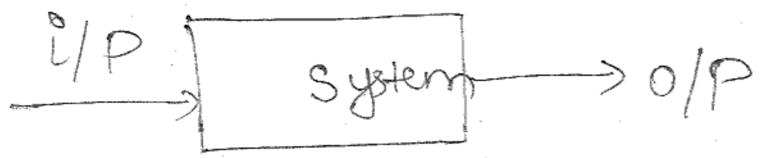
Control System

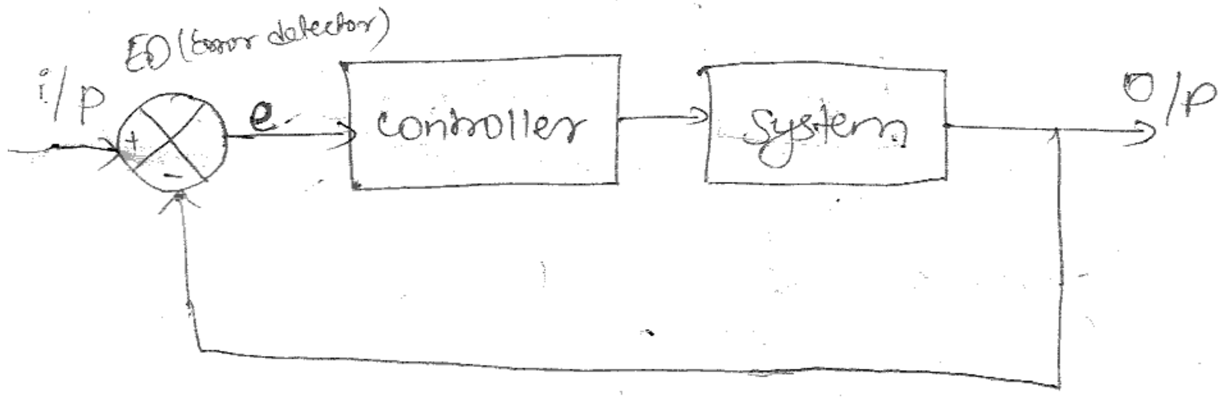
System :- It is the collection of physical Quantities in sequence to perform the desired task.

Types of CS

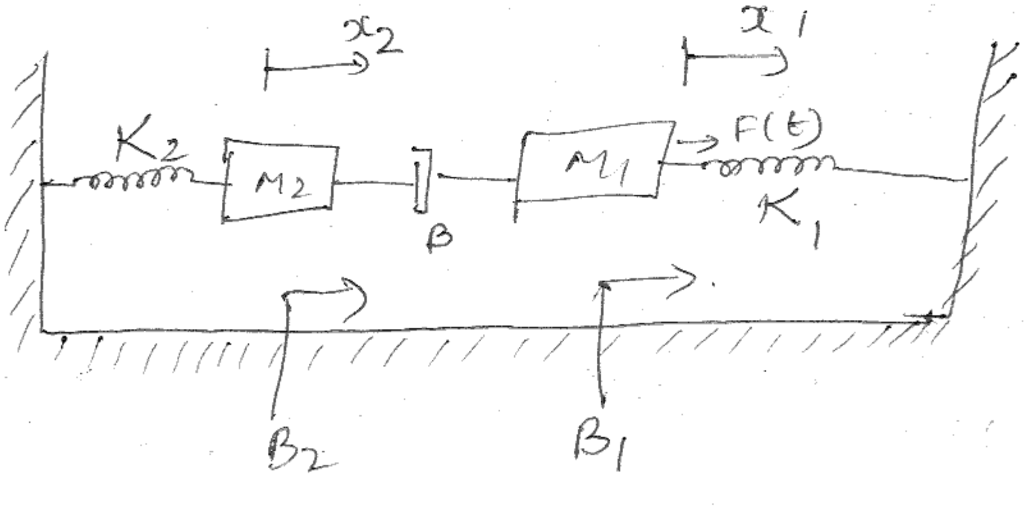
i) open loop system

2) Closed loop system





Transfer function of Mech system



$F(t)$ → input force

x → displacement

$\frac{dx}{dt} \rightarrow v \rightarrow$ velocity m/sec

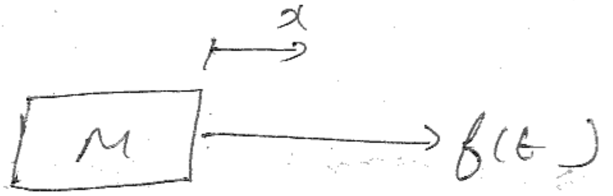
$\frac{dx^2}{dt^2} \rightarrow a \rightarrow$ accelerating m/sec^2

f_K → opposing force of Spring

f_B → " " " Dashpot

f_m → " " " mass

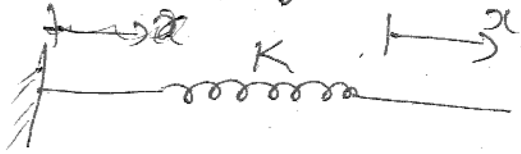
For mass



$$f_m \propto \frac{d^2x}{dt^2}$$

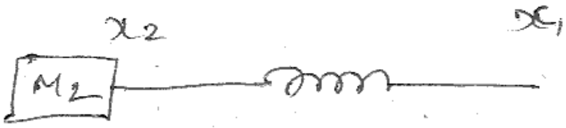
$$f_m = M \frac{d^2x}{dt^2}$$

For spring



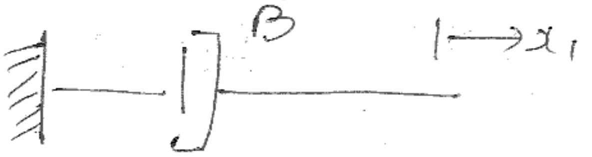
$$f_K \propto x,$$

$$f_K = Kx,$$



$$f_K \propto (x_2 - x_1)$$

For, Dash pot



$$f_B \propto \frac{dx_1}{dt}$$

$$f_B = B\frac{dx_1}{dt}$$

~~Taking laplace transform~~

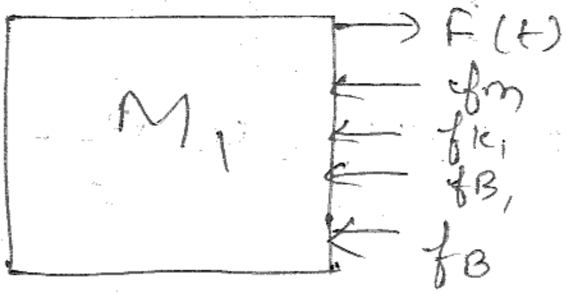
~~$L\{f_m\} = M s^2$~~

$L(x(t)=1) = \frac{1}{s}$

$L[x(t) = \frac{dx}{dt}] = s$

$L[x(t) = \frac{d^2x}{dt^2}] = s^2$

Now consider mass



According to Newton's third law

$$F(t) = f_m + f_{K_1} + f_{B_1} + f_B \quad \text{—} ①$$

$$f_m = M_1 \frac{d^2x_1}{dt^2} \qquad f_B = B\frac{d(x_1 - x_2)}{dt}$$

$$f_{k_1} = K x_1$$

$$f_{B_1} = B\frac{dx_1}{dt}$$

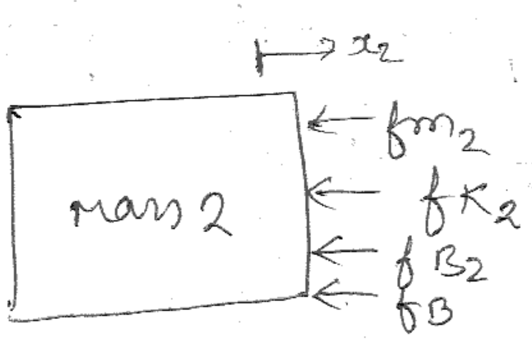
$$f(t) = M_1 \frac{d^2x_1}{dt^2} + Kx_1 + B_1 \frac{dx_1}{dt} + \frac{B\,d(x_1 - x_2)}{dt} \quad - ②$$

Apply laplace transform under zero initial condition

$$F(s) = M_1 S^2 x_1(s) + K\,x_1(s) + \cancel{B_1 S x_1(s) B} + B_1 S x_1(s) + B\,S(X_1(S) - X_2(S))$$

$$F(s) = [M_1 s^2 + K + B_1 S + BS] X_1(S) - BSX_2(S) \quad - ③$$

Consider Mass 2



$$0 = f_{m_2} + f_{K_2} + f_{B_2} + f_B$$

$$= M_2 \frac{d^2x_2}{dt} + Kx_2 + B_2 \frac{dx_2}{dt} + B \frac{d(x_2 - x_1)}{dt}$$

Apply laplace transform

$$= M_2 S^2 X_2(S) + K X_2(S) + B_2 S x_2(S) + BS(x_2(S) - X_1(S))$$

$$= [M_2 s^2 + K + B_2 S] + BS] X_2(S) - BSX_1(S) \quad - ④$$

$$X_2(s) = \frac{B s X_1(s)}{[M_2 s^2 + K + B_2 s + Bs]} \quad - ⑤$$

sub ⑤ in ③

$$F(s) = [M_1 s^2 + K + B_1 s + Bs] X_1(s) - \frac{Bs \cdot Bs X_1(s)}{M_2 s^2 + K + B_2 s + Bs}$$

$$\frac{F(s)}{X_1(s)} = M_1 s^2 + K + B_1 s + Bs - \frac{B^2 s^2}{M_2 s^2 + K + B_2 s + Bs}$$

$$\frac{F(s)}{X_1(s)} = \frac{(M_1 s^2 + K + B_1 s + Bs)(M_2 s^2 + K + B_2 s + Bs) - B^2 s^2}{M_2 s^2 + K + B_2 s + Bs}$$

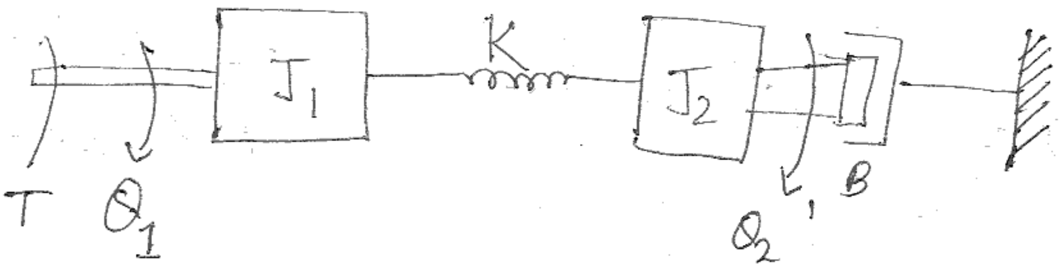
$$\frac{X_1(s)}{F(s)} = \xcancel{\frac{M_2 s^2 + K + B_2 s + Bs}{M_1 s^2 M_2 s^2 + M_1 s^2 K + M_1 s^2 B_2 s + M_1 s^2 Bs + K M_2 s^2 + K^2 + B_2 s K + BsK + B_1 s}}$$

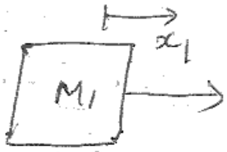
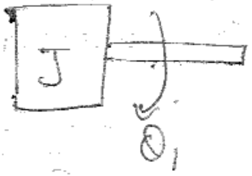
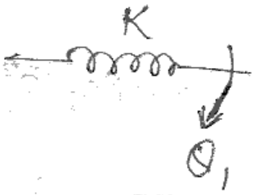
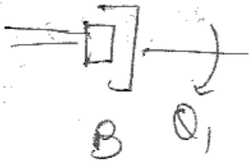
$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + B_2 s + Bs + K}{\left[(M_1 s^2 + K + B_1 s + Bs)(M_2 s^2 + K + B_2 s + Bs) - B^2 s^2\right]}$$

This is transfer function of Translational system.

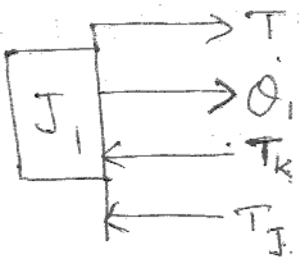
Angular (Rotational system)

To determine the transfer function for the System



	Translational system	Rotational system
Mass	 $f_m = M\frac{d^2x}{dt^2}$	 $T_J = J\frac{d^2\theta}{dt^2}$
Spring	 $f_K = Kx$	 $T_K = K\theta$
Dash POT	 $f_B = B\frac{dx}{dt}$	 $T_B = B\frac{d\theta}{dt}$

For body J_1



According to New

$$T = T_K + T_J$$

~~$T = K\theta_1$~~ $+ J\frac{d^2\theta_1}{dt^2}$

$$T = K(\theta_1 - \theta_2) + J_1\frac{d^2\theta_1}{dt^2}$$

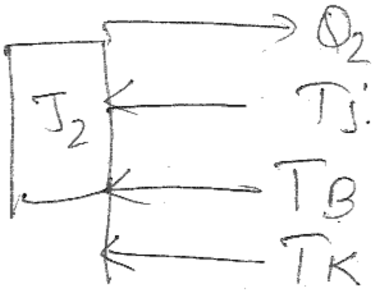
Taking laplace tranform

$$T(s) = K(\theta_1(s) - \theta_2(s)) + J_1 s^2 \theta_1(s)$$

$$T(s) = (K + J_1 s^2)\Theta_1(s) - K\Theta_2(s) \quad ①$$

for

For J_2



According to Newton's law

$$T_{J_2} + T_B + T_K = 0$$

$$J_2 \frac{d^2\Theta_2}{dt^2} + B\frac{d\Theta_2}{dt} + K(\Theta_2 - \Theta_1) = 0$$

$$J_2 s^2 \Theta_2(s) + Bs\Theta_2(s) + K\Theta_2(s) - K\Theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\Theta_2(s) - K\Theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\Theta_2(s) = K\Theta_1(s)$$

$$\Theta_2(s) = \frac{K\Theta_1(s)}{(J_2 s^2 + Bs + K)} \longrightarrow ②$$

Now we sub eqn ② in ①

$$T(s) = (K + J_1 s^2)\Theta_1(s) - \frac{K^2\Theta_1(s)}{(J_2 s^2 + Bs + K)}$$

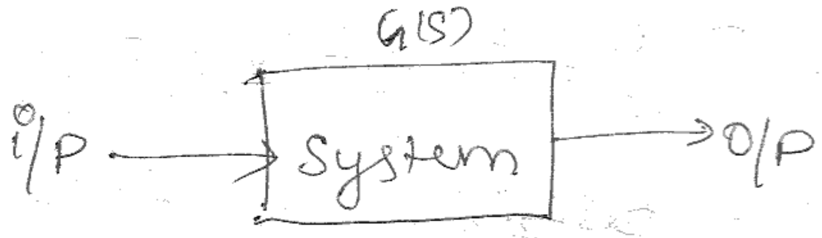
$$= \frac{(KJ_2 s^2 + KBs + K^2 + J_1 J_2 s^4 + J_1 s^3 B + J_1 s^2 K)\Theta_1(s) - K^2\Theta_1(s)}{(J_2 s^2 + Bs + K)}$$

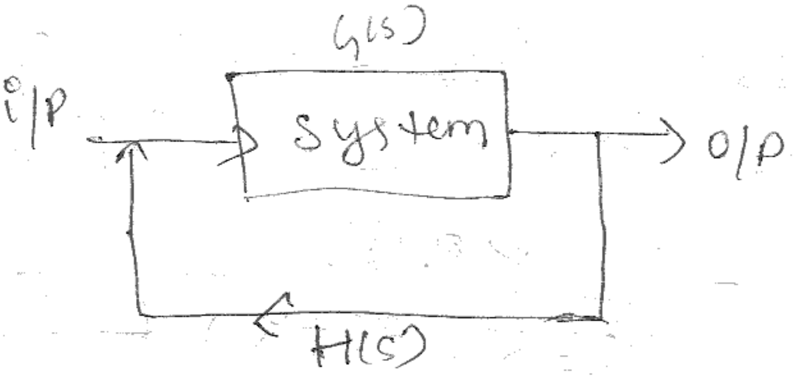
$$\frac{Q_1(S)}{T(S)} = \frac{J_2 S^2 + BS + K}{[(J_1 S^2 + K)(J_2 S^2 + BS + K) - K^2]}$$

State Variable Method

Signal flow graph

$$\text{Transfer function} = \frac{C(S) \rightarrow L[O/P]}{R(S) \rightarrow L[i/P]}$$





$$\text{Transfer function} = \frac{G(S)}{1+G(S)H(S)} \quad \{\text{closed loop}\}$$

Signal flow graph represents the System function by means of flow of Signals. It is invented by Mason's.

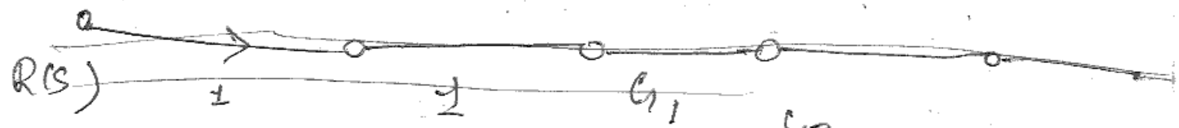
Tranfer function $T = \frac{1}{\Delta} \sum_{K=1}^{n} P_K \Delta_K$

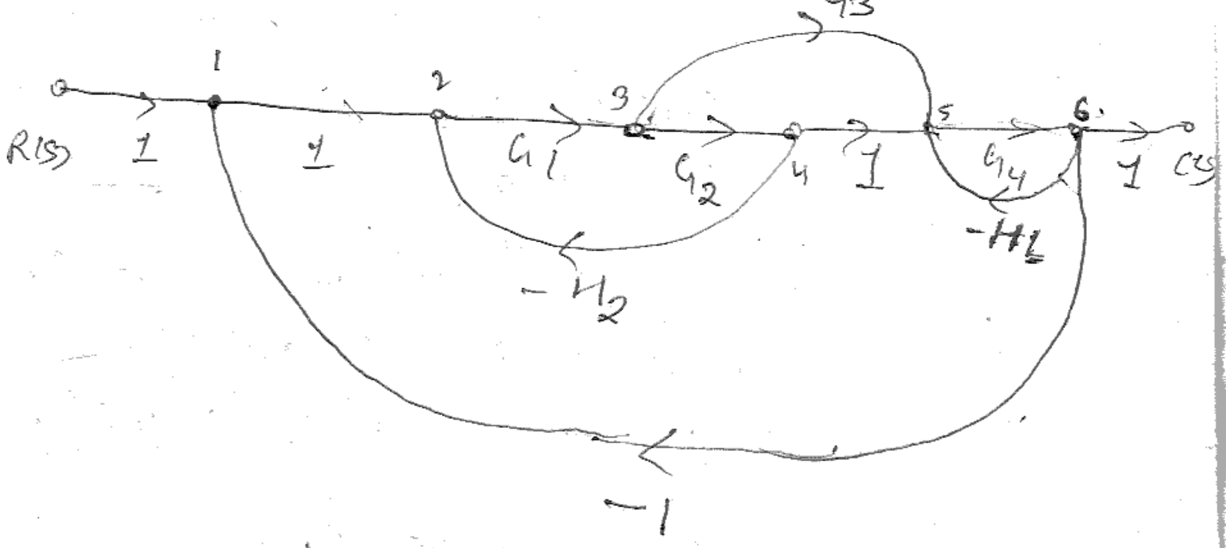
Δ = 1 - [Sum of individual loop gain] + [Sum of product of gain of all possible combination of two non-touching loops] −
[Sum of product of gain of all possible combination of three non-touching loops].

n = no of forward paths
K = which thing we want to choose
P_K = Forward path gain of K^{th} forward path.
Δ_K = is Δ for that part of non-touching loop for the K^{th} forward path.

30/06/11

Q. Find $\frac{C(s)}{R(s)}$ for signal flow graph





i) No of forward path = 2

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{K=1}^{2} P_K \Delta_K$$

P_1 and P_2 are two forward par

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right]$$

ii) $P_1 = G_1 G_2 G_4$

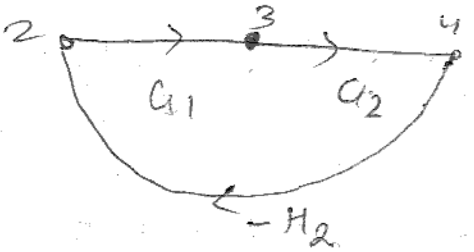
$P_2 = G_1 \cancel{G_2} G_3 G_4$

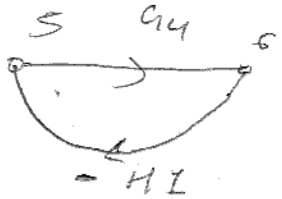
~~$\Delta = 1 - (G_1 G_2 G_4 + G_1 G_3 G_4) + ($~~

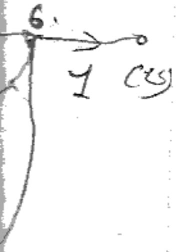
~~loop~~ 4

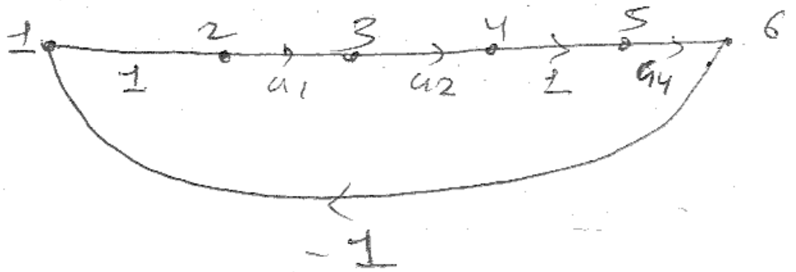
aph

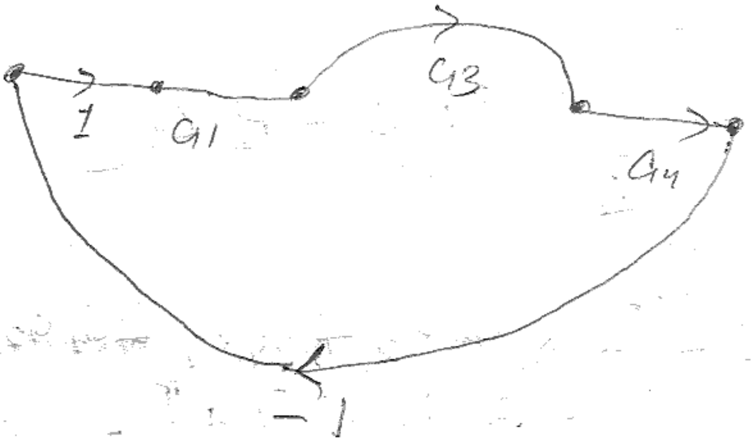
The individual loop











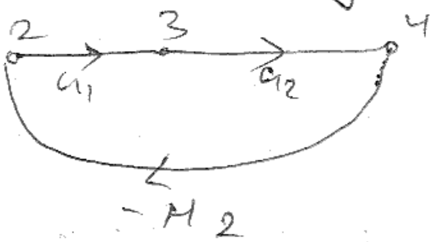
for loop 1 Gain = $-G_1G_2$ (P_{11})

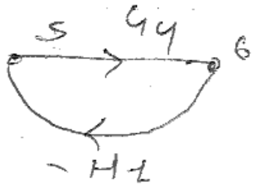
loop 2 Gain = $= G_4H_1$ (P_{21})

loop 3 " = $-G_1G_2G_4$ (P_{31})

loop 4 " = $-G_1G_3G_4$ (P_{41})

Non-touching loop





~~P_{22} = $\frac{G_1G_2H_2}{G_1 * G_2}$~~

$P_{12} = (-G_4 H_1) \times (-G_1 G_2 H_2)$

$\Delta =$

$$\Delta = 1 - [-G_1G_2 - G_4H_1 - G_1G_2G_4 - G_1G_3G_4] + [(-G_4H_1)(-G_1G_2H_2)] - 0$$

$$= 1 + [G_1G_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4] + [(-G_4H_1)(-G_1G_2H_2)]$$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$\therefore \frac{C(S)}{R(S)} = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$\Delta = 1 + [+G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4] + [(-G_4H_1)(-G_1G_2H_2)]$$

$$= 1 + [G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4] + [G_1G_2G_4H_1H_2]$$

$$= 1 + G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4 + G_1G_2G_4H_1H_2$$

$$T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_4 + G_1G_3G_4}{1 + [G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4] + [G_1G_2G_4H_1H_2]}$$