UNIT - II

Static Error constant

When a control system is excited with standard i/p s/l the steady state Error may be constant, zero or infinity. The value of steady state error depends upon the type, number & input signal. Type zero system we have a constant steady state Error and the input is step signal.

Type one system will have a constant steady state Error when the i/p signal is ramp i/p.

Type 2 system will have a constant steady state Error when i/p is parabolic s/l.

For the three cases mentioned above the steady state error is associated with one of the constant defined as follows:-

Positional Error constant $K_p = \underset{s \to 0}{Lt}\ G(s) \cdot H(s)$

Velocity " " $K_v = \underset{s \to 0}{Lt}\ s\,G(s) \cdot H(s)$

Acceleration " " $K_a = \underset{s \to 0}{Lt}\ s^2 G(s) H(s)$.

The steady state Error various inputs

Type	Steady State Error		
	unit step	unit Ramp	Parabolic
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$
3	0	0	0

*

Q.1 For a unity feedback system the open loop transfer function $G(s) = \frac{10(s+2)}{s^2(s+1)}$. Find static Error constant

a) the position, velocity & acceleration.

b) the steady state Error when the i/p is $R(s)$ where $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$.

$$G(s) = \frac{10(s+2)}{s^2(s+1)}$$

for unity feed back $H(s) = 1$

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)} \cdot 1$$

$$= \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)}$$

$$\boxed{K_p = \infty}$$

$$K_v = \lim_{s \to 0} s\, G(s)H(s)$$

$$= \lim_{s \to 0} s\left(\frac{10(s+2)}{s^2(s+1)}\right)$$

$$= \lim_{s \to 0} \frac{10(s+2)}{s(s+1)}$$

$$\boxed{K_v = \infty}$$

$$K_a = \lim_{s \to 0} s^2\left(\frac{10(s+2)}{s^2(s+1)}\right)$$

$$= \frac{20}{1}$$

$$\boxed{K_a = 20}$$

<u>Generalised Error - Coefficient Method</u>

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2 \ddot{r}(t)}{2!} + \cdots \frac{C_n}{n!} r^n(t).$$

where $C_0, C_1, C_2 \ldots C_n$ are the Error coefficient.

$$C_0 = \lim_{s \to 0} F(s)$$

$$F(s) = \frac{1}{1 + G(s)H(s)}$$

$$C_1 = \lim_{s \to 0} \frac{dF(s)}{ds}$$

$$C_2 = \lim_{s \to 0} \frac{d^2F(s)}{ds^2}$$

$$C_n = \lim_{s \to 0} \frac{d^nF(s)}{ds^n}$$

(Continue 1st problem)

$$L^{-1}[R(s)] = r(t) = L^{-1}\left[\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right]$$

$$r(t) = 3 - 2t + \frac{t^2}{6}$$

$$\dot{r}(t) = -2 + \frac{\cancel{2}t}{\cancel{6}\,3}$$

$$\ddot{r}(t) = \frac{2 \times \cancel{t}}{6} = \frac{1}{3}$$

$$\dddot{r}(t) = 0$$

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t)$$

$$C_0 = \lim_{s \to 0} F(s)$$

$$= \lim_{s \to 0} \left(\frac{1}{1 + G(s)H(s)} \right)$$

$$= \lim_{s \to 0} \left(\frac{1}{1 + \frac{10(s+2)}{s^2(s+1)}} \right)$$

$$= \lim_{s \to 0} \left(\frac{s^2(s+1)}{s^2(s+1) + 10(s+2)} \right)$$

$$C_0 = 0$$

$$C_1 = \lim_{s \to 0} \frac{d\,F(s)}{ds}$$

$$= \lim_{s \to 0} \frac{d \left(\frac{s^2(s+1)}{s^2(s+1) + 10(s+2)} \right)}{ds}$$

$$= \lim_{s \to 0} \frac{(3s^2+2s)(s^3+s^2+10s+20) - (s^3+s^2)(3s^2+2s+10)}{\left(s^2(s+1) + 10(s+2)\right)^2}$$

$$C_1 = 0$$

$$C_2 = \lim_{s \to 0} \frac{d^2 F(s)}{ds^2}$$

$$= \lim_{s \to 0} \frac{d}{ds} \left[\frac{\cancel{3s^5} + \cancel{2s^4} + 30s^3 + 60s^2 + \cancel{3s^4} + \cancel{2s^3} + 20s^2 + 40s - \cancel{3s^5} - \cancel{2s^4} - 10s^3 - \cancel{3s^4} - \cancel{2s^3} - 10s^2}{(s^3+s^2+10s+20)^2} \right]$$

$$= \lim_{s \to 0} \frac{d}{ds} \left[\frac{20s^3 + 70s^2 + 40s}{(s^3+s^2+10s+20)^2} \right]$$

$$= \lim_{s \to 0} \frac{(60s^2+140s+40)(s^3+s^2+10s+20)^2 - (20s^3+70s^2+40s)\left(2(s^3+s^2+10s+20)(3s^2+2s+10)\right)}{(s^3+s^2+10s+20)^4}$$

$$= \frac{16000}{160000}$$

$$= \frac{1}{10}$$

$$\boxed{c_2 = \frac{1}{10}}$$

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!} \ddot{r}(t)$$

$$= 0 + 0 + \frac{1}{10 \times 2} \times \frac{1}{3}$$

$$e(t) = \frac{1}{60}$$

$$\boxed{e(t) = \frac{1}{60}}$$

$\therefore$ Steady State Error $e_{ss} = \lim_{t \to \infty} e(t) = \lim_{t \to \infty} \frac{1}{60}$

$$\boxed{e_{ss} = \frac{1}{60}}$$

Method 2

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} s \left[\frac{\left[\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}\right]}{1 + \frac{10(s+2)}{s^2(s+1)} \cdot 1} \right]$$

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~~Effect of PROPORTI~~

~~In feed-back control system a controller may be introduced~~

$$= \lim_{s\to 0}\left[\frac{s\left[\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}\right]s^2(s+1)}{s^2(s+1)+10s+20}\right]$$

$$= \lim_{s\to 0}\left[\frac{s^3(s+1)\frac{3}{s}}{s^3+s^2+10s+20}-\frac{s^3(s+1)\frac{2}{s^2}}{s^3+s^2+10s+20}+\frac{\frac{1}{3s^3}\cdot s^3(s+1)}{s^3+s^2+10s+20}\right]$$

$$\Rightarrow \quad e_{ss} = 0 - 0 + \frac{\frac{1}{3}}{20}$$

$$\boxed{e_{ss} = \frac{1}{60}}$$

Q. The open loop transfer function of the system with unity feedback $G(s) = \frac{10}{s(0.1s+1)}$. Evaluate the

a) Static Error constant & also obtain the steady state error of the system with an i/p $r(t) = a_0 + a_1 t + \frac{a_2}{2}t^2$

$$G(s) = \frac{10}{s(0.1s+1)}$$

$$H(s) = 1$$

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{10}{s(0.1s+1)}$$

$$\boxed{K_p = \infty}$$

$$K_v = \lim_{s\to 0} s\,G(s)H(s)$$

$$= \lim_{s\to 0} s\left(\frac{10}{s(0.1s+1)}\right)$$

$$= \lim_{s\to 0} \frac{10s}{s(0.1s+1)}$$

$$\boxed{K_v = 10}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$= \lim_{s\to 0} s^2 \frac{10}{s(0.1s+1)}$$

$$\boxed{K_a = 0}$$

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + C_2 \ddot{r}(t)$$

$$r(t) = a_0 + a_1(t) + \frac{a_2}{2}t^2$$

$$\dot{r}(t) = a_1 + \frac{\cancel{2} a_2 t}{\cancel{2}}$$

$$= a_1 + a_2 t$$

$$\ddot{r}(t) = a_2$$

$$C_0 = \lim_{s\to 0} F(s)$$

$$= \lim_{s\to 0} \frac{1}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{1+\frac{10}{s(0.1s+1)}}$$

$$= \lim_{s\to 0} \frac{1(s(0.1s+1))}{s(0.1s+1)+10}$$

$$\boxed{C_0 = 0}$$

$$C_1 = \lim_{s\to 0} \frac{d\,F(s)}{ds}$$

$$= \lim_{s\to 0} \frac{d}{ds}\left(\frac{s(0.1s+1)}{0.1s^2+s+10}\right)$$

$$= \lim_{s\to 0} \frac{d}{ds}\left(\frac{0.1s^2+s}{0.1s^2+s+10}\right)$$

$$= \lim_{s\to 0} \frac{(0.2s+1)(0.1s^2+s+10)-(0.1s^2+s)(0.2s+1)}{0.1s^2+s+10}$$

$$= \frac{10}{100} = \frac{1}{10}$$

$$\boxed{C_1 = \frac{1}{10}}$$

$$C_2 = \lim_{s \to 0} \frac{d^2}{ds^2} \left(F(s) \right)$$

$$= \lim_{s \to 0} \frac{d}{ds} \frac{\left(0.02s^3 + 0.2s^2 + 2s + 0.1s^3 + s + 10 - 0.02s^3 - 0.1s^2 - 0.2s^2 - s \right)}{\left(0.1s^2 + s + 10 \right)^2}$$

$$= \lim_{s \to 0} \frac{d}{ds} \frac{2s + 10}{(0.1s^2 + s + 10)^2}$$

$$= \lim_{s \to 0} \frac{\left(2(0.1s^2 + s + 10)^2 - (2s+10)\left[2(0.1s^2 + s + 10)(0.2s + 1)\right] \right)}{(0.1s^2 + s + 10)^4}$$

$$= \frac{200 - 200}{10000}$$

$$\boxed{C_2 = 0}$$

$$e(t) = C_1 r(t)$$

$$= \frac{1}{10}(a_1 + a_2 t)$$

$$e(t) = \frac{a_1 + a_2 t}{10}$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} \frac{a_1 + a_2 t}{10}$$

$$\boxed{e_{ss} = \infty}$$

UNIT - III

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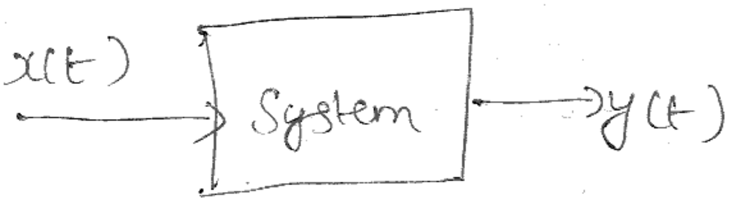
FREQUENCY RESPONSE :

It is the steady state response of a system when the i/p to the system is sinusoidal signal

Let $x(t)$ be an i/p sinusoidal signal. The response $y(t)$ is also a sinusoidal signal of same frequency but different magnitude and phase angle.



$$x(t) = X \sin \omega t$$
$$y(t) = Y \sin(\omega t + \phi)$$

The magnitude and phase relationship b/w sinusoidal i/p and the steady state o/p of the system is termed as frequency response.

The frequency response can be evaluated for both open loop system and closed loop system.

Open loop system

$$T.F = G(j\omega)H(j\omega) = |G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$$

Closed loop system

$$T.F = \frac{C(j\omega)}{R(j\omega)} = M(j\omega) = |M(j\omega)| \angle M(j\omega)$$

Frequency domain Specification

i) Resonant peak.
ii) Resonant frequency.
iii) Bandwidth.
iv) cutoff rate.
v) Gain margin } determine system is
vi) Phase margin } stable or not.

Gain margin :- Gain margin is denoted by K_g is defined as the magnitude of your open loop transfer function at phase cross over frequency.

The frequency at which phase of open loop transfer function is 180°.

$$K_g = \frac{1}{|G(j\omega_{pc})|}$$

$\omega_{pc} \rightarrow$ phase cross over frequency.

Phase Margin :- It is denoted by (γ)
The phase margin it that amount of additional phase lag at gain cross-over frequency required to bring the system to the verge of instability.

The gain cross-over frequency -
ω_{gc} is the frequency at which the magnitude of open loop transfer function is unity (or) the frequency at which dB magnitude is zero.

$$\gamma = 180^\circ + \phi_{gc}$$

Frequency Response plot
Frequency response analysis of control system can be carried either analytically or graphically.
The various graphical techniques are available for frequency response analysis are.

1) Bode plot
2) Polar plot
3) Nichols plot

* a) M and N circles

Polar plot

$$G(s) = \frac{1}{S(1+S)(1+2S)}$$

The open loop transfer function of unity feedback is given by $G(S)$. Sketch the polar plot & also determine the phase margin and gain margin.

Sub $S = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2}\sqrt{1^2+\omega^2}\sqrt{1^2+4\omega^2}}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}}$$

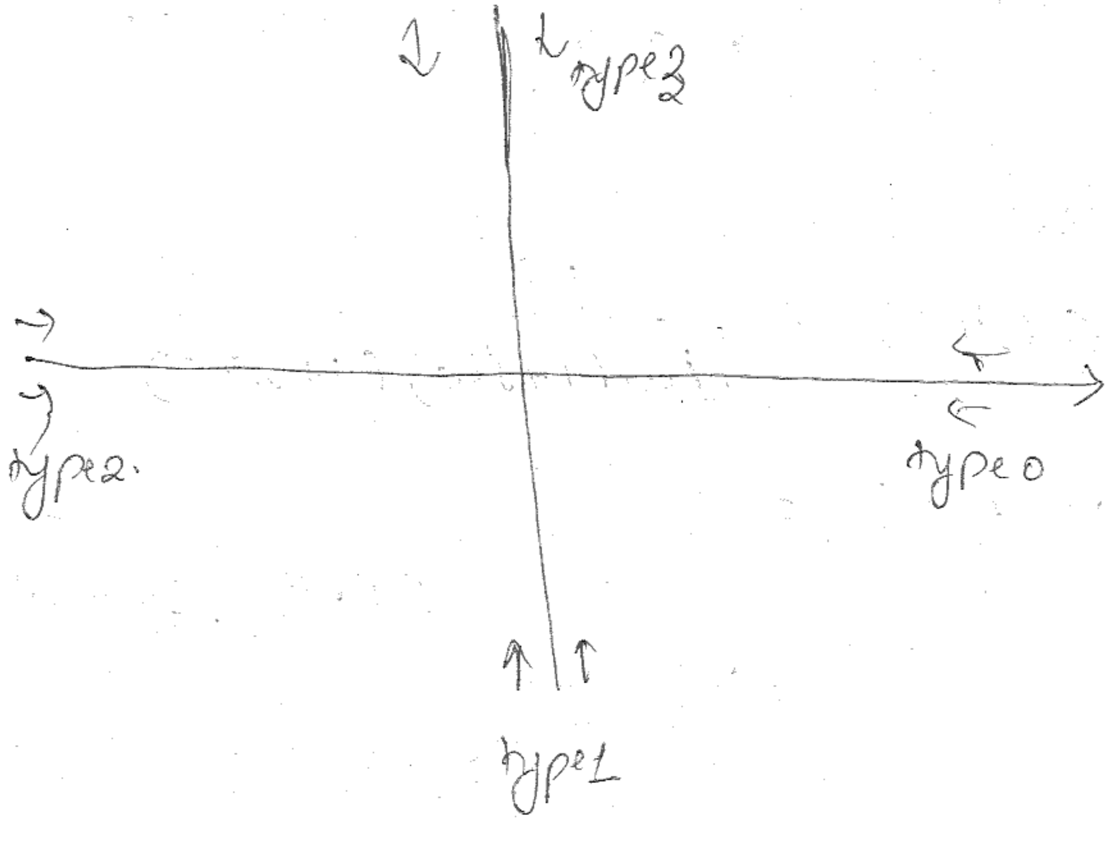
$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

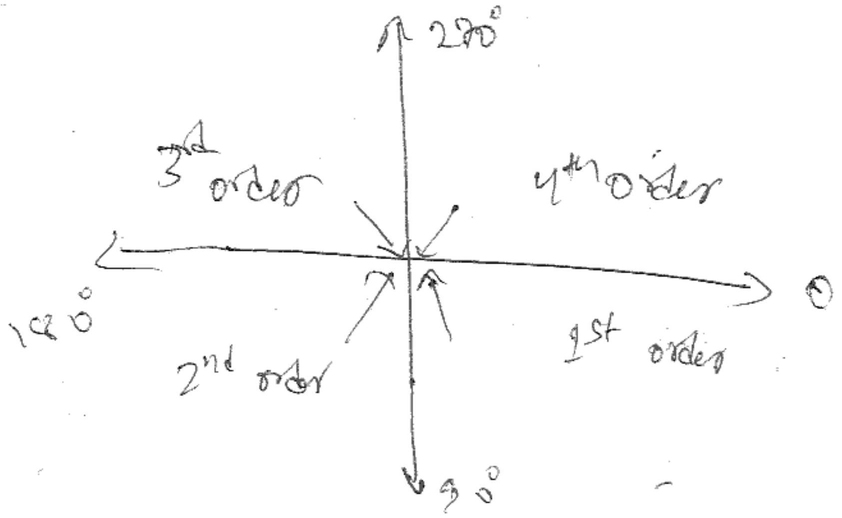
$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{2\omega}{1}\right)$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega).$$

$0.7 \rightarrow 0.680 \rightarrow -179.45$

ω	0.1	0.5	0.8	1	1.2	2	20
$\lvert G(j\omega)\rvert$	9.75	7.26	0.517	0.316	0.205	0.054	
$\angle G(j\omega)$	-107.02	-161.56	-186.65	-198.43	-207.57	229.39	





$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$s = j\omega$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$= \frac{1}{\cancel{-\omega^2(1+2j\omega+j\omega}}$$

$$|G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+j\omega}\sqrt{1+2j\omega}}$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$= \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}(2\omega)$$

ω	0.1	0.3	0.5	0.7	1	2	3
$\lvert G(j\omega)\rvert$	95.20	7.49	1.6	0.46	0.1	2.94×10^{-3}	3×10^{-4}
$\angle G(j\omega)$	-197.0	-227.6	-251.56	-269.45	-288.43	-319.39	[illegible]

-332.102

Bode plot

Procedure for plotting magnitude plot

1. Convert the transfer function into time Constant form.

$$G(S) = \frac{K(1+ST_1)}{S(1+ST_2)\left(1+\frac{S^2}{\omega_n^2}+2\xi\frac{S}{\omega_n}\right)}$$

$$G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)\left(1+\frac{(j\omega)^2}{\omega_n^2}+2\xi\frac{j\omega}{\omega_n}\right)}$$

Step 2 :-

Term	corner freq rad/sec	Slope dB/dec	change in slop
$\frac{K}{j\omega}$	—	-20	—
$(1+ST_1)$	0.2	20	0
$\frac{1}{1+ST_2}$	0.5	-20	-20

List the corner frequencies in the increasing order and prepare a table as shown above.

In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)$ for the first ~~term~~ term and remaining terms in the increasing order of corner frequency

$T_1 = 5$, $T_2 = 2$

$\frac{1}{T_1} = 0.2$ $\frac{1}{T_2} = 0.5$

Step 3 :- Choose an arbitrary frequency ω_{cl} which is lesser than first corner frequency ω_{c_1} and choose ω_{ch} which is higher than last corner frequency.

Step 4 :- Calculate the gain at every corner frequency one by one by using this formula.

$$A_1 = 20 \log (K/j\omega) \big|_{\omega = \omega_{cl}}$$

$$A_2 = 20 \log (K/j\omega) \big|_{\omega = \omega_{c_1}}$$

$$A_3 = (\text{change in slope from } \omega_x \text{ to } \omega_y) \times \log\left(\frac{\omega_y}{\omega_x}\right) + A_2 (\text{gain at } \omega_x \text{ previous gain})$$

Phase plot

$\angle G(j\omega) =$

Q. Sketch the Bode plot for the transfer function $G(S) = \dfrac{KS^2}{(1+0.2S)(1+0.02S)}$

Step 1.

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

Step 2: $G(j\omega) = \dfrac{4.(j\omega)^2}{..}$ Assume K = 1.

Term	corner freq	Slope	change in slope
$(j\omega)^2$	—	+40	—
$(1+0.2j\omega)$	$\omega_{c_1} = 5$	−20	20
$(1+0.02j\omega)$	$\omega_{c_2} = 50$	−20	+0

Magnitude

$$A_1 = 20\log\omega^2\Big|_{\omega=\omega_{cl}}$$

$$A_2 = 20\log\omega^2\Big|_{\omega=\omega_{c_1}}$$

$A_1 = \cancel{20\log}$

let us assume $\cancel{\omega_l = 0.5}$
$\omega_h = 100$

$A_1 = 20\log(0.25) = \cancel{6.020}$ $-12.041 \approx -12dB$

$A_2 = 20\log(25) = \cancel{55.91}$ $27.95 = 28$ db

$A_3 = 20\log\left(\frac{\omega c_2}{\omega c_1}\right) + A_2$ (Previous gain)

$= 20\log\left(\frac{50}{5}\right) + 28$

$= 20\log(10) + 28$

$= 48$

$A_4 = 0\log\left(\frac{\omega c_h}{\omega c_2}\right) + 48$

$A_4 = 48\,dB$

$\angle G(j\omega) = +180^\circ - \tan^{-1}(0.2\omega) - \tan^{-1}(0.02\omega)$

ω	0.5	5	10	50	100
$\angle G(j\omega)$	173.71	129.28	105.25	50.71	29.42

CLOSED LOOP RESPONSE FROM OPEN LOOP RESPONSE

The closed loop system transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

The sinusoidal transfer function is obtained by replacing of S by $j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)\cdot H(j\omega)}$$

$$\frac{C(j\omega)}{R(j\omega)} = M(j\omega) = |M| \angle \alpha$$

The magnitude and phase of closed loop system for various values of ω is evaluated by analytically or graphically.

The analytical method of determining the frequency response involves a tedious calculation.

Two graphical methods are available to determine the closed loop frequency response from open loop frequency response.

i) M and N circles
ii) ~~Nyquist plot~~ Nichols chart

M and N circles Method

The magnitude of closed loop transfer function with unity feedback, can be shown to be in the form of circle for every value of M. This circle is called M-circle.

If the phase of closed loop transfer function with unity feedback is α. Then it can be shown that $\tan\alpha$ will be in the form of circle for every value of α. This circle is called N circle.

M-Circle

Consider the closed loop transfer function is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} \quad (\because H(S)=1)$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)}$$

let $G(j\omega) = x + jy$

where $X \rightarrow$ Real part.
$Y \rightarrow$ imaginary part.

$$\frac{C(j\omega)}{R(j\omega)} = \frac{X+jY}{1+X+jY} = \frac{X+jY}{(1+X)+jY}$$

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

$$M = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}$$

$$M\sqrt{(1+X)^2+Y^2} = \sqrt{X^2+Y^2}$$

Take square both sides

$$M^2\left((1+x)^2+y^2\right) = x^2+y^2$$

$$M^2\left(1+x^2+2x+y^2\right) = x^2+y^2$$

$$M^2+M^2x^2+2M^2x+M^2y^2 = x^2+y^2$$

$$x^2(M^2-1)+(2x+1)M^2+y^2(M^2-1) = 0$$

when $M = 1$

the above equation becomes

$$2x+1 = 0$$

$$\boxed{x = -\frac{1}{2}}, \quad y = 0$$

Hence when $M=1$, the equation represents a straight line at the point of $(x=-\frac{1}{2}, y=0)$.

when $M \neq 1$

$$x^2(M^2-1)+y^2(M^2-1)+M^2+M^2 2x = 0$$

dividing by (M^2-1)

$$x^2+y^2+\frac{M^2}{M^2-1}+\frac{M^2}{M^2-1}\cdot 2x = 0$$

Adding $\frac{M^2}{(M^2-1)^2}$ on both side

$$x^2+y^2+\frac{M^2}{M^2-1}+\frac{M^2 2x}{M^2-1}+\frac{M^2}{(M^2-1)^2}=\frac{M^2}{(M^2-1)^2}$$

$$x^2+y^2+\frac{M^2(M^2-1)+M^2}{(M^2-1)^2}+\frac{M^2 2x}{(M^2-1)}=\frac{M^2}{(M^2-1)^2}$$

$$x^2+y^2+\frac{M^4-M^2+M^2}{(M^2-1)^2}+\frac{2x\cdot M^2}{M^2-1}=\frac{M^2}{(M^2-1)^2}$$

$$y^2+x^2+\frac{M^4}{(M^2-1)^2}+\frac{M^2 2x}{M^2-1}=\frac{M^2}{(M^2-1)^2}$$

$$\cancel{x^2+y^2+M^2}$$

$$y^2+\left(x+\frac{M^2}{M^2-1}\right)^2=\frac{M^2}{(M^2-1)^2}$$

The Equation of circle with the centre x_1, y_1 and radius r given by

$$(y-y_1)^2 + (x-x_1)^2 = r^2$$

$$y_1 = 0 \quad , \quad x_1 = -\frac{M^2}{M^2-1}$$

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N circle

$$\frac{C(j\omega)}{R(j\omega)} = \frac{x+jy}{1+x+jy} = \frac{x+jy}{(1+x)+jy}$$

$$\frac{\angle C(j\omega)}{\angle R(j\omega)} = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$\alpha = \angle\frac{C(j\omega)}{R(j\omega)} = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$N = \tan\alpha$$

$$N = \tan\left[\tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)\right]$$

$$N = \frac{\tan.\tan^{-1}\left(\frac{y}{x}\right) - \tan\tan^{-1}\left(\frac{y}{1+x}\right)}{1+\tan\tan^{-1}\left(\frac{y}{x}\right).\tan\tan^{-1}\left(\frac{y}{1+x}\right)}$$

$\because \tan \tan^{-1} A = A$

$$\therefore N = \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y}{x} \cdot \frac{y}{1+x}}$$

$$N = \frac{y(1+x) - yx}{x(1+x) + y^2}$$

$$N = \frac{\cancel{y} + \cancel{x}y - \cancel{yx}}{x + x^2 + y^2} = \frac{y}{x(1+x)+y^2}$$

$$N(x^2+y^2+x) = y$$

$$x + x^2 + y^2 = \frac{y}{N}$$

$$x + x^2 + y^2 - \frac{y}{N} = 0$$

Adding on both side $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$

$$\therefore x + x^2 + y^2 - \frac{y}{N} + \frac{1}{4} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(x + \frac{1}{4} + x^2\right) + \left(y^2 + \frac{1}{(2N)^2} - \frac{y}{N}\right) = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

The circle equation with centre (x_1, y_1)

$(x-x_1)^2 + (y-y_1)^2 = r^2$

$\therefore x_1 = -\frac{1}{2}$, $y_1 = \frac{1}{2N}$

Q. Sketch the bode plot for the open cloop transfer function & determine the phase margin & Gain margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100}$$

$$G(s) = \frac{75(1+0.2s)}{100s\left(\frac{s^2}{100} + \frac{16s}{100} + 1\right)}$$

Step 2.

$\boxed{s^2 + 2\xi\omega_n s + \omega_n^2}$ $\quad \omega_{c2} = \sqrt{\omega_n^2}$

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UNIT - IV

Stability Analysis

Stability :- The term stability refers stable working condition of control system. Every working system is designed to be stable, the different defination of stabilities are the following :-

1. A system is stable if its o/p is bounded for any bounded input. This is called BIBO stability.

2. A system is Asymptotically stable if in the absence ~~of~~ input, the output tends towards zero, irrespective of initial conditions.

3. A system is stable, if for a bounded distributing i/p signal, the o/p vanishes ultimately as $t \to \infty$.

4. A system is unstable, if for a bounded distorbing i/p signal the o/p of the infinite Amplitude.

ROUTH HURWITZ CRITERION

The closed loop transfer function of a System is a ratio of two polynomials in s. The denominator polynomial of closed loop system transfer function is called characteristics Equation of the System.

The Routh stability criterion is based on the ordering the coefficient of characteristics Equation

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n s^0 = 0$$

<u>condition</u>

$a_0 > 0$ into a schedule called Routh array as shown below.

$$\begin{array}{lllll}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_0 & b_1 & b_2 & b_3 \\
s^{n-3} & c_0 & c_1 & c_2 & c_3 \\
, & & & & \\
, & & & & \\
, & & & & \\
s^0 & & & &
\end{array}$$

$$b_0 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$\cancel{b_1 = \frac{a_3 a_4 - a_2 a_5}{a_3}}$$

$$b_1 = \frac{\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix}}{a_1}$$

$$b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_2 = \frac{\begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix}}{a_1}$$

$$b_2 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

Ill^{r}ly

$$c_0 = \frac{\begin{vmatrix} a_1 & a_3 \\ b_0 & b_1 \end{vmatrix}}{b_0} = \frac{b_0 a_3 - a_1 b_1}{b_0}$$

$$c_1 = \frac{\begin{vmatrix} a_1 & a_5 \\ b_0 & b_2 \end{vmatrix}}{b_0} = \frac{b_0 a_5 - a_1 b_2}{b_0}$$

$$c_2 = \frac{\begin{vmatrix} a_1 & a_7 \\ b_0 & b_3 \end{vmatrix}}{b_0} = \frac{b_0 a_7 - a_1 b_3}{b_0}$$

~~Conditions for~~

In The ~~necessary~~ process of Constructing the Routh array the missing terms are considered as zero. Also, all the Elements of any row can be multiplied or divided by a positive Constant to simplify the Computational work.

In the construction of Routh array one may come across the following cases.

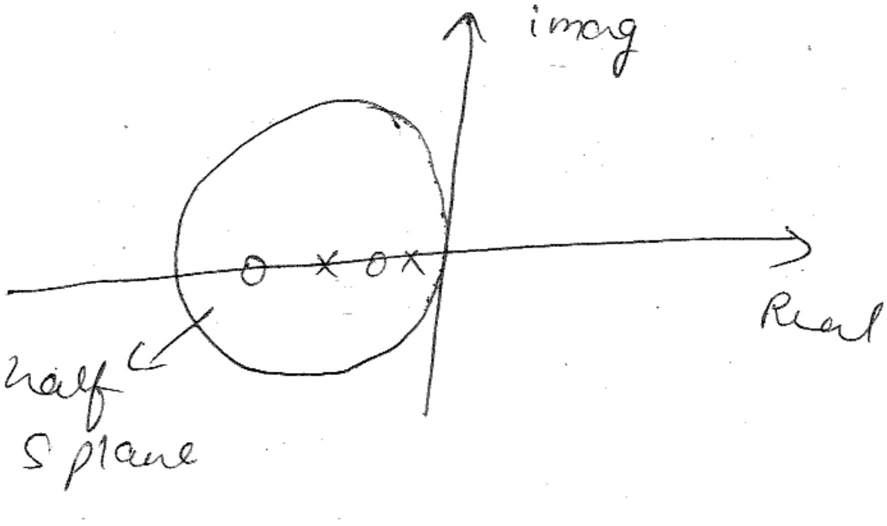
Case I: Normal Routh Array (Non-zero Elements in the first column of routh array)

Case II :- A row of all zeros.

Case III : First Element of row is Zero but some or other Elements are not zero.

Case 1 :-

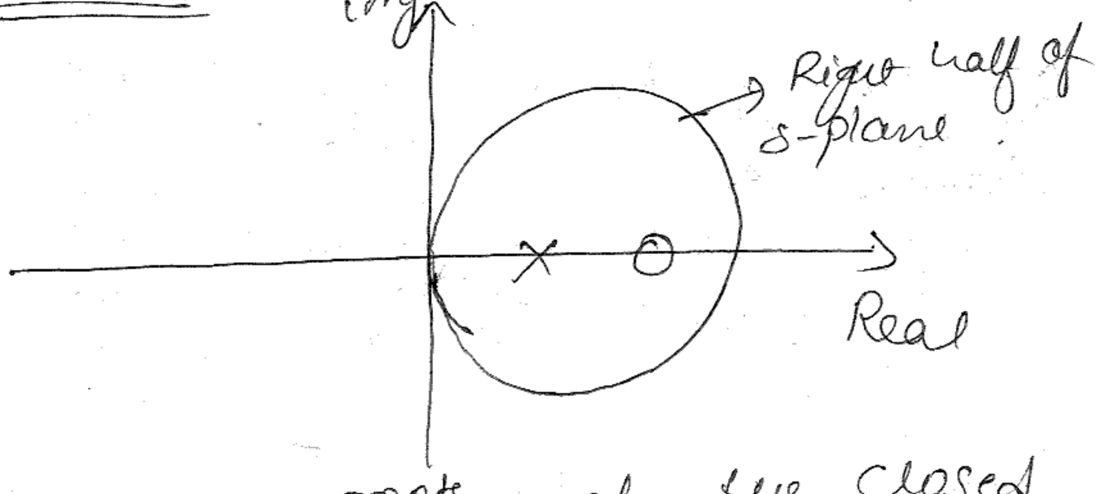
Stable



All Roots of closed loop transfer function is lie in the left side of s plane. Then the system must be stable for all working condition

Case 2 :-

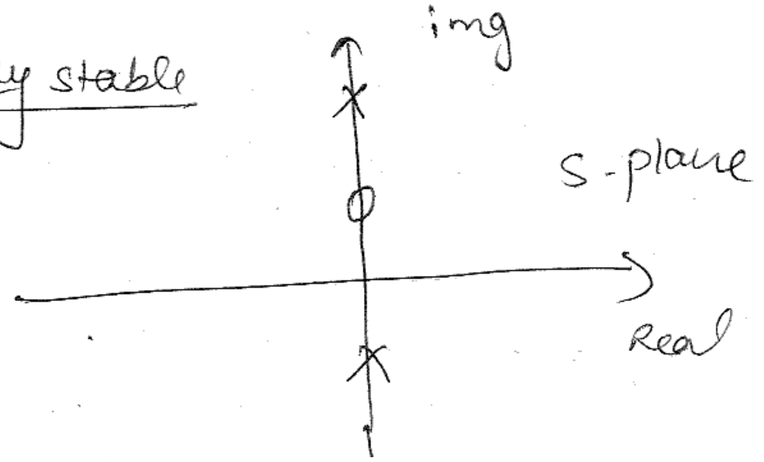
Unstable



If any one roots of the closed loop transfer function is lie in the Right half of s-plane. Then the system must be unstable for all working condition.

Case 3 :-

Marginally stable



If any one pole (or) zero of closed loop system transfer function is lie in the imaginary axis then the system is marginally stable for all condition.

Q1. Using Routh array, determine the stability of the system of the closed loop ~~standard~~ characteristic equation is given below.

$$S^4 + 8S^3 + 18S^2 + 16S + 5 = 0.$$

S^4	1	18	5
S^3	8	16	0
S^2	$16\left(\frac{8\times18 - 1\times16}{8}\right)$	5	0
S^1	$13.5\left(\frac{16\times16 - 8\times5}{16}\right)$	0	0
S^0	5		

i) The first row & first column are positive and Real, hence the system is stable.

ii) All roots lie on the left half of s plane.

Q2. Construct Routh Array & determine the stability of the system whose characteristics Equation is

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	
s^3	0	0		
s^2	0	0		
s^1	0			
s^0	0			

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Q. Determine the range of K for Stability of unity feedback system whose open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Sol^n.

closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} \quad (H(s)=1)$$

$$= \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}}$$

$$= \frac{\frac{K}{\cancel{s(s+1)(s+2)}}}{\frac{s(s+1)(s+2)+K}{\cancel{s(s+1)(s+2)}}}$$

$$= \frac{K}{s(s+1)(s+2)+K}.$$

characteristics equation is.

$$s(s+1)(s+2)+K = 0$$

$$s(s^2+3s+2)+K = 0$$

$$s^3+3s^2+2s+K = 0$$

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	0
s^0	K	

$\frac{6K-K^2}{3}$ $\frac{6K-K}{3}$ $\frac{K(6-K)}{6}$

$$\frac{6-K}{3} = 0 \Rightarrow K = 6$$

$$\boxed{0 < K < 6}$$

Q. The open loop transfer function of unity feedback control system is given by $G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$

The characteristic eqn. is

$$(s+2)(s+4)(s^2+6s+25) + K = 0$$

$$(s^2+6s+8)(s^2+6s+25) + K = 0$$

$$s^4 + 6s^3 + 25s^2 + 6s^3 + 36s^2 + 150s + 8s^2 + 48s + 200 + K$$

$$s^4 + 12s^3 + 69s^2 + 198s + 200 + K = 0$$

s^4	1	69	$200+K$
s^3	12	198	0
s^2	52.5	$\frac{39600+198K}{198} = 200+K$	
s^1	$\frac{7995-12K}{52.5}$		
s^0	$152.28 - 0.228K$		

52.5	$200+K$
$152.28 - 0.228K$	0

$30456 - 45.6K$

Root locus

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Step 1 :- ~~Determine no of poles & zeros.~~

Rules for constructing Root locus.

1. The root locus is symmetrical about the real axis.

2. Each branch of root locus originate from open loop pole, corresponding to $K = 0$ and terminates either on finite open loop zero, corresponding to $K = \infty$, the no of branches of root locus terminating on infinity is equal to $(n-m)$ i.e. the number of open loop poles - no of open loop zeros.

3. Segment of real axis having an odd number of real axis open loop poles + zeros, to their right or part of the root locus.

4. The $n-m$ root locus branches that tends to infinity go along a straight line asymptotes i.e. ~~ϕ_A~~

$$\phi_A = \frac{\pm 180^\circ (2q+1)}{(n-m)}$$

where $q = 0, 1, 2 \ldots\ldots (n-m)$.

5. The point of intersection of asymphits with real axis is called centroid & represented by σ_A

$$\sigma_A = \frac{\text{Sum of poles} - \text{Sum of zeros}}{(n-m)}$$

6. The break away & break ~~centroid~~ in points of root locus are determined ~~by~~ from the roots of the equation $\frac{dK}{ds} = 0$ if 'r' no. of branches of root locus meet at a point then the break away at an angle of $\pm \frac{180°}{r}$

7. Determine the angle of departure and angle of arrival.

The angle of arrival from a complex zeros given by $180° -$ sum of angle of vectors to the complex zero from all other zeros + sum of angle of vectors to the complex zero from other poles.

Angle of departure = 180° - sum of angle of vectors to the complex poles from all other poles + sum of angle of vectors to the complex pole from other zeros.

Q. A unity feedback control system has an open loop transfer function

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

. Sketch the root locus.

Soln.

$m \rightarrow 0$

$$s(s^2+4s+13) = 0$$

$$s^2 + 4s + 13 = 0$$

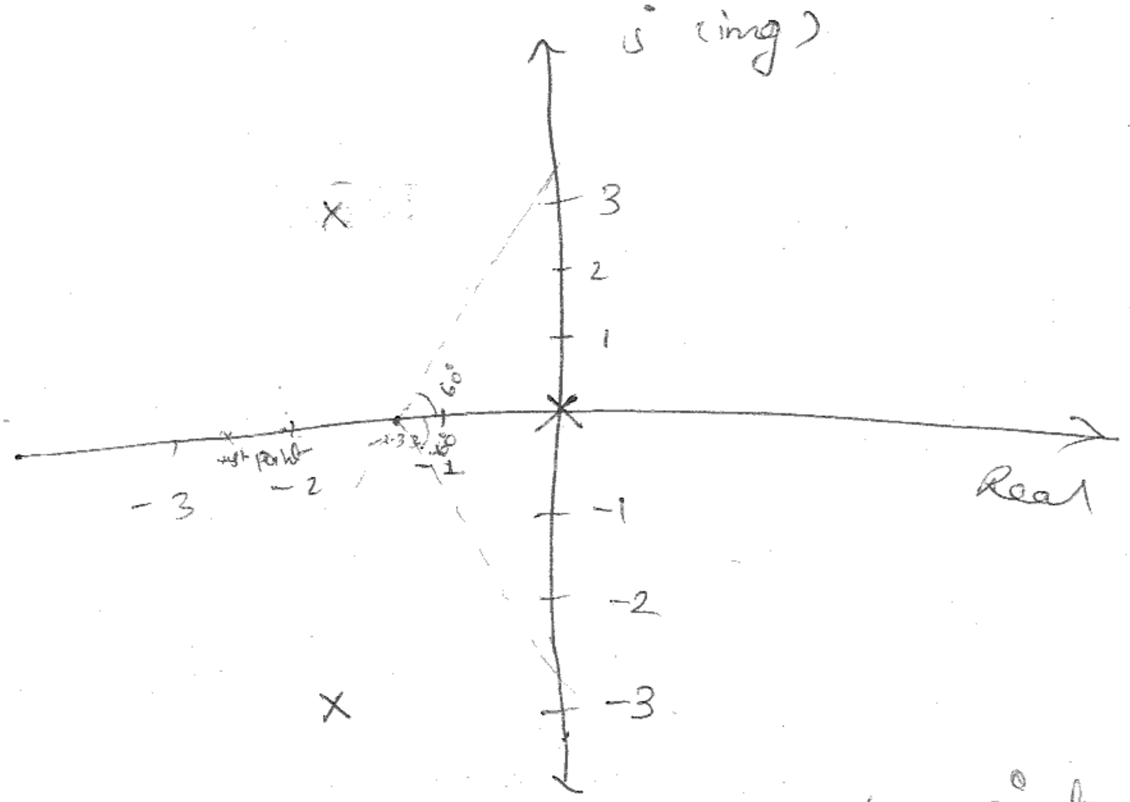
$$s = \frac{-4 \pm \sqrt{16-4\times13}}{2\times1}$$

13
$\times4$
52
-16
36

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$$= \frac{-4 \pm 6i}{2} = -2 \pm 3i$$

The poles are $0, -2+3j, -2-3j$



The path between origin and test point have odd no of pole so there is a root locus path.

b/w two poles. break away point
b/w two zero break in point

$$\phi_A = \frac{\pm 180^\circ (2q+1)}{n-m}$$

$q = 0, 1, 2, 3 \ldots (n-m)$

$q = 0, 1, 2, 3.$

$$\phi_A = \pm \frac{180^\circ (2 \times 0 + 1)}{3} = \pm \frac{\cancel{180}}{3} 60^\circ$$

$$\phi_A = \pm \frac{180^\circ \times 3}{3} = \pm 180^\circ$$

$$\phi_A = \pm \frac{\overset{60}{\cancel{180}}}{\cancel{3}} \times 5 = \pm 300^\circ = \pm 60^\circ$$

Centroid $\sigma_A = \dfrac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

$$= \frac{0 - 2 + 3j - 2 - 3j - 0}{3}$$

$$= -\frac{4}{3} = -1.33$$

$$\sigma_A = -1.33$$

$$G(s) = \frac{K}{S(S^2+4S+13)}$$

$$\frac{C(S)}{R(S)} = \frac{\dfrac{K}{S(S^2+4S+13)}}{1 + \dfrac{K}{S(S^2+4S+13)} \cdot 1}$$

$$= \frac{\dfrac{K}{\cancel{S(S^2+4S+13)}}}{\dfrac{S(S^2+4S+13)+K}{\cancel{S(S^2+4S+13)}}}$$

$$\frac{C(S)}{R(S)} = \frac{K}{S(S^2+4S+13)+K}$$

$$\frac{C(S)}{R(S)} = \frac{K}{S^3+4S^2+13S+K}$$

Characteristics equation is

$$S^3+4S^2+13S+K = 0$$

$$K = -S^3-4S^2-13S$$

$$\frac{dK}{dS} = -(3S^2+8S+13) = 0$$

$$3S^2+8S+13 = 0$$

$$S = \frac{-8 \pm \sqrt{64-4\times13\times3}}{2\times3}$$

$$= \frac{-8 \pm \sqrt{64-156}}{6}$$

$$= \frac{-8 \pm \sqrt{-92}}{6}$$

$$= \frac{-8 \pm 9.59j}{6}$$

$$S = -1.33 \pm 1.598j$$

$$S = -1.33 \pm 1.6j$$

$$3\overline{)39}$$
$$13$$

$$39 \times 4 = 156$$
$$156 - 64 = 92$$

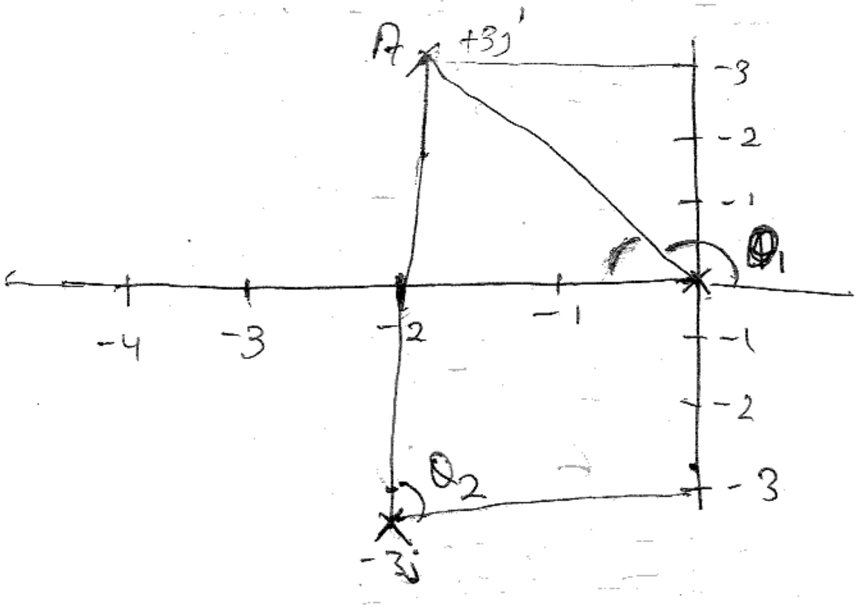
$$K = -\left((-1.33 \pm 1.6j)^3 + 4(-1.33+1.6j)^2 + 13(-1.33+6j)\right)$$

$K \neq$ Real & Positive.

Here no possiblities of break in and break away point.

Angle of departure

ϕ_d = 180° − [Sum of angle of vector of complex poles from all other poles] + [Sum of angle of vector of complex pole from zeros]



$$\phi_d(A) = 180 - (\phi_1 + \theta_2) + 0$$

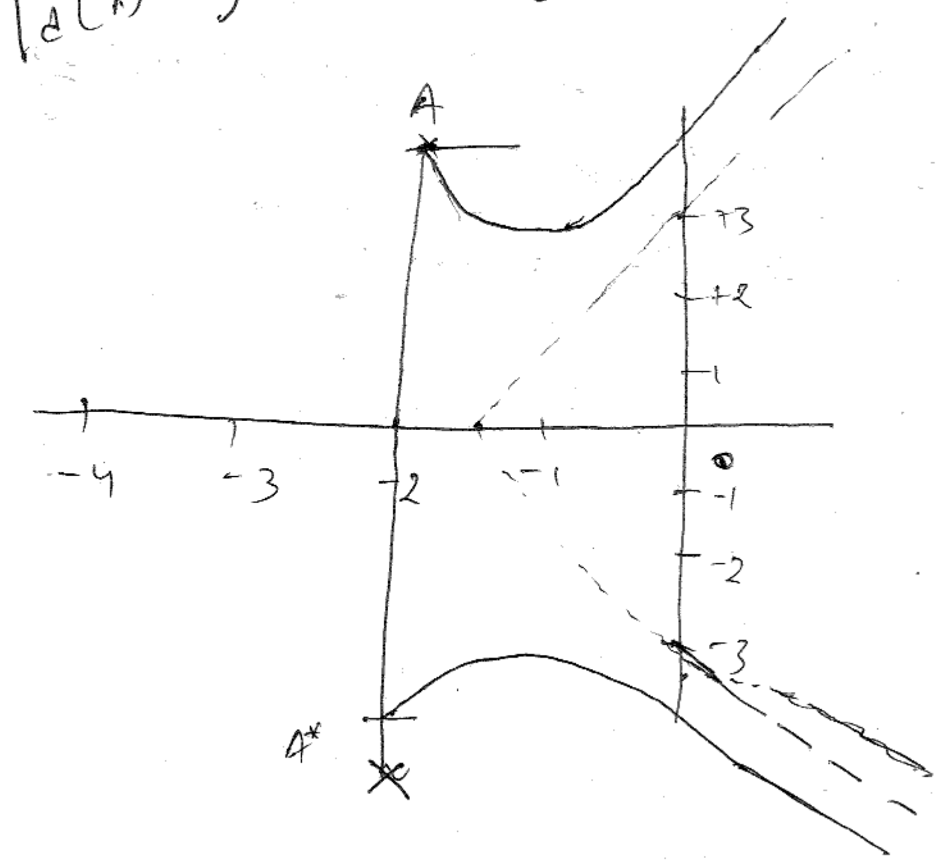
$$= 180° - [180 - \tan^{-1}(\tfrac{3}{2}) + 90°]$$

$$= \cancel{180} - \cancel{180°} + \tan^{-1}(\tfrac{3}{2}) - 90°$$

$$= \tan^{-1}(\tfrac{3}{2}) - 90°$$

$$\phi_d(A) = 56.30 - 90° = -33.69°$$

$\phi_d(A^*) = +33.7$.



To find the crossing of img axis & determine the gain K value.

chara eqn

$$s^3 + 4s^2 + 13s + k = 0$$

sub $s = j\omega$

$$j^3\omega^3 + 4j^2\omega^2 + 13j\omega + k = 0$$

$$-j\omega^3 - 4\omega^2 + 13j\omega + k = 0$$

$$(-j\omega^3 + 13j\omega) + (-4\omega^2 + k) = 0$$

Equating img & Real terms to zero

$$-4\omega^2 + k = 0 \Rightarrow k = 4\omega^2$$

$$-j\omega^3 + 13j\omega = 0$$

$$\cancel{j}\omega^{\cancel{3}2} = 13\cancel{j\omega}$$

$$\omega^2 = 13$$

$$\omega = \pm\sqrt{13}$$

$$\omega = \pm 3.6$$

$$K = 4(3.6)^2$$

$$\boxed{K = 51.84.}$$

$$\boxed{K \simeq 52}$$

Q. The open loop transfer function of a unity feedback System is given by

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$

Step 1: Find poles & zeros.

$s(s^2+4s+11) = 0$

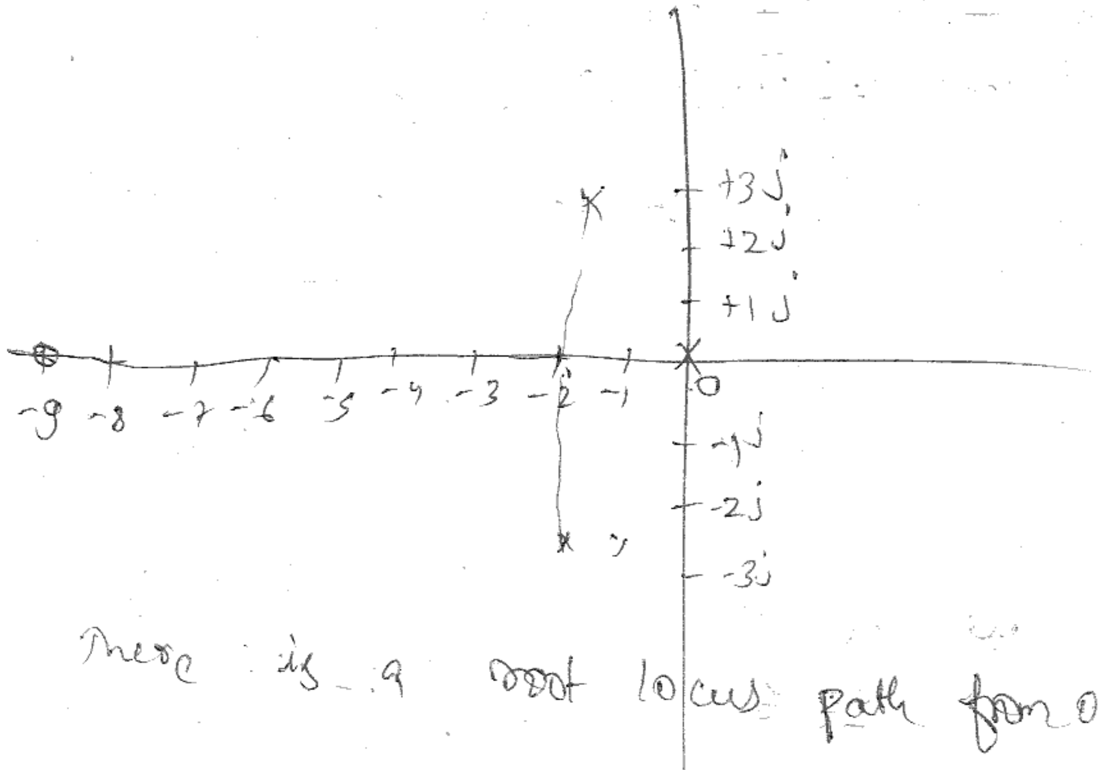
$s = 0$, $s^2+4s+11 = 0$

$$s = \frac{-4 \pm \sqrt{16-4\times11}}{2}$$

$$s = \frac{-4 \pm \sqrt{-28}}{2} = -2 \pm 2.64j$$

Poles are $s = 0, -2+2.64j, -2-2.64j$

Zeros is $s = -9$



There is a root locus path from 0 to -9.

3. To Find Angle of asymptotes = θ

$$\pm \frac{180(2q+1)}{n-m}$$

$q = 0, 1, 2$

$$\phi_A = \pm \frac{180(2q+1)}{2}$$

$$\phi_1 = \pm \frac{180}{2} = 90°$$

$$\phi_2 = \pm \frac{180 \times 3}{2} = \pm 270° = \pm 90°$$

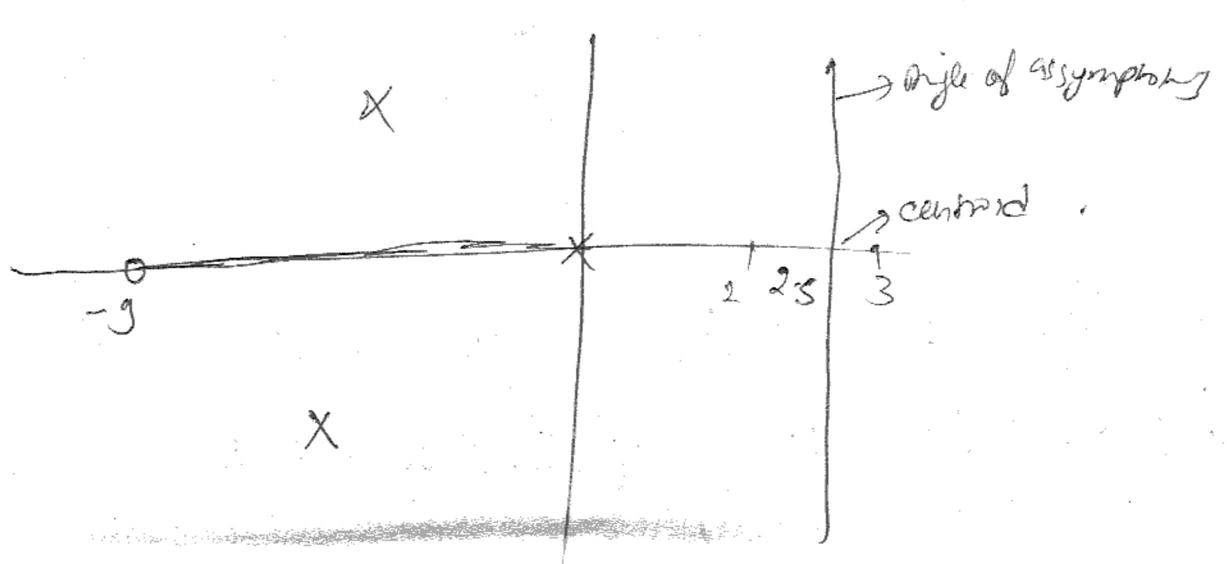
$$\phi_3 = \frac{180 \times 5}{2} = \pm 450° = \pm 90°$$

4. To Find the centroid

$$\text{centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$= \frac{0 - 2 + j2.64 - 2 - 2.64j + 9}{2}$$

$$= \frac{-4+9}{2} = \frac{5}{2} = 2.5$$



5. To find the breakaway & breakin point

There is no possibility of break away & break in point.

6.

The characteristic equn:

$$\frac{C(s)}{R(s)} = \frac{\frac{K(s+9)}{s(s^2+4s+11)}}{1+\frac{K(s+9)}{s(s^2+4s+11)}} = \frac{K(s+9)}{s(s^2+4s+11)+K(s+9)}$$

The char equn

$$s(s^2+4s+11)+K(s+9) = 0$$

$$s^3+4s^2+11s+Ks+9K = 0.$$

7. To find the crossing of img axis & determine the gain.

$$s^3+4s^2+11s+Ks+9K = 0$$

Sub $s = j\omega$.

$$-j\omega^3 - 4\omega^2 + 11j\omega + Kj\omega + 9K = 0$$

$$(-j\omega^3 + 11j\omega + Kj\omega) + (9K - 4\omega^2) = 0$$

$-j\omega^3 + 11j\omega + Kj\omega = 0$

$-\omega^2 + 11 + K = 0$

$\omega^2 = 11 + K$

and $\boxed{\omega = 4.44}$

$9K - 4\omega^2 = 0$

$4\omega^2 = 9K$

$\omega^2 = \frac{9K}{4}$.

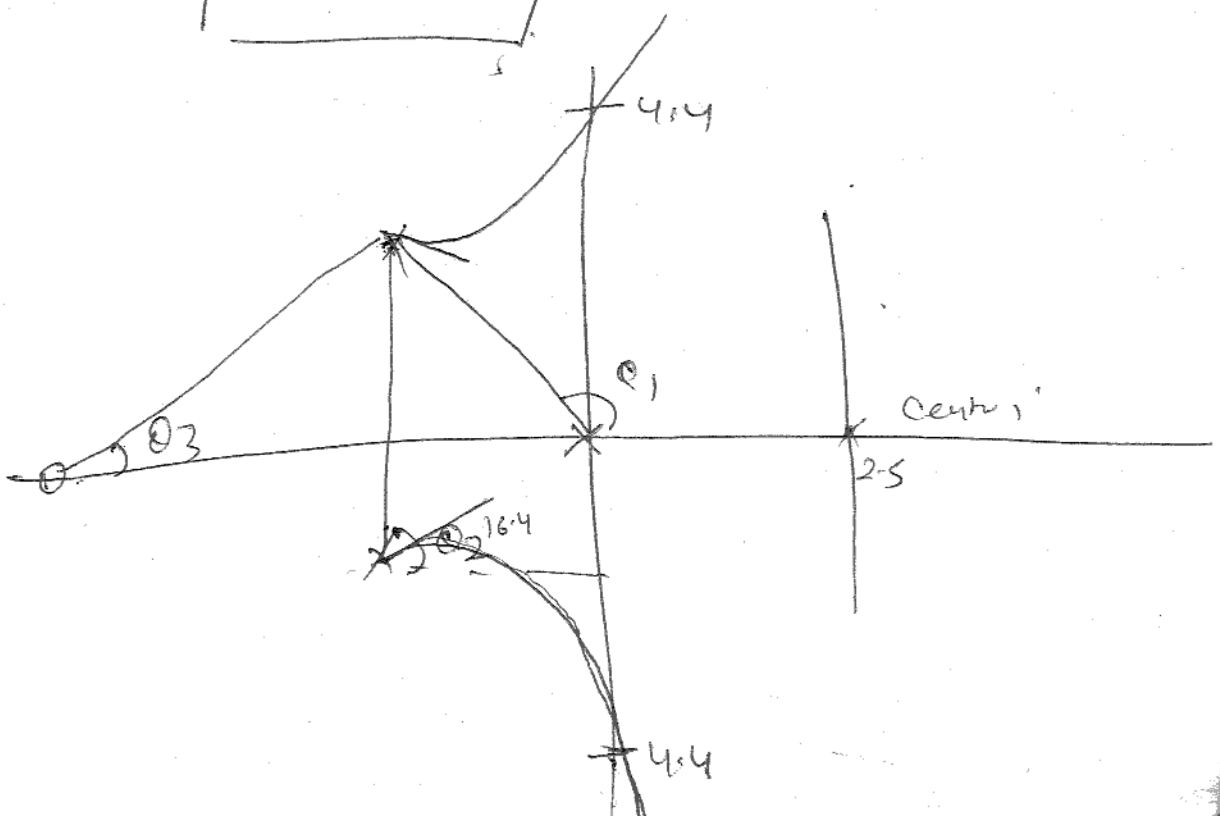
$\frac{9K}{4} = 11 + K$

$9K = 44 + 4K$

$5K = 44$

$K = \frac{44}{5} = 8.8$.

$\boxed{K = 8.8}$



Angle of departure $= 180° - (\phi_1 + \phi_2) + \phi_3$

$\phi_1 = 180° - \tan^{-1}\left(\frac{2.64}{2}\right)$

$= 127.1°$

$\phi_2 = 90°$

$\phi_3 = \tan^{-1}\left(\frac{2.64}{7}\right) = 20.7$

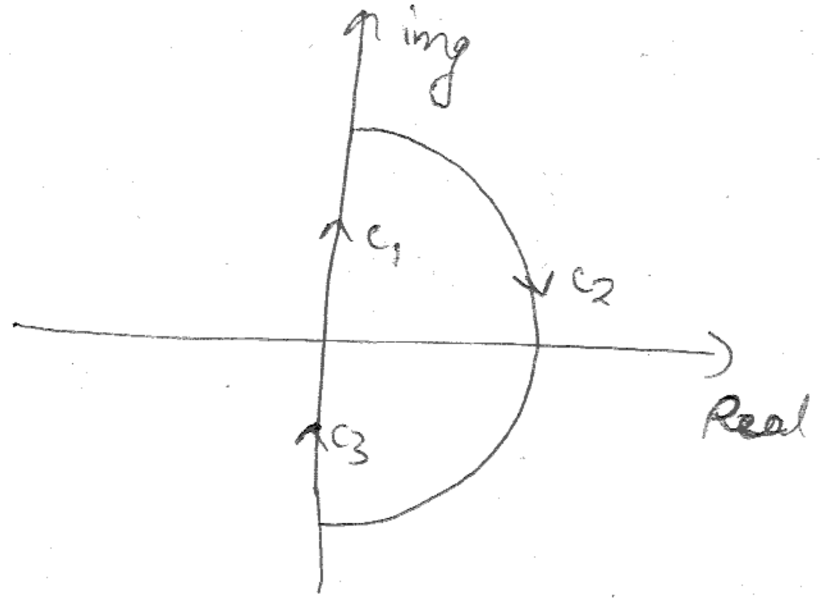
$\phi_d(A) = 180° - (127.1 + 90°) + 20.7$

$= 180° - 217.1 + 20.7$

$= -37.1 + 20.7$

$\phi_d(A) = -16.4°$

NYQUIST STABILITY CRITERION



1) All phase - ω 0 to α
2) minimum phase system - $\omega \to \alpha$ to $-\alpha$
3) Non minimum " " - $\omega = -\alpha$ to 0

No of contour circle = $P - Z$

Let us choose an arbitrary contour in the S-plane which encircles the right half zeros and poles of $F(s)$. The principle of argument states that, the corresponding contour in $F(s)$ plane will encircles the ~~splane~~ origin of $F(s)$ plane, n times in the anticlock wise direction.

where $N = P - Z$ — (1)

Here P = no of poles of $F(s)$ lying on right half S plane.

Z = No of zeros of F(S) lying On right half S-plane.

For the stability of the system, the roots of char equ^n and so the zeros of F(S) should not lie on the right half of S plane.

Hence for a stable system Z = 0.

Hence from equ^n (1),

when Z = 0, we get, N = P - (2)

when Z ≠ 0, N ≠ P - (3)

In order to investigate, the presence of poles of G(S)·H(S) on the right half of S-plane. A contour C is choosen such that it encloses entire S half of S plane as shown in fig.

Such a contour C is called nyquist Contour.

The nyquist contour is directed clockwise direction and comprises two segments.

i) An infinite line segment C_1 along the imag axis

iii) A semicircle C_2 of infinite radius.

Along C_1 sub $s = j\omega$ which varying from $-j\alpha$ to $+j\alpha$.

Along C_2, $S = \lim_{R \to \infty} Re^{j\theta}$ with θ varying from $+\frac{\pi}{2}$ to $-\frac{\pi}{2}$.

Eq 1.

$$G(s) = \frac{K(s+4)}{(s-1)(s-2)}$$

$K = 8$.

sub $s = j\omega$

$$G(j\omega) = \frac{8(j\omega+4)}{(j\omega-1)(j\omega-2)} = \frac{8(0.25j\omega+4)}{(1+j\omega)(1+0.5}$$

Terms	Corner freq	Slope	Chag of Slop
8	0	~~20~~ 0	
$0.25j\omega+1$	4	20	
$1+j\omega$	1	-20	
$1+0.5j\omega$	2	-20	

Q2.

$$G(s) = \frac{(1+0.2S)(1+0.025S)}{S^3(1+0.005S)(1+0.001S)}$$

Polar plot and find phase margin

Sub $S = j\omega$

$$G(j\omega) = \frac{(1+0.2j\omega)(1+0.025j\omega)}{(j\omega)^3(1+0.005j\omega)(1+0.001j\omega)}$$

$$|G(j\omega)| = \frac{\sqrt{1+(0.2)^2\omega^2}\sqrt{1+(0.025)^2\omega^2}}{-\omega^3\sqrt{1+}}$$