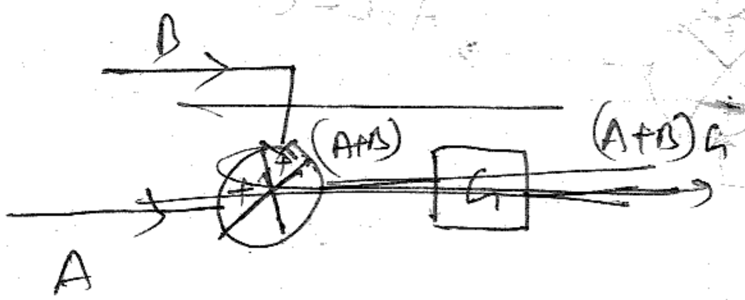
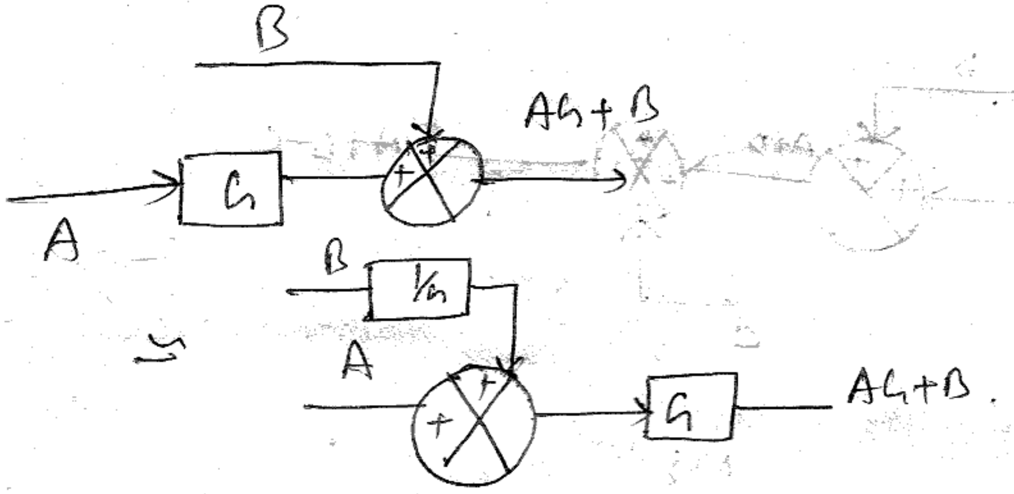
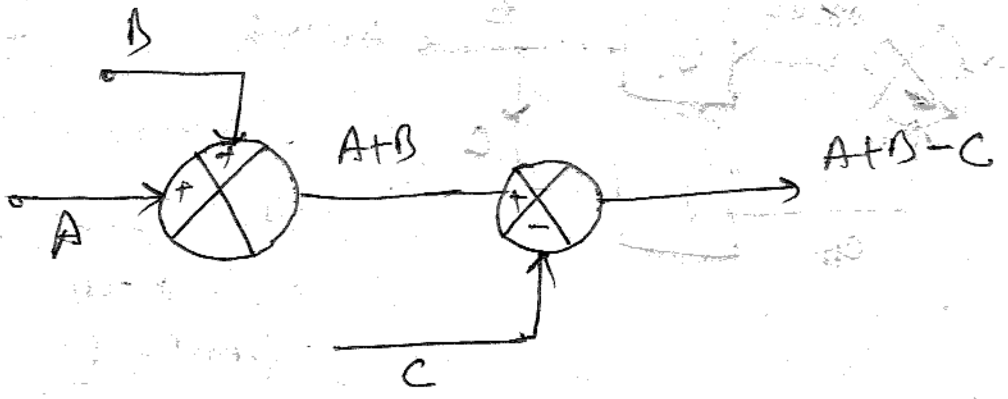
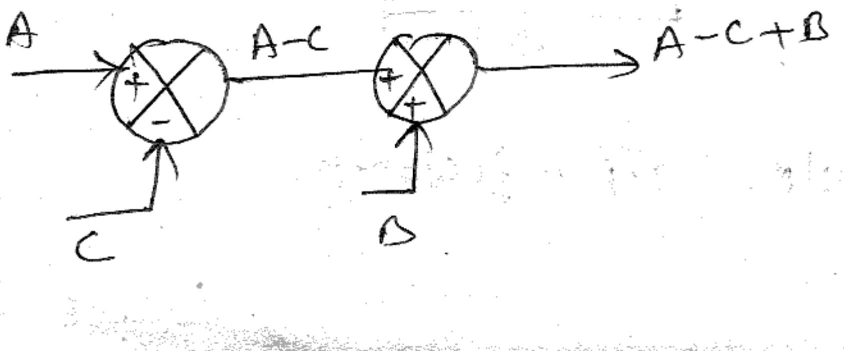


⑥ Moving summer before the block.

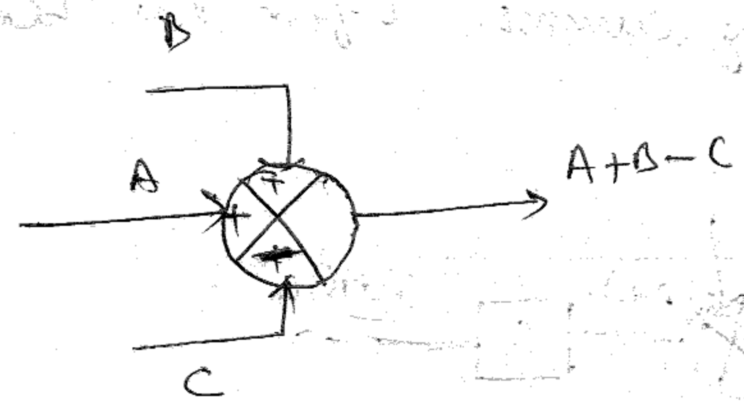






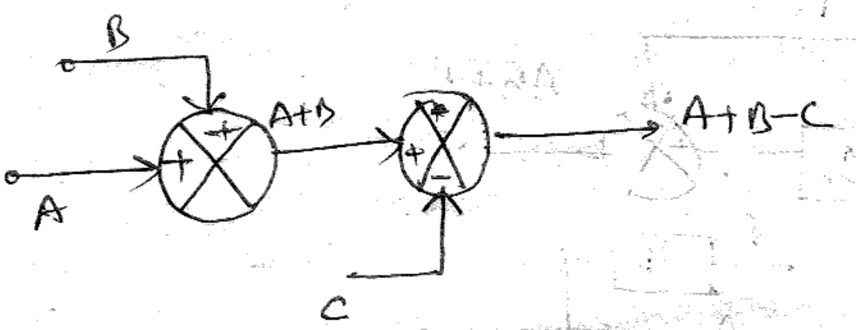


8.

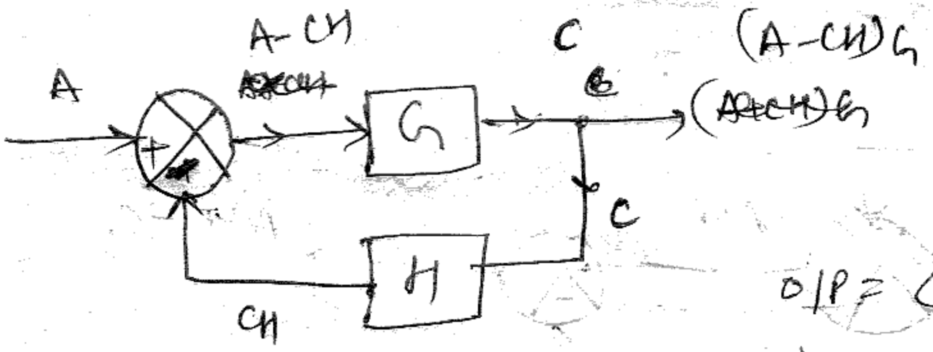


8.

⇒

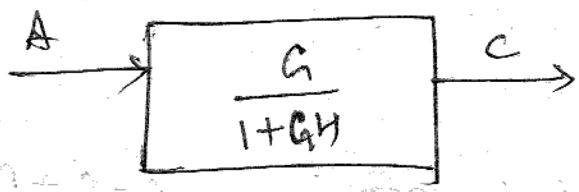


9 Eliminating the Negative feedback :-



O/P = C
I/P = A − CH
feedb^k = (

⇒



O/P = I/P × feedback.

$$C = (A - CH)G$$

$$= AG - CHG$$

$$C + CGH = AG$$

$$C(1 + GH) = AG$$

$$\Rightarrow \boxed{C = \frac{AG}{1+GH}}$$

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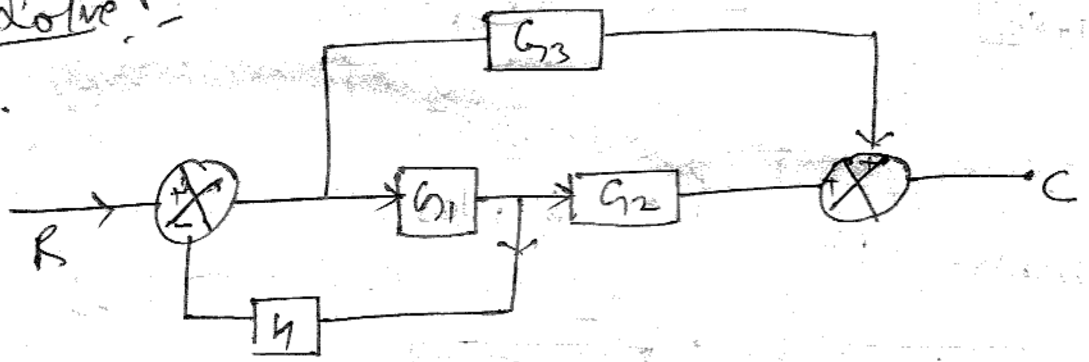
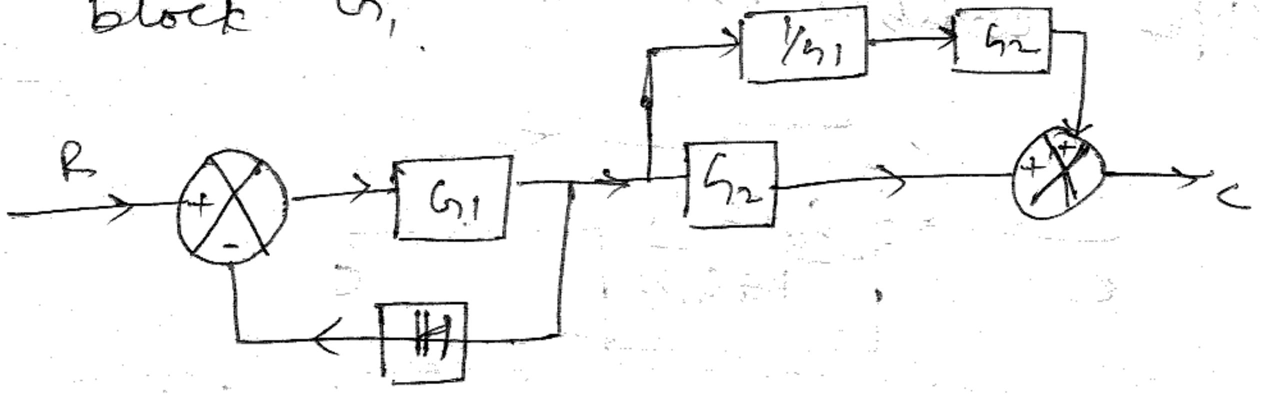
Solve :-

1).

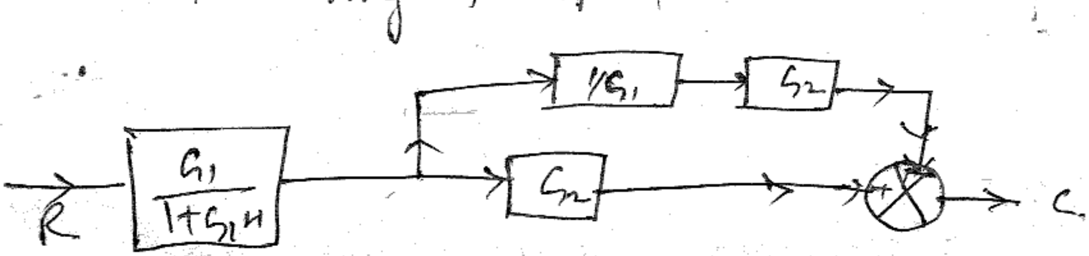


Sol:-

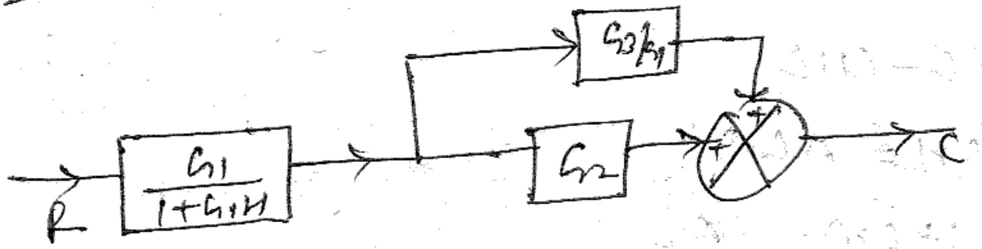
Step 1 Moving branch point ahead the block G_1.



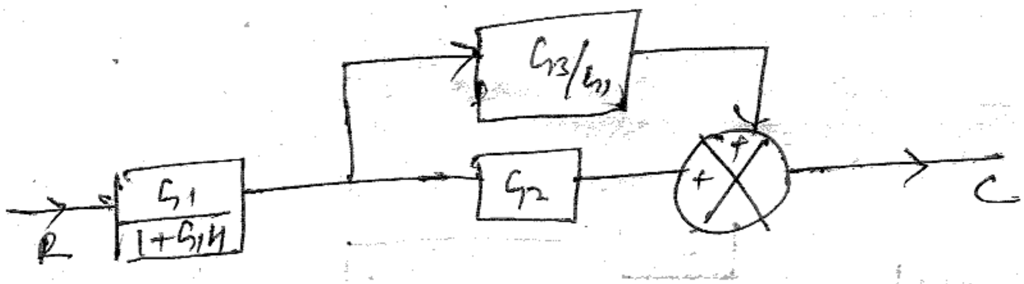
Step 2 eliminating the feedback H.



Step 3 Combine the cascade blocks.

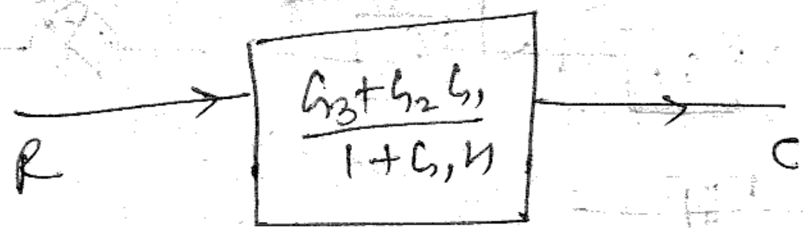


Step 3 Combine the cascade blocks.



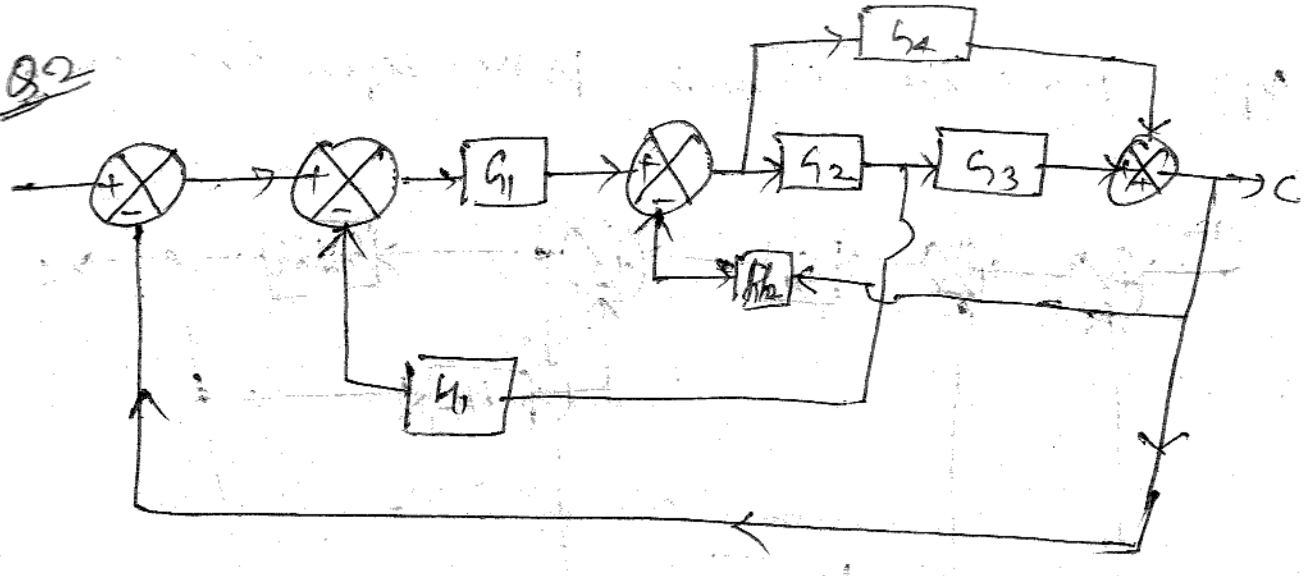
Step 4 Combine the ‖ᵈ blocks.

Step 5 Combine the series blocks.

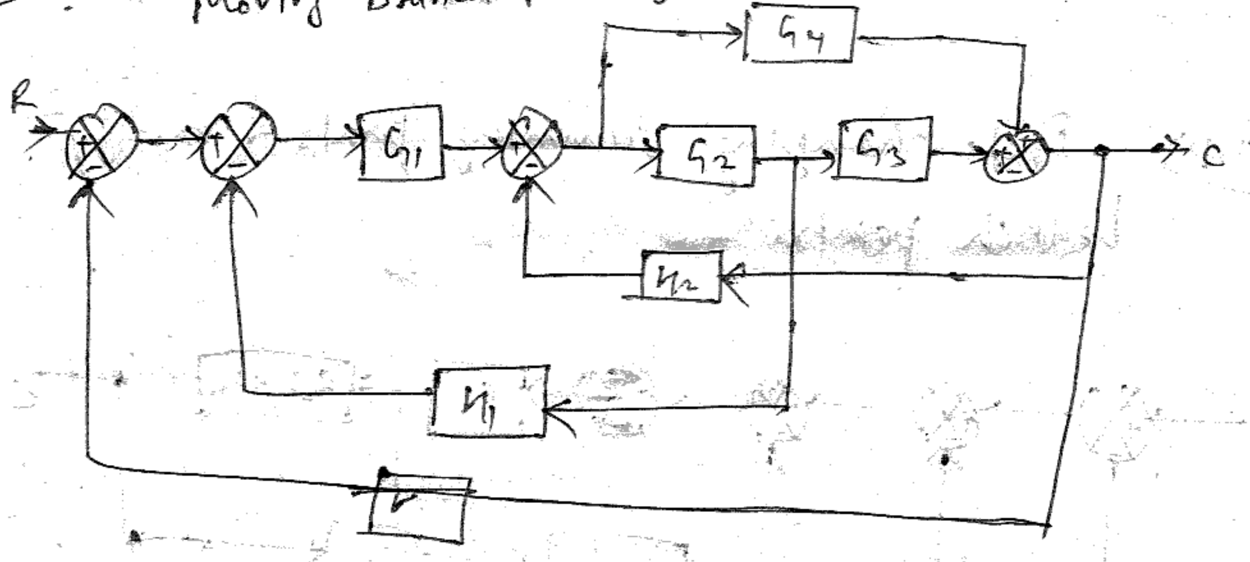


$$\boxed{\text{Transfer function} = \frac{C}{R} = \frac{G_3+G_2G_1}{1+G_1H}}$$

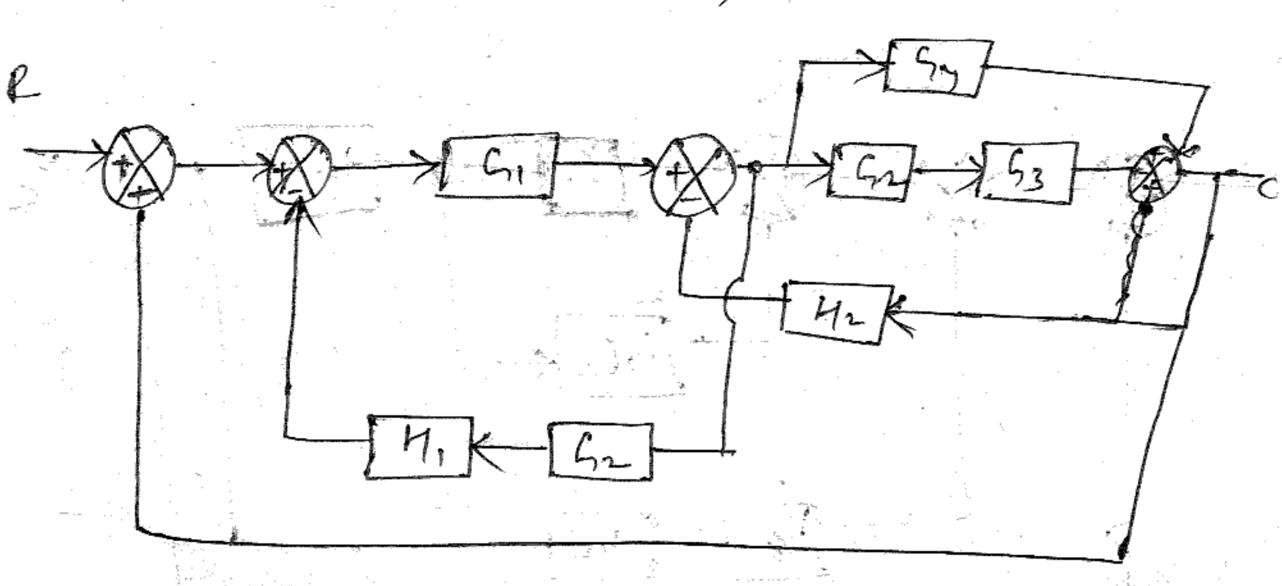
Q2



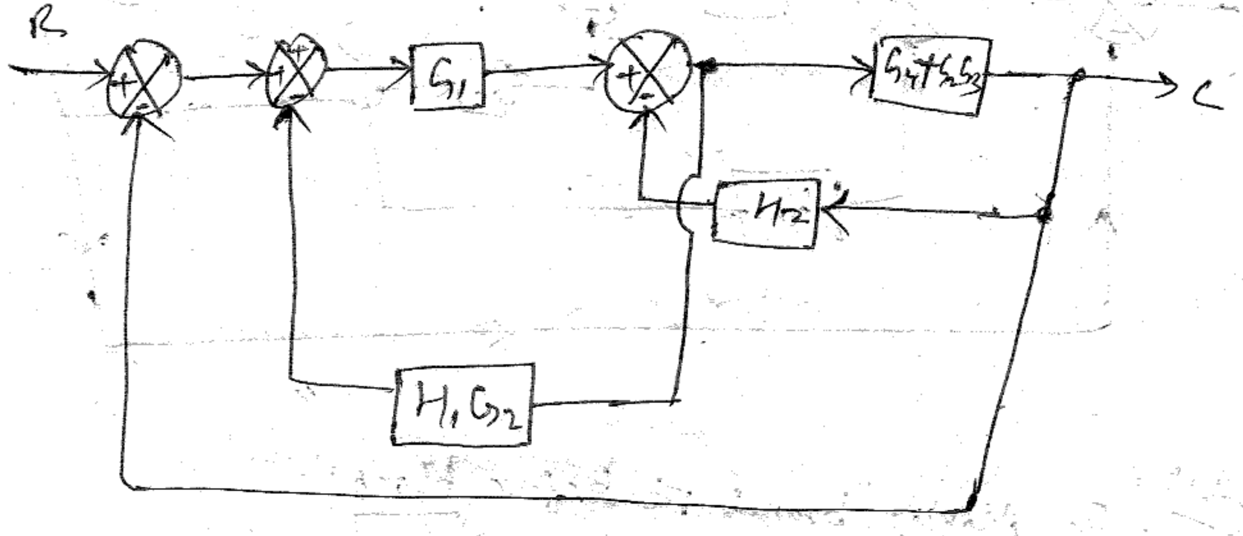
Sol^n :- Moving branch pt before block:-



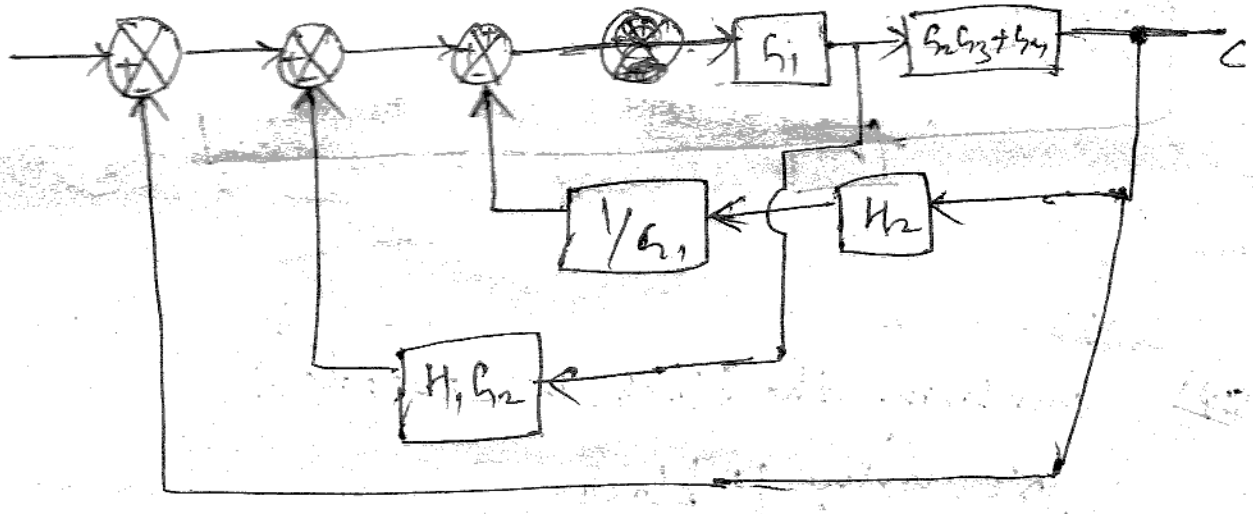
Step 2 combining the blocks in Cascade & removing || blocks,

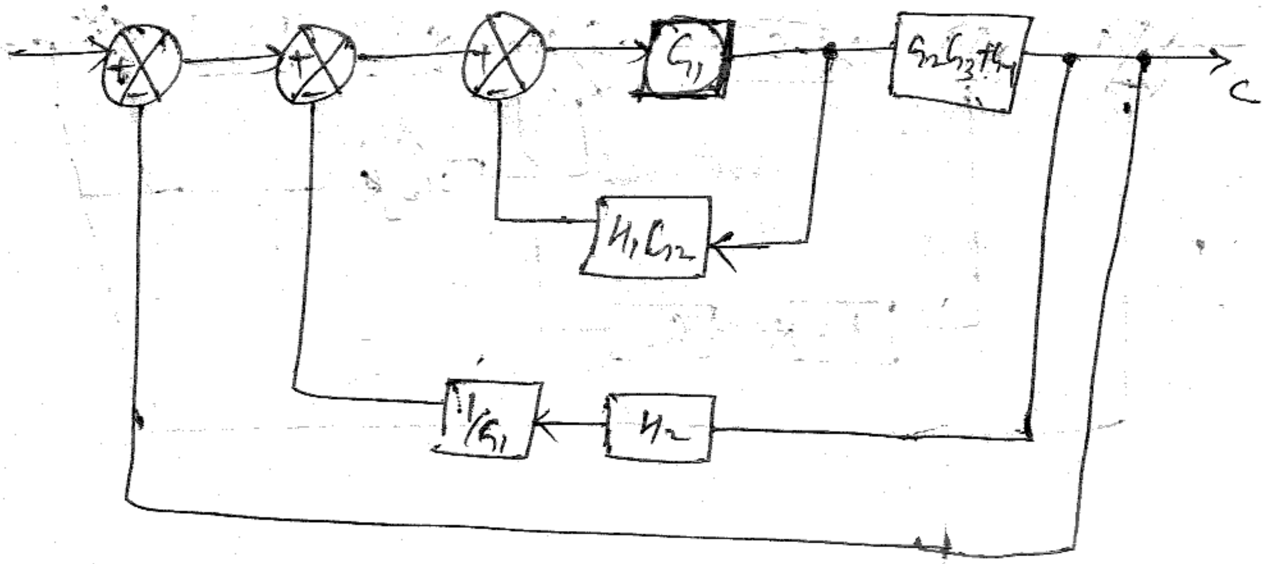


Step 3 Move the summer point before the block,

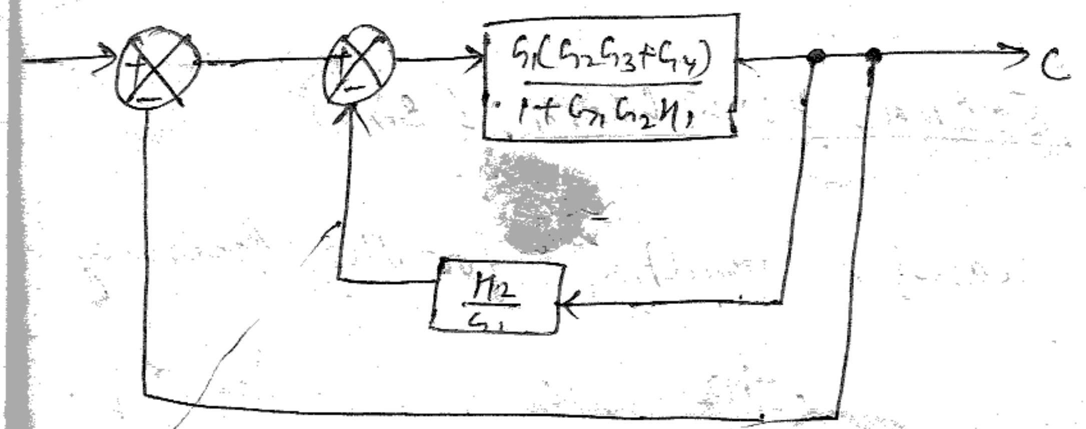


Step 4. Interchanging summing points & modify branch points.





Step 6

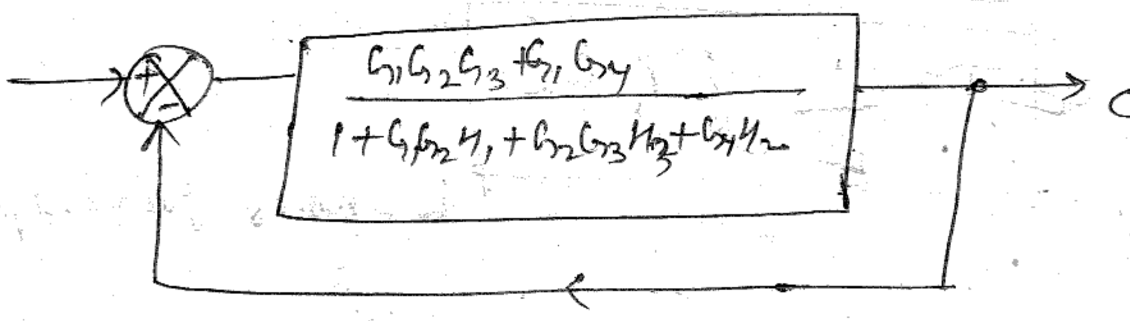


$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\left(\dfrac{H_2}{G_1}\right)} = \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}}$$

$$\Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_3+G_4H_2}$$

$\frac{C}{R}$

Step 7



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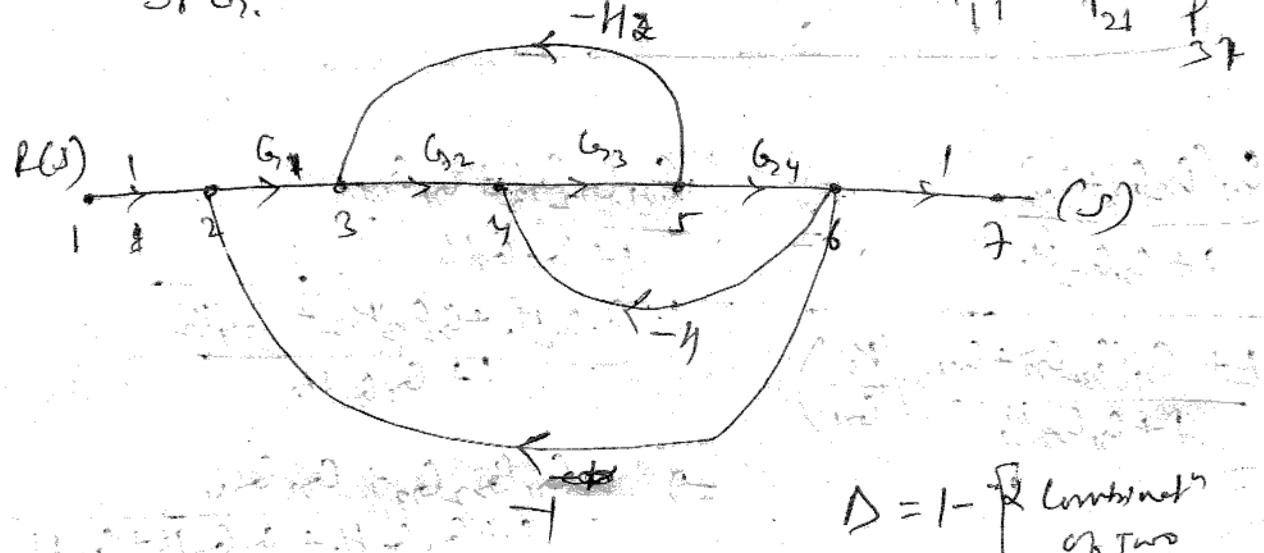
SIGNAL FLOW GRAPH :- (SFG)

Q1 Find the transfer function for the following SFG.

P_{11} P_{21} P_{37}



$\Delta = 1 - \{$ combination of two forward path $+3$ " $\}$.

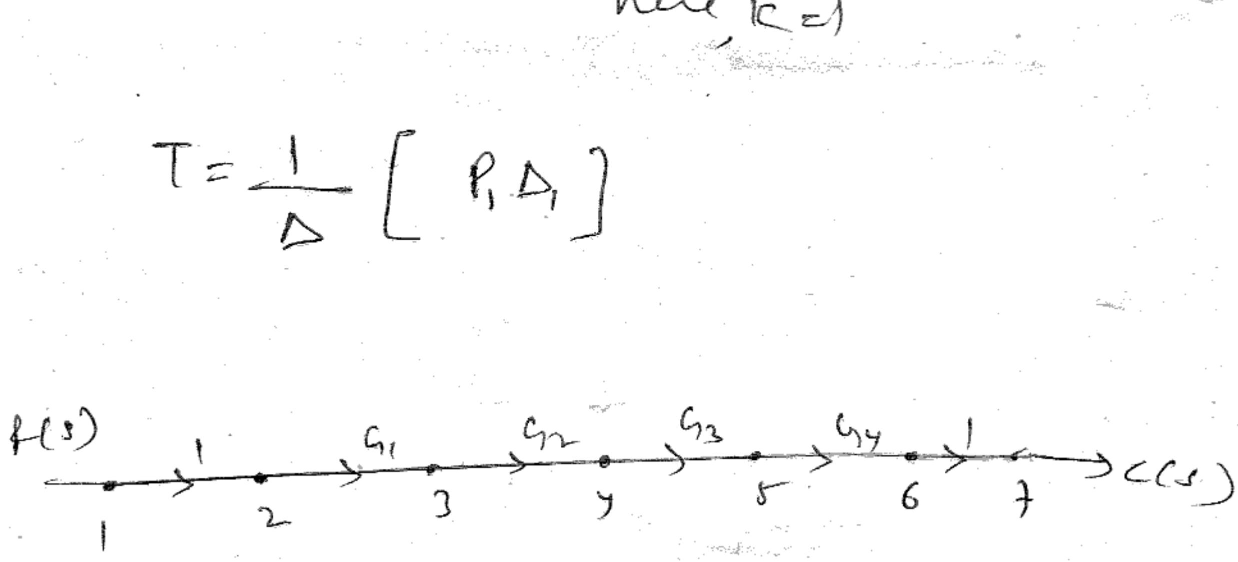
Mason formula :-

$$\boxed{T = \frac{1}{\Delta} \sum_k P_k \Delta_k}$$

k = no. of forward path

here, $k = 1$

$$T = \frac{1}{\Delta} \left[P_1 \Delta_1 \right]$$



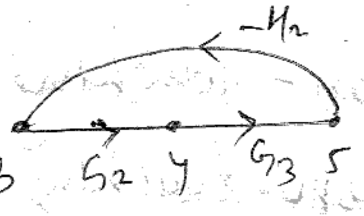
gain of the forward path

$= 1 \times G_1 \times G_2 \times G_3 \times G_4 \times 1 = G_1 G_2 G_3 G_4$.

Steps 3 :-

Individual gain for loop 1 (3 to 5) ← Consisting of feedback

P_{11}

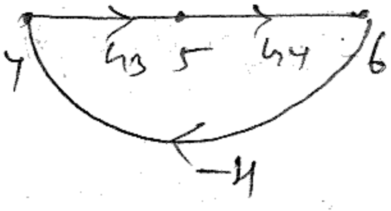


First individual loop with 1st path

gain $= (G_2)(G_3)(-H_2)$

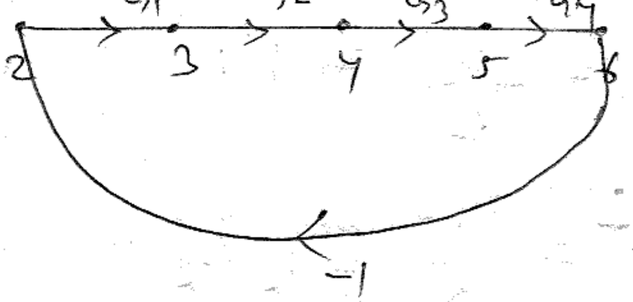
$= -G_2 G_3 H_2$

$P_{21} =$



gain $= -G_3 G_4 H$

$P_{31} =$



gain $= -G_1 G_2 G_3 G_4$

Step 4 :- gain products of non-touching loops

⇒ There are no possible combinations of two non-touching loops and three non-touching loops.

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$

$$= 1 - (-G_2 G_3 H_2 + (-G_3 G_4 H) + (-G_1 G_2 G_3 G_4$$

$$= 1 + G_2 G_3 H_2 + G_3 G_4 H + G_1 G_2 G_3 G_4$$

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

but $k = 1$

here $\Delta_k = 1$

⇒ Since no touching loops.

$$\therefore T = \frac{1}{\Delta} \Sigma (P_1 \Delta_1)$$

$$\frac{C(s)}{R(s)} = T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

$$= \frac{1}{\Delta} \Sigma P_1 \Delta_1$$

$$\boxed{T = \frac{1 \; (G_1 G_2 G_3 G_4) \times 1}{1 + G_2 G_3 H_2 + G_3 G_4 H + G_1 G_2 G_3 G_4}}$$

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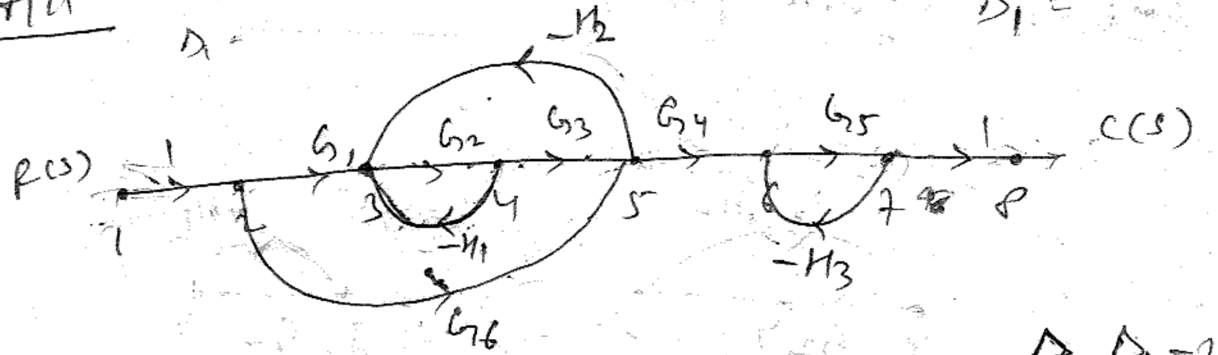
$\Delta = 1$ $\quad \Delta_1 = ??$

$\Delta_1 =$

Q2



$\Delta, \Delta_k = ?$

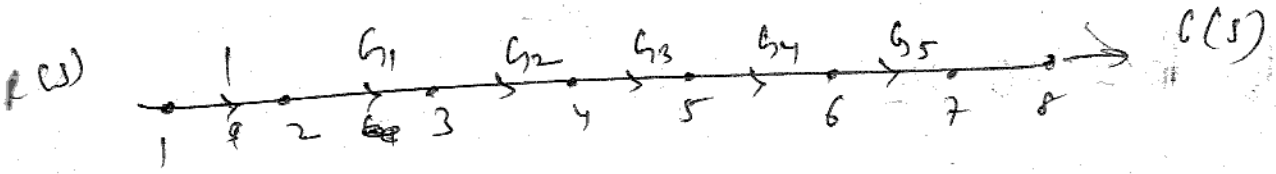
K = no. of forward path = 2.

$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

$$= \frac{1}{\Delta} \left[P_1 \Delta_1 + P_2 \Delta_2 \right]$$

$1 + G_2H_1 + G_2G_3H_2 + G_5H_3$
$+ G_2H_1 \cdot G_5H_3$
$+ G_2G_3H_2 \cdot G_5H_3$

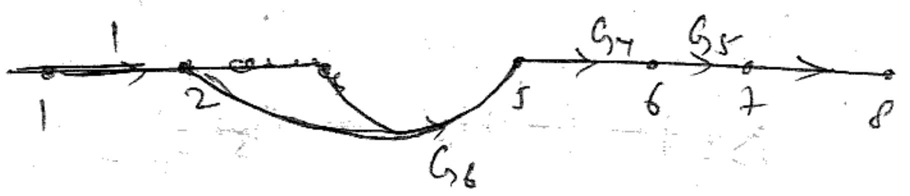
gain for ~~for~~ 1st forward path.



$P_1 = 1 \times G_1 G_2 G_3 G_4 G_5$

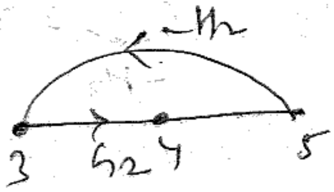
$$\boxed{P_1 = G_1 G_2 G_3 G_4 G_5}$$

gain for 2nd forward path

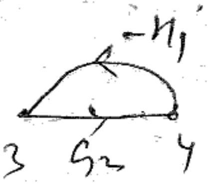


$$\boxed{P_2 = G_4 G_5 G_6}$$

Step 2: gain for individual loop.

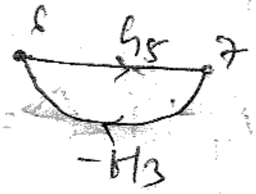


(i) P_{12}



(ii) P_{11}

(iii)



P_{21}

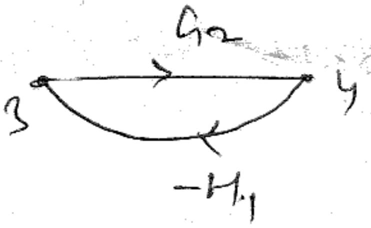
$1 + G_2H_1 + G_2H_2 + G_5H_3$

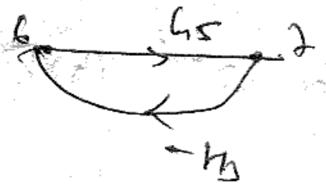
$P_a = P_{11} = \cancel{-G_2} - G_2H_1$

$P_b = P_{12} = -G_2H_2$

$P_c = P_{21} = -G_5H_3$

Step 3: Combination of two non touching loops





$\cancel{\Delta = 1 - (-H_1G_2 - G_5H_3)}$

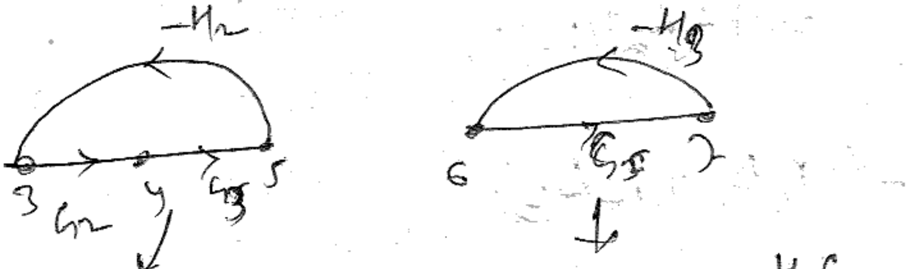
$\cancel{= 1 + H_1G_2 + G_5H_3}$

gain for loop (3 to 4) $= -H_1G_2$

gain for loop (6 to 7) = $-H_3 G_5$

overall gain = $G_2 G_5 H_1 H_3$.

<u>another Non touching loop</u>



gain $= -G_2 H_2 G_3$ gain $= -H_3 G_5$

$$\boxed{P_{12} = \cancel{G_2 G_5 H_2 H_3}\ G_2 G_3 G_5 H_2 H_3.}$$

$\Delta_1 =$

loops * <u>MESSON'S GAIN FORMULA</u>:-

The oval gain $\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$

where, T = transfer funct^n of system

P_k = the forward path gain or, gain of k^{th} forward path

$\Delta = 1 -$ {sum of individual loop gains + Combination of two non touching loops − Combin^n of 3 non touching loops − ...}.

$\Delta_k = \Delta$ for that part of the graph for k^{th} forward path.

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - (-G_2 H_1)$$
$$= 1 + G_2 H_1$$

$$T = \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$\Delta = 1 - (P_a + P_b + P_c)$$
$$= 1 + G_2 H_1 + G_2 H_2 + G_5 H_3$$

$$T = \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$\boxed{T = \frac{(G_1 G_2 G_3 G_4 G_5) + (G_4 G_5 G_6)(1 + G_2 H_1)}{(1 + G_2 H_1 + G_2 H_2 + G_5 H_3) + G_2 H_1 G_5 H_3 + G_2 G_3 H_2 G_5 H_3}}$$

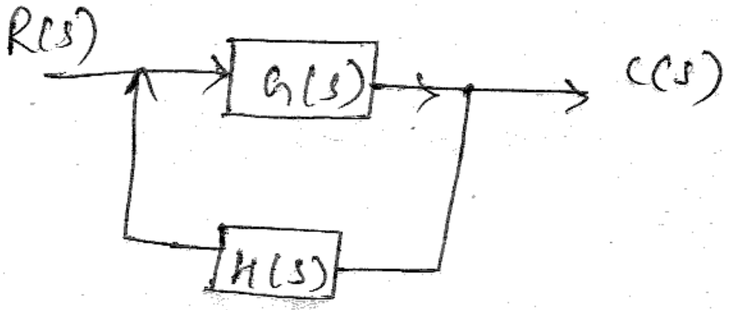
19/7/14

Units (Nagoor Kani)

IInd Unit

TIME RESPONSE ANALYSIS

Time response of the system is the o/p of the closed loop system as the function of time ; denoted by c(t).



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$\Rightarrow \quad C(s) = R(s).\frac{G(s)}{1+G(s).H(s)}$$

Applying inverse laplace transform

$$\boxed{c(t) = L^{-1}\left\{ R(s).\frac{G(s)}{1+G(s)H(s)} \right\}}$$

Time Response :-

Time Response

- Transient Response
- Steady State response.

~~Resp~~ Response :- o/p.

~~Resp~~ TEST SIGNAL :-

i) Step signal
ii) Ramp signal
iii) Parabolic signal.
iv) Impulse Signal.
v) Sinusoidal Signal.